CP violation in the cubic coupling of neutral gauge bosons

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We investigate the CP-violating form factor of the ZZZ and $ZZ\gamma$ vertices in the pair production of Z^0 bosons. Useful observables in azimuthal distributions are constructed to probe CP nonconservation which may originate from these vertices. A simple two-Higgs-doublet model of CP violation is used as an illustration.

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I. INTRODUCTION

In the near future, with the availability of experimental data at energies around the electroweak breaking scale, one expects to learn about the structure of the cubic and quartic self-interactions of gauge bosons. So far, these interactions have not been directly tested in any experiments.

One exciting possibility is that such interactions will give new insight into CP violation, whose physical origin has not been understood with satisfaction yet. The observation of CP violation in the kaon system can be explained in various ways within the framework of gauge theories, and choosing between them requires additional observation of CP violation. With this in mind, it is interesting to look for CP-violating signals which may be induced by the self-interactions of gauge bosons. We discuss here one such possibility, where the coupling of three neutral gauge bosons has a CP-violating term in it. We first did a model-independent discussion based on the most general form factors. Then a simple model, the two-Higgs-doublet model, is used as an illustration of how the form factors may arise in a realistic CP-violating theory.

II. HELICITY AMPLITUDES

Such a CP-odd term is indeed allowed in general on fundamental grounds, as is obvious from the general parametrization of the cubic coupling of gauge bosons [1–3]. Most theoretical studies along this direction have been done [4–6] only for the process $e^-e^+ \to W^-W^+$. The effect of CP violation in $e^-e^+ \to Z^0Z^0$ has not been thoroughly carried through [7] and there is the need for a detailed analysis. This motivates us to perform a careful model-independent study. In Sec. V, a simple two-Higgs-doublet model is used as an illustration. We follow the helicity formalism for Z^0 pair production, $e^-(\sigma)e^+(\bar\sigma) \to Z^0(\lambda)Z^0(\lambda')$, outlined in Appendix D of Ref. [3]. Here we include explicitly effects from the form factors f_4 and f_5 which describe the vertex $V(P) \to Z(q)Z(q')$ for outgoing on-shell Z^0 bosons, where the incoming particle V is either another Z boson or a photon:

$$ie\Gamma^{\mu\alpha\beta}_{V\to ZZ} = ie\,\frac{s-m_V^2}{M_Z^2}\left[if_4^V(P^\alpha g^{\mu\beta}+P^\beta g^{\mu\alpha}) + if_5^V\epsilon^{\mu\alpha\beta\rho}(q-q')_\rho\right] \quad (V=Z,\gamma)\,, \tag{1}$$

where $s=P^2$. Note that f_4 term is CP odd. The f_5 term, although CP even, is included for completeness. The helicity amplitudes are given by

$$\mathcal{M}_{\sigma,\bar\sigma;\lambda,\lambda'}(\Theta) = 4\sqrt{2}\,e^2\; d^{\max(|\Delta\sigma|,|\Delta\lambda|)}_{\Delta\sigma,\Delta\lambda}(\Theta)\left[\frac{(g_{\Delta\sigma})^2A_{\lambda,\lambda'}(\Theta)}{4\beta^2\sin^2\Theta+\gamma^{-4}} + \sum_{i=4,5}\gamma^2(g_{\Delta\sigma}f_i^Z - f_i^\gamma)A^{(i)}_{\lambda,\lambda'}\right]\,. \tag{2}$$

