Uncertainties from long range effects in $B \to K^*\gamma$

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We reconsider the "long-range" component of the radiative transition $B \to K^*\gamma$. A careful analysis of the vector-dominance amplitude $B \to V_1V_2 \to V_1\gamma$ is carried out, with emphasis on the role of gauge invariance. The procedure for incorporating phenomenological $B \to V_1V_2$ data is identified, and polarization data, only recently available, are employed to estimate the magnitude of the vector dominance effect. We summarize uncertainties in the $B \to K^*\gamma$ radiative transition produced by long-range effects and provide suggestions for further experimental work.

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I. INTRODUCTION

Some time ago, we considered the possibility that the flavor-changing radiative transition $B \to K^*\gamma$ might experience a contribution from a so-called "long-range" component [1]. Among the possible contributions studied were those in Fig. 1. We concluded that the vector-meson-dominance (VMD) diagram of Fig. 1(a) was most likely the largest such contributor, with the dominant process being $B \to K^*\Psi \to K^*\gamma$. We then estimated the relative magnitude of the short-distance and VMD amplitudes. Since data on exclusive hadronic B decays were practically nonexistent at that time, we used theoretical estimates for the VMD amplitude. Part of the motivation for this paper is to update our original analysis in light of today's improved database. In addition, we wish to present a careful justification for using the VMD concept in flavor-changing radiative decays and also to explicitly show how phenomenology of the decay $B \to V_1V_2$ can be adopted via the VMD process to the radiative transition $B \to V\gamma$. Hopefully, this will clarify some confusion on this subject.

It is well chronicled in the literature just how active the study of the $B \to K^*\gamma$ mode has become, particularly with the recent experimental detection of this mode [2]:

$$B_{B\to K^*\gamma} = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5} \ . \quad (1)$$

As is well know, this determination is in accord (within errors) with expectations of the standard model prediction based upon the "short-distance" electromagnetic (EM) penguin transition. Of course, marked progress in lowering the present 39% uncertainty in the observed signal is anticipated. As this happens, an increased burden will fall upon theorists to properly interpret the experimental finding.

Although there is qualitative agreement regarding the importance of the EM penguin effect, the current theoretical situation is far from resolved in at least two respects. As pointed out by Buras and co-workers [4], scale dependence occurring at leading-order (LO) in QCD radiative corrections produces an uncertainty at the 25% level in the inclusive branching ratio $B_{b\to s\gamma}$. The dependence on scale would be reduced in a complete next-to-leading order (NLO) calculation, but analysis at this level has yet to be completed and formidable calculational complexities lie ahead.

Moreover, precise determination of the ratio $\Gamma_{B\to K^*\gamma}/\Gamma_{b\to s\gamma}$ of exclusive to inclusive decay rates is still a somewhat controversial subject. Presumably the very recent CLEO result [3]

$$B_{b\to s\gamma} = (2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4} \ , \quad (2)$$

which combined with Eq. (1) implies

$$\frac{B_{B\to K^*\gamma}}{B_{b\to s\gamma}} = 0.194 \pm 0.094 \ , \quad (3)$$

will begin the process of finally resolving this issue. The range of theoretical predictions, spanning almost 2 orders of magnitude, which appears in the literature is distressingly large. The situation is perhaps not surprising in view of the array of methods employed, from potential models to lattice-theoretic simulations. Although it is encouraging that the spread in lattice-based estimates is not as large, recent results ranging from 6 to 23 % indi-

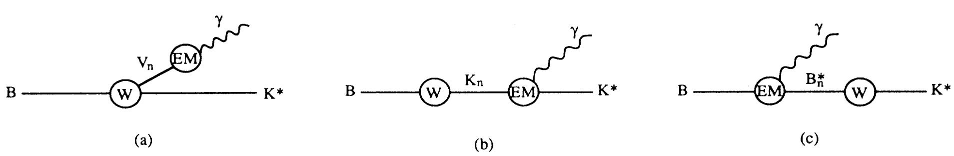

FIG. 1. Long-range effects.

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cate that more work is needed [5–7].

Despite the above uncertainties, it is clear that studies of the $B \to K^*\gamma$ decay have attained an impressive level of maturity. We have every reason to expect that the physics of this reaction will ultimately be understood. We feel that part of this understanding should involve the role of long-range effects. Let us now summarize the contents to follow. In Sec. II, we address the VMD effect by analyzing the topic of vector-vector final states in B meson decay ($B \to V_1V_2$) and the vector-meson–photon (V_γ) conversion process. In particular, we show how to extract relevant information form the $B \to V_1V_2$ amplitude and we also review the current status of the database. Then in Sec. III, we consider other possible long-range contributions such as pole diagrams, which are induced by the weak mixing of pseudoscalar and/or vector B mesons with non-b-flavored states. Our conclusions and recommendations for future study are given in Sec. IV.

