Automatically R-conserving supersymmetric SO(10) models and mixed light Higgs doublets

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In automatic R-parity-conserving supersymmetric (SUSY) SO(10) models, the simplest way to accommodate realistic fermion masses is to demand that the light Higgs doublets are linear combinations of the $\{10\}$ and $\{\overline{126}\}$ grand unified Higgs representations. We study the realization of this mixed light Higgs property consistent with doublet-triplet splitting in a minimal R-conserving SUSY SO(10) model. We then discuss predictions for neutrino masses and mixings in this model as well as its implications for proton decay.

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I. INTRODUCTION

The supersymmetric (SUSY) SO(10) model has a number of desirable features that make it an ideal candidate to describe physics beyond the standard model in a grand unified framework. They are (a) unification of all fermions of each generation into a single spinor representation [1] restoring quark-lepton symmetry to weak interactions, (b) a natural implementation of the seesaw mechanism [2] for understanding small neutrino masses, which in the minimal version of the model are of the right order of magnitude to explain the solar neutrino puzzle via the Mikheyev-Smirnov-Wolfenstein (MSW) [3] mechanism, and (c) a simple mechanism for explaining the origin of matter starting with a zero baryon and lepton asymmetry of the Universe for temperature above the grand unification scale [4].

In this paper, we wish to discuss a subclass of SUSY SO(10) models which have another highly desirable feature: automatic R-parity conservation which leads to natural conservation of baryon and lepton number symmetries prior to symmetry breaking. As is well known, this property is not present in the SUSY standard model nor in the SUSY SU(5) model [5], where extra symmetries have to be imposed by hand to ensure R-parity invariance. On the other hand, in the SO(10) model if all Higgs representations are chosen to have congruence number zero (such as **45, 54, 210**, etc.) and two (such as **10, 120, 126, $\overline{126}$**, etc.), the R-parity symmetry is automatic. Two possible minimal models with this property are given below.

Model A. The Higgs particles belong to representations $\{210\}$, $\{126\} \oplus \{\overline{126}\}$ and $\{10\}$[6]. The role of $\{210\}$ is to break SO(10) down to $SU(2)_L \times SU(2)_R \times SU(4)_C$; that of $\{\overline{126}\}$ (denoted by $\overline{\Delta}$) is to break $SU(2)_R \times SU(4)_C$ down to $U(1)_Y \times SU(3)_C$ while at the same time giving heavy Majorana mass to the right-handed neutrino (ν_R) to implement the seesaw mechanism for neutrino masses; the role of $\{126\}$ (denoted by Δ) is to cancel the $\overline{\Delta}$ contribution to the D term so that supersymmetry is maintained down to the weak scale; the role of $\{10\}$ (denoted by H) is of course to break the

$SU(2)_L \times U(1)_Y$ to $U(1)_{em}$ and generate fermion masses. The model is minimal in the sense that omitting any one of these multiplets will leave extra undesirable local symmetries at low energies or break SUSY at GUT scale.

Model B. The Higgs particles belong to the multiplets $\{45\} \oplus \{54\}$ (denoted by A and S respectively), and $\{126\} \oplus \{\overline{126}\} \oplus \{10\}$. Apart from being more economical compared to model A, this model has another difference from model A: i.e., here SO(10) is broken down to $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ before breaking to $U(1)_Y \times SU(3)_C$.

It is worth pointing out that in general superstring models, the $\{126\} \oplus \{\overline{126}\}$ (or higher) SO(10) representations do not emerge below the Planck scale after compactification [7]. Therefore, several SO(10) models discussed in the literature [8] have only $\{45\}$, $\{16_H + \overline{16}_H\}$, and $\{10\}$, where $\{45\} \oplus \{16_H\} \oplus \{\overline{16}_H\}$ are used to break SO(10) down to the standard model. In these models, the vacuum expectation values(VEV's) of $\{16_H\} \oplus \{\overline{16}_H\}$ lead to R-parity-breaking terms at low energies. For instance, a term of the form $\Psi\Gamma_a\Psi\Psi\Gamma_a\Psi_H/M_{Pl}$ will lead to a B-violating term of the form $\langle \bar{\nu}^c \rangle (u^c d^c d^c + QLd^c)/M_{Pl}$, which can lead to catastrophic baryon number violation.[1] One must invoke additional symmetries to prevent the R-noninvariant terms. In fact, one may take the point of view that only those compactification schemes where such symmetries emerge are acceptable.

Let us now discuss the question of fermion masses in the R-parity-conserving SO(10) models. It is well known [9] that if only a complex **10** Higgs representation contributes to fermion masses, then one gets the undesirable mass relations between leptons and quarks, e.g., $m_d/m_s = m_e/m_\mu$, which is a factor of 10 off compared to observations. One way to correct it is to have the Higgs doublets contained in the $\overline{126}$ contribute to the fermion masses through their direct dimension-3 couplings. How-

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[1]The existence of the $u^c d^c d^c$ terms was pointed out to R.N.M. by A. Farragi.

ever, if we naively added a separate $\overline{\bf 126}$, for this purpose, there would be an extra pair of Higgs doublets at low energies which is unacceptable from the point of view of gauge coupling unification [10]. Moreover, the usual problem of doublet-triplet splitting will also get worse. It was pointed out in Ref. [6] that there is a simple solution to this problem: the same $\overline{\bf 126}$, which contributes to the breaking of $B - L$ symmetry and to the seesaw mechanism, can have its weak doublets acquire an induced VEV without leaving any extra light doublet and without any fine-tuning provided there is a coupling between the $\bf 10$ and the $\overline{\bf 126}$ in the superpotential via other Higgs multiplets that only have VEV's of order of the grand unified theory (GUT) scale. (Such couplings can arise, for instance, if there is a $\bf 210$ multiplet in the theory.) The induced VEV then will correct the bad charged fermion mass relations.