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The kinematic variables are defined as usual, $\gamma^{-2} = 1-\beta^2 = 4M_Z^2/s$. The amplitude for the initial helicity configuration $\bar{\sigma} = \sigma$ is highly suppressed due to the helicity argument in the high energy limit $\sqrt{s} \gg m_e$. Therefore we are only interested in the cases for which $\Delta\sigma \equiv \frac{1}{2}(\sigma - \bar{\sigma}) = \pm 1$. The relevant Wigner d functions appearing in Eq. (2) are listed below:

$$\begin{aligned} d^2_{1,\pm 2}(\Theta) &= -d^2_{-1,\mp 2}(\Theta) = \pm\frac{1}{2}(1 \pm \cos\Theta)\sin\Theta, \\ d^1_{1,\pm 1}(\Theta) &= d^1_{-1,\mp 1}(\Theta) = \frac{1}{2}(1\pm\cos\Theta), \\ d^1_{1,0}(\Theta) &= -d^1_{-1,0}(\Theta) = -\frac{1}{\sqrt{2}}\sin\Theta. \end{aligned} \tag{3}$$

In the standard electroweak model at the tree level, the elements $A_{\lambda,\lambda'}(\Theta)$ come from the t-channel exchange diagram. The electron couplings $g_{\Delta\sigma}$ to the Z^0 boson are specified by

$$\begin{aligned} g_- = g_L &= \left(\frac{1}{\sin\theta_W \cos\theta_W}\right)(-\frac{1}{2} + \sin^2\theta_W), \\ g_+ = g_R &= \left(\frac{1}{\sin\theta_W\cos\theta_W}\right)(\sin^2\theta_W). \end{aligned} \tag{4}$$

After simplification, we summarize the result for various cases $\Delta\lambda = \lambda - \lambda'$ as follows:

$\Delta\lambda$	$\lambda\ \lambda'$	$A_{\lambda\lambda'}(\Theta)$	$A^{(4)}_{\lambda\lambda'}$	$A^{(5)}_{\lambda\lambda'}$
± 2	$\pm\ \mp$	$-\sqrt{2}(1+\beta^2)$	0	0
± 1	$\pm\ 0$	$(1/\gamma)[\Delta\sigma\Delta\lambda(1+\beta^2) - 2\cos\Theta]$	$+i\gamma\beta$	$-\Delta\lambda\gamma\beta^2$
± 1	$0\ \pm$	$(1/\gamma)[\Delta\sigma\Delta\lambda(1+\beta^2) - 2\cos\Theta]$	$-i\gamma\beta$	$-\Delta\lambda\gamma\beta^2$
0	$\pm\ \pm$	$-(1/\gamma^2)\cos\Theta$	0	0
0	0 0	$-(2/\gamma^2)\cos\Theta$	0	0

(5)

When the kinematic variables of the two identical Z^0 bosons are interchanged, i.e.,

$$(\lambda, \lambda') \leftrightarrow (\lambda', \lambda), \quad \Theta \leftrightarrow \pi - \Theta, \quad \Phi \leftrightarrow \pi + \Phi\ , \tag{6}$$

the amplitude is unchanged because of Bose symmetry, if one includes a negative sign coming from the azimuthal Φ rotation $\exp(i\Delta\sigma\pi)$.

The usual CP transformation is

$$(\lambda,\lambda') \to (-\lambda, -\lambda'), \quad \Theta \to \pi - \Theta, \quad \Phi \to \pi + \Phi\ . \tag{7}$$

However, we can simplify this CP transformation by incorporating Bose symmetry in Eq. (6). The resulting CP transformation becomes

$$(\lambda, \lambda') \to (-\lambda', -\lambda), \quad \Theta,\ \Phi \text{ unchanged.} \tag{8}$$

The situation now becomes very similar to our previous analysis [6] in the process $e^-e^+ \to W^-W^+$.

If CP is conserved (when f_4's are turned off), we have the following relation for the amplitudes in our phase convention:

$$\mathcal{M}_{\sigma,\bar{\sigma};\lambda,\lambda'}(\Theta) = \mathcal{M}_{\sigma,\bar{\sigma};-\lambda',-\lambda}(\Theta)\ . \tag{9}$$

This equality will be destroyed by the presence of CP-violating form factors f_4 in channels $(\lambda, \lambda') = (0, \pm)$ or $(\pm, 0)$.