II. VECTOR DOMINANCE AMPLITUDE

We wish to consider long-range contributions to the transition

$$B(p) \to K^*(\mathbf{k},\lambda) + \gamma(\mathbf{q},\sigma) \ . \tag{4}$$

The transition amplitude can be written in gauge-invariant form as

$$\mathcal{A}_{B\to K^*\gamma} = \epsilon_\mu^\dagger(k,\lambda)\epsilon_\nu^\dagger(q,\sigma) \times \left[\bar{B}(p^\mu p^\nu - g^{\mu\nu} q\cdot p) + i\bar{C}\epsilon^{\mu\nu\alpha\beta}k_\alpha p_\beta \right] , \tag{5}$$

where the overbars denote working in the B rest frame for the $B \to K^*\gamma$ process. Note the presence of the two independent amplitudes $\bar{B}$ and $\bar{C}$, which carry the dimension of inverse energy and are respectively parity violating and parity conserving. In general, both amplitudes are required because the weak interaction does not respect parity invariance. [8]

The $B \to K^*\gamma$ decay rate is given by

$$\Gamma_{B\to K^*\gamma} = \frac{|\mathbf{q}|^3}{4\pi}[|\bar{B}|^2 + |\bar{C}|^2] , \tag{6}$$

where $\mathbf{q}$ is the decay momentum in the B rest frame,

$$|\mathbf{q}| = \frac{m_B^2 - m_{K^*}^2}{2m_B} \ . \tag{7}$$

The branching ratio of Eq. (1) together with the average B lifetime value [3,9],

$$\tau_B = (1.63 \pm 0.07) \times 10^{-12} \text{ sec} , \tag{8}$$

implies that the transition amplitude has a magnitude

$$\left|\mathcal{A}_{B\to K^*\gamma}^{\text{expt}}\right| \equiv \sqrt{|\bar{B}|^2 + |\bar{C}|^2} = \left[\frac{4\pi\Gamma_{B\to K^*\gamma}}{|\mathbf{q}|^3}\right]^{1/2} = (3.68 \pm 0.72) \times 10^{-9} \text{ GeV}^{-1} \ . \tag{9}$$

A branching ratio determination alone does not distinguish between the parity-conserving and parity-violating amplitudes. Polarization data are required to disentangle them.

A. The $B \to V_1V_2$ transition

Application of the VMD concept to flavor-changing radiative decays is a subtle issue. This is partly because VMD is not a basic tenet of the standard model. That is, it is not explicitly present in the fundamental description of how quarks couple to gluons or to the electroweak gauge bosons nor is it associated with the Higgs sector. Rather it is a product of phenomenology, having been originally motivated by the similarity between photon-hadron scattering processes and purely hadronic reactions. Actually, this original application of the VMD concept resembles its proposed use here in weak radiative decays since the photon-hadron scattering and radiative decays both involve physical external-leg photons [10]. At any rate, the task is to formulate a radiative transition amplitude which respects basic principles and which utilizes VMD parameters (the $\{f_V\}$) which are determined from $V \to \ell^+\ell^-$ data.

Although we are primarily concerned with the decay $B \to K^*\gamma$, much of what follows is true for a more general transition $B \to M\gamma$, where M is a meson of nonzero spin. The VMD contribution to such a general flavor-changing radiative decay is depicted in Fig. 2, where (i) the pseudoscalar meson B decays weakly into meson M and a virtual neutral vector meson V, followed by (ii) the electromagnetic VMD conversion of V into a photon. There are two main issues, whether such a VMD amplitude is "really there" and if so, how to properly use $B \to MV$ data as input.

Consider the decay $B \to V_1V_2$ of the B into two vector mesons, where V_2 is electrically neutral. The constraint of angular-momentum conservation states,

$$\mathbf{J}_B = \mathbf{J}_{V_1V_2} \ \text{ with } \mathbf{J}_{V_1V_2} = \mathbf{L} + \mathbf{S} \ , \tag{10}$$

where $\mathbf{S}$ and $\mathbf{L}$ are the V_1V_2 total spin and orbital angular momentum. Since meson B is spinless, we have $\mathbf{J}_B = 0$ and so the three possible total spins $S = 0,1,2$ of the V_1V_2 state must be accompanied by the three orbital angular momenta $L = 0,1,2$. Thus there are three independent amplitudes $\mathcal{M}^{(\ell)}$ ($\ell = 0,1,2$). One could instead use helicity amplitudes $\mathcal{M}_{\lambda_1\lambda_2}$. As indicated in Fig. 3, these correspond to the three independent choices $\lambda_1\lambda_2 = ++, --, 00$. Throughout this paper, we shall classify these three configurations either as "transverse" T (for $++, --$) or "longitudinal" L (for 00).

Calculations are most easily performed in terms of

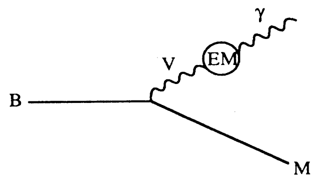

FIG. 2. VMD amplitude for $B\to M+V$.

Lorentz-covariant kinematic variables and their corresponding invariant amplitudes. Following Valencia [11], one can denote the three invariant amplitudes as a, b, c and write

$$
\begin{aligned}
\mathcal{M}_{\lambda_1\lambda_2} &= {}_{\rm out}\langle V_1(k_1,\lambda_1)V_2(k_2,\lambda_2)|B(p)\rangle_{\rm in} \\
&= \epsilon^\dagger_\mu(k_1,\lambda_1)\epsilon^\dagger_\nu(k_2,\lambda_2) \\
&\quad\times\left[ag^{\mu\nu}+\frac{b}{m_1m_2}p^\mu p^\nu\right. \\
&\quad\left.+i\frac{c}{m_1m_2}\epsilon^{\mu\nu\alpha\beta}k_{1\alpha}p_\beta\right] . \qquad (11)
\end{aligned}
$$