This property of inducing VEV for $(\bf 2,2,15)$ of $\overline{\Delta}$ can also be stated in another way. When SO(10) breaks down to the standard model at M_U, there are two light doublets (say ϕ_u and ϕ_d). To obtain realistic quark lepton mass relations, these doublets must not be pure and must not emerge from a single $\bf 10$ multiplet. In the specific model under consideration, they must be a linear combination of the doublets in $\{\bf 10\}$ and $\{\overline{\bf 126}\}$. In general, they could be doublets residing in two different $\bf 10$'s or may be mixtures of doublets in $\bf 16$ and $\overline{\bf 16}$, etc. In the rest of the paper, we will call this "mixed light Higgs property"(MLHP). We will call a model to have a "pure light Higgs property" (PLHP) if the two light Higgs doublets in the low energy theory arise from a single 10-dimensional Higgs multiplet. It is simple to see that models with MLHP do not have the property of Yukawa unification widely discussed in recent literature [11]. This property (MLHP) does impose nontrivial constraints on model building. For instance, maintaining MLHP while implementing the doublet-triplet splitting (DTS) is rather nontrivial in general. In the Dimopoulos-Wilczek scheme [12] for DTS, an example was constructed [14], where this property emerges consistent with some discrete symmetries. In fact, it was suggested in Ref. [14] that any new DTS scheme must have this property. In this paper, we will demonstrate a simple SO(10) model which has this good light Higgs property and study the consequences of this general class of SO(10) models with the additional minimality criterion.

It is important to point out that the requirement of the light Higgs doublets at low energies being mixed applies to all SUSY SO(10) models, whereas the specific linear combination chosen in this paper is model dependent. For instance, if the model had $\bf 16_H + \overline{\bf 16}_H$ in order to break the SO(10) symmetry instead of $\bf 126 + \overline{\bf 126}$, then the mixed light Higgs doublet could have a piece from them. The light Higgs doublets could even be purely from $\bf 10$, as long as they are from two different $\bf 10$'s. Second, the doublet-triplet splitting mechanism presented here is specific to this model, whereas the form the doublet mass matrix as a way of obtaining the MLHP property could be used in any model. Finally, the predictions for neutrino masses and mixings discussed here are specific

to the minimal model we are considering.

This paper has been organized as follows: in Sec. II, we argue in favor of the necessity of the MLHP in R-conserving SO(10) models; in Sec. III, we discuss the superpotential of the model B and introduce the doublet-triplet splitting mechanism with MLHP; in Sec. IV, we study the quark and lepton mass matrices and discuss their implications for neutrino masses and mixings; in Sec. V, we discuss the implications for proton decay; Sec. VI is devoted to an R-conserving SO(10) model where the Higgsino mediated contributions to proton decay is naturally suppressed while maintaining MLHP; in Sec. VII, we summarize our results and conclude; in an appendix, we discuss the minimum of the Higgs potential consistent with the desired symmetry-breaking pattern.

II. MIXED VERSUS PURE LIGHT HIGGS

In this section, we explore to what extent it is an absolute necessity to have mixed light Higgs doublets in an automatically R-parity conserving SO(10) model: i.e., could the light Higgs below GUT scale be purely light Higgs (PLH) arising solely from a single complex $\bf 10$? It is obvious that if nonrenormalizable Planck induced terms are not included in the superpotential, then PLH scheme will not work since it will lead to the bad fermion mass relation $m_d/m_s = m_e/m_\mu$ already mentioned. However, once the nonrenormalizable Planck induced terms are included, the result is not obvious. To see what happens, let us consider a model where the PLH scheme is realized in the manner of Ref. [12]. As has been noted in [13], for complete symmetry breaking to occur in this model, one must have other multiplets. Since we are interested in automatically R-conserving models, we will use $\bf 126 + \overline{\bf 126}$ for this purpose. Then the only leading order contribution to fermion masses will come from terms of the form $\psi_a\psi_b AH$ (where A stands for the $\bf 45$ dimension multiplet). The resulting mass matrices look as follows:

$$M_u = (h + h')\kappa_u,$$
$$M_d = (h + h')\kappa_d,$$
$$M_l = (h - 3h')\kappa_d. \qquad (1)$$

In the above equation, h denotes the dimension-3 $\bf 10$ coupling in the superpotential whereas the h' stands for the induced higher dimensional couplings. The h' of order $M_U/M_{\rm Pl}$. This leads to vanishing quark mixing angles and is therefore not realistic.

One could ask at this stage whether contributions from still higher order Planck induced terms (i.e., $\simeq M_{\rm Pl}^{-2}$) could not fix this problem. A typical order of magnitude for such terms is $\simeq (M_U/M_{\rm Pl})^2(m_b/m_t)v_{\rm wk} \simeq 0.1$ MeV (assuming $M_{\rm Pl} \simeq 1.8\times10^{18}$ GeV and $M_U \simeq 2\times10^{16}$ GeV) and therefore they are unlikely to help solve this problem. We therefore feel that it is highly unlikely that a pure light Higgs possibility would be realistic. On the other hand, in the MLHP scenario, the presence of additional multiplets allows additional parameters and therefore it is possible to accommodate realistic fermion masses and mixings.

III. MIXED LIGHT HIGGS IN MODEL B

In this section, we present a simple minimal SO(10) model, where the light Higgs doublets have the desirable property (MLHP) described in the introduction. As already mentioned, the key step is to have a term in the superpotential that couples the **10** Higgs representation with the $\overline{\mathbf{126}}$ Higgs representation(s) via Higgs fields that have VEV's of order of the GUT scale. If we do not allow for Planck scale induced nonrenormalizable terms, then we need a **210** Higgs representation to achieve this goal; but as shown in [14], achieving this together with doublet-triplet splitting is not very simple, although we did manage to construct an example which is technically natural. In this paper, we will take the point of view that one should include nonrenormalizable Planck scale induced terms and we will keep only the lowest order Planck induced terms. As we will see this allows for the construction of a rather simple model with MLHP.

As usual, we assign the fermions to the 16-dimensional spinor representation of SO(10). We denote them by Ψ_a (where $a = 1, 2, 3$ stands for generations) and we use the following *minimal* set of Higgs bosons needed for complete symmetry breaking: (i) $\{\mathbf{45}\} \oplus \{\overline{\mathbf{54}}\}$ (denoted by A and S respectively) to break SO(10) down to the left-right symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$; (ii) $\{\mathbf{126}\} \oplus \{\overline{\mathbf{126}}\}$ (denoted by $\Delta + \overline{\Delta}$) to break the $SU(2)_R \times U(1)_{B-L}$ symmetry down to $U(1)_Y$ while keeping supersymmetry in fact down to the M_W scale; (iii) a single $\{\mathbf{10}\}$ (denoted by H) to break the $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$.