III. SPIN-DENSITY MATRICES

To avoid studying complicated event topology in the four-fermion final configuration from the decays of the Z^0 pair, we concentrate our attention to the decay of a single Z^0. This strategy is equivalent to the study of the density matrix for one of the Z^0 bosons.

We only look at the Z^0 boson at the scattering angle Θ and temporarily ignore the recoiling one, which is considered as being produced at the scattering angle π-Θ. The polar angle ψ and the azimuthal angle ϕ are defined in the Z^0 rest frame for the lepton ℓ^- in the decay $Z^0 \to \ell^-\ell^+$. We define the axes of the rest frame of Z^0 as follows. The z axis is along the direction of motion of Z^0 in the e^-e^+ c.m. frame. The x axis lies on the reaction plane and toward the direction where Θ increases. The y axis is given by the right-hand rule.

The angular distribution of ℓ^- from the $Z^0 \to \ell^-\ell^+$ decay is specified by the the spin-density matrix $\rho_{i,j}$ of the Z^0 boson:

$$\rho(\Theta)_{i,j} = \mathcal{N}(\Theta)^{-1} \sum_{\sigma,\bar{\sigma},\lambda'} \mathcal{M}_{\sigma,\bar{\sigma};i,\lambda'}(\Theta)\mathcal{M}^*_{\sigma,\bar{\sigma};j,\lambda'}(\Theta)\ . \tag{10}$$

Here $\mathcal{N}$ is the normalization such that $\mathrm{Tr}\rho = 1$. ρ is Hermitian by definition. The normalized distribution for ℓ^- is given by

$$\begin{aligned} \frac{dN(\ell^-,\Theta)}{d\phi\, d\cos\psi} = \frac{1}{4\pi}\frac{3}{4}\sum_{h=\pm} w_h \Big[&(1+h\cos\psi)^2\rho(\Theta)_{++} + (1-h\cos\psi)^2\rho(\Theta)_{--} + 2\rho(\Theta)_{00}\sin^2\psi \\ &-2\sqrt{2}\mathrm{Re}\rho(\Theta)_{+,0}(1+h\cos\psi)\sin\psi\cos\phi + 2\sqrt{2}\mathrm{Im}\,\rho(\Theta)_{+,0}(1+h\cos\psi)\sin\psi\sin\phi \\ &-2\sqrt{2}\mathrm{Re}\rho(\Theta)_{-,0}(1-h\cos\psi)\sin\psi\cos\phi - 2\sqrt{2}\mathrm{Im}\,\rho(\Theta)_{-,0}(1-h\cos\psi)\sin\psi\sin\phi \\ &+2\mathrm{Re}\ \rho(\Theta)_{+,-}(1-\cos^2\psi)\cos 2\phi - 2\mathrm{Im}\,\rho(\Theta)_{+,-}(1-\cos^2\psi)\sin 2\phi \Big]\ . \end{aligned} \tag{11}$$

The two contributions come from helicity configurations $\ell^-_R(h=1)$ and $\ell^-_L(h=-1)$, with different weights:

$$w_- = g_L^2/(g_L^2+g_R^2)\ , \quad w_+ = g_R^2/(g_L^2+g_R^2)\ , \quad w_- + w_+ = 1\ . \tag{12}$$

In our present phase convention, if CP were conserved (i.e., when $f_4 = 0$), we would have the identities

$$\rho(\Theta)_{\lambda,\lambda'} = \rho(\pi-\Theta)_{-\lambda,-\lambda'}\ , \tag{13}$$

based on the transformation in Eq. (7). Similar expressions were first noticed in Ref. [4] on the process $e^-e^+ \to W^-W^+$ and in Ref. [8] on the process $e^-e^+ \to t\bar{t}$.

IV. *CP*-VIOLATING OBSERVABLES

Under CP conjugation, we change variables $\Theta \to \pi - \Theta$, $\psi \to \pi - \psi$, and $\phi \to -\phi$. The distribution in Eq. (11) is transformed into itself if we assume CP conservation as in Eq. (13). In the presence of the CP-violating term f_4, our analysis of CP-violating observables in Ref. [6] can be easily applied here.