Note that the a, b, c amplitudes each carry the dimension of energy. The set of helicity amplitudes is constructed by writing explicit representations of the V_1, V_2 polarization vectors. We summarize the results here for convenience:

$$
\begin{aligned}
\mathcal{M}_{++} &= a-\sqrt{x^2-1}c\ , \\
\mathcal{M}_{--} &= a+\sqrt{x^2-1}c\ , \qquad (12)\\
\mathcal{M}_{00} &= -xa+(x^2-1)b\ ,
\end{aligned}
$$

where x is defined by

$$
x\equiv\frac{k_1\cdot k_2}{m_1m_2}=\frac{m_B^2-m_1^2-m_2^2}{2m_1m_2} \qquad (13)
$$

and obeys

$$
x^2=1+\frac{m_B^2|\mathbf{k}|^2}{m_1^2m_2^2}\ . \qquad (14)
$$

In Ref. [11], amplitude c is called the P-wave amplitude, while a and b are the S-wave and D-wave amplitudes. This nomenclature becomes apparent from the decay rate for $B\to V_1V_2$:

$$
\begin{aligned}
\Gamma_{B\to V_1V_2} &= \frac{|\mathbf{k}|}{8\pi m_B^2}\left[2|a|^2+|xa+(x^2-1)b|^2\right. \\
&\quad\left.+2(x^2-1)|c|^2\right]\ , \qquad (15)
\end{aligned}
$$

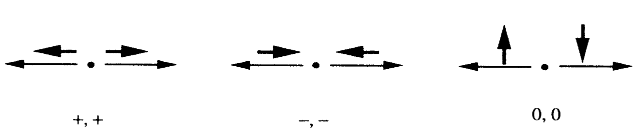

FIG. 3. Helicity configurations in $B\to V_1V_2$.

upon using Eq. (14). The $|\mathbf{k}|$, $|\mathbf{k}|^3$, and $|\mathbf{k}|^5$ dependences of the a, c, and b amplitudes mirror the threshold behaviors expected of S, P, and D waves, respectively. Decay rates corresponding to helicity configurations are obtained from the helicity amplitudes of Eq. (12):

$$
\Gamma_{B\to V_1V_2}=\frac{|\mathbf{k}|}{8\pi m_B^2}[|\mathcal{M}_{++}|^2+|\mathcal{M}_{--}|^2+|\mathcal{M}_{00}|^2]\ . \qquad (16)
$$

As expected, the various helicity contributions are decoupled since they are physically distinct.

B. The VMD amplitude

Let us consider the construction of a VMD amplitude using $B\to V_1V_2$ as input. There are problems with taking a theoretical model for $B\to V_1V_2$ since even if the model is arranged to fit the $B\to V_1V_2$ transition rate, the vector-meson polarizations may well not agree with experiment [12]. Therefore, we adopt a phenomenological approach. Suppose all the invariant amplitudes a, b, c have been determined in terms of experimental data from $B\to V_1V_2$ measurements. The next step is then to continue the $B\to V_1V_2$ decay amplitude from $k_2^2=m_2^2$ to $k_2^2=0$ such that the meson V_2 propagates as a massless virtual particle before converting to a photon. Throughout, however, the mass parameter m_2 in Eq. (11) will remain fixed at its physical value since it is present in the definition of amplitudes b,c simply for dimensional reasons. Using the *full* $B\to V_1V_2$ amplitude in the VMD calculation results in the $B\to V_{1\gamma}$ amplitude

$$
\begin{aligned}
\mathcal{A}_{\rm VMD} &= \frac{e}{\bar f_V}\epsilon^\dagger_\mu(k,\lambda)\epsilon^\dagger_\nu(q,\sigma) \\
&\quad\times\left[\bar a g^{\mu\nu}+\frac{\bar b}{m_1m_2}p^\mu p^\nu\right. \\
&\quad\left.\times i\frac{\bar c}{m_1m_2}\epsilon^{\mu\nu\alpha\beta}k_\alpha p_\beta\right] \qquad (17)
\end{aligned}
$$

where we denote the kinematics of the radiative process with overbars and take $k_1\to k, k_2\to q$ as well. The quantity $\bar f_V$ is the continuation to $k_2^2=0$ of a coupling, whose determination we shall discuss shortly, which occurs in the $V_2\to\gamma$ conversion vertex.

Then, under a gauge transformation as implemented by $\epsilon^\nu\to q^\nu$, the amplitude of Eq. (17) responds as

$$
\mathcal{A}_{\rm VMD}\to\frac{e}{\bar f_V}\left[\bar a\epsilon_1\dagger\cdot q+\frac{\bar b}{m_1m_2}q\cdot p\epsilon_1\dagger\cdot p\right]\ . \qquad (18)
$$

The new term must vanish if gauge invariance is to be maintained. Upon noting

$$
p=k+q\Longrightarrow\epsilon_1\dagger\cdot p=\epsilon_1\dagger\cdot q\ , \qquad (19)
$$

we obtain the constraint

$$
\bar a+\frac{\bar b}{m_1m_2}q\cdot p=0\ . \qquad (20)
$$

This condition means that we cannot use the full set of invariant functions a, b, c in the VMD amplitude for $B \to V_1\gamma$. The combination appearing in Eq. (20) must be avoided.