The superpotential of the model is chosen to consist of the following parts:

$$W = W_f + W_s + W_p, \qquad (2)$$

where

$$W_f = h_{ab}\Psi_a\Psi_b H + f_{ab}\Psi_a\Psi_b\overline{\Delta}, \qquad (3)$$

$$W_s = (\mu_H + \lambda S)HH + \mu_s S^2 + \lambda_s S^3 + \mu_A A^2 + \lambda' A^2$$
$$+ \mu_\Delta \Delta\overline{\Delta} + \lambda_\Delta \Delta A\overline{\Delta} + \lambda_S(S\Delta\Delta + S\overline{\Delta}\overline{\Delta}), \qquad (4)$$

$$W_p = \frac{\sqrt{8\pi}\lambda_p}{M_{\mathrm{Pl}}}\Delta A^2 H. \qquad (5)$$

As noted in Ref. [6], if we show that the light doublets in the model are linear combinations of the doublets in $\{\mathbf{10}\}$ and $\{\overline{\mathbf{126}}\}$ multiplets, then W_f can accommodate a realistic charged fermion spectrum for all generations. This is, of course, intimately connected with the question

of the doublet-triplet splitting.

It is easy to see (see Appendix), using the above superpotential, that the vanishing of F terms at the scale M_U and v_R (which are equal to fit low energy LEP data [8]), is guaranteed by the following choice of VEV's for the Higgs fields S, A, Δ, and $\overline{\Delta}$ which therefore correspond to a minimum of the potential:

$$\langle S \rangle = \mathrm{diag}(1, 1, 1, 1, 1, 1, -\tfrac{3}{2}, -\tfrac{3}{2}, -\tfrac{3}{2}, -\tfrac{3}{2})M_U,$$
$$\langle A \rangle = i\tau_2 \otimes \mathrm{diag}(b, b, b, c, c),$$
$$\langle \Delta \rangle_{(1,3,\overline{10})} = \langle \overline{\Delta} \rangle_{(1,3,10)} = v_R. \qquad (6)$$

Using this, we find that the mass matrix for the fermionic doublets in H, Δ, and $\overline{\Delta}$ can be written as

$$\begin{array}{c} \\ \overline{\Delta}_d \\ \Delta_d \\ H_d \end{array}
\begin{array}{c} \overline{\Delta}_u \qquad \Delta_u \qquad H_u \\ \left[\begin{array}{ccc} 0 & \mu_\Delta + \lambda_\Delta c & 0 \\ \mu_\Delta - \lambda_\Delta c & 0 & \mu_1 \\ 0 & \mu_2 & 0 \end{array}\right] \end{array}, \qquad (7)$$

where we have fine-tuned $\mu_H - (3/2)\lambda M_U \approx M_W$ [assumed to be zero in writing Eq. (7)]. This fine-tuning can receive higher order Planck corrections; we have assumed that there exists a clever choice of discrete symmetries that suppresses these corrections.

In Eq. (7), the rows and columns denote the down- and up-type doublets contained in H, Δ, and $\overline{\Delta}$ respectively. The $\Delta A^2 H$ entries are induced by the Planck scale corrections to W_p leading to $\mu_i \approx \lambda_p\sqrt{8\pi}M_U^2/M_{\mathrm{Pl}}$ ($i = 1, 2$) which is of order $10^{-1}M_U$ to $10^{-2.5}M_U$. Note that the $(\mathbf{1,3,1})$ VEV in $\langle A \rangle$ makes the entry $\Delta_u H_d$ different from $\Delta_d H_u$ (i.e., $\mu_1 \neq \mu_2$).

It is easy to see from Eq. (7) that the light Higgs doublets (denoted by ϕ_u and ϕ_d) are given by

$$\phi_u = \cos\alpha H_u + \sin\alpha \overline{\Delta}_u,$$
$$\phi_d = \cos\gamma H_d + \sin\gamma \overline{\Delta}_d, \qquad (8)$$

where $\tan\alpha = \mu_1/(\mu_\Delta - \lambda_\Delta c)$ and $\tan\gamma = \mu_2/(\mu_\Delta + \lambda_\Delta M_2)$. It is important to note that in general $\alpha \neq \gamma$. We find this to be a rather simple way to get the light doublets with the correct group theoretical property at low energies for fermion masses. Furthermore, we expect α and γ to be much smaller than one so that the departure from strict Yukawa unification [11] is small.

The triplet mass matrix is a four-by-four matrix as follows:

$$\begin{array}{c} \\ H \\ \overline{\Delta} \\ \Delta \\ \Delta_R \end{array}
\begin{array}{c} H \qquad\qquad \Delta \qquad\qquad \overline{\Delta} \qquad\qquad \overline{\Delta}_R \\ \left[\begin{array}{cccc} 2\mu_H + \lambda M_U & q_1(-b^2 + \alpha'c^2) & 0 & 0 \\ 0 & \mu_\Delta - \lambda_\Delta\beta'b & 0 & 0 \\ q_1(-b^2 + \alpha'c^2) & 0 & \mu_\Delta + \lambda_\Delta\beta'b & 0 \\ q_2 bc & 0 & 0 & \mu_\Delta + \lambda_\Delta\gamma'b \end{array}\right] \end{array}. \qquad (9)$$

In Eq. (9), the first three rows denote the anti-quark-type Higgsinos contained in $(\mathbf{1,1,6})$ of H, $\overline{\Delta}$, Δ, and the last row denotes the same in the $(\mathbf{1,3,\overline{10}})$ of Δ respectively; similarly, the first three columns denote the quark-type Higgsinos contained in $(\mathbf{1,1,6})$ of H, Δ, $\overline{\Delta}$, and the last column the same in $(\mathbf{1,3,10})$ of $\overline{\Delta}$. And the primed symbols represent nonzero group theoretical factors and q_i are proportional to μ_i. It is certain that all eigenvalues

are of order M_U. This solves the doublet-triplet splitting problem.

