

## Effective Lagrangian approach to weak radiative decays of heavy hadrons

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Motivated by the observation of the decay  $\bar{B} \rightarrow \bar{K}^* \gamma$  by the CLEO Collaboration, we have systematically analyzed the two-body weak radiative decays of bottom and charmed hadrons. There exist two types of weak radiative decays: One proceeds through the short-distance  $b \rightarrow s \gamma$  transition and the other occurs through  $W$  exchange accompanied by a photon emission. Effective Lagrangians are derived for the  $W$ -exchange bremsstrahlung processes at the quark level and then applied to various weak electromagnetic decays of heavy hadrons. Predictions for the branching ratios of  $\bar{B}^0 \rightarrow D^{*0} \gamma$ ,  $\Lambda_b^0 \rightarrow \Sigma_c^0 \gamma$ ,  $\Xi_b^0 \rightarrow \Xi_c^0 \gamma$ , and  $\Xi_b^0 \rightarrow \Xi_c^{\prime 0} \gamma$  are given. In particular, we find  $\mathcal{B}(\bar{B}^0 \rightarrow D^{*0} \gamma) \approx 0.9 \times 10^{-6}$ . Order of magnitude estimates for the weak radiative decays of charmed hadrons,  $D^0 \rightarrow \bar{K}^{*0} \gamma$ ,  $\Lambda_c^+ \rightarrow \Sigma^+ \gamma$ , and  $\Xi_c^0 \rightarrow \Xi^0 \gamma$ , are also presented. Within this approach, the decay asymmetry for antitriplet to antitriplet heavy baryon weak radiative transitions is uniquely predicted by heavy quark symmetry. The electromagnetic penguin contribution to  $\Lambda_b^0 \rightarrow \Lambda \gamma$  is estimated by two different methods and its branching ratio is found to be of the order of  $1 \times 10^{-5}$ . We conclude that weak radiative decays of bottom hadrons are dominated by the short-distance  $b \rightarrow s \gamma$  mechanism.

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### I. INTRODUCTION

The recent observation of the weak radiative decay  $\bar{B} \rightarrow \bar{K}^* \gamma$  by the CLEO Collaboration [1] with the branching ratio  $(4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$  confirms the standard-model expectation that this decay mode is dominated by the short-distance electromagnetic penguin transition  $b \rightarrow s \gamma$ . Naively, it is tempting to think that  $\bar{B} \rightarrow D^* \gamma$  will be the dominant weak radiative decay of the  $\bar{B}$  meson as it is not suppressed by quark mixing angles. However, owing to the large top quark mass, the amplitude of  $b \rightarrow s \gamma$  is neither quark mixing nor loop suppressed. Moreover, it is largely enhanced by QCD corrections. As a consequence, the short-distance contribution due to the electromagnetic penguin diagram dominates over the  $W$ -exchange bremsstrahlung. This phenomenon is quite unique to the bottom hadrons which contain a heavy  $b$  quark; such a magic short-distance enhancement does not occur in the systems of charmed and strange hadrons. For example, it is known that the mechanism  $s \rightarrow d \gamma$  plays only a minor role in the radiative decays of kaons and hyperons.

In Ref. [2] we have systematically studied the flavor-conserving electromagnetic decays of heavy mesons and heavy baryons. Various photon coupling constant are related through the usage of heavy quark symmetry. For example, the  $\bar{B}^* \bar{B}^* \gamma$  coupling, which is very difficult to measure in practice, is related to the  $\bar{B}^* \bar{B} \gamma$  coupling

via heavy-quark spin symmetry. The coupling constants appearing in the Lagrangians depend only on the light quarks and can be calculated in the nonrelativistic quark model. Consequently, the dynamics of the electromagnetic transitions for emission of soft photons and pions is completely determined by heavy quark and chiral symmetry, supplemented by the quark model. The purpose of this paper is to extend our previous work to the weak radiative decays of heavy hadrons.

At the quark level, there are three different types of processes which can contribute to the weak radiative decays of heavy hadrons, namely, single-, two-, and three-quark transitions [3]. The single-quark transition mechanism comes from the so-called electromagnetic penguin diagram. Since the penguin process  $c \rightarrow u \gamma$  is very suppressed, it plays no role in charmed hadron radiative decays. We will thus focus on the two-body radiative decays of bottom hadrons proceeding through the electromagnetic penguin mechanism  $b \rightarrow s \gamma$ :

$$\begin{aligned} \bar{B} &\rightarrow \bar{K}^* \gamma, \quad \bar{B}_s \rightarrow \phi \gamma, \\ \Lambda_b^0 &\rightarrow \Sigma^0 \gamma, \quad \Lambda^0 \gamma, \\ \Xi_b^0 &\rightarrow \Xi^0 \gamma, \quad \Xi_b^- \rightarrow \Xi^- \gamma, \quad \Omega_b^- \rightarrow \Omega^- \gamma. \end{aligned} \quad (1.1)$$

There are two contributions from the two-quark transitions: one from the  $W$ -exchange diagram accompanied by a photon emission from the external quark (see, for example, Fig. 1), and the other from the same  $W$ -exchange diagram but with a photon radiated from the  $W$  boson. The latter is typically suppressed by a factor of  $m_q k / M_W^2$  ( $k$  being the photon energy) as compared to the former bremsstrahlung process [4]. For bottom hadrons, the dominant decays which occur through the quark-quark bremsstrahlung  $b\bar{d} \rightarrow c\bar{u} \gamma$  or  $bu \rightarrow cd \gamma$  are

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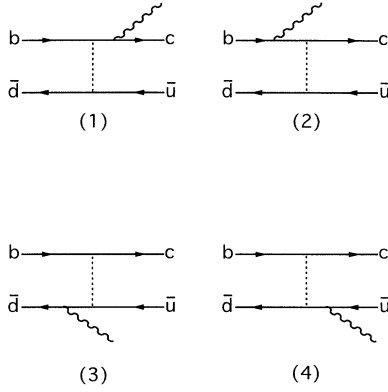


FIG. 1.  $W$ -exchange diagrams contributing to the quark-quark bremsstrahlung process  $b + \bar{d} \rightarrow c + \bar{u} + \gamma$  induced by the four-quark operator  $O_A$  defined in Eq. (2.6).

$$\bar{B}^0 \rightarrow D^{*0}\gamma, \quad \Lambda_b^0 \rightarrow \Sigma_c^0\gamma, \quad \Xi_b^0 \rightarrow \Xi_c^0\gamma, \quad \Xi_b^{\prime 0} \rightarrow \Xi_c^{\prime 0}\gamma, \quad (1.2)$$

where we have followed the convention that a  $\bar{B}$  meson contains a  $b$  quark and that  $\Xi_Q$  ( $\Xi_Q'$ ) denote antitriplet (sextet) heavy baryons. For charmed hadrons, the Cabibbo-allowed decay modes via  $c\bar{u} \rightarrow s\bar{d}\gamma$  or  $cd \rightarrow us\gamma$  are

$$D^0 \rightarrow \bar{K}^{*0}\gamma, \quad \Lambda_c^+ \rightarrow \Sigma^+\gamma, \quad \Xi_c^0 \rightarrow \Xi^0\gamma. \quad (1.3)$$

Note that some decay modes in (1.1) also receive contributions from  $W$ -exchange bremsstrahlung, but they are suppressed by quark mixing angles. Finally, the three-quark transition involving  $W$ -exchange between two quarks and a photon emission by the third quark is quite suppressed because of very small probability of finding three quarks in adequate kinematic matching with the baryons [3,5].

To summarize, the two important mechanisms for weak radiative decays of heavy hadrons are  $W$ -exchange bremsstrahlung and the electromagnetic penguin transition  $b \rightarrow s\gamma$ . Since the effective Lagrangian for the latter is known, the calculation for the radiative amplitude induced by the penguin diagram appears easier at first sight.

The  $W$ -exchange bremsstrahlung effect is usually evaluated under the pole assumption; that is, its amplitude is saturated by one-particle intermediate states. When dealing with weak radiative decays of heavy hadrons, one encounters two predicaments. First, the hadronic matrix elements for the processes (1.1) are evaluated at  $q^2 = 0$  for a real photon emission, whereas heavy quark symmetry and the quark model are known to be more reliable at zero recoil kinematic point where  $q^2$  is maximum. (The quark-model wave functions best resemble the hadron states in the frame where both hadrons are *static*.) Second, the intermediate states appearing in the pole diagrams for the processes (1.2) or (1.3) are very far

from their mass shell. For example, the four-momentum squared of the  $D$  pole in the decay  $\bar{B} \rightarrow D^*\gamma$  is  $m_B^2$  (see Fig. 3). This means that the residual momentum of the  $D$  meson defined by  $P_\mu = m_D v_\mu + k_\mu$  must be of order  $m_B$ , so the approximation  $k/m_D \ll 1$  required by the heavy quark effective theory is no longer valid. Also, the quark-model prediction for the photon coupling constants is presumably reliable only when both hadrons are nearly on their mass shell. The question is then how to extrapolate the hadronic matrix elements from zero recoil to maximal recoil, and the photon couplings from the on-shell point to off-shell? In principle, one can treat the intermediate state as an on-shell particle and then assume that off-shell effects of the pole can be parametrized in terms of form factors. Such form factors are basically unknown, though they are expected to become smaller as the intermediate state moves away from its mass shell due to less overlap of initial and final hadron wave functions. Consequently, based on heavy quark symmetry and the nonrelativistic quark model, at best we can only predict the upper bound of the decay rates for the radiative decays in the category of (1.2).

We will present in this paper a different but more powerful approach to the  $W$ -exchange bremsstrahlung processes. The fact that the intermediate quark state in these processes is sufficiently off shell (see, e.g., Fig. 1) and the emitted photon is hard suggests that the possibility of analyzing these processes by perturbative QCD. As a first step in this direction we study the tree amplitudes responsible for these processes and derive a gauge invariant effective five-point interaction for the quark-quark bremsstrahlung  $bu \rightarrow cd\gamma$  or  $b\bar{d} \rightarrow c\bar{u}\gamma$ .

The physical mass of a heavy hadron differs from the heavy quark mass by an amount of order  $\Lambda_{\text{QCD}}$ . This difference is due to the presence of the light quark(s). It is therefore reasonable to assign a constituent mass of order  $\Lambda_{\text{QCD}}$  to the light quark(s) inside a heavy hadron. In addition, the light quarks move, on average, with the same velocity as the heavy quark. We will make the simplifying assumption of neglecting the relative Fermi motion. Thus, the heavy quark and the light quark(s) in a heavy hadron move with equal four-velocity. This momentum parametrization has the advantage that the resulting effective interaction is local and manifestly gauge invariant. In Sec. III we will show explicitly for the meson case that the effective Lagrangian and the pole model approaches are indeed equivalent, but the former is much simpler and provides information on the form factors.

Armed with the effective Lagrangian for the  $W$ -exchange bremsstrahlung we are able to study various radiative decay modes of bottom and charmed hadrons listed in (1.2) and (1.3), bearing in mind that this approach presumably works better when both the initial and final hadrons contain a heavy quark. We will use the factorization method, which is known to work well for nonleptonic weak decays of heavy mesons, to evaluate the mesonic matrix elements. As for the baryon radiative decays we will demonstrate that heavy quark symmetry leads to a nontrivial prediction for antitriplet to antitriplet heavy baryon transitions: The ratio of parity-conserving and parity-violating amplitudes is uniquely

predicted. Baryonic matrix elements will be calculated using the MIT bag model.

This paper is organized as follows. Local effective Lagrangians for the quark-quark bremsstrahlung processes are derived in Sec. II. We then apply this approach to weak radiative decays of heavy mesons in Sec. III and to various bottom decays in Sec. IV. Discussion and conclusion are presented in Sec. V. Some preliminary results have been reported earlier [6].

## II. EFFECTIVE LAGRANGIANS FOR WEAK RADIATIVE DECAYS

In this section we will present the effective Lagrangians for the penguin transition  $b \rightarrow s + \gamma$  and for the  $W$ -exchange bremsstrahlung processes  $b + u \rightarrow c + d + \gamma$  and  $b + \bar{d} \rightarrow c + \bar{u} + \gamma$ .