However, as the coupling of $\ell\bar{\ell}Z^0$ is almost purely axial vectorial, there is approximate charge symmetry C, which assigns this vertex even C parity, with the f_4 term also even as well. Any C-odd observable will be suppressed.

We find out that the most prominent effect of CP nonconservation resides in the elements $(+,-)$ or $(-,+)$ of the spin-density matrix:

$$\mathrm{Im}\,\rho(\Theta)_{+,-} - \mathrm{Im}\,\rho(\pi-\Theta)_{-,+} = \frac{32e^4}{\mathcal{N}(\Theta)} \sum_{\Delta\sigma=\pm} (g_{\Delta\sigma})^2\,(\Delta\sigma)\,\gamma^2(\beta+\beta^3)\sin^2\Theta \frac{\mathrm{Re}\,(f_4^\gamma - g_{\Delta\sigma}f_4^Z)}{4\beta^2\sin^2\Theta+\gamma^{-4}}\ . \tag{14}$$

This particular location in the density matrix produces the azimuthal dependence in the form of $\sin 2\phi$. If we integrate ψ and ϕ over quadrants, we expect that CP nonconservation appears in the folded asymmetry, $\mathcal{A}''(\Theta)$, which is

$$\frac{[dN(\ell,\Theta,\mathrm{I{+}III}) + dN(\ell,\pi-\Theta,\mathrm{I{+}III})] - [dN(\ell,\Theta,\mathrm{II{+}IV}) + dN(\ell,\pi-\Theta,\mathrm{II{+}IV})]}{[dN(\ell,\Theta,\mathrm{I{+}II{+}III{+}IV}) + dN(\ell,\pi-\Theta,\mathrm{I{+}II{+}III{+}IV})]}\ . \tag{15}$$

Here the range of the azimuthal angle has been divided into the four usual quadrants I, II, III, and IV. It turns out that this observable $\mathcal{A}''$ is C even and thus it is not subjected to the suppression from approximate C symmetry.

We can show that

$$\mathcal{A}''(\Theta) = -\frac{1}{\pi}\Big[\mathrm{Im}\,\rho(\Theta)_{+,-} - \mathrm{Im}\,\rho(\pi-\Theta)_{-,+}\Big]\ . \tag{16}$$

It is interesting [9] to note that we do not need to know the charge of ℓ as the events are collected over quadrants I+III or II+IV. We can use this fact to apply our formula even to the larger sample of jet events from the Z^0 pair without tagging the charges of the primary partons. Our formalism can be easily translated for the process $q\bar{q} \to Z^0Z^0$ in the hadron collider.

In Fig. 1, we show the CP-odd asymmetry in the density matrix versus the scattering angle Θ per unit of small Re f_4^Z at various energies, $\sqrt{s} = 200$, 250, and 300 GeV in the e^+e^- collider. Observation of this asymmetry is a genuine signal CP violation, as it is not faked by the final state interaction.

At CERN e^+e^- collider LEP II energy $\sqrt{s} = 200$ GeV, the Z^0Z^0 production cross section is about 1.4 pb (see Fig. 2) which can provide about 700 Z^0Z^0 pairs per year for the design luminosity of 5×10^{31} cm^{-2} s^{-1}. As we have shown above, it is possible to test CP symmetry in purely charged leptonic, purely hadronic, or mixed channels of the two Z^0 boson decays. However, we may require that at least one of the Z^0 decay into the charged leptons in order to avoid backgrounds from the W^+W^- production. The branching ratio of a single Z^0 decaying into all charged leptonic channels $(e^+e^- + \mu^+\mu^- + \tau^+\tau^-)$ is about 10%. Therefore, we estimate about 140 tagged events per year in this category, and it is possible to measure $\mathcal{A}''(\Theta)$ at a sensitivity about 0.1, which translates to the level of f_4^Z about 0.4, according to Fig. 1. At higher energy $\sqrt{s} > 300$ GeV, such as that in the Next Linear Collider (NLC), we can greatly improve the measurement of CP asymmetry. In fact, the contribution $\Delta\sigma$