We want to point out that the specific form of the superpotential $W_S + W_p$ can be derived by requiring invariance under a Z_2 symmetry, under which S, H, and Δ are even and A, $\overline{\Delta}$ are odd. This symmetry for instance forbids the $\overline{\Delta}A^2H$ term. Coming to the matter part of the superpotential, W_f, the $\Psi\Psi\overline{\Delta}$ term is forbidden but there is an allowed Planck induced term $f'\Psi\Psi A\overline{\Delta}$ which essentially plays the same role as the second term in W_f. The effective coupling in the mass matrix is then given by $f'v_R\sqrt{8\pi}/M_{\mathrm{Pl}}$.

Before turning to a discussion of the fermion masses in the model, we wish to emphasize two points: first, in discussing the fermion masses in any R-conserving SO(10) model, one must first ensure that the light Higgs doublets have the correct MLHP consistent with the doublet-triplet splitting. The absence of color singlet Higgs doublets in **45** representation makes our mechanism a good starting point for model building; second, this property of the mixed light Higgs doublets is not trivial to ensure while keeping all color triplet Higgsinos heavy. For instance, if instead of {**45**}, we used a {**210**} to break SO(10), the doublet mass matrix becomes a four-by-four matrix since it includes the (**2,2,10**) submultiplet of {**210**} and several elements in this matrix must be engineered to zero value to attain the MLHP goal [14]. Of course, one could double the number of {**126**} ⊕ {**126**} multiplets such that one set contributed to the light Higgs doublets while the other breaks $B - L$ symmetry and the two remain totally separate. However, in this case, one must worry about the possibility of unwanted pseudo Goldstone supermultiplets which spoil gauge unification.

IV. THE FERMION SECTOR AND PREDICTIONS FOR THE NEUTRINOS

Let us now turn to the fermion mass matrices in this model. It was shown that in Ref. [6], the fermion mass matrices in this model are characterized by 12 parameters, which can all be evaluated given the six quark masses, three charged lepton masses, and three Cabibbo-Kobayashi-Maskawa (CKM) angles. The neutrino masses and mixings are then completely predicted. We repeat this discussion with two differences from Ref. [6]. First, we take the effect of the superpartners on the running of the gauge and Yukawa couplings. Second, we take the effect of the top quark Yukawa couplings on the running of the masses [15]. We will follow Naculich [15] below. The low energy superpotential for the model is given by

$$W_0 = h_u Q\phi_u u^c + h_d Q\phi_d d^c + h_e L\phi_d e^c + \mu\phi_u\phi_d, \quad (10)$$

where h_u, h_d, and h_e are three-by-three matrices expressible in terms of the SO(10) coupling matrices h and f in Eq. (3) as follows:

$$h_u = h\cos\alpha + f\sin\alpha, \quad (11)$$
$$h_d = h\cos\gamma + f\sin\gamma, \quad (12)$$
$$h_e = h\cos\gamma - 3f\sin\gamma. \quad (13)$$

As emphasized earlier, $\alpha \neq \gamma$ is required. Otherwise all CKM angles vanish. Moreover, in a strict supergravity framework, where supersymmetry breaking is implemented in the hidden sector, at $\mu = M_{\mathrm{Pl}}$, we have $m_{\phi_u}^2 = m_{\phi_d}^2 = m_0^2$. Their extrapolation down to the electroweak scale is governed predominantly [16] by $h_{u,33}$ and $h_{d,33}$. Correct symmetry-breaking pattern (i.e., $\tan\beta > 1$) also requires that α and γ be different from each other. The electroweak symmetry is then broken radiatively so that

$$\langle\phi_u^0\rangle = v\sin\beta,$$
$$\langle\phi_d^0\rangle = v\cos\beta. \quad (14)$$

The mass matrices at GUT scale can then be written as $(\mu = M_U)$

$$\overline{M}_u = (\overline{h} + \overline{f})v,$$
$$\overline{M}_d = (\overline{h}r_1 + \overline{f}r_2)v,$$
$$\overline{M}_l = (\overline{h}r_1 - 3\overline{f}r_2)v, \quad (15)$$

where

$$\overline{h} = h\cos\alpha\sin\beta, \quad \overline{f} = f\sin\alpha\sin\beta,$$
$$r_1 = \frac{\cos\gamma}{\cos\alpha}\cot\beta, \quad r_2 = \frac{\sin\gamma}{\sin\alpha}\cot\beta. \quad (16)$$

This is now in the same notation as in Ref. [6]. In order to evaluate the matrices $\overline{h}$ and $\overline{f}$, r_1 and r_2, we use the following sum rule derived in Ref. [6]: i.e.,

$$\overline{M}_l = \frac{4r_2r_1}{r_2 - r_1}\overline{M}_u - \frac{3r_2 + r_1}{r_2 - r_1}\overline{M}_d. \quad (17)$$

We then take Tr $\overline{M}_l$, Tr $\overline{M}_l^2$, and Tr $\overline{M}_l^3$ to obtain r_2 and r_1 [17]. The light neutrino mass matrix is given by the seesaw formula [3] to be [6]

$$M_\nu = -M_{\nu D}M_{\nu M}^{-1}M_{\nu D}^T, \quad (18)$$

where

$$M_{\nu D} = \frac{3r_1 + r_2}{r_2 - r_1}\overline{M}_u - \frac{4}{r_2 - r_1}\overline{M}_d, \quad (19)$$

$$M_{\nu M} = -\frac{1}{R}\left[\frac{r_1}{r_2 - r_1}\overline{M}_u - \frac{1}{r_2 - r_1}\overline{M}_d\right], \quad (20)$$

with

$$R = \frac{v\sin\alpha\,\sin\beta}{v_R}.$$

Note that this light neutrino mass matrix defined at v_R needs to be extrapolated to the weak scale; but since ν_R decouples below v_R, there are only some overall anomalous dimensions of the effective light Majorana neutrino mass [18]. This effect is small and we will ignore it.

In order to carry out the numerical analysis, we choose the following values for the running masses:

$$m_u = 5.1 \text{ MeV}, \quad m_c = 1.27 \text{ GeV}, \quad m_t(m_t) = 166 \text{ GeV},$$
$$m_d = (8.9 + ds) \text{ MeV}, \quad m_s = (0.175 + ss) \text{ GeV}, \quad m_b = 4.25 \text{ GeV},$$
$$m_e = 0.51 \text{ MeV}, \quad m_\mu = 105.6 \text{ MeV}, \quad m_\tau = 1.784 \text{ GeV}. \tag{21}$$

The $m_t(m_t)$ is obtained by taking the Collider Detector at Fermilab (CDF) [19] mean value of 174 GeV for the m_t pole. The symbols ds and ss are left free to be fixed by the sum rule in Eq. (17) along with r_1 and r_2. The CKM angles are parametrized in terms of s_{12}, s_{23}, and s_{13}, with $s_{12} = -0.221$, $s_{23} = 0.043$, and $s_{13} = 0.0045$ as our choice corresponding to the mean values from experiments [20].

