

**Possible tests for  $b \rightarrow sg$  penguin-diagram decays via inclusive  $K$  distributions and exclusive processes**

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We discuss experimental signatures capable of nearly immediate study that would discern and/or constrain new physics manifested via enhanced gluonic penguin-diagram decays of the  $b$ .

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**I. INTRODUCTION**

Despite the large  $B$  mass, it appears that a systematic expansion in  $\alpha_s$  and  $1/m_b$  fails by about 15% [1] to predict the semileptonic branching ratio

$$\mathcal{B}^{sl} = \frac{\Gamma(B \rightarrow e\bar{\nu}_e X)}{\Gamma_B^{tot}}. \tag{1}$$

This discrepancy presumably requires enhancement of the nonleptonic decay width and most likely will be resolved within the framework of the standard model. For example, it could be that local “quark-hadron duality” is not as good at the  $B$  scale as originally hoped. In particular, one recent suggestion [2] is that nonperturbative effects could significantly enhance the  $b \rightarrow c\bar{c}s$  rate (with relatively slow charm quarks in the final state) beyond the naive  $\Gamma(b \rightarrow c\bar{c}s)/\Gamma(b \rightarrow c\bar{u}d) \approx 1/3$  phase space prediction.

Alternatively it has been pointed out that an enhanced  $b \rightarrow sg$  rate via penguin amplitudes could help resolve the discrepancy [3]. These will contribute only to the nonleptonic decay width  $\Gamma^{nl}$ . These decays do not lead to charm production (and hence the reason that their contribution could be added incoherently). Therefore such enhanced penguin decays would not generate an unwanted charm excess [2].

A branching of  $\Gamma(b \rightarrow sg)/\Gamma_B^{tot} \approx 20\%$  is needed to explain completely the semileptonic deficiency. To leading order in  $\alpha_s$ , the gluonic penguin decay rate is given by

$$\Gamma(b \rightarrow sg) = \frac{8\alpha_s}{\pi} \Gamma_0 \left( \frac{V_{bt}V_{ts}^* C_M(\mu)}{V_{bc}} \right)^2. \tag{2}$$

$\Gamma_0 = m_b^5 G_F^2 V_{bc}^2 / 192\pi^3$  is the lowest order, perturbative result for the semileptonic decay  $b \rightarrow ce\bar{\nu}_e$  (excluding phase-space modifications due to the finite mass of charm quark), and  $C_M(\mu)$  is the coefficient of the magnetic penguin operator entering the effective Hamiltonian ( $H_{eff}$ ),

$$O_g^M = \frac{g_s}{16\pi^2} m_b \bar{s} \tau^a G_{\mu\nu}^a \sigma^{\mu\nu} \frac{1}{2} (1 + \gamma_5) b, \tag{3}$$

evaluated at a mass scale  $\mu$  appropriate to the decay at hand. Working within the standard model and taking

$\mu = m_b/2$ , one obtains, using the evolution equations for  $C_M$  found in Ref. [4] (wherein the recent controversy [5,6] concerning the apparent scheme dependence of this evolution has been resolved) that

$$\mathcal{B}(b \rightarrow sg) \approx 2 \times 10^{-3}. \tag{4}$$

We have taken  $V_{ts} = 0.045$ , the lifetime  $\tau_B = 1.3$  picoseconds [7] and we have equated the bottom quark mass with that of the  $B$  meson, i.e.,  $m_b = M_B$ . Using  $m_b = M_B - 0.5$  yields a rate 60% of the above.

A significant enhancement above standard model predictions is thus required for gluonic penguin-diagram decays to play a role in the semileptonic decay rate issue. Admittedly this scenario may seem unlikely in view of the recent measurement by CLEO [8] of the inclusive photo-penguin decay rate,  $b \rightarrow s\gamma$ , found with a branching rate of only  $\mathcal{B}(b \rightarrow s\gamma) \approx 2.2 \times 10^{-4}$ , and in general agreement with standard model predictions [9]. Nonetheless, a preferred enhancement of the gluonic penguin by some appropriate supersymmetric (SUSY) extension of the standard model relative to its electromagnetic analogue is conceivable. Even an enhanced branching ratio  $\mathcal{B}(b \rightarrow sg)$  of just a few percent could be part of a “cocktail” [1] solution for the semileptonic problem.