The effective Lagrangian for the short-distance  $b \rightarrow s\gamma$  transition including QCD corrections reads [7]

$$\mathcal{L}_{\text{eff}}(b \rightarrow s\gamma) = \frac{G_F}{2\sqrt{2}} \frac{e}{8\pi^2} F_2 V_{tb} V_{ts}^* \bar{s} \sigma \cdot F [m_b(1 + \gamma_5) + m_s(1 - \gamma_5)] b, \quad (2.1)$$

where  $\sigma \cdot F \equiv \sigma_{\mu\nu} F^{\mu\nu}$ , and we have neglected contributions which vanish for a real photon emission,  $V_{ij}$  is the quark mixing matrix element,  $F_2 \cong F_2(x_t) - F_2(x_c) \cong F_2(x_t)$  with  $x_i = m_i^2/M_W^2$ , and

$$F_2(x) = \rho^{-16/23} \left\{ \bar{F}_2(x) + \frac{116}{27} \left[ \frac{1}{5} (\rho^{10/23} - 1) + \frac{1}{14} (\rho^{28/23} - 1) \right] \right\}, \quad (2.2)$$

with

$$\bar{F}_2(x) = \frac{(8x^2 + 5x - 7)x}{12(x-1)^3} - \frac{(3x-2)x^2}{2(x-1)^4} \ln x, \quad (2.3a)$$

$$\rho = \frac{\alpha_s(m_b^2)}{\alpha_s(M_W^2)} = 1 + \frac{23}{12\pi} \alpha_s(m_b^2) \ln \left( \frac{M_W^2}{m_b^2} \right). \quad (2.3b)$$

It is easily seen that  $F_2$  is a smooth function of the top quark mass. For  $m_t = 174$  GeV and  $\Lambda_{\text{QCD}} = 200$  MeV, we find  $\bar{F}_2(x_t) = 0.39$  and  $F_2(x_t) = 0.65$ , so that the radiative decay  $b \rightarrow s\gamma$  is enhanced by QCD corrections by a factor of 2. The radiative decays  $\bar{B} \rightarrow \bar{K}^* \gamma$  and  $\bar{B} \rightarrow \phi \gamma$ , mediated by this penguin mechanism, have been studied extensively in the literature. In Sec. IV we will apply the effective Lagrangian (2.1) to the decay  $\Lambda_b \rightarrow \Lambda \gamma$ .

We next turn to the  $W$ -exchange bremsstrahlung processes  $b\bar{d} \rightarrow c\bar{u}\gamma$  and  $bu \rightarrow cd\gamma$ . The difficulty associ-

ated with highly off-shell intermediate states mentioned in the Introduction is easily overcome at the quark level. The propagator of the highly virtual quark can be reduced to a constant by energy-momentum conservation. The above photon emission reactions are then described by an effective five-point local interaction which is also gauge invariant. To begin with we note that the relevant QCD-corrected effective weak Hamiltonian is given by

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{2\sqrt{2}} V_{cb} V_{ud}^* (c_+ O_+ + c_- O_-) + \text{H.c.}, \quad (2.4)$$

with

$$O_{\pm} = O_A \pm O_B \quad (2.5)$$

and

$$O_A = (\bar{c}b)(\bar{d}u), \quad O_B = (\bar{c}u)(\bar{d}b), \quad (2.6)$$

where  $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_{\mu} (1 - \gamma_5) q_2$ . The Wilson coefficient functions  $c_{\pm}$ , evaluated at the scale  $\mu = m_b$ , have the values

$$c_+(m_b) \simeq 0.85, \quad c_-(m_b) \simeq 1.38. \quad (2.7)$$

We first consider the photon emission process  $b\bar{d} \rightarrow c\bar{u}\gamma$ . The amplitudes mediated by the operator  $O_A$  (see Fig. 1) are

$$A_1 = ee_c \bar{u}_c \gamma^{\mu} \frac{1}{\not{p}_c + \not{k} - m_c} \gamma^{\nu} (1 - \gamma_5) u_b \bar{v}_d \gamma_{\nu} (1 - \gamma_5) v_{\mu}, \quad (2.8a)$$

$$A_2 = ee_b \bar{u}_c \gamma^{\nu} (1 - \gamma_5) \frac{1}{\not{p}_b - \not{k} - m_b} \gamma^{\mu} u_b \bar{v}_d \gamma_{\nu} (1 - \gamma_5) v_{\mu}, \quad (2.8b)$$

$$A_3 = ee_d \bar{u}_c \gamma^{\nu} (1 - \gamma_5) u_b \bar{v}_d \gamma^{\mu} \frac{1}{-\not{p}_d + \not{k} - m_d} \gamma_{\nu} (1 - \gamma_5) v_{\mu}, \quad (2.8c)$$

$$A_4 = ee_u \bar{u}_c \gamma^{\nu} (1 - \gamma_5) u_b \bar{v}_d \gamma_{\nu} (1 - \gamma_5) \frac{1}{-\not{p}_{\bar{u}} - \not{k} - m_u} \gamma^{\mu} v_{\mu}, \quad (2.8d)$$

where  $k$  is the photon momentum.

We will parametrize the quark momenta in terms of velocities; this is more suitable when dealing with heavy quark symmetry:

$$p_b = m_b v, \quad p_{\bar{d}} = m_d v, \quad p_c = m_c v', \quad p_{\bar{u}} = m_u v'. \quad (2.9)$$

Since the  $b$  quark is heavy we have set the velocity of the  $\bar{d}$  quark to be the same as the  $b$  quark so that they will move together to form a bound meson state. Likewise,  $v_{\bar{u}} = v_c = v'$ . At this point we wish to emphasize that the light quark masses appearing in (2.8) and (2.9) are of the constituent type. This is attributed to the fact the typical Fermi momentum of the quarks in a hadron is of order  $\Lambda_{\text{QCD}}$ . Consequently, although the current quark masses of the light  $u$  and  $d$  quarks are only of order 10 MeV, their off shellness is of order  $\Lambda_{\text{QCD}}$ . We thus choose to have the light quarks close to their mass shell, so that  $p_q \approx m_q v$  with  $v^2 = 1$  and  $m_q$  being the constituent

quark mass. Obviously, this parametrization (2.9) does not provide a complete description of the Fermi motion inside the bound state. Nevertheless, it does take into account its average effect by giving a constituent mass of order  $\Lambda_{\text{QCD}}$  to the light quarks. This parametrization greatly simplifies the calculation by eliminating the photon's coupling to the convection currents and making the effective interaction local and manifestly gauge invariant.

With the momentum parameterization given by (2.9), it is easily seen that the contributions from the convection current add up to zero and the amplitude arises entirely from the magnetic moments of the quarks:

$$\begin{aligned}
A &= A_1 + A_2 + A_3 + A_4 \\
&= -\frac{ie}{m_i^2 - m_f^2} \left[ e_c \frac{m_f}{m_c} \bar{u}_c \sigma^{\mu\lambda} k_\lambda \gamma^\nu (1 - \gamma_5) u_b \bar{v}_d \gamma_\nu (1 - \gamma_5) v_u - e_b \frac{m_i}{m_b} \bar{u}_c \gamma^\nu (1 - \gamma_5) \sigma^{\mu\lambda} k_\lambda u_b \bar{v}_d \gamma_\nu (1 - \gamma_5) v_u \right. \\
&\quad \left. - e_d \frac{m_i}{m_d} \bar{u}_c \gamma^\nu (1 - \gamma_5) u_b \bar{v}_d \sigma^{\mu\lambda} k_\lambda \gamma_\nu (1 - \gamma_5) v_u + e_u \frac{m_f}{m_u} \bar{u}_c \gamma^\nu (1 - \gamma_5) u_b \bar{v}_d \gamma_\nu (1 - \gamma_5) \sigma^{\mu\lambda} k_\lambda v_u \right], \quad (2.10)
\end{aligned}$$

where  $m_i = m_b + m_d$  and  $m_f = m_c + m_u$ . The amplitude corresponding to Fig. 2 induced by the operator  $O_B$  can be obtained from Eq. (2.10) by the substitution  $u_b \leftrightarrow v_u$ . The above amplitudes can be further simplified by considering the commutator and anticommutator relations

$$\frac{1}{2} \{ \sigma_{\mu\nu}, \gamma_\lambda \} = \epsilon_{\mu\nu\lambda\alpha} \gamma^\alpha \gamma_5, \quad (2.11a)$$

$$\frac{1}{2} [ \sigma_{\mu\nu}, \gamma_\lambda ] = -i (g_{\mu\lambda} \gamma_\nu - g_{\nu\lambda} \gamma_\mu). \quad (2.11b)$$

We find that for the photon emission process  $b\bar{d} \rightarrow c\bar{u}\gamma$  we can simply replace the operator  $O_\pm$  in (2.4) by  $O_\pm^F$  so that the effective Hamiltonian is given by

$$\mathcal{H}_{\text{eff}}(b\bar{d} \rightarrow c\bar{u}\gamma) = \frac{G_F}{2\sqrt{2}} V_{cd} V_{ud}^* (c_+ O_+^F + c_- O_-^F), \quad (2.12)$$

with

$$O_\pm^F(b\bar{d} \rightarrow c\bar{u}\gamma) = \frac{e}{m_i^2 - m_f^2} \left\{ \left( e_c \frac{m_f}{m_c} + e_d \frac{m_i}{m_d} \right) (\tilde{F}_{\mu\nu} + iF_{\mu\nu}) O_\pm^{\mu\nu} - \left( e_u \frac{m_f}{m_u} + e_b \frac{m_i}{m_b} \right) (\tilde{F}_{\mu\nu} - iF_{\mu\nu}) O_\mp^{\mu\nu} \right\}, \quad (2.13)$$

where

$$O_\pm^{\mu\nu} = \bar{c} \gamma^\mu (1 - \gamma_5) b \bar{d} \gamma^\nu (1 - \gamma_5) u \pm \bar{c} \gamma^\mu (1 - \gamma_5) u \bar{d} \gamma^\nu (1 - \gamma_5) b, \quad (2.14a)$$

$$\tilde{F}_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}. \quad (2.14b)$$

Similarly, for the  $W$ -exchange bremsstrahlung  $bu \rightarrow cd\gamma$  we have

$$O_\pm^F(bu \rightarrow cd\gamma) = \frac{e}{m_i^2 - m_f^2} \left\{ \left( e_c \frac{m_f}{m_c} - e_d \frac{m_f}{m_d} \right) (\tilde{F}_{\mu\nu} + iF_{\mu\nu}) O_\pm^{\mu\nu} + \left( e_u \frac{m_i}{m_u} - e_b \frac{m_i}{m_b} \right) (\tilde{F}_{\mu\nu} - iF_{\mu\nu}) O_\mp^{\mu\nu} \right\}, \quad (2.15)$$

where now  $m_i = m_b + m_u$ ,  $m_f = m_c + m_d$ .

Strictly speaking, terms in (2.13) and (2.15) proportional to  $1/m_c$  and  $1/m_b$  should have been dropped since we have not included corrections of the same order to those proportional to  $1/m_u$  or  $1/m_d$ . These  $1/m_c$  and  $1/m_b$  terms are kept mainly for phenomenological reasons. We encounter a similar situation in the electromagnetic decays of heavy hadrons [2,8] in which the com-

mon practice is to retain the  $1/m_Q$  contributions due to the heavy quark's magnetic moments but to neglect the  $1/m_Q$  corrections to the matrix elements of the light quark's electromagnetic currents. Numerically, the  $1/m_c$  terms are especially significant because  $m_c$  is not very large and the  $c$  quark carries  $2/3$  units of charge. The effective Lagrangians (2.13) and (2.15) are the main results in this section. We will apply them and (2.1) to the weak

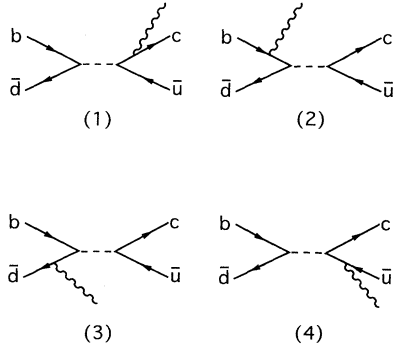


FIG. 2. Same as Fig. 1 except for the operator  $O_B$  defined in Eq. (2.6).

radiative decays of heavy mesons and heavy baryons in Secs. III and IV, respectively.