We then extrapolate all masses to the SUSY breaking scale [15]. The extrapolation factors are defined as $\eta_i = m_i(m_i \text{ or } 1 \text{ GeV})/m_i(\mu_{\text{SUSY}})$. They are

$$\eta_u = 2.17, \quad \eta_c = 1.89, \quad \eta_t = 1, \quad \eta_d = 2.16,$$
$$\eta_s = 2.16, \quad \eta_b = 1.47, \quad \eta_e = \eta_\mu = \eta_\tau = 1.02.$$

In order to extrapolate from μ_{SUSY} to M_U, we need to know $\tan\beta$. We follow Naculich [15] and assume $\tan\beta < 40$ so that effects of all Yukawa couplings except that of the top quark can be ignored. In this limit, the top Yukawa coupling effect is accounted for by the factor $B_t = 0.88647$. The GUT scale values of various masses (denoted with bars) are given by

$$m_u = \overline{m}_u \eta_u A_u B_t^3, \quad m_c = \overline{m}_c \eta_c A_u B_t^3, \quad m_t = \overline{m}_t \eta_t A_u B_t^6,$$
$$m_d = \overline{m}_d \eta_d A_d B_t^0, \quad m_s = \overline{m}_s \eta_s A_d B_t^0, \quad m_b = \overline{m}_b \eta_b A_d B_t^1,$$
$$m_e = \overline{m}_e \eta_e A_e B_t^0, \quad m_\mu = \overline{m}_\mu \eta_\mu A_e B_t^0, \quad m_\tau = \overline{m}_\tau \eta_\tau A_e B_t^0, \tag{22}$$

where A factors are the contributions of the gauge groups to the extrapolation and are numerically given by (choosing $\mu_{\text{SUSY}} = 170$ GeV)

$$A_u = 3.21, \quad A_d = 3.13, \quad A_e = 1.48. \tag{23}$$

Some of the mixing angles are also extrapolated and one has (for $ij = 13$ and 23)

$$s_{ij} = \overline{s}_{ij} B_t^{-1}. \tag{24}$$

In order to predict neutrino masses, we need the value of the parameter $R = v \sin\alpha \sin\beta / v_R$. The value of $\sin\alpha$ and $\sin\beta$ are arbitrary, whereas $v = 246$ GeV and v_R is fixed by unification of the gauge couplings. In the absence of the heavy particle threshold corrections, one has $v_R \approx M_U \approx 2 \times 10^{16}$ GeV [15]; but as has been noted for the case of non-SUSY SO(10) models [21], the threshold corrections can easily introduce uncertainty of a factor of $10^{\pm 1}$ in M_U and v_R. We will therefore assume that $v_R \approx 10^{15}$ to 10^{16} GeV in what follows.

In our input, the signs of the fermion masses can be arbitrary. There are many possibilities. Below we give the results for the choice of signs for masses that satisfy all the constraints of the model.

Case I. All masses chosen positive. In this case, we find $ds = -1.581$, $ss = -0.03533$, $r_1 = 0.00952$, and $r_2 = 0.00479$. The Eq. (17) for this choice of $\{r_1, r_2\}$ leads to the values of $\{\overline{m}_e, \overline{m}_\mu, \overline{m}_\tau\} = \{0.0003377, 0.0695543, 1.18177\}$ to be compared with extrapolated values: $\{ 0.0003378, 0.0695548, 1.18177\}$. The predictions for neutrino masses and mixing for this case are given by

$$M_\nu = R\{-0.679348, 43.2421, 704.332\},$$
$$V_l = \begin{pmatrix} 0.999238 & -0.0381721 & 0.00819604 \\ 0.0379246 & 0.998875 & 0.0284804 \\ -0.00927398 & -0.0281479 & 0.999561 \end{pmatrix}. \tag{25}$$

A natural value for $R \approx 10^{-14}$ to 10^{-13} (since $v \sin\alpha \sin\beta \approx 10^2$ GeV) depending on whether v_R is M_U or $M_U/10$, we get $m_{\nu_\tau} \approx 7 \times 10^{-3}$ to 7×10^{-2} eV, and $m_{\nu_\mu} \approx 4 \times 10^{-4}$ to 4×10^{-3} eV. The $\theta_{e\tau}$ mixing angle in this case is rather small; but $\sin^2 2\theta_{e\mu} \approx 5 \times 10^{-3}$, which is of right order of magnitude to resolve the solar neutrino puzzle via the MSW mechanism [22].

Case II. For the choice of all signs for masses to be negative, we get a consistent fit to all charged fermion masses for $r_1 = 0.00637635$, and $r_2 = 0.0187798$. The predictions for the neutrino sector in this case are

$$M_\nu = R\{-0.910023, -21.4474, -547.607\},$$
$$V_l = \begin{pmatrix} 0.995493 & 0.0921015 & -0.0225942 \\ -0.0939442 & 0.990303 & -0.102345 \\ 0.012949 & 0.104006 & 0.994492 \end{pmatrix}. \tag{26}$$

In this case, both $\theta_{e\tau}$ and $\theta_{e\mu}$ mixing angles are outside the MSW two neutrino solution given by Hata and Langacker [22]. Therefore, if the solar neutrino deficit situation continues to remain as it is now, this solution will be ruled out.

Case III. We have found another fit to the sum rule in Eq. (17), for the choice of masses, where m_c, m_d, and $m_s < 0$ whereas all the remaining masses are chosen positive. The values of r_1 and r_2 are: $r_1 = 0.0100082$ and $r_2 = 0.072028$. The predictions for neutrino masses and lepton mixing for this case are given by

$M_\nu = R\{0.0662987, 4.60097, -2041.97\}$,

$$V_l = \begin{pmatrix} 0.722321 & 0.684518 & 0.0984244 \\ 0.69126 & -0.718838 & -0.0736991 \\ 0.0203028 & 0.121271 & -0.992412 \end{pmatrix}. \quad (27)$$

Again, here all (mass difference)^2 are outside the range of the small angle as well as the large angle MSW solutions to the solar neutrino puzzle. Again, this solution can be tested by the solar neutrino data.