There is a simple physical interpretation of the above rule. Consider the decay of the pseudoscalar B into two longitudinally polarized vector mesons. Expressed in terms of the invariant amplitudes from Eq. (11), the corresponding amplitude is

$$\mathcal{M}_{00} = -\frac{1}{m_1 m_2}\left[(E_1E_2 + \mathbf{k}^2)a + \frac{m_B^2\mathbf{k}^2}{m_1m_2}b\right] . \quad (21)$$

In the $k_2^2 \to 0$ limit relevant to the VMD amplitude, we can reexpress this as

$$\mathcal{M}_{00} = -\frac{1}{m_1 m_2} q\cdot p\left[\bar{a} + \frac{\bar{b}}{m_1m_2}q\cdot p\right] . \quad (22)$$

Thus, the condition obtained in Eq. (20) from gauge invariance is equivalent to demanding that a vanishing contribution to $\mathcal{A}_{\rm VMD}$ coming from the $k_2^2 = 0$ off-shell extension of $\mathcal{M}_{00}$. That is, if the $B \to V_1V_2$ amplitudes are to be used as input to a VMD calculation, then one must not use the "00" helicity amplitude—it must be discarded. This result is entirely natural when viewed in terms of vector-meson–photon mixing. The helicity of a physical photon must have unit magnitude, so conversion from a vector meson with helicity zero is forbidden. As a corollary, it follows that if the physical $B \to V_1V_2$ decay consists entirely of the 00 helicity mode, then the corresponding VMD amplitude for radiative decay will vanish.

Next we turn to consideration of the VMD conversion vertex $V_2 \to \gamma$. It too must be constructed in a manner consistent with electromagnetic gauge invariance. This amounts to demanding that any mixing experienced by a photon propagating with squared momentum q^2 should not lead to a nonzero mass when the photon is on shell ($q^2 = 0$). The simplest possible conversion vertex involves just the field strength tensors $F_{\mu\nu}$ and $V^{\mu\nu}$ in the Lorentz-invariant combination $F_{\mu\nu}V^{\mu\nu}$ where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \text{ and } V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu . \quad (23)$$

However, an interaction of this type implies vanishing matrix elements at $q^2 = 0$ and would rule out application to any process with external photons. Among others, it was most notably Sakurai [13] who argued that the above relation, although gauge invariant, is too restrictive and should be extended to

$$\mathcal{L}_{\rm VMD} = \frac{e}{f_V}\left[\frac{1}{2}F_{\mu\nu}V^{\mu\nu} + J_\mu^V A^\mu\right] , \quad (24)$$

where J_μ^V is the (conserved current to which the vector meson V couples and f_V is defined by

$$\langle 0|eJ^\mu_{\rm em}(0)|V(k,\lambda)\rangle \equiv \frac{em_V^2}{f_V}\epsilon^\mu(k,\lambda) . \quad (25)$$

The dimensionless quantity f_V is typically determined in terms of $V \to \ell^+\ell^-$ data,

$$\Gamma_{V\to\ell^+\ell^-} = \frac{4\pi\alpha^2}{3}\frac{m_V}{f_V^2}\left[1 - \frac{4m_\ell^2}{m_V^2}\right]^{1/2}\left[1 + \frac{2m_\ell^2}{m_V^2}\right] . \quad (26)$$

Numerical values extracted in this manner are displayed in Table I, where the unit of energy is GeV.

It is clear from the above discussion that each f_V is determined at the physical kinematic value of $k^2 = m_V^2$. In the VMD amplitude, however, the kinematics is changed to $k^2 = 0$. In recognition of this, we have denoted the off-shell extension appearing in the VMD amplitude as $\bar{f}_V$.

C. Phenomenology

To proceed further, we must make a phenomenological determination of the invariant amplitudes from $B \to V_1V_2$ data. Table II displays some relevant branching fractions, taken from a very recent CLEO analysis of B decays [14]. As Eq. (15) for the $B \to V_1V_2$ width reminds us, measurement of the decay rate yields only partial information. To extract the amplitudes a, b, c requires, in addition, polarization measurements. The current status of such data is, to the best of our knowledge, summarized in Table III [14]. Note that existing data only distinguish between longitudinal and transverse polarization in the V_1V_2 final state.

For the remainder of this section, we shall restrict our discussion to $B \to K^*\Psi$ decay and take into account only the Ψ intermediate state. The vector meson Ψ is the only one for which relevant date are available and at the same time gives rise to the largest VMD amplitude. From Table II and the lifetime value in Eq. (8), we find

$$\Gamma^{(L)}_{B\to K^*\Psi} = (5.60 \pm 1.55)\times 10^{-13}\text{ MeV} ,$$
$$\Gamma^{(T)}_{B\to K^*\Psi} = (1.40 \pm 0.39)\times 10^{-13}\text{ MeV} , \quad (27)$$

where neutral and charged modes are averaged over.

On the other hand, the theoretical decay rates of transversely (summed over the $++$ and $--$ helicity configurations) and longitudinally polarized particles are

$$\Gamma^{(T)}_{B\to K^*\Psi} = \frac{|\mathbf{k}|}{4\pi m_B^2}\left[|a|^2 + (x^2-1)|c|^2\right] ,$$
$$\Gamma^{(L)}_{B\to K^*\Psi} = \frac{|\mathbf{k}|}{8\pi m_B^2}|xa + (x^2-1)b|^2 . \quad (28)$$

Without more detailed polarization data, we cannot determine all three a, b, c amplitudes. Therefore, let us first

TABLE I. The coefficients f_V.

V	$\Gamma_{V\to e^+e^-}$	m_v	f_V	e/f_V
ρ^0	6.77×10^{-6}	0.768	5.03	0.06
ω^0	6.03×10^{-7}	0.782	17.1	0.018
ϕ^0	1.37×10^{-6}	1.019	12.9	0.024
Ψ	5.36×10^{-6}	3.097	11.3	0.027
Ψ'	2.14×10^{-6}	3.686	19.6	0.015
Ψ''	0.26×10^{-6}	3.770	56.9	0.005

TABLE II. $B \to V_1V_2$ branching fractions.