### III. APPLICATIONS TO HEAVY MESON DECAYS

As shown in the Introduction, the radiative decay modes of interest for  $B$  mesons are  $\bar{B} \rightarrow \bar{K}^* \gamma$ ,  $\bar{B}_s \rightarrow \phi \gamma$  which receive short-distance contributions from the electromagnetic penguin  $b \rightarrow s \gamma$  transition, and  $\bar{B} \rightarrow D^* \gamma$

proceeding through the  $W$  exchange accompanied by a photon emission. The general amplitude of weak radiative decay with one real photon emission is given by

$$A[\bar{B}(p) \rightarrow P^*(q)\gamma(k)] = i\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu k^\nu \epsilon^{*\alpha} q^\beta f_1(k^2) + \epsilon^\mu [\epsilon_\mu^*(m_B^2 - m_{P^*}^2) - (p+q)_\mu \epsilon^* \cdot k] f_2(k^2), \quad (3.1)$$

where  $\epsilon$  and  $\epsilon^*$  are the polarization vectors of the photon and the vector meson  $P^*$ , respectively, the first (second) term on the right-hand side (RHS) is parity conserving (violating), and  $k^2 = 0$ . The decay width implied by the amplitude (3.1) is

$$\Gamma(\bar{B} \rightarrow P^* \gamma) = \frac{1}{32\pi} \frac{(m_B^2 - m_{P^*}^2)^3}{m_B^3} (|f_1|^2 + 4|f_2|^2). \quad (3.2)$$

#### A. Effective Lagrangian approach for $\bar{B}^0 \rightarrow D^{*0} \gamma$

Since the radiative decay  $\bar{B} \rightarrow \bar{K}^* \gamma$  has been discussed extensively in the literature we will only focus on the second-type mode, namely,  $\bar{B}^0 \rightarrow D^{*0} \gamma$ . Our goal is to see if the tree-level  $W$  exchange with a photon emission is comparable with the short-distance  $b \rightarrow s \gamma$  mechanism. We shall use the factorization method (for a review, see Ref. [9]) to evaluate the hadronic matrix elements. It follows from Eq. (2.12) that

$$A(\bar{B}^0 \rightarrow D^{*0} \gamma) = -\langle D^{*0} \gamma | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle = -\frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_2 \frac{e}{m_i^2 - m_f^2} \langle D^{*0} | \bar{c} \gamma_\mu (1 - \gamma_5) u | 0 \rangle \langle 0 | \bar{d} \gamma_\nu (1 - \gamma_5) b | \bar{B}^0 \rangle \times \left[ \tilde{F}^{\mu\nu} \left( e_c \frac{m_f}{m_c} + e_d \frac{m_i}{m_d} + e_u \frac{m_f}{m_u} + e_b \frac{m_i}{m_b} \right) + iF^{\mu\nu} \left( e_c \frac{m_f}{m_c} + e_d \frac{m_i}{m_d} - e_u \frac{m_f}{m_u} - e_b \frac{m_i}{m_b} \right) \right], \quad (3.3)$$

with<sup>1</sup>

$$a_2 = \frac{1}{2}(c_- - c_+), \quad (3.4)$$

and  $m_i = m_b + m_d \approx m_B$ ,  $m_f = m_c + m_u \approx m_{D^*}$ . The one-body matrix elements appearing in (3.3) have the expressions

$$\langle 0 | A_\mu | P(p) \rangle = i f_P p_\mu, \quad (3.5a)$$

$$\langle 0 | V_\mu | P^*(p, \epsilon^*) \rangle = i f_V m_{P^*} \epsilon_\mu^*. \quad (3.5b)$$

Therefore,

<sup>1</sup>In the conventional vacuum insertion method  $a_2$  is equal to  $(2c_+ - c_-)/3$ , while it is  $(c_+ - c_-)/2$  in the large- $N_c$  approximation in which the Fierz-transformed contributions characterized by the color factor  $1/N_c$  are dropped [9]. The leading  $1/N_c$  expansion is known to work well for nonleptonic weak decays of charmed mesons. In bottom meson decays, the magnitude of  $a_2$  determined from the measured  $\bar{B} \rightarrow \psi \bar{K}, \psi \bar{K}^*$  rates is in agreement with that predicted by the large- $N_c$  approach [10–12]. However, contrary to that expected from the same approach, the sign of  $a_2$  is found to be positive by recent CLEO measurements of  $\bar{B} \rightarrow D\pi, D\rho, D^*\pi, D^*\rho$  decays [11–13]. Thus we take  $a_2$  to be that given by (3.4).

$$\langle D^{*0}(p_D) | \bar{c} \gamma_\mu (1 - \gamma_5) u | 0 \rangle \langle 0 | \bar{d} \gamma_\nu (1 - \gamma_5) b | \bar{B}^0(p_B) \rangle = -f_B f_{D^*} m_{D^*} \varepsilon_{\mu\nu}^* p_{B\nu} . \quad (3.6)$$

With the substitution of (3.6), (3.3) has the amplitude structure indicated by (3.1). We find

$$f_1 = 2\eta e \left[ \left( \frac{e_u}{m_u} + \frac{e_c}{m_c} \right) \frac{m_{D^*}}{m_B} + \left( \frac{e_d}{m_d} + \frac{e_b}{m_b} \right) \right] \frac{m_B m_{D^*}}{m_B^2 - m_{D^*}^2} , \quad (3.7a)$$

$$f_2 = \eta e \left[ \left( \frac{e_u}{m_u} - \frac{e_c}{m_c} \right) \frac{m_{D^*}}{m_B} - \left( \frac{e_d}{m_d} - \frac{e_b}{m_b} \right) \right] \frac{m_B m_{D^*}}{m_B^2 - m_{D^*}^2} , \quad (3.7b)$$

with

$$\eta = \frac{G_F}{2/\sqrt{2}} V_{cb} V_{ud}^* a_2 f_B f_{D^*} . \quad (3.8)$$

Substituting (3.7) into (3.2) gives the decay rate<sup>2</sup>

$$\begin{aligned} \Gamma(\bar{B}^0 \rightarrow D^{*0} \gamma) &= \frac{\alpha}{32} G_F^2 f_B^2 f_{D^*}^2 |V_{cb} V_{ud}^*|^2 m_{D^*}^2 m_B \left( 1 - \frac{m_{D^*}^2}{m_B^2} \right) (c_- - c_+)^2 \\ &\times \left\{ e_u^2 \frac{m_{D^*}^2}{m_B^2} \left( \frac{1}{m_u^2} + \frac{1}{m_c^2} \right) + e_d^2 \left( \frac{1}{m_d^2} + \frac{1}{m_b^2} \right) + 2e_u e_d \frac{m_{D^*}}{m_B} \left( \frac{1}{m_u m_b} + \frac{1}{m_d m_c} \right) \right\} . \end{aligned} \quad (3.9)$$

In order to have a numerical estimate we adopt the following mass parameters<sup>3</sup>:

$$\begin{aligned} m_{B^0} &= 5279.0 \text{ MeV} , \quad m_{D^{*0}} = 2006.7 \text{ MeV} , \\ m_u &= 338 \text{ MeV} , \\ m_d &= 322 \text{ MeV} , \quad m_s = 510 \text{ MeV} , \end{aligned} \quad (3.10)$$

from the Particle Data Group (PDG) [15], and  $m_c = 1.6$  GeV,  $m_b = 5.0$  GeV. Using  $f_{D^*} = 200$  MeV,  $f_B = 190$  MeV,  $V_{cb} = 0.040$  [16], and  $\tau(B^0) = 1.50 \times 10^{-12} s$  [15] we obtain the branching ratio

$$\mathcal{B}(\bar{B}^0 \rightarrow D^{*0} \gamma) = 0.92 \times 10^{-6} . \quad (3.11)$$

It is evident that the weak radiative decays of the  $\bar{B}$  mesons are dominated by the electromagnetic penguin diagram. The suppression of  $\bar{B} \rightarrow D^* \gamma$  relative to  $\bar{B} \rightarrow \bar{K}^* \gamma$  is mainly attributed to the smallness of the decay constants  $f_{D^*}$  and  $f_B$  occurred in weak transitions.

In the factorization approximation the three decay amplitudes  $\bar{B}^0 \rightarrow D^{*0} \gamma$ ,  $\bar{B}^{*0} \rightarrow D^{*0} \gamma$ , and  $\bar{B}^{*0} \rightarrow D^0 \gamma$  are all related since  $f_P = f_V$  by heavy quark symmetry. However, the branching ratios for the latter two reactions are expected to be much smaller than that for  $\bar{B}^0 \rightarrow D^{*0} \gamma$  as  $\bar{B}^{*0}$  has a dominant electromagnetic decay.

## B. Pole model approach for $\bar{B}^0 \rightarrow D^{*0} \gamma$

Before proceeding further we would like to compare our present formalism with the conventional long-distance pole mechanism which has been applied to the decay  $\bar{B} \rightarrow \bar{K}^* \gamma$  before [17]. Note that the intermediate  $\bar{B}$  state is absent in Fig. 3(d) as the  $\bar{B} \bar{B} \gamma$  coupling is prohibited. If we just focus on the low-lying intermediate states in the pole diagrams depicted in Fig. 3, the amplitudes of the first three pole diagrams are

$$M_a = \langle D^{*0}(q) | \mathcal{L}_{\text{em}} | D^0(p) \rangle \frac{1}{m_B^2 - m_D^2} \langle D^0(p) | \mathcal{L}_W | \bar{B}^0(p) \rangle , \quad (3.12a)$$

$$M_b = \langle D^{*0}(q) | \mathcal{L}_W | \bar{B}^{*0}(q) \rangle \frac{1}{m_{D^*}^2 - m_{B^*}^2} \langle \bar{B}^{*0}(q) | \mathcal{L}_{\text{em}} | \bar{B}^0(p) \rangle , \quad (3.12b)$$

$$M_c = \langle D^{*0}(q) | \mathcal{L}_{\text{em}} | D^{*0}(p) \rangle \frac{1}{m_B^2 - m_{D^*}^2} \langle D^{*0}(p) | \mathcal{L}_W | \bar{B}^0(p) \rangle , \quad (3.12c)$$

where the amplitudes  $M_a$  and  $M_b$  are parity conserving, while  $M_c$  is parity violating. Since the intermediate pole states are far from their mass shell we write the photon couplings in Fig. 3 as

<sup>2</sup>The result (3.9) was also obtained by Mendel and Stierski [14] in a different approach except for the Wilson factor  $a_2$  being replaced by  $(2c_+ - c_-)/3$  in the latter work.

<sup>3</sup>The values of the constituent quark masses are given on p. 1729 of [15].

$$g_{DD^*\gamma}(q^2 = m_B^2) = g_D(q^2)g_{DD^*\gamma}, \quad g_{B^*B\gamma}(q^2 = m_D^2) = g_{B^*}(q^2)g_{B^*B\gamma}, \quad g_{D^*D^*\gamma}(q^2 = m_B^2) = g_{D^*}(q^2)g_{D^*D^*\gamma}, \quad (3.13)$$

where  $g_{DD^*\gamma}$ ,  $g_{B^*B\gamma}$ , and  $g_{D^*D^*\gamma}$  are on-shell photon coupling constants which can be calculated in the nonrelativistic quark model. In Eq. (3.13),  $g_D$ ,  $g_{B^*}$ , and  $g_{D^*}$  are form factors accounting for off-shell effects. They are normalized to unity when mesons are on shell; for example,  $g_D(m_D^2) = 1$ ,  $g_{B^*}(m_{B^*}^2) = 1$ .