In the following we argue that inclusive measurements of the kaon spectrum (presumably  $K_S^0$  for experimental feasibility) near the region of  $\tilde{u} = P_K/P_{Kmax} \rightarrow 1$  would be sensitive to  $b \rightarrow sg$  rates in the range 5–20%. We also discuss exclusive decay modes, in particular  $B \rightarrow K\pi$ , first indications of which were possibly seen last year at CLEO [10]. Here too enhancements would naively be likely, although as we will see, bound state effects could complicate such expectations.

**II. INCLUSIVE  $K$  SPECTRUM**

For a  $B$  meson at rest, the inclusive kaon momentum spectrum for kaons emerging from the cascade

$$\begin{aligned} B &\rightarrow D + X \\ &\hookrightarrow K + X' \end{aligned} \tag{5}$$

can be readily calculated from the available data. The inclusive momentum distributions for  $B \rightarrow D(P) + X$  [11] and  $D \rightarrow K(q) + X'$  [12] have been experimentally measured. A good overall fit to these distributions can be given by the rather simple parametrizations

$$\begin{aligned} \rho_{D/B}(x) &= 6x(1-x), \\ \rho_{K/D}(y) &= \frac{6}{(1-y_{\min})^3}(y-y_{\min})(1-y), \end{aligned} \quad (6)$$

where

$$x \equiv P/P_{\max} \quad y \equiv q/q_{\max} \quad y_{\min} \approx 0.14 \quad (7)$$

and all momentum are defined in the rest frame of the decaying heavy meson. Note that the crucial regions of interest,  $x, y \rightarrow 1$  are particularly well fit by Eq. (6). For simplicity we have also normalized each distribution to one.

The energy  $E_K$  of the kaon in the  $B$  decay frame is obtained from the energy and momentum in the  $D$  decay frame via a boost transformation:

$$M_D E_K = P^0 q^0 + |\vec{P}||\vec{q}|z, \quad (8)$$

$$\rho_{K/B}(u) = \int_0^1 dx \rho_{D/B}(x) \int_{y_{\min}}^1 dy \rho_{K/D}(y) \int_{-1}^1 dz \delta\left(a \left[ \sqrt{x^2 + \alpha^2} \sqrt{y^2 + \beta^2} + xyz \right] - u\right). \quad (12)$$

Integrating over the  $\delta$  function using the  $z$  integral, Eq. (12) becomes

$$\begin{aligned} \rho_{K/B}(u) &= \int_0^1 dx \frac{\rho_{D/B}(x)}{x} \int_{y_{\min}}^1 dy \frac{\rho_{K/D}(y)}{y} \Theta\left(a \left[ \sqrt{x^2 + \alpha^2} \sqrt{y^2 + \beta^2} + xy \right] - u\right) \\ &\quad \times \Theta\left(u - a \left[ \sqrt{x^2 + \alpha^2} \sqrt{y^2 + \beta^2} - xy \right]\right). \end{aligned} \quad (13)$$

Figure 1 displays the resulting distribution  $\rho_{K/B}(\tilde{u})$  which for conformity with the rest of the literature, [11,12], we have plotted not as a function of the energy variable  $u$ , but in terms of the kaon's momentum:

$$\tilde{u} = P_K/P_{K\max}. \quad (14)$$

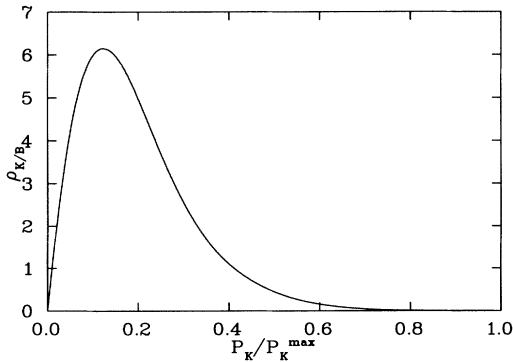


FIG. 1. The expected distribution of kaons (normalized to one) with momenta coming from the cascade decay Eq. (5) of the  $B$  meson.