Mode	$B_{B\to V_1V_2}$
$B^0 \to \bar{K}^{*0}\Psi$	$(1.69 \pm 0.031 \pm 0.018) \times 10^{-3}$
$B^- \to \bar{K}^{*-}\Psi$	$(1.78 \pm 0.051 \pm 0.023) \times 10^{-3}$
$B^- \to D^{*0}\rho^-$	$(1.68 \pm 0.21 \pm 0.25 \pm 0.12) \times 10^{-2}$
$B^0 \to D^{*+}\rho^-$	$(0.74 \pm 0.10 \pm 0.14 \pm 0.03) \times 10^{-2}$

assume that the entire decay of transversely polarized particles comes from the P-wave amplitude c. If so, we have the relation

$$\frac{|c|}{m_{K^*}m_\Psi} = \left[\frac{4\pi\Gamma^{(T)}_{B\to K^*\Psi}}{|\mathbf{k}|^3}\right]^{1/2} . \tag{29}$$

The other extreme, where transversely polarized particles arise from the a amplitude, implies

$$|a| = m_B\left[\frac{4\pi\Gamma^{(T)}_{B\to K^*\Psi}}{|\mathbf{k}|}\right]^{1/2} . \tag{30}$$

For the sake of completeness, we note in passing that the rate for longitudinal polarization then implies a value for the amplitude b, up to a phase ambiguity:

$$b_\pm = \frac{m^2_{K^*}m^2_\Psi}{m^2_B|\mathbf{k}|^2}\left[-xa \pm \sqrt{8\pi\Gamma^{(L)}_{B\to K^*\Psi}/|\mathbf{k}|}\right] . \tag{31}$$

Finally, we consider the VMD process $B \to K^*\Psi \to K^*\gamma$. This involves both implementing the phenomenological input obtained above and taking account of possible effects from the off-shell extrapolation procedure. As regards the former point, recall that we are constrained by gauge invariance to work only with the amplitudes for transversely polarized particles. Upon comparing Eq. (5) with Eq. (17), we obtain, for the scenario of pure parity-conserving (PC) decay,

$$\begin{aligned}\left|\mathcal{A}^{(\mathrm{PC})}_{\mathrm{VMD}}\right| \equiv \bar{C}_{\mathrm{VMD}} &= \frac{e}{\bar{f}_\Psi}\frac{|\bar{c}|}{m_{K^*}m_\Psi} \\ &\simeq \eta\frac{e}{f_\Psi}\frac{|c|}{m_{K^*}m_\Psi} \\ &= 3.68 \times 10^{-10}\ \mathrm{GeV}^{-1} .\end{aligned} \tag{32}$$

The parameter η incorporates modifications encountered in the kinematic extrapolation $k_2^2 = m_\Psi^2 \to k_2^2 = 0$. Fortunately, some relevant phenomenological guidance is already present in the literature [15] and we take $\eta = 0.63$. We shall return to this matter in the Conclusion. In magnitude, the ratio of the above parity-conserving amplitude to that of the full empirical amplitude of Eq. (9) is

TABLE III. Helicity content of $B \to V_1V_2$ transitions.

Mode	$\Gamma_L/(\Gamma_L + \Gamma_T)$
$B \to \bar{K}^*\Psi$	$0.80 \pm 0.08 \pm 0.05$
$B^0 \to D^{*+}\rho^-$	$0.93 \pm 0.05 \pm 0.05$

$$\left|\frac{\mathcal{A}^{(\mathrm{PC})}_{\mathrm{VMD}}}{\mathcal{A}^{(\mathrm{expt})}_{B\to K^*\gamma}}\right| \simeq 0.10 . \tag{33}$$

Following a similar procedure for a purely parity-violating (PV) amplitude yields the result

$$\begin{aligned}\left|\mathcal{A}^{(\mathrm{PV})}_{\mathrm{VMD}}\right| \equiv \bar{B}_{\mathrm{VMD}} &= \frac{e}{\bar{f}_\Psi}\frac{|\bar{a}|}{m_BE_\gamma} \\ &\simeq \eta\frac{e}{f_\Psi}\frac{|a|}{m_BE_\gamma} \\ &= 2.26 \times 10^{-10}\ \mathrm{GeV}^{-1},\end{aligned} \tag{34}$$

which implies

$$\left|\frac{\mathcal{A}^{(\mathrm{PV})}_{\mathrm{VMD}}}{\mathcal{A}^{(\mathrm{expt})}_{B\to K^*\gamma}}\right| \simeq 0.06 . \tag{35}$$

We shall discuss these findings in Sec. V. Next we address the estimation of other long-range effects in $B \to K^*\Psi$.

III. LONG-DISTANCE POLE CONTRIBUTIONS

By "long-distance pole contributions" we mean processes of the type shown in Figs. 1(b) and 1(c). Such amplitudes contain three essential ingredients: (1) a weak mixing in which the nonleptonic weak Hamiltonian (depicted by the circled W) converts the B meson to a non-bottom-flavored meson; (2) the radiation of a photon, which occurs independently of the weak mixing; (3) propagation of an off-shell meson whose flavor depends on the relative order of the above two items. In Figs. 1(a) and 1(b), there are two particles in the final state, a photon and a meson M, which we shall take as K^* or ρ. In principle, however, meson M can have any spin other than spin zero. The intermediate state in Fig. 1(b), denoted as P_n, is some non-bottom-flavored pseudoscalar meson. The subscript n indicates that we are to sum over all pseudoscalar mesons of the appropriate flavor. The intermediate state in Fig. 1(c) has an analogous meaning, except now one sums over all excited mesons B_n^* except for spin zero.