For  $P, P^* = (Q\bar{q})$  we find from Eqs. (2.19) and (2.30) of Ref. [2] that

$$\langle D^{*0}(q) | \mathcal{L}_{\text{em}} | D^0(p) \rangle = ig_D(m_B^2)g_{D^0D^*0\gamma}\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu k^\nu p^\alpha \epsilon^{*\beta}, \quad (3.14a)$$

$$\langle \bar{B}^{*0}(q) | \mathcal{L}_{\text{em}} | \bar{B}^0(p) \rangle = ig_{B^*}(m_D^2)g_{\bar{B}^{*0}\bar{B}^0\gamma}\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu k^\nu p^\alpha \epsilon^{*\beta}, \quad (3.14b)$$

$$\langle D^{*0}(q) | \mathcal{L}_{\text{em}} | D^{*0}(p) \rangle = g_{D^*}(m_B^2)g_{D^*0D^*0\gamma}(k_\mu\epsilon_\nu - k_\nu\epsilon_\mu)\epsilon^{*\nu}(p)m_{D^*}. \quad (3.14c)$$

Because the intermediate state is far from its mass shell, we have introduced the form factors defined in Eq. (3.13). In the heavy quark effective theory, the on-shell  $P^*P\gamma$  and  $P^*P^*\gamma$  coupling constants are related to each other and are given by [2]

$$\begin{aligned} g_{PP^*\gamma} &= -2 \left( \frac{m_{P^*}}{m_P} \right)^{1/2} (e_q d + e_Q d') \\ &\equiv e \left( \frac{m_{P^*}}{m_P} \right)^{1/2} (e_q \beta + e_Q \beta'), \end{aligned} \quad (3.15a)$$

$$\begin{aligned} g_{P^*P\gamma} &= -2 \left( \frac{m_P}{m_{P^*}} \right)^{1/2} (e_q d + e_Q d') \\ &\equiv e \left( \frac{m_P}{m_{P^*}} \right)^{1/2} (e_q \beta + e_Q \beta'), \end{aligned} \quad (3.15b)$$

$$\begin{aligned} g_{P^*P^*\gamma} &= -2(e_q d - e_Q d') \\ &\equiv e(e_q \beta - e_Q \beta'), \end{aligned} \quad (3.15c)$$

where  $e_q$  is the charge of the light quark  $q$  (not  $\bar{q}$ ) and  $e_Q$  is the charge of the heavy quark  $Q$ . The coupling  $\beta'$  (or  $d'$ ) is fixed by heavy quark symmetry to be  $1/m_Q$ , while  $\beta$  (or  $d$ ) is independent of heavy quarks and cannot be determined by heavy quark symmetry alone. In the

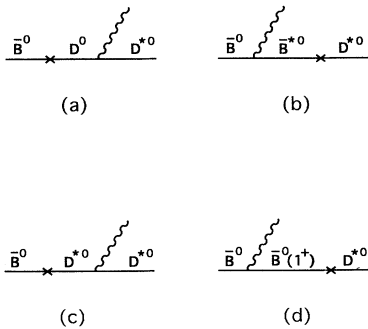


FIG. 3. Possible pole diagrams contributing to the radiative decay  $\bar{B}^0 \rightarrow D^{*0}\gamma$ .

constituent quark model,  $\beta$  is given by [2]

$$\beta = \frac{1}{m_q}. \quad (3.16)$$

Therefore,

$$g_{D^0D^*0\gamma} = e \left( \frac{e_u}{m_u} + \frac{e_c}{m_c} \right), \quad (3.17a)$$

$$g_{\bar{B}^{*0}\bar{B}^0\gamma} = e \left( \frac{e_d}{m_d} + \frac{e_b}{m_b} \right), \quad (3.17b)$$

$$g_{D^*0D^*0\gamma} = e \left( \frac{e_u}{m_u} - \frac{e_c}{m_c} \right), \quad (3.17c)$$

where the small difference between  $m_P$  and  $m_{P^*}$  has been neglected. Note that it is important to keep the contribution from the magnetic moment of the charmed quark since it is not particularly heavy. In general, it is expected that the form factors appearing in Eq. (3.14) become smaller as the hadron is more away from its mass shell owing to less overlap between initial and final meson wave functions.

Using the factorization method we obtain

$$\langle D^0(p) | \mathcal{L}_W | \bar{B}^0(p) \rangle = \frac{G_F}{2\sqrt{2}} V_{cb} V_{ud}^* (c_- - c_+) f_D f_B m_B^2, \quad (3.18)$$

and use has been made of  $p^2 = m_B^2$ . Likewise, we obtain

$$\begin{aligned} \langle D^{*0}(q) | \mathcal{L}_W | \bar{B}^{*0}(q) \rangle \\ = \frac{G_F}{2\sqrt{2}} V_{cb} V_{ud}^* (c_- - c_+) f_D f_B m_D m_B (\epsilon_B^* \cdot \epsilon_D^*), \end{aligned} \quad (3.19a)$$

$$\begin{aligned} \langle D^{*0}(p) | \mathcal{L}_W | \bar{B}^0(p) \rangle \\ = -\frac{G_F}{2\sqrt{2}} V_{cb} V_{ud}^* (c_- - c_+) f_D f_B m_{D^*} (p \cdot \epsilon_D^*). \end{aligned} \quad (3.19b)$$

Putting everything together and using the relation

$$\sum_{\lambda} \varepsilon_{\mu}^{*}(q) \varepsilon_{\mu}^{*}(q) = -g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{m^2}, \quad (3.20)$$

we finally find

$$f_1(0) = 2\eta \frac{f_D}{f_{D^*}} \left[ \frac{m_B^2}{m_B^2 - m_D^2} g_{D^0 D^{*0} \gamma} g_D(m_B^2) + \frac{m_D m_B}{m_{B^*}^2 - m_{D^*}^2} g_{B^{*0} B^0 \gamma} g_{B^*}(m_D^2) \right], \quad (3.21a)$$

$$f_2(0) = -\eta \frac{f_D}{f_{D^*}} g_{D^{*0} D^0 \gamma} g_{D^*}(m_B^2). \quad (3.21b)$$

Comparing (3.21) with (3.7) yields

$$g_D(m_B^2) = \frac{m_D^2}{m_B^2}, \quad g_{B^*}(m_D^2) = 1, \quad (3.22)$$

$$g_{D^*}(m_B^2) = -\frac{m_{D^*}^2}{m_B^2 - m_{D^*}^2},$$

where use of the heavy quark symmetry relations  $m_{B^*} = m_B$ ,  $m_{D^*} = m_D$ , and  $f_{D^*} = f_D$  has been made. We note that in the effective Lagrangian approach there is an additional term proportional to  $(e_d/m_d - e_b/m_b)$  in  $f_2$ . What is the counterpart of this term in the pole model? Evidently, it must come from a  $p$ -wave  $1^+ \bar{B}$  resonance state [see Fig. 3(d)]: The  $E1 \bar{B}^0(1^+) \bar{B}^0(0^-) \gamma$  transition coupling is proportional to  $(e_d/m_d - e_b/m_b)$  provided that  $\bar{B}(1^+)$  is a  $p$ -wave spin-singlet, while  $\bar{B}(1^+) - D^*$  weak transition is parity violating. We thus see that both approaches are consistent with each other. However, the effective Lagrangian approach is simpler and it also provides the information on the form factors, as shown in (3.22).

#### IV. APPLICATIONS TO HEAVY BARYON DECAYS

In this section we will first focus our attention on the short-distance penguin effect in the decays  $\Lambda_b \rightarrow \Sigma \gamma$  and  $\Lambda_b \rightarrow \Lambda \gamma$  and then turn to the weak radiative decays of the antitriplet bottom baryons, namely,  $\Xi_b^0 \rightarrow \Xi_c^0(\Xi_c^{\prime 0}) \gamma$  and  $\Lambda_b^0 \rightarrow \Sigma_c^0 \gamma$ . Since the weak radiative decay of  $\bar{B}$  mesons is dominated by the short-distance  $b \rightarrow s \gamma$  transition, it is natural to expect that the same mechanism

also works for bottom baryons. The general amplitude of baryon weak radiative decay reads

$$A(B_i \rightarrow B_f \gamma) = i \bar{u}_f (a + b \gamma_5) \sigma_{\mu\nu} \varepsilon^{\mu} k^{\nu} u_i, \quad (4.1)$$

where  $a$  and  $b$  are parity-conserving and -violating amplitudes, respectively. The corresponding decay rate is

$$\Gamma(B_i \rightarrow B_f \gamma) = \frac{1}{8\pi} \left( \frac{m_i^2 - m_f^2}{m_i} \right)^3 (|a|^2 + |b|^2). \quad (4.2)$$

#### A. Penguin-induced baryon radiative decays

The electromagnetic penguin transition  $b \rightarrow s + \gamma$  at the quark level is well understood in QCD, and its effective Lagrangian is given by (2.1). However, its hadronic matrix elements depend on the nature of the bound states, and there are no known methods to calculate them from first principles. In this section we will use two different methods to estimate the rates for the decay  $\Lambda_b \rightarrow \Lambda + \gamma$ . In the first method, both the  $b$  and  $s$  quarks are treated as heavy. Heavy quark symmetry then gives model independent prediction. Although we are able to compute the correction of order  $m_s/m_b$  and  $1/m_s$ , higher order corrections are needed for a realistic comparison with experiment. They are in principle well defined in QCD; but it is impossible to compute them all. In the second method, only the  $b$  quark is treated as heavy so the form factors of tensor currents needed are related by heavy quark symmetry to those of vector and axial vector currents, which are then evaluated in the MIT bag model. The MIT bag model includes the  $m_s$  effects to all orders, but the reliability of the model is hard to assess. It is clear that neither of the two approaches is very satisfactory. Together, however, they provide a more or less consistent order of magnitude estimate of these weak radiative decays.

In the first method we treat the  $s$  quark as a heavy quark and then take into account the  $1/m_s$  and QCD corrections. Despite that the effective mass of the  $s$  quark in hyperons is only of order 500 MeV, it is not small compared to the QCD scale and we thus expect to see some vestiges of heavy quark symmetry. In the heavy  $s$  quark limit, the hyperon  $\Lambda$  behaves as an antitriplet heavy baryon  $B_{\bar{3}}$ , while  $\Sigma^0$  as a sextet baryon  $B_6$ . From Eq. (2.1) we obtain

$$A(\Lambda_b^0 \rightarrow \text{hyperon} + \gamma)_{\text{SD}} = i \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} F_2 V_{tb} V_{ts}^* m_b \varepsilon^{\mu} k^{\nu} \langle \text{hyperon} | \bar{s} \sigma_{\mu\nu} [(1 + \gamma_5) + \frac{m_s}{m_b} (1 - \gamma_5)] b | \Lambda_b^0 \rangle. \quad (4.3)$$

Using the interpolating fields [18]

$$B_{\bar{3}}(v, s) = \bar{u}(v, s) \phi_v h_v, \quad (4.4a)$$

$$B_6(v, s) = \bar{B}_{\mu}(v, s) \phi_v^{\mu} h_v, \quad (4.4b)$$

where  $\phi_v$  and  $\phi_v^{\mu}$  are the  $0^+$  and  $1^+$  diquarks, respec-

tively, which combine with the heavy quark  $h_v$  of velocity  $v$  to form the appropriate heavy baryon, and the relations [18]

$$\langle 0 | \phi_v \cdot \phi_v^{\dagger} | 0 \rangle = \zeta (v \cdot v'), \quad (4.5a)$$



$$\langle 0 | \phi_v^\mu \phi_v^\dagger | 0 \rangle = 0 , \quad (4.5b)$$

where  $\zeta(v \cdot v')$  is a universal baryonic Isgur-Wise function normalized to unity at the zero recoil  $v \cdot v' = 1$ , we find<sup>4</sup>

$$\begin{aligned} \langle \Lambda(v', s') | \bar{s} \sigma_{\mu\nu} (1 \pm \gamma_5) b | \Lambda_b^0(v, s) \rangle \\ = \bar{u}_\Lambda \sigma_{\mu\nu} (1 \pm \gamma_5) u_{\Lambda_b} \zeta(v \cdot v') , \end{aligned} \quad (4.6a)$$

$$\langle \Sigma^0(v', s') | \bar{s} \sigma_{\mu\nu} (1 \pm \gamma_5) b | \Lambda_b^0(v, s) \rangle = 0 . \quad (4.6b)$$