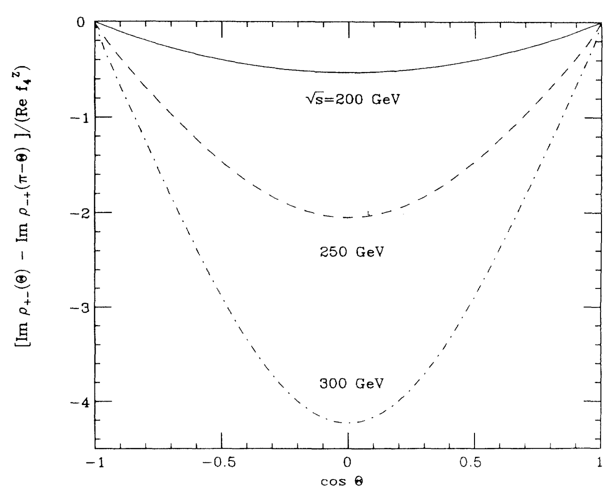

FIG. 1. The CP-odd asymmetry in the density matrix versus the scattering angle Θ per unit of small Re f_4^Z at various energies, $\sqrt{s} = 200$, 250, and 300 GeV.

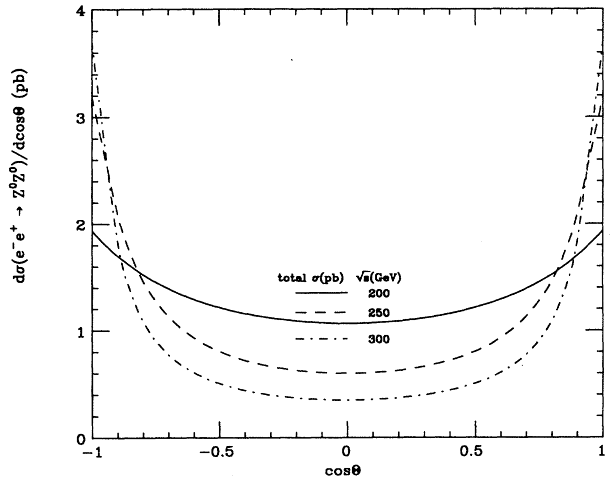

FIG. 2. Differential cross section $d\sigma/d\cos\Theta$ for $e^-e^+ \to Z^0Z^0$ at various energies $\sqrt{s}$= 200 (solid curve), 250 (dashed curve), and 300 GeV (dot-dashed curve), predicted by the standard model. The horizontal lines indicate the level of the corresponding total cross sections.

to the total cross section from the anomalous form factor f_4^Z grows rapidly as the energy increases. Figure 3 compares the standard model prediction $\sigma_{\rm SM}$ (solid curve) with $\Delta\sigma$ (dashed curve). Around $\sqrt{s} \simeq 450$ GeV, $\Delta\sigma$ becomes comparable to $\sigma_{\rm SM}$ even for a small $f_4^Z \simeq 0.05$, and a large CP effect can appear.

V. TWO-HIGGS-DOUBLET MODEL

Cubic couplings among neutral gauge bosons do not appear at the tree level in the standard model gauge group of $SU(2)_L \times U(1)_Y$. But they can be induced at the loop level. In the minimal standard model with just one Higgs doublet, such amplitudes do not have any CP violation even at the one-loop level, as will be clear from our analysis below. We therefore perform the calculation of CP-violating effects in these trilinear couplings when there are two-Higgs-doublets [10] present in the model, which is a popular model in its own right. Among the possibilities which open up with the two doublets are spontaneous CP violation [11], incorporation of the Peccei-Quinn symmetry [12] to solve the strong CP problem, and incorporation of supersymmetry.