For definiteness, we shall use the effective weak Hamiltonian of Bauer, Stech, and Wirbel [16] (BSW). The transition operator appropriate for our purposes is

$$\mathcal{H}_w^{(\mathrm{eff})} = -\frac{G_F}{\sqrt{2}}V_{ub}V^*_{us}[a_1 : (\bar{u}b)(\bar{s}u) : +a_2 : (\bar{s}b)(\bar{u}u) :] , \tag{36}$$

where the colons denote normal ordering. In this paper, we employ the following numerical values for the Cabibbo-Kobayashi-Maskawa (CKM) parameters [3]:

$$\begin{aligned}|V_{us}| &= 0.22 , \\ |V_{ub}| &= 0.08 * |V_{cb}| = 0.08 * 0.040 \simeq 0.003 .\end{aligned} \tag{37}$$

The quark fields occur in left-handed combinations, denoted by

$$(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1+\gamma_5) q_2 \ , \tag{38}$$

and a_1, a_2 are free parameters determined by fitting to two-body B decays [17]:

$$\begin{aligned} a_1 &= 0.98 \pm 0.03 \pm 0.04 \pm 0.09, \\ a_2 &= 0.25 \pm 0.013 \pm 0.006 \pm 0.02 \ . \end{aligned} \tag{39}$$

It is important to remember that in the BSW description, the effective Hamiltonian is to be interpreted such as Fierz reordering is not allowed. Thus color-mismatched matrix elements are forbidden.

A. Pole amplitudes of type I

For amplitudes of this type, the weak mixing occurs prior to photon emission, as in Fig. 1(b). For $B \to K^*\gamma$, the virtual particle P_n which propagates will be a kaon or one of its pseudoscalar recurrences. Data availability forces us to consider just the kaon here. The decay amplitude $\mathcal{A}^{(\mathrm{I})}_{\mathrm{pole}}$ for the transition $B \to \gamma + M$ has the general form

$$\mathcal{A}^{(\mathrm{I})}_{\mathrm{pole}} = \sum_n g_{M\gamma P_n} \frac{1}{m_B^2 - m_{P_n}^2} \langle P_n | \mathcal{H}_w^{(\mathrm{eff})} | B \rangle \ . \tag{40}$$

With Fig. 1(b) as a guide, the notation should be self-evident.

The calculation of weak-mixing matrix element $\langle P_n | \mathcal{H}_W^{(\mathrm{eff})} | B \rangle$ of B with the kaon is straightforward in the factorization approach:

$$\langle K | \mathcal{H}_w^{(\mathrm{eff})} | B \rangle \simeq a_1 V_{ub} V_{us} f_K f_B m_B^2 G_F / \sqrt{2} \ . \tag{41}$$

For the decay constant of the kaon, we take

$$f_K = 161 \ \mathrm{MeV} \ . \tag{42}$$

The present situation for the decay constant f_B is somewhat problematic in that only theoretical estimates exist. These occur in three categories: lattice theoretic [18], QCD sum rule [19], and the quark model [20]. Estimates fall in the range $104 < f_B(\mathrm{MeV}) < 229$. We shall adopt the value

$$f_B \simeq f_K \ , \tag{43}$$

which is influenced most heavily by the lattice estimates.

The only other ingredient needed is the radiative coupling constant $g_{K^*K_\gamma}$. This is obtained from data on the transition $K^* \to K_\gamma$:

$$\Gamma_{K^* \to K_\gamma} = \frac{g^2_{K^*K_\gamma}}{12\pi} \mathbf{q}^3 \ , \tag{44}$$

where $\mathbf{q}$ is the decay momentum in the K^* rest frame. We find

$$g_{K^*K_\gamma} = 0.318 \ \mathrm{GeV}^{-1} \ , \tag{45}$$

where we have taken the average of the K^{*-} and $\bar{K}^{*0}$ radiative decays. We note in passing the rather large difference (40%) between the charged and neutral K^* modes. The types-I poles would be a possible source of isospin splitting in the B^- and $\bar{B}^0$ radiative decays, were such an effect detected.

Substituting in all the above values, we obtain

$$\mathcal{A}^{(\mathrm{I})}_{\mathrm{pole}} \simeq 3.77 \times 10^{-11} \ \mathrm{GeV}^{-1} \tag{46}$$

or equivalently

$$\left| \frac{\mathcal{A}^{(\mathrm{I})}_{\mathrm{pole}}}{\mathcal{A}_{\mathrm{expt}}} \right| \simeq 0.01 \ . \tag{47}$$

B. Pole amplitudes of type II

As mentioned above, a type-II pole amplitude is one in which the electromagnetic transition occurs before the weak mixing, cf. Fig. 1(c). Thus, we write

$$\mathcal{A}^{(\mathrm{II})}_{\mathrm{pole}} = \sum_n \langle V | \mathcal{H}_w^{(\mathrm{eff})} | B_n^* \rangle \frac{1}{m_{B_n^*}^2 - m_V^2} g_{B_n^* B_\gamma} \ . \tag{48}$$

For a phenomenological approach, the type-II transitions are less accessible than those of type I due mainly to a scarcity of data. However, with some theoretical input, we shall be able to consider the contribution from the $B^*(5325)$ intermediate state in detail.