Therefore, no weak  $B_{\bar{3}} - B_6$  transition can be induced by the  $b \rightarrow s\gamma$  mechanism in the heavy quark limit. So, the first prediction we have is

$$\Gamma(\Lambda_b^0 \rightarrow \Sigma^0 \gamma) \ll \Gamma(\Lambda_b^0 \rightarrow \Lambda^0 \gamma) . \quad (4.7)$$

We will follow Ref. [19] to treat the  $1/m_s$  corrections to  $\Lambda_b^0 \rightarrow \Lambda^0 \gamma$ . First, from the relationship between the  $s$  quark field and the effective field  $h_{v'}^{(s)}$ ,

$$s(x) = e^{-im_s v' \cdot x} \left[ 1 + \frac{1 - \not{v}'}{2} \frac{i \not{D}}{2m_s} \right] h_{v'}^{(s)} , \quad (4.8)$$

we get

$$\bar{s} \sigma_{\mu\nu} (1 \pm \gamma_5) b \rightarrow \bar{h}_{v'}^{(s)} \left( 1 - \frac{i \not{D}}{2m_s} \right) \sigma_{\mu\nu} (1 \pm \gamma_5) h_v^{(b)} . \quad (4.9)$$

Applying the result [19]

$$\langle \Lambda(v', s') | \bar{h}_{v'}^{(s)} i \overleftarrow{D} \Gamma h_v^{(b)} | \Lambda_b^0(v, s) \rangle = \bar{\Lambda} \zeta(v \cdot v') \frac{v_\mu - (v \cdot v') v'_\mu}{1 + v \cdot v'} \bar{u}_\Lambda(v', s') \gamma^\mu \Gamma u_{\Lambda_b}(v, s) , \quad (4.10)$$

with the new parameter  $\bar{\Lambda}$  being

$$\bar{\Lambda} = m_{\Lambda_b} - m_b = m_{\Lambda_c} - m_c = m_\Lambda - m_s \approx 700 \text{ MeV} , \quad (4.11)$$

we obtain

$$\langle \Lambda(v', s') | \bar{h}_{v'}^{(s)} i \overleftarrow{D} \sigma_{\mu\nu} \varepsilon^\mu k^\nu (1 \pm \gamma_5) h_v^{(b)} | \Lambda_b^0(v, s) \rangle = h \bar{\Lambda} \zeta(v \cdot v') \bar{u}_\Lambda(v', s') \sigma_{\mu\nu} \varepsilon^\mu k^\nu (1 \pm \gamma_5) u_{\Lambda_b}(v, s) , \quad (4.12)$$

with  $h = (m_\Lambda/m_{\Lambda_b} - v \cdot v')/(1 + v \cdot v')$ , where use has been made of

$$v = \frac{m_\Lambda}{m_{\Lambda_b}} v' + \frac{1}{m_{\Lambda_b}} k , \quad \not{k} \not{k} = - \not{k} \not{k} . \quad (4.13)$$

It follows from Eqs. (4.3), (4.6), (4.9), and (4.12) that

$$a = \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} F_2 m_b V_{tb} V_{ts}^* \left( 1 + \frac{m_s}{m_b} - \frac{\bar{\Lambda}}{2m_s} h \right) \zeta(v \cdot v') , \quad (4.14a)$$

$$b = \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} F_2 m_b V_{tb} V_{ts}^* \left( 1 - \frac{m_s}{m_b} - \frac{\bar{\Lambda}}{2m_s} h \right) \zeta(v \cdot v') . \quad (4.14b)$$

Including QCD corrections gives rise to [20]

$$\zeta(v \cdot v') = C(\mu) \zeta_0(v \cdot v', \mu) , \quad (4.15)$$

where

$$C(\mu) = \left( \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{-6/25} \left( \frac{\alpha_s(m_c)}{\alpha_s(m_s)} \right)^{-6/27} \left( \frac{\alpha_s(m_s)}{\alpha_s(\mu)} \right)^{a_L} , \quad (4.16a)$$

$$a_L(\omega) = \frac{8}{29} \left( \frac{\omega}{\sqrt{\omega^2 - 1}} \ln(\omega + \sqrt{\omega^2 - 1}) - 1 \right) , \quad (4.16b)$$

with  $\omega \equiv v \cdot v'$ . There is no obvious choice for the normalization scale in Eq. (4.15). It is expected that there will be no large parameters in the function  $\zeta_0(\omega, \mu)$  if the renormalization scale is low [19]. However, since perturbation theory will break down at very low scales, we thus choose  $\mu \sim m_s$  so that  $\alpha_s(\mu) \sim 1$ , and  $C(\mu) \simeq 1.23$ . For the decay  $\Lambda_b \rightarrow \Lambda \gamma$ ,  $\omega = 2.63$ . It is easily seen that the  $1/m_s$  correction to the  $\Lambda_b \rightarrow \Lambda \gamma$  amplitude is about 50% for  $m_s = 510$  MeV, which is quite sizable. This implies that it is important to include higher order  $1/m_s$  corrections. However, this is beyond the scope of this paper.

In order to estimate the decay rate of  $\Lambda_b \rightarrow \Lambda \gamma$  we employ two recent models for  $\zeta(v \cdot v')$ :

$$\zeta_0(\omega) = 0.99 \exp[-1.3(\omega - 1)] \quad (\text{soliton model [21]}), \quad (4.17a)$$

$$\zeta_0(\omega) = \left( \frac{2}{\omega + 1} \right)^{3.5+1.2/\omega} \quad (\text{MIT bag model [22]}). \quad (4.17b)$$

Hence,  $\zeta_0(\omega = 2.63)$  ranges from 0.09 to 0.12. Substituting (4.14) into (4.2) using the good approximation  $V_{tb} V_{ts}^* \cong -V_{cb} V_{cs}^*$  and the lifetime  $\tau(\Lambda_b) = 1.07 \times 10^{-12}$  s [15], we find

$$\begin{aligned} B(\Lambda_b^0 \rightarrow \Lambda^0 \gamma) &= 1.04 \times 10^{-3} |\zeta_0(\omega)|^2 \\ &= (0.8-1.5) \times 10^{-5} . \end{aligned} \quad (4.18)$$

<sup>4</sup>In the heavy  $s$  quark limit, a  $\Lambda$  is made of just a strange quark and a scalar diquark with  $(ud)$  quantum numbers. Contrary to some claims made in the literature, it is not necessary to include a Clebsch-Gordan coefficient  $1/\sqrt{3}$  in Eq. (4.6a).

Since there is only one strange quark in  $\Xi_b$  but two strange quarks in  $\Omega_b$  and  $\Xi$ , it is not clear to us how to generalize the above heavy  $s$  quark method to the radiative decays  $\Xi_b \rightarrow \Xi\gamma$  and  $\Omega_b \rightarrow \Omega\gamma$ .

In the second method, only the  $b$  quark is treated as heavy. Since  $\gamma_0 b = b$  in the static limit of the  $b$  quark we have the relation

$$\langle \Lambda | \bar{s} i \sigma_{0i} (1 \pm \gamma_5) b | \Lambda_b \rangle = \langle \Lambda | \bar{s} \gamma_i (1 \mp \gamma_5) b | \Lambda_b \rangle. \quad (4.19)$$

In the heavy  $b$ -quark limit, there are only two independent form factors in the  $\gamma_\mu$  and  $\gamma_\mu \gamma_5$  matrix elements [23]:

$$\begin{aligned} & \langle \Lambda(p) | \bar{s} \gamma_\mu (1 \mp \gamma_5) b | \Lambda_b(v) \rangle \\ &= \bar{u}_\Lambda [F_1^{\Lambda_b \Lambda}(v \cdot p) + \not{v} F_2^{\Lambda_b \Lambda}(v \cdot p)] \gamma_\mu (1 \mp \gamma_5) u_{\Lambda_b}. \end{aligned} \quad (4.20)$$

Recall that  $F_1 = 1$  at zero recoil and  $F_2 = 0$  if both  $\Lambda_b$  and  $\Lambda$  are treated as heavy. The relationship between  $F_{1,2}$  and the standard form factors defined by

$$\begin{aligned} & \langle \Lambda(p) | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b(p') \rangle \\ &= \bar{u}_\Lambda [f_1(q^2) \gamma_\mu + i f_2(q^2) \sigma_{\mu\nu} q^\nu + f_3(q^2) q_\mu \\ &\quad - (g_1(q^2) \gamma_\mu + i g_2(q^2) \sigma_{\mu\nu} q^\nu + g_3(q^2) q_\mu) \gamma_5] u_{\Lambda_b}, \end{aligned} \quad (4.21)$$

reads

$$f_1(q^2) = g_1(q^2) = F_1(q^2) + \frac{m_\Lambda}{m_{\Lambda_b}} F_2(q^2), \quad (4.22)$$

$$f_2(q^2) = g_2(q^2) = f_3(q^2) = g_3(q^2) = \frac{1}{m_{\Lambda_b}} F_2(q^2).$$

It follows from (4.3), (4.19), and (4.20) that

$$a = \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} F_2 m_b V_{tb} V_{ts}^* \left( 1 + \frac{m_s}{m_b} \right) [F_1^{\Lambda_b \Lambda}(0) - F_2^{\Lambda_b \Lambda}(0)], \quad (4.23a)$$

$$b = \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} F_2 m_b V_{tb} V_{ts}^* \left( 1 - \frac{m_s}{m_b} \right) [F_1^{\Lambda_b \Lambda}(0) + F_2^{\Lambda_b \Lambda}(0)], \quad (4.23b)$$

where  $F(0)$  here means  $F(q^2 = 0)$ .

In principle, the Isgur-Wise form factors  $F_{1,2}(0)$  can be obtained by first computing the form factors  $f_i$  and  $g_i$  at zero recoil in a quark model<sup>5</sup> and then extrapolating to  $q^2 = 0$  under some assumption on their  $q^2$  dependence. In the literature, form factors  $f_i$  and  $g_i$  have been calculated for  $\Lambda_c \rightarrow \Lambda$  transition in two different quark models: the nonrelativistic quark model and the MIT bag model [24]. The model calculation can be tested by

<sup>5</sup>This is because the quark model calculations are presumably most reliable at zero recoil where both baryons are static.

a recent CLEO measurement [25]. Assuming a dipole  $q^2$  dependence of the form factors

$$F_i(q^2) = \frac{F_i(0)}{(1 - q^2/m_*^2)^2}, \quad (4.24)$$

where  $m_*$  is a pole mass,<sup>6</sup> the form factor ratio  $R \equiv F_2^{\Lambda_c \Lambda}/F_1^{\Lambda_c \Lambda}$  is found to be

$$R = -0.33 \pm 0.16 \pm 0.15 \quad (4.25)$$

by CLEO for  $m_* = 2.11$  GeV [25]. From Tables IV and VI of Ref. [24] we find<sup>7</sup>

$$R = \begin{cases} -0.34 & \text{MIT bag model,} \\ -0.23 & \text{nonrelativistic quark model.} \end{cases} \quad (4.26)$$

It is clear that while both models' predictions are consistent with experiment, the bag model gives a better agreement.