Case IV. This case is obtained by changing the signs of masses in case III. This corresponds to the choices $r_1 = 0.0175744$ and $r_2 = 0.00699614$. The predictions for neutrino masses and lepton mixing in this case are

$M_\nu = R\{-0.460917, -30.2621, -663.422\}$,

$$V_l = \begin{pmatrix} 0.99973 & 0.0226327 & 0.00523126 \\ -0.0227722 & 0.999339 & 0.0283497 \\ -0.00458617 & -0.0284611 & 0.999584 \end{pmatrix}. \quad (28)$$

Here again the mixing angles are outside the range required by the MSW analysis of the present solar neutrino data.

Case V. In this case, all masses are chosen positive except the electron mass and we find $ds = -1.581$, $ss = -0.03833$, $r_1 = 0.00955$, and $r_2 = 0.00500$. The predictions for neutrino masses and mixing are given by

$M_\nu = R\{-0.945754, 42.8837, 714.726\}$,

$$V_l = \begin{pmatrix} 0.999542 & -0.029543 & 0.00655202 \\ 0.0294036 & 0.999359 & 0.0204401 \\ -0.00715168 & -0.020238 & 0.99977 \end{pmatrix}. \quad (29)$$

A natural value for $R \approx 10^{-14}$ to 10^{-13} (since $v \sin\alpha \sin\beta \approx 10^2$ GeV) depending on whether v_R is M_U or $M_U/10$, we get $m_{\nu_\tau} \approx 7 \times 10^{-3}$ to 7×10^{-2} eV, and $m_{\nu_\mu} \approx 4 \times 10^{-4}$ to 4×10^{-3}eV. The $\theta_{e\mu}$ and $\theta_{e\tau}$ mixing angles in this case are rather small for the MSW mechanism to work.

Case VI. If we choose all masses to be negative except the electron mass, we get a fit consistent with all charged fermion masses for $r_1 = 0.00656483$, and $r_2 = 0.0187798$. The predictions for the neutrino sector in this case are

$M_\nu = R\{-1.02584, -23.7002, -616.621\}$,

$$V_l = \begin{pmatrix} 0.993278 & 0.112199 & -0.0284463 \\ -0.114706 & 0.98706 & -0.112052 \\ 0.015506 & 0.114562 & 0.993295 \end{pmatrix}. \quad (30)$$

In this case, both $\theta_{e\tau}$ and $\theta_{e\mu}$ mixing angles are outside the MSW two neutrino solution [22]. Therefore, if the solar neutrino deficit situation continues to remain as it is now, this solution will also be ruled out.

If we set aside prejudices towards mixing angles coming from solar neutrinos, then our ν_μ-ν_τ mixing angles are in the interesting ranges to be testable in the next generation of proposed acceleration ν_μ-ν_τ oscillation experiments. In Fig. 1, we compare our predictions with the domains of Δm^2 and $\sin^2 2\theta_{\nu_\mu \nu_\tau}$ angles to be explored in the proposed CERN and Fermilab experiments.
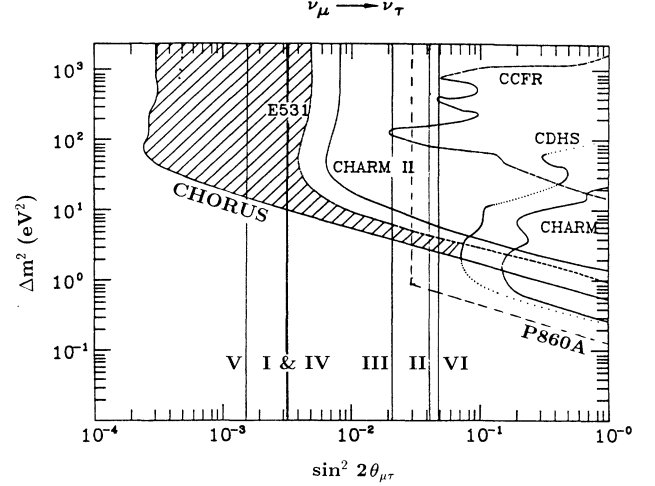

FIG. 1. This figure shows the present limits on $\nu_\mu \nu_\tau$ oscillation parameters (Δm^2 and $\sin^2 2\theta$) and future possibilities on two proposed experiments CHORUS at CERN and P860A at Fermilab. The solid vertical lines are the predictions of the minimal SO(10) model described in this paper for the six allowed parameter ranges, denoted as cases I through VI in the text.

V. PROTON DECAY

One of the key predictions of grand unified theories is the lifetime of the proton and its decay mode. In non-SUSY GUT models, the dominant decay of the proton arises from the exchange of superheavy gauge bosons and the operators responsible for this have dimension six. The primary decay mode is $p \to e^+ \pi^0$. On the other hand, in SUSY GUT models, in addition to the above dimension-6 operators, there also exist dimension-5 operators, and in simple SUSY SU(5) or SUSY SO(10) models, the latter graphs dominate. The resulting dominant decay mode is $p \to \bar{\nu}_\mu K^+$, which can be used to distinguish between the SUSY GUT theories from non-SUSY GUT ones.

Proton decay in SUSY SU(5) model has been extensively studied [23, 24] and it has been established that in this case, one requires the superheavy color-triplet Higgsino ($\tilde{H}_3$) mass $M_{\tilde{H}_3} \gg M_U$ in order to be consistent with the existing lower bounds on the $\tau_{p \to \bar{\nu}_\mu K^+}$ [20]. We will show below that in the SUSY SO(10) model dimension-5 proton decay operators receive contributions from two diagrams: one involving $\{10\}$ Higgs representations and the other involving $\{\overline{126}\}$ Higgs representations and these graphs could interfere destructively, thereby reducing the effective p-decay amplitude. This in turn can relax the constraints on color-triplet Higgsino masses.