Finally, there is the weak mixing between B^* and K^*:

$$\langle K^* | \mathcal{H}_w^{(\mathrm{eff})} | B^* \rangle \simeq a_1 V_{ub} V_{us} g_{K^*} g_{B^*} G_F / \sqrt{2} \ . \tag{49}$$

The "decay constant" for a vector meson V^b is defined by the matrix element

$$\langle 0 | V_\mu^a(0) | V^b(p,\lambda) \rangle = \delta^{ab} g_{V^b} \epsilon_\mu(p,\lambda) \ . \tag{50}$$

For the K^*, we use the SU(3) estimate

$$g_{K^*} \simeq g_\rho \simeq \sqrt{2} \frac{m_\rho^2}{f_\rho} \simeq 0.166 \ \mathrm{GeV}^2 \ , \tag{51}$$

where f_ρ is given in Table I. The B^* decay constant is estimated from the heavy-quark-symmetry relation

$$g_{B^*} = m_B f_B \simeq 0.845 \ \mathrm{GeV}^2 \ , \tag{52}$$

where we employ the numerical estimate for f_B given in Eq. (43).

Although there is not sufficient experimental data to infer the radiative coupling constant $g_{B^*B\gamma}$ phenomenologically, this quantity has been estimated in Ref. [21]. These authors correctly identify the $B^* \to B\gamma$ decay as a magnetic dipole transition and thus express the radiative coupling in terms of the B^* magnetic moment:

$$g_{B^*B\gamma} = e\left(\frac{Q_b}{m_b}+\frac{Q_q}{m_q}\right) \simeq 0.61\ \mathrm{GeV}^{-1}\ . \quad (53)$$

Thus we conclude

$$\mathcal{A}^{(\mathrm{II})}_{\mathrm{pole}} \simeq 1.79\times 10^{-11}\ \mathrm{GeV}^{-1}\ , \quad (54)$$

which is roughly half the size of the type-I amplitude. What about higher B^* excitations? A consequence of the BSW approach is that only states with $J = 1$ can contribute. States with $J > 1$ would not have a nonzero matrix element with the vacuum via the current $\bar{q}\gamma_\mu(1+\gamma_5)b$. The possibility of an intermediate bottomlike meson with $J = 0$ is disallowed since it could only mix with a final state $J = 0$ particle and the decay of a spinless particle to another spinless particle plus a photon is forbidden.

The values arrived at in this section should be considered as upper bounds for the following reason. We have considered just the lightest possible intermediate states, because only for these particles in there sufficient data for making a reasonable phenomenological determination. However, for the type-I amplitude, the kaon intermediate state propagates far off shell. Instead of having a squared momentum near $q^2 = m_K^2$, the kaon carries $q^2 = m_b^2 \gg m_K^2$. This effect should suppress the transition amplitude by an unknown amount. In principle, one is to sum over intermediate states. Contributions from excited states should be less affected by this suppression. Although there is not sufficient data to make a numerical estimate of their effect, we can anticipate that (i) the propagator contribution will indeed be larger, but (ii) the weak mixing between a ground-state B meson and a radially excited meson P_n will be wave-function suppressed, and (iii) the radiative coupling constant $g_{M_\gamma P_n}$ will be relatively smaller due to phase space competition with other decay modes of meson M. Qualitatively, the net effect of these considerations would be expected to decrease the overall radiative amplitude.

IV. CONCLUSION

In general, there can be no doubt that any study of long-distance effects for the heavy-meson transition $B \to K^* + \gamma$ is a very difficult task. For example, even a standard technique such as dispersion theory faces a host of contributing multiparticle intermediate states, and there exists no rigorous approximation scheme for dealing with these. Our feeling is that the most theoretically and empirically accessible long-distance contribution is the VMD amplitude, and our study of its quantitative role in $B \to K^* + \gamma$ is probably the most secure of our results. Interestingly it turns out to be the largest of the effects that we considered. As regards non-VMD contributions, we restricted our attention to pole diagrams. It is only for these that we have sufficient knowledge of the underlying parameters to do the field theory calculation with any confidence.

The analysis in Sec. II of the VMD decay amplitude had two aspects. First, there was the demonstration that a gauge-invariant formulation is possible provided that input from the $B \to V_1V_2$ transition is restricted to the transversely polarized part of the V_1V_2 final state. The phenomenological study which followed indicated a VMD component in the range

$$\left|\frac{\mathcal{A}^{(\mathrm{VMD})}_{B\to K^*\gamma}}{\mathcal{A}^{(\mathrm{expt})}_{B\to K^*\gamma}}\right| \le 0.1\ . \quad (55)$$

Such a term would affect the decay rate mainly through interference with the EM-penguin amplitude. We cannot be precise about the size of the interference because we do not know *a priori* the relative phases of the interfering amplitudes. However, if the VMD amplitude is purely parity conserving, it follows from Eq. (33) and our knowledge of the experimental amplitude that the interference effect is 15% in the rate. A purely parity-violating VMD amplitude would give rise to an interference term of 10%, and if the EM-penguin and VMD amplitudes have a common phase, the effect is 16%. In summary, we estimate the interference effect in the decay rate as induced by the VMD amplitude to lie between 10% and 16 %.