As for the  $\Lambda_b \rightarrow \Lambda$  form factors, our strategy is as follows. We will use the bag model to compute  $f_1^{\Lambda_b \Lambda}$  at zero recoil, and then employ the heavy-flavor-symmetry relation

$$F_2^{\Lambda_c \Lambda}(q_{\Lambda_c}^2)/F_1^{\Lambda_c \Lambda}(q_{\Lambda_c}^2) = F_2^{\Lambda_b \Lambda}(q_{\Lambda_b}^2)/F_1^{\Lambda_b \Lambda}(q_{\Lambda_b}^2), \quad (4.27)$$

where  $q_{\Lambda_c} = m_{\Lambda_c} v - q$  and  $q_{\Lambda_b} = m_{\Lambda_b} v - q$ , together with the CLEO data for  $R$ , to determine  $F_{1,2}^{\Lambda_b \Lambda}$  at  $q_m^2 \equiv (m_{\Lambda_b} - m_\Lambda)^2$ , which can be extrapolated to  $q^2 = 0$  using Eq. (4.24). In the MIT bag model [27],  $f_1$  at zero recoil is given by<sup>8</sup>

$$f_1^{\Lambda_b \Lambda}(q_m^2) = \int d^3r [u_b(r) u_s(r) + v_b(r) v_s(r)], \quad (4.28)$$

where  $u(r), v(r)$  are, respectively, the large and small components of the  $1S_{1/2}$  quark spatial wave function. Numerically, we find  $f_1^{\Lambda_b \Lambda}(q_m^2) = 0.95$  (see, e.g., Ref. [28] for the technique). It follows from (4.22), (4.25), and (4.27) that

<sup>6</sup>Beyond the heavy quark limit, the pole masses  $m_V$  and  $m_A$  for form factors  $f_i$  and  $g_i$ , respectively, are different. In general, the pole mass is taken to be the mass of the nearest spin-one meson with the right quantum number. For  $\Lambda_c \rightarrow \Lambda$  transition,  $m_V = m_{D_s(1^-)} = 2.11$  GeV,  $m_A = m_{D_s(1^+)} = 2.536$  GeV, while for  $\Lambda_b \rightarrow \Lambda$ :  $m_V = m_{B_s(1^-)} \cong 5.42$  GeV,  $m_A = m_{B_s(1^+)} \cong 5.86$  GeV.

<sup>7</sup>Note that the definition of the form factors  $f_i$  and  $g_i$  in [24] follows that in [26] and is different from ours. In terms of our notation, the bag model predictions shown in Table VI for  $\Lambda_c \rightarrow \Lambda$  form factors evaluated at  $q^2 = 0$  with dipole approximation are  $f_1 = 0.46$ ,  $f_2 = -0.19/m_{\Lambda_c}$ ,  $g_1 = 0.50$ ,  $g_2 = -0.05/m_{\Lambda_c}$ , while the nonrelativistic quark model predicts  $f_1 = 0.35$ ,  $f_2 = -0.09/m_{\Lambda_c}$ ,  $g_1 = 0.61$ ,  $g_2 = -0.04/m_{\Lambda_c}$ . The difference between  $f_i$  and  $g_i$  is attributed to  $1/m_c$  corrections. The form factor ratio calculated in (4.26) comes from the form factors  $f_1$  and  $f_2$ .

<sup>8</sup>A small and negligible correction to (4.28) is shown in Eq. (5a) of [24].

$$F_1^{\Lambda_b \Lambda}(q_m^2) = 1.02, \quad F_2^{\Lambda_b \Lambda}(q_m^2) = -0.34, \quad (4.29)$$

where only the central values are presented. Since the form factor ratio is measured by CLEO using the dipole  $q^2$  dependence, it is natural to employ the same dipole behavior for  $F_i^{\Lambda_b \Lambda}$  for the sake of consistency. Note that the dipole form

$$G(q^2) = \frac{(1 - q_m^2/m_*^2)^2}{(1 - q^2/m_*^2)^2} \quad (4.30)$$

plays the role of the baryonic Isgur-Wise function  $\zeta(v \cdot v')$ . Using  $m_* = 5.42$  GeV we find  $G(0) = 0.092$ , in accordance with  $\zeta(2.63) = 0.09 \sim 0.12$ . Putting everything together we finally obtain

$$\mathcal{B}(\Lambda_b \rightarrow \Lambda + \gamma) = 0.4 \times 10^{-5}. \quad (4.31)$$

This model result is close to the previous prediction (4.18).

We should accentuate the nature of the above two calculations. The first method is model independent; but since the effective  $s$  quark in the baryon is only of order 500 MeV, it is questionable to apply the heavy quark effective theory to hyperons, as evidenced by the sizable  $1/m_s$  corrections shown above. The second method takes care of the  $m_s$  effects to all orders, but it is hard

to assess its reliability. For example, quark model prediction for the form factors  $f_i$  and  $g_i$  depend on the model content; the form factor  $q^2$  dependence is very sensitive to the choice of the pole behavior: monopole or dipole. Nevertheless, we see that both approaches provide a consistent order of magnitude estimate; that is,  $\mathcal{B}(\Lambda_b \rightarrow \Lambda \gamma) \sim 1 \times 10^{-5}$ .

## B. Heavy quark symmetry predictions for $\Xi_b^0 \rightarrow \Xi_c^0 \gamma$

Before embarking on quark model calculations for the radiative decays  $\Xi_b^0 \rightarrow \Xi_c^0 \gamma$ ,  $\Xi_c^0 \gamma$ , and  $\Lambda_b^0 \rightarrow \Sigma_c^0 \gamma$ , we would like to see what we can learn from applying the heavy quark symmetry to these decays. It turns out that for the antitriplet to antitriplet radiative transition  $\Xi_b^0 \rightarrow \Xi_c^0 \gamma$ , heavy quark symmetry implies a nontrivial model independent prediction for  $a/b$ , the ratio of the parity-conserving and parity-violating amplitudes.<sup>9</sup>

Let us denote

$$O_{1\mu\nu} = \bar{c}\gamma_\mu(1 - \gamma_5)b\bar{d}\gamma_\nu(1 - \gamma_5)u, \quad (4.32a)$$

$$O_{2\mu\nu} = \bar{c}\gamma_\mu(1 - \gamma_5)u\bar{d}\gamma_\nu(1 - \gamma_5)b, \quad (4.32b)$$

and apply the interpolating field (4.4a) to the antitriplet heavy baryons to get

$$\begin{aligned} \langle \Xi_c^0(v') | O_{1\mu\nu} | \Xi_b^0(v) \rangle &= \langle 0 | \bar{u}_f(v', s') \phi_{v'} c_{v'} \bar{c}_{v'} \gamma_\mu (1 - \gamma_5) b_v \bar{d}_{v'} \gamma_\nu (1 - \gamma_5) u \bar{b}_v \phi_v^\dagger u_i(v, s) | 0 \rangle \\ &= \bar{u}_f(v', s') \frac{1 + \not{v}'}{2} \gamma_\mu (1 - \gamma_5) \frac{1 + \not{v}}{2} u_i(v, s) \langle 0 | \phi_{v'} \bar{d}_{v'} \gamma_\nu (1 - \gamma_5) u \phi_v^\dagger | 0 \rangle. \end{aligned} \quad (4.33)$$

Lorentz invariance implies that

$$\langle 0 | \phi_{v'} \bar{d}_{v'} \gamma_\nu (1 - \gamma_5) u \phi_v^\dagger | 0 \rangle = A(v \cdot v') v_\nu + B(v \cdot v') v'_\nu. \quad (4.34)$$

Therefore,

$$\langle \Xi_c^0(v') | O_{1\mu\nu} | \Xi_b^0(v) \rangle = \bar{u}_f(v', s') \gamma_\mu (1 - \gamma_5) u_i(v, s) [A(v \cdot v') v_\nu + B(v \cdot v') v'_\nu]. \quad (4.35)$$

Likewise, the matrix element of  $O_{2\mu\nu}$  is

$$\langle \Xi_c^0(v') | O_{2\mu\nu} | \Xi_b^0(v) \rangle = \bar{u}_f(v', s') \gamma_\mu (1 - \gamma_5) \langle 0 | \phi_{v'} u \bar{d}_{v'} \phi_v^\dagger | 0 \rangle \gamma_\nu (1 - \gamma_5) u_i(v, s). \quad (4.36)$$

Again, Lorentz invariance demands that

$$\langle 0 | \phi_{v'} u \bar{d}_{v'} \phi_v^\dagger | 0 \rangle = A'(v \cdot v') \not{v} + B'(v \cdot v') \not{v}' + C'(v \cdot v') \not{v}' \not{v} + D(v \cdot v'). \quad (4.37)$$

Our next task is to recast (4.35) and (4.36) into a more suitable form. Since  $O_{1\mu\nu}$  and  $O_{2\mu\nu}$  are multiplied by  $F^{\mu\nu}$  and  $\tilde{F}^{\mu\nu}$  [see (2.15)], only the antisymmetric part will contribute. Thus we write

$$\langle \Xi_c^0(v') | O_{1\mu\nu} | \Xi_b^0(v) \rangle = \frac{1}{2} \bar{u}_f(v', s') [A(\gamma_\mu v_\nu - \gamma_\nu v_\mu) + B(\gamma_\mu v'_\nu - \gamma_\nu v'_\mu)] (1 - \gamma_5) u_i(v, s). \quad (4.38)$$

By virtue of the equation of motion  $\not{v}u(v, s) = u(v, s)$ , two useful relations can be derived:

<sup>9</sup>We have checked explicitly that heavy quark symmetry alone does not lead to any useful predictions for other decays such as  $\Xi_b^0 \rightarrow \Xi_c^0 \gamma$ ,  $\Lambda_b^0 \rightarrow \Sigma_c^0 \gamma$ .

$$(\gamma_\mu v_\nu - \gamma_\nu v_\mu)(1 - \gamma_5) = -\frac{i}{2}\sigma_{\mu\nu}(1 + \gamma_5) + \frac{i}{2}\not{v}\sigma_{\mu\nu}(1 - \gamma_5), \quad (4.39a)$$

$$(\gamma_\mu v'_\nu - \gamma_\nu v'_\mu)(1 - \gamma_5) = \frac{i}{2}\sigma_{\mu\nu}(1 - \gamma_5) - \frac{i}{2}\sigma_{\mu\nu}(1 + \gamma_5)\not{v}', \quad (4.39b)$$

which can be further simplified by applying the analogue of Eq. (4.13) to the  $\Xi_b \rightarrow \Xi_c$  transition. As a consequence of (4.33) and (4.39) we obtain

$$\langle \Xi_c^0(v') | O_{1\mu\nu} \Xi_b^0(v) \rangle = -\frac{i}{4} \left( A + B \frac{m_{\Xi_b}}{m_{\Xi_c}} \right) \bar{u}_f(v', s') [\sigma_{\mu\nu}(1 + \gamma_5) - \frac{m_{\Xi_c}}{m_{\Xi_b}} \sigma_{\mu\nu}(1 - \gamma_5)] u_i(v, s). \quad (4.40)$$

As for the matrix element of  $O_{2\mu\nu}$ , (4.36) leads to

$$\begin{aligned} \langle \Xi_c^0(v') | O_{2\mu\nu} | \Xi_b^0(v) \rangle &= \bar{u}_f(v', s') [A'(\gamma_\mu \not{v}'\gamma_\nu - \gamma_\nu \not{v}'\gamma_\mu) + B'(\gamma_\mu \not{v}'\gamma_\nu - \gamma_\nu \not{v}'\gamma_\mu)] (1 - \gamma_5) u_i(v, s) \\ &= i \bar{u}_f(v', s') \{ A' [(1 + \not{v}')\sigma_{\mu\nu} + (1 - \not{v}')\sigma_{\mu\nu}\gamma_5] + B' \sigma_{\mu\nu} [(1 + \not{v}') - \gamma_5(1 - \not{v}')] \} u_i(v, s). \end{aligned} \quad (4.41)$$

Applying the analog of (4.13) again to (4.41) leads to

$$\langle \Xi_c^0(v') | O_{2\mu\nu} | \Xi_b^0(v) \rangle = i \left( A' + \frac{m_{\Xi_b}}{m_{\Xi_c}} B' \right) \bar{u}_f(v', s') \sigma_{\mu\nu} \left[ (1 + \gamma_5) + \frac{m_{\Xi_c}}{m_{\Xi_b}} (1 - \gamma_5) \right] u_i(v, s). \quad (4.42)$$

Finally, substituting (4.40) and (4.42) into (2.15) we obtain

$$\begin{aligned} \langle \Xi_c^0(v') | O_{\pm}^F | \Xi_b^0(v) \rangle &= \frac{2e}{m_i^2 - f_f^2} \left[ \frac{1}{4} \left( A + B \frac{m_{\Xi_b}}{m_{\Xi_c}} \right) \mp \left( A' + B' \frac{m_{\Xi_b}}{m_{\Xi_c}} \right) \right] \\ &\quad \times \bar{u}_f(v', s') \sigma \cdot F \left\{ \left[ e_c \frac{m_f}{m_c} - e_d \frac{m_f}{m_d} + \left( e_u \frac{m_i}{m_u} - e_b \frac{m_i}{m_b} \right) \frac{m_{\Xi_c}}{m_{\Xi_b}} \right] \right. \\ &\quad \left. + \left[ e_c \frac{m_f}{m_c} - e_d \frac{m_f}{m_d} - \left( e_u \frac{m_i}{m_u} - e_b \frac{m_i}{m_b} \right) \frac{m_{\Xi_c}}{m_{\Xi_b}} \right] \gamma_5 \right\} u_i(v, s). \end{aligned} \quad (4.43)$$

From (4.43) we see that although the  $\bar{3} \rightarrow \bar{3} + \gamma$  transition depends on the unknown parameters  $A$ ,  $B$ ,  $A'$ , and  $B'$ , a unique tree-level prediction the ratio of  $a/b$  based on heavy quark symmetry is nevertheless accomplished:

$$\frac{a}{b} \Big|_{\Xi_b^0 \rightarrow \Xi_c^0 \gamma} = \frac{e_c \frac{m_f}{m_c} - e_d \frac{m_f}{m_d} + \left( e_u \frac{m_i}{m_u} - e_b \frac{m_i}{m_b} \right) \frac{m_{\Xi_c}}{m_{\Xi_b}}}{e_c \frac{m_f}{m_c} - e_d \frac{m_f}{m_d} - \left( e_u \frac{m_i}{m_u} - e_b \frac{m_i}{m_b} \right) \frac{m_{\Xi_c}}{m_{\Xi_b}}}. \quad (4.44)$$

This ratio can be tested by measuring the asymmetry parameter  $\alpha$  as will be discussed later.