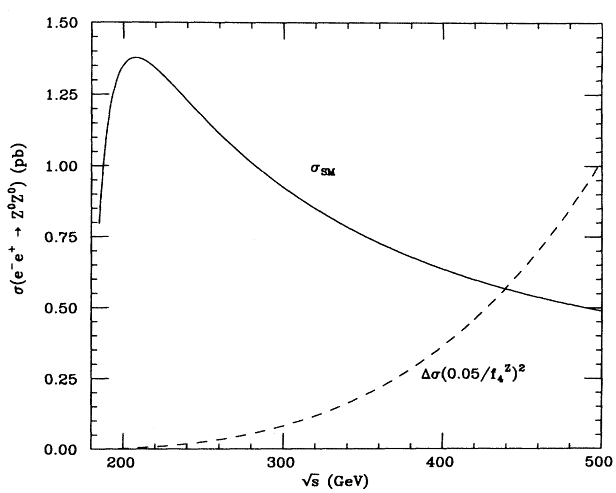

FIG. 3. Standard model prediction of the cross section $\sigma_{\rm SM}$ (solid curve) is compared with the deviation $\Delta\sigma$ (dashed curve) due to the anomalous form factor Re f_4^Z which is set to be 0.05.

At the one-loop level, cubic coupling obviously comes from triangle diagrams. If the internal lines are fermions, no CP-violating effect is generated at the one-loop level, because the Z or photon couplings with fermions are flavor diagonal and CP conserving. There are also triangle diagrams with internal W lines. In the Feynman–'t Hooft gauge, it can be shown that they do not contribute to the form factors as shown in Eq. (1). Thus, for our purpose, we need to calculate only the diagrams involving Higgs bosons in the loop. Obviously, such diagrams can never involve the antisymmetric ε symbol, and so one can only obtain a nonzero f_4^Z. This term has been shown to be nonzero for WWZ coupling at the one-loop level for the model at hand [13]. We want to extend their calculation for the case of V^*ZZ couplings, where V^* can be either an off-shell Z boson or photon, and the other two Z bosons are assumed to be on shell.

To set up the notation, we call the two Higgs multiplets to be φ_1 and φ_2. Usually, they are assumed to have special transformation properties with respect to some discrete symmetries in order to avoid flavor-changing neutral currents at the tree level. We assume that such discrete symmetries are not imposed on the soft terms in the Higgs potential; otherwise, CP violation would be eliminated in the Higgs sector of the model. Without any loss of generality, we can take the vacuum expectation values (VEV's) of φ_1 and φ_2 to be $v_1 \exp(i\vartheta)$ and v_2. One can then define a linear combination φ of the two multiplets which has a VEV, $v = \sqrt{v_1^2 + v_2^2}$, and the orthogonal one φ' has a vanishing VEV. The components of these doublets can then be written as

$$\varphi = \begin{pmatrix} w^+ \\ \frac{1}{\sqrt{2}}(v + \phi_1 + iz) \end{pmatrix}, \qquad \varphi' = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\phi_2 + i\phi_3) \end{pmatrix}. \tag{17}$$

The fields shown here are complex combinations of the fields in the φ_1-φ_2 basis. The components $w^\pm$ and z are absorbed by the gauge bosons and disappear from the physical spectrum. There are four physical spinless bosons in the model. One of them is the complex field H^+. The other three are, in general, superpositions of the fields ϕ_1, ϕ_2, and ϕ_3. We define the eigenstates by H_A, where

$$\phi_a = \sum_{A=1}^{3} O_{aA} H_A\,, \tag{18}$$

O being an orthogonal mixing matrix.