The color-triplet Higgsinos that mediate proton decay are part of a four-by-four matrix, given in Eq. (9). One can always find two four-by-four unitary matrices V and U, for the triplet mass matrix Eq. (9), such that

$$(V^\dagger)_{ik}(M_T)_{kl}U_{lj} = M_i \delta_{ij}.$$

Then, the dimension-5 operators at the GUT scale, which are to be turned into baryon number violating four-fermion interactions by gaugino- or Higgsino-dressing at the electroweak scale, have the common factor

$$C_{abcd} = C_0 \sum_{i=1}^{4} \frac{(h_{ab}U_{1i} + f_{ab}f_0 U_{3i})(h_{cd}V_{1i}^* + f_{cd}f_0 V_{2i}^*)}{M_i},$$

(31)

where the early Latin indices are family ones and run over 1,2,3; C_0 and f_0 represent some overall factors (Clebsch-Gordan coefficients).

From the definition Eq. (15), we obtain

$$\overline{h} = \frac{1}{v} \frac{r_2 \overline{M}_u - \overline{M}_d}{-r_1 + r_2},$$

$$\overline{f} = \frac{1}{v} \frac{-r_1 \overline{M}_u + \overline{M}_d}{-r_1 + r_2}.$$

The first point to note is that since for all our solutions in Sec. IV, $r_1 + r_2 \approx 5 \times 10^{-2}$ to 5×10^{-3}, the proton decay amplitudes receive an extra enhancement factor ≈ 20 to 200 (depending on the cases) compared to minimal SU(5). However, unlike the SU(5) case, we have two separate contributions to the proton decay amplitude and we can hope to invoke parameters for which there is a cancellation. To see if this is possible, we choose the special case where $b^2 = \alpha' c^2$. For convenience, we define

$$\eta_{ab} = \frac{h_{ab}}{f_{ab}}.$$

We, then, get

$$C_{abcd} = C_0 h_{ab} f_{cd} \sum_{i=1}^{3} \frac{U_{1i}(\eta_{cd}V_{1i}^* + f_0 V_{2i}^*)}{M_i}.$$

(Note that one of the triplet pairs completely decouples from the proton decay amplitude.) We have $\eta_{11} \neq \eta_{22} \neq \eta_{12}$. We, thus, have three equations involving the three mixing angles that characterize the matrices V_{1i} and V_{2i} and we can therefore expect a solution for which the proton decay is suppressed. It must however be pointed out that this does require a fine-tuning of parameters.

VI. FURTHER SUPPRESSION OF PROTON DECAY AMPLITUDE

In the previous section, we showed that the minimal R-parity-conserving SO(10) model of Sec. II tends to predict higher strengths for the dimension-5 proton decay operators compared to minimal SUSY SU(5); however, unlike the minimal SUSY SU(5) model, here there is the possibility of cancellation if one allows fine-tuning among the mixings and masses for the color-triplet Higgsinos. It is, however, worth emphasizing that the predictions for the neutrino sector and realization of realistic quark-lepton masses are logically independent of the proton life-time predictions. The question, therefore, arises as to whether it is possible to suppress the Higgsino-mediated

proton decay amplitude while at the same time keeping the discussion of light Higgs doublets with correct group theoretical properties to give the mass matrix structure of Sec. IV. A discussion of how to suppress such amplitudes in a different class of SO(10) models has recently been given in Ref. [13]. We present a different example that has the additional property of "mixed light Higgs doublet" property (as defined in the introduction).

We extend the model by addition of one more Higgs in the {10} representation (denoted by H_2). Note that automatic R conservation is maintained in the model. We choose the superpotential to be

$$W' = \mu_2 H_2^2 + \mu_{12} H_1 H_2 + \lambda_s S H_1 H_2$$
$$+ \mu_s S^2 + \lambda_s' S^3 + \mu_A A^2$$
$$+ \lambda_A S A^2 + \mu_\Delta \Delta \overline{\Delta} + \lambda_\Delta \Delta A \overline{\Delta} + \frac{\lambda_p}{M_{\rm Pl}} \Delta A^2 H_1.$$

(32)

We require all $\mu \approx M_U$ and the fine-tuning condition $\mu_{12} + \lambda_s(-3/2)M_U \approx M_W$. It is then clear that the light doublet mass matrix in Eq. (7) is preserved.

In Table I, we present a discrete Z_3 symmetry under which all but dimension-2 terms are invariant, providing a symmetry basis for this idea. Because of the presence of dimension-2 terms, this symmetry is softly broken, which in the supersymmetric context means that even after SUSY-breaking terms are included no hard dimension-4 term would be generated with infinite coefficient that breaks the symmetry (thereby helping to maintain our conclusion). This symmetry has the implication that it allows the following matter couplings that will eventually lead to realistic fermion masses and mixings if we assign $\Psi_a \Psi_b$ to be ω under the symmetry:

$$W_m' = h_{ab} \Psi_a \Psi_b H_1 + \frac{f_{ab}}{M^2} \Psi_a \Psi_b A^2 \overline{\Delta},$$

(33)

where $M = M_{\rm Pl}/\sqrt{8\pi}$. The second term leads to both $\{\overline{126}\}$, $\{120\}$ as well as $\{10\}$ type couplings. The mass matrices are therefore less predictive than before.

VII. SUMMARY AND CONCLUSIONS

In summary, we have studied the question of how to get realistic fermion masses in automatically R-parity-conserving SUSY SO(10) models. We have argued that an economical way to do this is to have the light Higgs doublets contain a piece from the $\overline{126}$ Higgs multiplet that is responsible for the breaking of the $B - L$ sym-

TABLE I. Z_3 symmetry quantum numbers for various fields. ($w = e^{i2\pi/3}$.)

Fields	Z_3
S	ω
A	ω
H_1	ω^2
H_2	1
Δ	ω^2
$\overline{\Delta}$	1

metry. Strict Yukawa unification does not hold in these models [11]. We have given two examples of such models and studied their predictions for neutrino masses and mixings as well as proton decay. We find that all the models discussed in this paper are testable by the solar neutrino experiments as well as the proposed accelerator experiments. (We of course do not mean that the predictions for neutrino masses and mixings apply to arbitrary MLHP models.) In the proton decay sector, the situation is less predictive than in the case of the minimal SU(5) model because of the presence of two different contributions. We have also given a model in which the Higgsino-mediated proton decay can indeed be arbitrarily suppressed.