### C. Bag model calculations for $\Xi_b^0 \rightarrow \Xi_c^0(\Xi_c^{\prime 0})\gamma$ and $\Lambda_b^0 \rightarrow \Sigma_c^0\gamma$

Recall from Sec. II that the effective Hamiltonian responsible for the weak radiative decays of heavy baryons is given by

$$\mathcal{H}_{\text{eff}}(bu \rightarrow cd\gamma) = \frac{G_F}{2\sqrt{2}} V_{cb} V_{ud}^* (c_+ O_+^F + c_- O_-^F), \quad (4.45)$$

where [cf. Eq. (2.15)]

$$O_{\pm}^F = d_f (\tilde{F}_{\mu\nu} + iF_{\mu\nu}) O_{\pm}^{\mu\nu} + d_i (\tilde{F}_{\mu\nu} - iF_{\mu\nu}) O_{\mp}^{\mu\nu}, \quad (4.46)$$

with  $O_{\pm}^{\mu\nu} = O_1^{\mu\nu} \pm O_2^{\mu\nu}$ , and

$$d_i = m_i \left( \frac{e_u}{m_u} - \frac{e_b}{m_b} \right) \frac{e}{m_i^2 - m_f^2}, \quad (4.47a)$$

$$d_f = m_f \left( \frac{e_c}{m_c} - \frac{e_d}{m_d} \right) \frac{e}{m_i^2 - m_f^2}, \quad (4.47b)$$

and  $m_i = m_b + m_u$ ,  $m_f = m_c + m_d$ . Writing

$$\langle B_f(v') | O_{1,2}^{\mu\nu} | B_i(v) \rangle = i \bar{u}_f(v', s') (a_{1,2} + b_{1,2}\gamma_5) \sigma^{\mu\nu} u_i(v, s), \quad (4.48)$$

we get

$$\begin{aligned} \langle B_f(v') | O_{\pm}^F | B_i(v) \rangle = & -\bar{u}_f(v', s') \{ d_f(a_{\pm} + b_{\pm}) - d_i(a_{\mp} - b_{\mp}) \\ & + [d_f(a_{\pm} + b_{\pm}) + d_i(a_{\mp} - b_{\mp})] \gamma_5 \} \sigma \cdot F u_i(v, s), \end{aligned} \quad (4.49)$$

where  $a_{\pm} = a_1 \pm a_2$ ,  $b_{\pm} = b_1 \pm b_2$ , corresponding to the parity-conserving and parity-violating matrix elements of  $O_{\pm}^{\mu\nu}$ . It follows from Eqs. (4.45) and (4.1) that

$$a = -\frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \{ c_+ [d_f(a_+ + b_+) - d_i(a_- - b_-)] + c_- [d_f(a_- + b_-) - d_i(a_+ - b_+)] \}, \quad (4.50a)$$

$$b = -\frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \{ c_+ [d_f(a_+ + b_+) + d_i(a_- - b_-)] + c_- [d_f(a_- + b_-) + d_i(a_+ - b_+)] \}. \quad (4.50b)$$

We shall employ the MIT bag model [27] to evaluate the four baryon matrix elements  $a_{\pm}$  and  $b_{\pm}$ . Since the quark-model wave functions best resemble the hadronic states in the frame where both baryons are static, we thus adopt the static bag approximation for the calculation. Because

$$\bar{u}_f^{\uparrow} \sigma^{xy} u_i^{\uparrow} = 1, \quad \bar{u}_f^{\uparrow} \sigma^{0z} \gamma_5 u_i^{\uparrow} = i, \quad (4.51)$$

for  $\mathbf{v}_f = \mathbf{v}_i = 0$ , it follows from (4.48) that

$$a_{1,2} = -i \langle B_f \uparrow | O_{1,2}^{xy} | B_i \uparrow \rangle, \quad (4.52a)$$

$$b_{1,2} = -\langle B_f \uparrow | O_{1,2}^{0z} | B_i \uparrow \rangle. \quad (4.52b)$$

Matrix elements  $a_{1,2}$  and  $b_{1,2}$  in the MIT bag model can be expressed in terms of four-quark overlap bag integrals (see, e.g., Ref. [28] for the method). To do this we need the spin-color wave functions of the baryons involved such as

$$\Xi_b^0 = \frac{1}{\sqrt{6}} [(bus - bsu) \chi_A + (12) + (13)], \quad (4.53)$$

$$\Sigma_c^0 = \frac{1}{\sqrt{3}} [cdd \chi_s + (12) + (13)],$$

where  $abc\chi_s = (2a^{\uparrow}b^{\uparrow}c^{\uparrow} - a^{\uparrow}b^{\downarrow}c^{\downarrow} - a^{\uparrow}b^{\downarrow}c^{\uparrow})/\sqrt{6}$ ,  $abc\chi_A = (a^{\uparrow}b^{\uparrow}c^{\downarrow} - a^{\uparrow}b^{\downarrow}c^{\uparrow})/\sqrt{2}$ , and  $(ij)$  means permutation for the quark in place  $i$  with the quark in place  $j$ . As an example of the bag model evaluation we look at the decay  $\Xi_b^0 \rightarrow \Xi_c^0 \gamma$ . With the wave functions given by (4.53) we find

$$\langle \Xi_c^0 \uparrow | b_{1c}^{\dagger} b_{1b} b_{2d}^{\dagger} b_{2u} \sigma_1^z | \Xi_b^0 \uparrow \rangle = \frac{1}{6}, \quad (4.54a)$$

$$\langle \Xi_c^0 \uparrow | b_{1c}^{\dagger} b_{1b} b_{2d}^{\dagger} b_{2u} \sigma_2^z | \Xi_b^0 \uparrow \rangle = 0, \quad (4.54b)$$

$$\langle \Xi_c^0 \uparrow | b_{1c}^{\dagger} b_{1b} b_{2d}^{\dagger} b_{2u} (\sigma_{1+} \sigma_{2-} - \sigma_{1-} \sigma_{2+}) | \Xi_b^0 \uparrow \rangle = 0, \quad (4.54c)$$

$$\langle \Xi_c^0 \uparrow | b_{1c}^{\dagger} b_{1u} b_{2d}^{\dagger} b_{2b} \sigma_1^z | \Xi_b^0 \uparrow \rangle = \frac{1}{12}, \quad (4.54d)$$

$$\langle \Xi_c^0 \uparrow | b_{1c}^{\dagger} b_{1u} b_{2d}^{\dagger} b_{2b} \sigma_2^z | \Xi_b^0 \uparrow \rangle = \frac{1}{12}, \quad (4.54e)$$

$$\langle \Xi_c^0 \uparrow | b_{1c}^{\dagger} b_{1u} b_{2d}^{\dagger} b_{2b} (\sigma_{1+} \sigma_{2-} - \sigma_{1-} \sigma_{2+}) | \Xi_b^0 \uparrow \rangle = \frac{1}{12}. \quad (4.54f)$$

For the decay  $\Xi_b^0 \rightarrow \Xi_c^0 \gamma$  we find finally

$$\begin{aligned} a_1 = -b_2 &= \frac{4\pi}{3} \int_0^R r^2 dr (v_c u_b + u_c v_b) (v_d u_u - u_d v_u), \\ b_1 = -a_2 &= -\frac{2\pi}{3} \int_0^R r^2 dr [(3u_c u_b - v_c v_b) (u_d u_u + v_d v_u) \\ &+ (u_c v_b - v_c u_b) (u_d v_u - v_d u_u)]. \end{aligned} \quad (4.55)$$

Consequently,

$$a_{\pm} = \frac{1}{2} I_{\pm}, \quad b_{\pm} = \mp \frac{1}{2} I_{\pm}, \quad (4.56)$$

with

$$\begin{aligned} I_+ &= \frac{4\pi}{3} \int_0^R r^2 dr [(u_c v_b + 3v_c u_b) (v_d u_u - u_d v_u) \\ &+ (3u_c u_b - v_c v_b) (u_u u_d + v_u v_d)], \end{aligned} \quad (4.57a)$$

$$\begin{aligned} I_- &= \frac{4\pi}{3} \int_0^R r^2 dr [(3u_c v_b + v_c u_b) (v_d u_u - u_d v_u) \\ &- (3u_c u_b - v_c v_b) (u_u u_d + v_u v_d)]. \end{aligned} \quad (4.57b)$$

From Eqs. (4.50) and (4.56) we obtain

$$a = -\frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* c_- (d_f I_- - d_i I_+), \quad (4.58a)$$

$$b = -\frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* c_- (d_f I_- + d_i I_+). \quad (4.58b)$$

Several remarks are in order. (i) We have explicitly confirmed that the operator  $O_{\pm}^F$  does not contribute to the baryon transition matrix elements as the baryon-color wave function is totally antisymmetric. This is ascribed to the fact that  $O_{\pm}^F$  is symmetric in color indices, as can be seen by applying the Fierz transformation to the effective operator for the amplitude (2.10) and by noting that the photon interaction is color singlet. From Eq. (4.43) we conclude that  $A = 4A'$  and  $B = 4B'$ . (ii) In the isospin symmetry limit we have  $I_+ = -I_-$ , so there is only one independent bag integral for the decay ampli-

tude of  $\Xi_b^0 \rightarrow \Xi_c^0 \gamma$ . In the same limit we find that

$$\frac{a}{b} \Big|_{\Xi_b^0 \rightarrow \Xi_c^0 \gamma} = \frac{e_c \frac{m_f}{m_c} - e_d \frac{m_f}{m_d} + \left( e_u \frac{m_i}{m_u} - e_b \frac{m_i}{m_b} \right)}{e_c \frac{m_f}{m_c} - e_d \frac{m_f}{m_d} - \left( e_u \frac{m_i}{m_u} - e_b \frac{m_i}{m_b} \right)}. \quad (4.59)$$

Comparing this with (4.44), it appears that the bag model does not predict correctly the ratio  $a/b$ . This seeming inconsistency comes from the static bag approximation we have adopted. In the rest frame of the initial baryon one can show that the ratio of the masses  $m_i$  and

$m_f$  is given by

$$r \equiv \frac{m_i}{m_f} = \left( \frac{1 + v_f/c}{1 - v_f/c} \right)^{1/2}. \quad (4.60)$$