The size of the VMD effect given in this paper is smaller than the one we gave earlier [1], and it is instructive to see why. Three different experimental effects each turn out to decrease the VMD effect: (1) increase in B lifetime value (from 1.1 to 1.63 psec); (2) decrease in $B_{B\to K^*\Psi}$ (from 3.6 $\times 10^{-3}$ to 1.73 $\times 10^{-3}$); and (3) newly available polarization data in $B \to K^*\Psi$ which sharply limit decay into transversely polarized particles.

The first of these decreases the overall B decay rate and affects all modes equally. The latter two are specific to the VMD amplitude and suppress it relative to the experimental signal. The overall effect is a reduction of about 15 in the "transverse" $B \to K^*\Psi$ decay rate.

Although the VMD contribution obtained here has been inferred from the polarization data of just one experiment [14], the errors are encouragingly small and we expect our phenomenological finding to be stable. Indeed, the very recent Collider Detector at Fermilab (CDF) Collaboration announcement [22] regarding the helicity content in $B \to K^*\Psi$,

$$\frac{\Gamma_L}{\Gamma_L+\Gamma_T} = 0.66 \pm 0.10^{+0.08}_{-0.10}\ , \quad (56)$$

taken with the CLEO value in Table III implies the weighted average

$$\left\langle \frac{\Gamma_L}{\Gamma_L+\Gamma_T}\right\rangle = 0.75 \pm 0.08\ . \quad (57)$$

This change hardly affects the inequality in Eq. (55), raising the right-hand side to 0.11. There is also the question of the effect that the off-shell extrapolation procedure has on the VMD amplitude. We argued in Sec. II that a suppression will occur, which we are able to estimate by using an analysis relating inclusive ΨN and γN scattering [15]. Although the $\Psi \to \gamma$ conversion is common to that process and the radiative decay studied here, the hadronic matrix elements differ. We know of no rigorous means of determining this latter dependence, but have no reason to believe that the effect is a dominant one.

Suppose we accept at face value our findings regarding the smallness of long-distance effects in $B \to K^*\gamma$. What does this imply for subsequent studies of this decay? Within the context of the standard model, it emphasizes the dominance of the electromagnetic-penguin amplitude. We expect theorists to focus on reducing remaining uncertainties in the standard model prediction for this process. Since it will take a while for this to happen, we recommend prudence in avoiding overly strong claims. It has been suggested that opportunities exist for detecting the presence of physics beyond the standard model in $b \to s\gamma$ [23]. We concur, but at the same time caution that allowance be made for standard model uncertainties, e.g., such as those discussed here. That is, the inclusive branching ratio sums over exclusive processes, and each of these is (to a greater or lesser extent) itself subject to the influence of long-range effects [24].

As for experimental studies, we stress that as valuable as the $B \to K^*\gamma$ branching ratio determination has become, polarization studies of the $K^*\gamma$ final state would yield significant additional information. The chiral structure of the EM-penguin operator, up to $O(m_s/m_b)$, predicts in the notation of Eq. (5) that, $\bar{B} = \bar{C}$. As a consequence, the ratio of helicity amplitudes is

$$\left|\frac{\mathcal{M}_{--}}{\mathcal{M}_{++}}\right| = \frac{m_B^2}{m_{K^*}^2} \ . \tag{58}$$

This is an even firmer prediction of the short-distance amplitude than is the branching ratio.

We hope that experimental efforts to improve the already impressive accuracy of the helicity-dependent transition rates for $B \to K^*\Psi$ will continue. Of course, accurate decay-rate and helicity-content determinations of transitions such as $B \to K^*\Psi', K^*\rho, \ldots$ would likewise be welcome. Only with such information could we extend our phenomenological VMD analysis beyond the one given here. If the domination of longitudinal helicities seen in $B \to K^*\Psi$ continues to hold for the other transitions, we would expect the VMD chain $B \to K^*\Psi \to K^*\gamma$ studied here to be the largest. Based on the current experimental bound for $B \to K^*\Psi'$, the Ψ' VMD amplitude is estimated to be roughly 0.4 of the Ψ contribution and perhaps even smaller. Since expectations are not bright for even observing the $B \to K^*\rho$ transition in the immediate future, neglect of the ρ VMD amplitude appears well justified.

As we pointed out some time ago [1], isospin invariance is a consequence (in the spectator model) of describing the $B \to K^*\gamma$ decay solely in terms of the short-distance EM-penguin amplitude. That is, in this approximation, the rates for $B^- \to K^{*-}\gamma$ and $\bar{B}^0 \to \bar{K}^{*0}\gamma$ should be equal. Such is not the case for all other possible contributions. For example, there is a large isospin violation in the system of $K^* \to K\gamma$ transitions which would be manifested in the pole contributions of Sec. III. Further isospin dependence might be expected from dynamical interactions between the b quark and the light antiquark (i.e., wave-function effects).

Two additional radiative transitions of experimental interest are $B \to \rho\gamma$ and $B \to \omega\gamma$. From our vantage, these cannot be analyzed with the phenomenological method described here because the appropriate data does not yet exist. Therefore, purely theoretical models must be employed, and as a result the predictions for these decays will be rather model dependent. This work will be described in a separate publication.

Finally, we note that a forthcoming paper will deal with long-range effects in charm meson radiative decays [25]. This is especially interesting because, for charm transitions, the magnitude of the penguin short-distance contribution is greatly suppressed. Thus, it should be possible to experimentally probe the long-distance sector much more cleanly. Fortunately, experimental sensitivity is beginning to reach meaningful levels [26].

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