ACKNOWLEDGMENTS

APPENDIX

In this appendix, we will discuss the symmetry breaking of the SO(10) model down to the standard model for the choice of Higgs multiplets S {54}, A {45}, Δ {126} $\oplus \overline{\Delta}$ {$\overline{126}$}. The relevant part of the superpotential is given by W_s in Eq. (4). To this we add the Lagrange multiplier term $g\,\mathrm{Tr}\,S$ so that we can carry out the variation of all elements of the ten-by-ten symmetric matrix representing {54} $\oplus$ {1}. We will go to a basis where S is diagonal without loss of generality. Let us now write down the constraints implied by all F terms being zero. We will look for solutions with

$$\langle S\rangle = \mathrm{diag}(1,1,1,1,1,1,-\tfrac{3}{2},-\tfrac{3}{2},-\tfrac{3}{2},-\tfrac{3}{2})M_U,$$
$$\langle A\rangle = i\tau_2 \otimes \mathrm{diag}(b,b,b,c,c),$$
$$\langle\Delta\rangle_{\nu^c\nu^c} = \langle\overline{\Delta}\rangle_{\bar\nu^c\bar\nu^c} = v_R,$$
$$\langle H\rangle = 0. \tag{A1}$$

From the equation $F_s = 0$,

$$2\mu_s M_U + 3\lambda_s M_U^2 - \lambda_A b^2 + g = 0, \tag{A2}$$
$$2\mu_s(-\tfrac{3}{2}M_U) + 3\lambda_s(\tfrac{9}{4}M_U^2) - \lambda_A c^2 + g = 0. \tag{A3}$$

Demanding that $\mathrm{Tr}S = 0$ determines g as

$$g = \left[-\frac{9}{2}\lambda_s M_U^2 + \frac{1}{5}\lambda_A(3b^2 + 2c^2)\right]. \tag{A4}$$

The other constraints are from vanishing of F_A, F_Δ and $F_{\overline{\Delta}}$ respectively:

$$-2\mu_A b - 2\lambda_A b M_U + \lambda_\Delta x_0 d^2 = 0, \tag{A5}$$
$$-2\mu_A c + 3\lambda_A c M_U + \lambda_\Delta d^2 = 0, \tag{A6}$$
$$\mu_\Delta d + \lambda_\Delta(x_0 b + c)d = 0, \tag{A7}$$

where x_0 is a Clebsch-Gordan (CG) coefficient.
Using the above constraints, we find the doublet

Higgsino matrix with quantum numbers (2,1,1) or (2,−1,1) [under SU(2)$_L\times$ U(1)$_Y\times$ SU(3)$_C$] to be that given in Eq. (7). As already mentioned, it contains only one pair of light doublets whose bosonic partners will be used to break the SU(2)$_L\times$ U(1)$_Y$. For the color-triplet Higgsino matrix with quantum numbers (1,−2/3,3) or (1,2/3,3*) to be that given in Eq. (9). As already mentioned, it does not have zero eigenvalues.

The Goldstone modes are contained in the following mixings:

(2,1/3,3), (2,−1/3,3*); $S_{2,2,6}$, $A_{2,2,6}$, $\Delta_{2,2,15}$, $\overline{\Delta}_{2,2,15}$,
(2,−5/3,3), (2,5/3,3*); $S_{2,2,6}$, $A_{2,2,6}$,

(1,4/3,3); $A_{1,1,15}$, $\overline{\Delta}_{1,3,10}$
(1,−4/3,3*); $A_{1,1,15}$, $\Delta_{1,3,\overline{10}}$,
(1,2,1); $A_{1,3,1}$, $\Delta_{1,3,\overline{10}}$,
(1,−2,1); $A_{1,3,1}$, $\overline{\Delta}_{1,3,10}$,
(1,0,1); $S_{1,1,1}$, $A_{1,1,15}$, $A_{1,3,1}$, $\Delta_{1,3,\overline{10}}$, $\overline{\Delta}_{1,3,10}$.

Now, only two mixings are left. They are
(1,0,8); $S_{1,1,20'}$, $A_{1,1,15}$,
(3,0,1); $S_{3,3,1}$, $A_{3,1,1}$.

We have exhausted all the submultiplets contained in A. The other submultiplets of S, Δ, and $\overline{\Delta}$ cannot be mixed. The remaining submultiplets in S are
(1,−4/3,6), (1,4/3,6*); $S_{1,1,20'}$,

(3,2,1), (3,−2,1); $S_{3,3,1}$.
Their masses are given by

$$|2\mu_s + 6\lambda_s c_i M_U|, \tag{A8}$$

where c_i are CG coefficients. We have checked that their masses are of order M_U, using Eqs. (A2)–(A4).

The remaining submultiplets are contained in Δ or $\overline{\Delta}$, and they are unmixed. Their masses have the form

$$|\mu_\Delta + \lambda_\Delta(x_i b + y_i c)|, \tag{A9}$$

where x_i and y_i are CG coefficients. To confirm that all the submultiplets are heavy, it is sufficient to show that x_i in Eq. (A9) cannot be x_0 in Eq. (A7) or that y_i in Eq. (A9) cannot be 1. From simple group theoretical consideration, $y_i = 0$ for the submultiplets in $\overline{\Delta}_{1,1,6}$, and $\overline{\Delta}_{3,1,\overline{10}}$. For the submultiplets which have the quantum numbers (1,3,−2,1), (1,3,−2/3,3), and (1,3,2/3,6) [under SU(2)$_L\times$ SU(2)$_R\times$ U(1)$_{B-L}\times$ SU(3)$_C$], contained in $\overline{\Delta}_{1,3,10}$, the ratios [25] of x_i are 1:1/3:1/3. The submultiplet with the quantum numbers (1,−4,1) in the (1,3,−2,1) has the value $y_i = -1$. Considering tensor indices of $\overline{\Delta}_{2,2,15}$ and the (1,0,1) in $A_{1,1,15}$, we find that $x_i = 0$ for the submultiplets in $\overline{\Delta}_{2,2,15}$. The same is true for the corresponding submultiplets in Δ. Thus it is guaranteed by Eq. (A7) that all submultiplets whose masses are given by Eq. (A9) are superheavy. Also, we have checked that except the Goldstone modes all submultiplets involved in the above mixings have superheavy masses.

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