The ratio  $a/b$  is in principle a function of the quark masses, the bag parameters and the velocity  $v_f$ . But in the static bag approximation, we always have  $r = 1$ . In order to get the heavy quark symmetry prediction for  $a/b$ , we thus need to utilize a moving bag to describe the recoil effect of the final baryon state. The net effect should be that (4.58) is modified to

$$a = -\frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* c_- I_- \frac{e}{m_i^2 - m_f^2} \left[ m_f \left( \frac{e_c}{m_c} - \frac{e_d}{m_d} \right) + m_i \left( \frac{e_u}{m_u} - \frac{e_b}{m_b} \right) \frac{m_{\Xi_c}}{m_{\Xi_b}} \right], \quad (4.61a)$$

$$b = -\frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* c_- I_- \frac{e}{m_i^2 - m_f^2} \left[ m_f \left( \frac{e_c}{m_c} - \frac{e_d}{m_d} \right) - m_i \left( \frac{e_u}{m_u} - \frac{e_b}{m_b} \right) \frac{m_{\Xi_c}}{m_{\Xi_b}} \right], \quad (4.61b)$$

as implied by Eq. (4.43). Later we will use (4.61) rather than (4.58) to compute the decay rate and branching ratio for the decay  $\Xi_b^0 \rightarrow \Xi_c^0 \gamma$ . (iii) Experimentally, the ratio  $a/b$  can be determined by measuring the asymmetry parameter  $\alpha$  when the initial baryon is polarized with the polarization vector  $\mathbf{s}_i$ :

$$\frac{d\Gamma(B_i \rightarrow B_f \gamma)}{d\Omega} = \frac{1}{4\pi} \Gamma(B_i \rightarrow B_f \gamma) (1 + \alpha \mathbf{s}_i \cdot \hat{\mathbf{p}}_f), \quad (4.62)$$

where

$$\alpha = \frac{2 \operatorname{Re}(a^* b)}{|a|^2 + |b|^2}. \quad (4.63)$$

For completeness we shall write down the results for the remaining two decay modes of the bottom baryon. For  $\Xi_b^0 \rightarrow \Xi_c^{\prime 0} \gamma$  we get

$$a_1 = -b_2 = \frac{4\pi}{3\sqrt{3}} \int_0^R r^2 dr [u_c u_b (3u_u u_d - v_u v_d) + 2v_c u_b (u_u v_d + v_u u_d) - v_c v_b (u_u u_d + v_u v_d)],$$

$$b_1 = -a_2 = -\frac{4\pi}{6\sqrt{3}} \int_0^R r^2 dr [v_c u_b (5u_u v_d - v_u u_d) - u_c v_b (3v_u u_d + u_u v_d) + (u_c u_b + v_c v_b) (3u_u u_d - v_u v_d)]. \quad (4.64)$$

As a result,

$$a_{\pm} = \frac{1}{2} I'_{\pm}, \quad b_{\pm} = \mp \frac{1}{2} I'_{\pm}, \quad (4.65)$$

with

$$I'_+ = \frac{4\pi}{3\sqrt{3}} \int_0^R r^2 dr [3(u_c u_d + v_c v_d) (3u_u u_b - v_u v_b) + (3u_b v_u + v_b u_u) (u_d v_c - v_d u_c)], \quad (4.66a)$$

$$I'_- = \frac{4\pi}{3\sqrt{3}} \int_0^R r^2 dr [(3u_c u_d - v_c v_d) (u_u u_b + v_u v_b) + (u_b v_u - v_b u_u) (5u_d v_c - v_d u_c)]. \quad (4.66b)$$

The resulting amplitudes for the decay  $\Xi_b^0 \rightarrow \Xi_c^{\prime 0} \gamma$  are

$$a = -\frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* c_- (d_f I'_- - d_i I'_+), \quad (4.67a)$$

$$b = -\frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* c_- (d_f I'_- + d_i I'_+). \quad (4.67b)$$

As for the transition  $\Lambda_b^0 \rightarrow \Sigma_c^0 \gamma$ , we find that its am-

plitude is the same as that of  $\Xi_b^0 \rightarrow \Xi_c^{\prime 0} \gamma$  except for a different overall normalization factor. More precisely,

$$a(\Lambda_b^0 \rightarrow \Sigma_c^0 \gamma) = \sqrt{2} a(\Xi_b^0 \rightarrow \Xi_c^{\prime 0} \gamma), \quad (4.68a)$$

$$b(\Lambda_b^0 \rightarrow \Sigma_c^0 \gamma) = \sqrt{2} b(\Xi_b^0 \rightarrow \Xi_c^{\prime 0} \gamma). \quad (4.68b)$$

Finally, we come to numerical estimates. For the bag

parameters we use [27]<sup>10</sup>

$$\begin{aligned} m_u = m_d = 0, \quad m_s = 0.279 \text{ GeV}, \\ m_c = 1.551 \text{ GeV}, \quad m_b = 5.0 \text{ GeV}, \end{aligned} \quad (4.69)$$

$$\begin{aligned} x_u = 2.043, \quad x_s = 2.488, \quad x_c = 2.948, \\ x_b = 3.079, \quad R = 5.0 \text{ GeV}^{-1}, \end{aligned}$$

where  $R$  is the bag radius. The eigenvalues  $x_q$ 's are determined by the transcendental equation

$$\tan x = \frac{x}{1 - mR - (x^2 + m^2 R^2)^{1/2}}. \quad (4.70)$$

Numerically, the relevant bag integrals are found to be

$$\begin{aligned} I_+ = -I_- = 2.443 \times 10^{-3} \text{ GeV}^3, \\ I'_+ = 3.720 \times 10^{-3} \text{ GeV}^3, \\ I'_- = 1.267 \times 10^{-3} \text{ GeV}^3. \end{aligned} \quad (4.71)$$

Putting everything together and using  $m_{\Xi_b} = 5809 \text{ MeV}$  [29],  $m_{\Xi'_c} = 2573 \text{ MeV}$ , we finally obtain the decay rates

$$\Gamma(\Xi_b^0 \rightarrow \Xi_c^0 \gamma) = 3.95 \times 10^{-20} \text{ GeV}, \quad (4.72a)$$

$$\Gamma(\Xi_b^0 \rightarrow \Xi'_c \gamma) = 3.54 \times 10^{-19} \text{ GeV} \quad (4.72b)$$

$$\Gamma(\Lambda_b^0 \rightarrow \Sigma_c^0 \gamma) = 6.65 \times 10^{-19} \text{ GeV}, \quad (4.72c)$$

the branching ratios

$$\mathcal{B}(\Xi_b^0 \rightarrow \Xi_c^0 \gamma) = 6.4 \times 10^{-8}, \quad (4.73a)$$

$$\mathcal{B}(\Xi_b^0 \rightarrow \Xi'_c \gamma) = 5.7 \times 10^{-7}, \quad (4.73b)$$

$$\mathcal{B}(\Lambda_b^0 \rightarrow \Sigma_c^0 \gamma) = 1.2 \times 10^{-6}, \quad (4.73c)$$

for  $\tau(\Xi_b^0) \sim \tau(\Lambda_b^0) = 1.07 \times 10^{-12} \text{ s}$  [15], and the decay asymmetry

$$\alpha(\Xi_b^0 \rightarrow \Xi_c^0 \gamma) = -0.47, \quad (4.74a)$$

$$\alpha(\Xi_b^0 \rightarrow \Xi'_c \gamma) = -0.98, \quad (4.74b)$$

$$\alpha(\Lambda_b^0 \rightarrow \Sigma_c^0 \gamma) = -0.98. \quad (4.74c)$$

Note that the prediction of  $\alpha$  for the decay mode  $\Xi_b^0 \rightarrow \Xi_c^0 \gamma$  is based on heavy quark symmetry [cf. Eq. (4.44)]. Since the baryonic matrix elements are evaluated under the static bag approximation which amounts to a maximal overlap of wave functions, the decay rates and branching ratios given by (4.72) and (4.73) for  $\Xi_b^0 \rightarrow \Xi'_c \gamma$  and  $\Lambda_b^0 \rightarrow \Sigma_c^0 \gamma$  ought to be regarded as the most optimistic estimates, and the respective decay asymmetry parameters as order of magnitude estimate. Nevertheless, the sign of  $\alpha$  given in (4.74) is more trustworthy. Finally, a comparison of (4.73) with (4.18) leads to the conclu-

sion that the weak radiative decays of bottom baryons are indeed dominated by the electromagnetic penguin mechanism.

## V. CONCLUSIONS

Nonpenguin weak radiative decays of heavy hadrons are characterized by emission of a hard photon and the presence of a highly virtual intermediate quark between the electromagnetic and weak vertices. We have argued that these features should make possible to analyze these processes by perturbative QCD.

In this work we have found in tree approximation that these processes are describable by an effective local and gauge invariant Lagrangian. This Lagrangian leads to a unique heavy quark symmetry prediction for the asymmetry parameter in the decay  $\Xi_b^0 \rightarrow \Xi_c^0 \gamma$ . Other interesting results are obtained by making use of the effective Lagrangian in conjunction with the factorization approximation for heavy meson decays and the MIT bag model for heavy baryon decays. In particular, the branching ratio for  $\bar{B}^0 \rightarrow D^{*0} \gamma$  is found to be  $0.9 \times 10^{-6}$ . This is very important for the experimental interpretation of the inclusive measurement of  $\bar{B}^0 \rightarrow \gamma + \text{anything}$  and its relation to the penguin dominated decay  $\bar{B}^0 \rightarrow \bar{K}^{*0} \gamma$ . In Sec. IV we have presented two different methods for estimating the rates of  $\Lambda_b \rightarrow \Lambda \gamma$ . Both approaches provide a consistent order of magnitude estimate, namely,  $\mathcal{B}(\Lambda_b \rightarrow \Lambda \gamma) \sim 1 \times 10^{-5}$ . We conclude that weak radiative decays of bottom hadrons are dominated by the short-distance  $b \rightarrow s \gamma$  mechanism.

The factorization method has been known to be reliable for nonleptonic decays of heavy mesons. So the prediction for  $\bar{B}^0 \rightarrow D^{*0} \gamma$  based on this method should also be reliable. For the heavy baryon sector we have to resort to the static bag approximation even though the initial and final heavy baryons move with substantially different velocities. This is a serious drawback in our work. The bag results obtained in Sec. IV can only be regarded as order of magnitude's rough estimates. It remains an important theoretical question how to incorporate the relative motion between two bags in the MIT bag model.

Cabibbo-allowed weak radiative decays of charmed hadrons give rise to strange hadron in the final states. The constituent  $s$  quark, whose mass is of order 500 MeV, is not very heavy. We can make rough estimates of the branching ratios for the aforementioned decays by treating the  $s$  quark as heavy and applying the formalism developed in this paper. Even though the  $1/m_s$  corrections are expected to be (50–100)%, they should not alter the order of magnitudes. In this way we find

$$\mathcal{B}(D^0 \rightarrow \bar{K}^{*0} \gamma) = 1.1 \times 10^{-4}, \quad (5.1)$$

$$\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \gamma) = 4.9 \times 10^{-5}, \quad (5.2)$$

$$\begin{aligned} \mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \gamma) &= 3.1 \times 10^{-5}, \\ \alpha(\Lambda_c^+ \rightarrow \Sigma^+ \gamma) &= -0.86, \end{aligned} \quad (5.3)$$

$$\alpha(\Xi_c^0 \rightarrow \Xi^0 \gamma) = -0.86,$$

<sup>10</sup>It should be stressed that except for the bag quark masses used in (4.69), all the light quark masses employed in the present paper are of the constituent type [see Eq. (3.10)], as explained in Sec. II.

where we have used the lifetimes for  $D$ ,  $\Lambda_c$ ,  $\Xi_c$  from the PDG [15]. The branching ratio for  $D^0 \rightarrow \bar{K}^{*0} + \gamma$  is quite sizable. These decays should be searched for experimentally.

Finally, we observe that in the weak radiative decays of bottom hadrons the highly virtual quark's squared invariant mass is of order  $m_b^2$  or smaller. It is therefore appropriate to employ the renormalization group improved weak interaction Hamiltonian (2.4) with a renormalization scale  $\mu = m_b$  as a starting point. If one wishes to use a renormalization scale smaller than  $m_b$ , one must reanalyze the one loop corrections in a heavy quark ef-

fective theory including diagrams with the photon inside the loop. However, we have not considered such an analysis.

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