The coupling of these neutral Higgs bosons with the Z boson looks very simple in the ϕ basis:

Vertex	Feynman rule
$\phi_1(p) \xrightarrow{Z_\mu} z(q)$	$\frac{g}{2\cos\theta_W}(p+q)_\mu$
$\phi_2(p) \xrightarrow{Z_\mu} \phi_3(q)$	$\frac{g}{2\cos\theta_W}(p+q)_\mu$

(19)

Using Eq. (18), it is trivial to rewrite these Feynman rules in terms of the mass eigenstates of neutral Higgs bosons:

Vertex	Feynman rule
$H_A(p) \xrightarrow{Z_\mu} z(q)$	$\frac{g}{2\cos\theta_W}O_{1A}(p+q)_\mu$
$H_A(p) \xrightarrow{Z_\mu} H_B(q)$	$\frac{g}{2\cos\theta_W}(O_{2A}O_{3B}-O_{2B}O_{3A})(p+q)_\mu$

(20)

Using the orthogonality of the mixing matrix O, we can write

$$O_{2A}O_{3B}-O_{2B}O_{3A}=\sum_C \epsilon_{ABC}O_{1C}\,, \qquad (21)$$

which simplifies the form of the Z coupling with two physical Higgs bosons. Notice that the Z coupling between two physical Higgs bosons is necessarily flavor changing, which opens up the possibility for CP violation at the one-loop level. For the reason that the photon field preserves flavors at the tree level, there is no f_4^γ form factor at the one-loop calculation in the two-Higgs-doublet model.

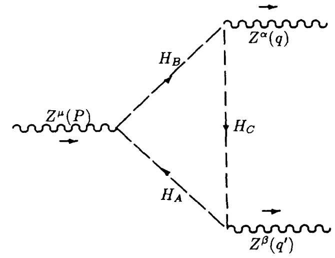

FIG. 4. Triangle diagrams with internal scalar lines which give rise to the Z^*ZZ coupling.

These cubic couplings appear in the triangle diagrams shown in Fig. 4. Notice that, in the figure, the Higgs boson lines have been denoted with subscripts i, j, k, which run from 0 to 3, where H_0 is identified with the unphysical Higgs boson z which appears as intermediate lines since we adopt the Feynman–'t Hooft gauge. A straightforward calculation now shows that the form factor f_4^Z from these diagrams can be written in the form

$$ef_4^Z=-\frac{1}{128\pi^2}\left(\frac{e}{\sin\theta_W\cos\theta_W}\right)^3\frac{M_Z^2}{P^2-M_Z^2}\sum_{i,j,k}\lambda_{ijk}I(M_i,M_j,M_k)\,. \qquad (22)$$

Here, λ_{ijk} is a factor coming from vertices which will be discussed below, and the loop integral $I(M_i,M_j,M_k)$ is equal to

$$2!\int\int(x-y)\,\ln\frac{\Lambda^2}{xM_i^2+yM_j^2+wM_k^2-w(1-w)M_Z^2-xyP^2-i0^+}dxdy\,, \qquad (23)$$

where the positive Feynman parameters x and y are restricted within the integration domain $x+y\le 1$ and also $w=1-x-y$. Λ is a cutoff which disappears in the expression for f_4^Z, as we will show below. When one of the particles denoted by i, j, or k is the unphysical Higgs boson, the corresponding mass should be interpreted to be M_Z, because the propagator of the unphysical Higgs boson has a pole for this value of mass in the gauge we use. For future purposes, notice that

$$I(M_i,M_j,M_k)=-I(M_j,M_i,M_k)\,, \qquad (24)$$

which follows from the definition in Eq. (23).

Let us now discuss the factor λ_{ijk}. First, consider the case when all the Higgs bosons in the loop are physical ones. Because of the antisymmetry of the coupling of $H_AH_BZ_\mu$ from Eq. (20), all the Higgs bosons in the loop must be different. If, following the direction of the momentum arrow in Fig. 4, we encounter the mass eigenstates H_1, H_2, and H_3 in that order, it is easy to see that the factor coming from the vertices is

$$\lambda_{123}=O_{11}O_{12}O_{13}\equiv\lambda\,. \qquad (25)$$

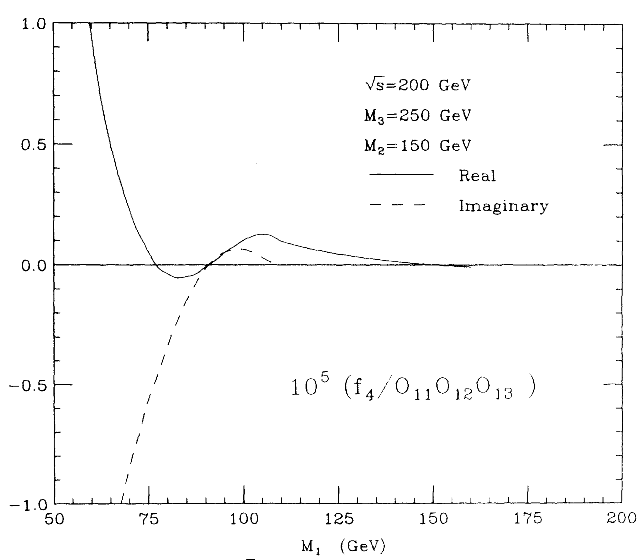

FIG. 5. The size of $f_4^Z/O_{11}O_{12}O_{13}$ versus the lightest Higgs boson mass at $\sqrt{s}=200$ GeV, for the cases M_2=150 GeV and $M_3=250$ GeV. The real and the imaginary parts are given by the solid and the dashed lines, respectively.

Obviously, there are three such diagrams, and their total contribution is

$$\lambda \{I(M_1, M_2, M_3) + I(M_2, M_3, M_1) + I(M_3, M_1, M_2)\} \ . \tag{26}$$

On the other hand, if we encounter the eigenstates in the reverse order, we obtain a factor $-\lambda$ from the vertices. However, this term will be multiplied by

$$\{I(M_2, M_1, M_3) + I(M_3, M_2, M_1) + I(M_1, M_3, M_2)\} \ .$$

By virtue of Eq. (24), the product of the two is the same as the contribution of Eq. (26).

Next we consider diagrams where one of the internal lines is the unphysical neutral Higgs boson z. Note that since there is no coupling of the Z boson with two unphysical Higgs bosons, at most one internal line can be the unphysical Higgs boson. In this case, one can derive

$$\lambda_{120} = \lambda_{230} = \lambda_{310} = -\lambda \ , \tag{27}$$

and the same value for any even permutation of subscripts, but opposite sign for an odd permutation. Therefore, the last factor of summation in Eq. (22) becomes

$$\begin{aligned}\sum_{i,j,k} \lambda_{ijk} I(M_i, M_j, M_k) = 2\lambda\Big\{ &+ I(M_1, M_2, M_3) + I(M_2, M_3, M_1) + I(M_3, M_1, M_2) \\ &- I(M_1, M_2, M_Z) - I(M_2, M_3, M_Z) - I(M_3, M_1, M_Z) \\ &- I(M_Z, M_1, M_2) - I(M_Z, M_2, M_3) - I(M_Z, M_3, M_1) \\ &+ I(M_Z, M_1, M_3) + I(M_Z, M_2, M_1) + I(M_Z, M_3, M_2)\Big\} \ .\end{aligned} \tag{28}$$

One can see that the cutoff Λ dependence is canceled by pairs in Eq. (28). We also note that f_4^Z remains finite when $P^2 = M_Z^2$ as noted in Ref. [3].

Figure 5 shows the extremely tiny size ($\sim 10^{-6}$) of f_4^Z for typical choices of parameters. We only use this two-Higgs-doublet model as an illustration of how CP violation occurs even in a purely bosonic sector.

VI. CONCLUSION

We have demonstrated possible CP-violating effects in the process $e^+e^- \to ZZ$. While the event statistics probably will not be large enough to test some of the popular alternative gauge models of CP violation, such as the two-Higgs-doublet model, it is nevertheless sufficient to provide nontrivial constraints on the CP-odd form factors in the three gauge boson couplings.

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