where  $z$  is the cosine of the angle in the rest frame of the  $D$  meson between the decay direction of the kaon and the boost direction of the  $D$ . Defining

$$u = E_K/E_{K\max}, \quad (9)$$

and

$$\begin{aligned} \alpha &= \frac{M_B}{P_{\max}} = \frac{2M_B M_D}{M_B^2 - M_D^2}, \\ \beta &= \frac{M_K}{q_{\max}} = \frac{2M_D M_K}{M_D^2 - M_K^2}, \\ a &= \frac{P_{\max} q_{\max}}{M_D E_{K\max}} = \frac{(M_B^2 - M_D^2)(M_D^2 - M_K^2)}{2M_D^2(M_B^2 + M_D^2)} \end{aligned} \quad (10)$$

we have then that

$$u = a \left[ \sqrt{x^2 + \alpha^2} \sqrt{y^2 + \beta^2} + xyz \right]. \quad (11)$$

Using the fact that the  $D$  is spinless so that the angular distribution  $z$  is uniform, one obtains that the inclusive energy distribution  $\rho_{K/B}(u)$ , of the cascade kaons is given simply by

The two remaining integrations in Eq. (13) were performed numerically and the entire distribution was again normalized to integrate to one.

We note that at the present only preliminary data [13] on the  $K_S^0$  inclusive spectrum at the  $\Upsilon^{4s}$  is available. Nevertheless this does conform to the expected overall cascade form of Fig. 1. Our main interest here, however, is in the end point region,  $u \approx \tilde{u} \rightarrow 1$  for which much more precise data will be required and should be available from CLEO II. In this endpoint region one can directly show that

$$\lim_{u \rightarrow 1} \rho_{K/B}(u) \propto (1-u)^4. \quad (15)$$

For  $u = 1$ , both momentum variables  $x, y$  must also approach their upper limit. The explicit distributions  $\rho_{D/B}(x)$  and  $\rho_{K/D}(y)$  thus yield two of the four powers in Eq. (15). The remaining two powers arise from phase space constraints generated by the  $\Theta$  functions which force  $x, y \geq 1 - \eta$  if  $u = 1 - \eta$  and  $\eta \rightarrow 0$ . The importance of this result is that  $\rho_{K/B}(u)$  is severely suppressed near  $u = 1$  compared to the much harder function for an  $s$  quark jet to fragment into a kaon. From general counting rules [14,15] one can show from perturbative QCD that

the fragmentation function for an  $s \rightarrow K^{+,0}$  behaves as

$$\lim_{u \rightarrow 1} D_{K^{+,0}/s}(u) \sim (1-u)^2. \quad (16)$$

Combining Eqs. (13) and (16), the complete distribution,  $\rho_{K/B}^{\text{tot}}(u)$ , of kaons from  $B$  decays, is given by

$$\rho_{K/B}^{\text{tot}}(u) = (1-\epsilon)\rho_{K/B}(u) + \epsilon D_{K^{+,0}/s}(u), \quad (17)$$

where  $\epsilon$  is the  $b \rightarrow sg$  branching fraction. From the previous discussion, one knows that for any finite  $\epsilon$ , at some sufficiently large  $u \rightarrow 1$ , the  $s$  quark fragmentation function must dominate. The issue now is to better quantify this result with a reasonable parametrization for  $D_{K^{+,0}/s}(u)$ .

The new CLEO  $B \rightarrow X_s + \gamma$  data [16] could in principle yield  $s \rightarrow K$  information in a setting which appears to be kinematically similar to that in  $b \rightarrow sg$ . Nearly 30% of all the  $B \rightarrow X_s + \gamma$  inclusive decays occur through the  $K^*$  resonance. If we simply focus on the kaons produced through the decay of the  $K^*$ , one finds that the kaon spectrum is very hard as  $u \rightarrow 1$ , with a typical value of  $u = 0.7$  [replacing  $M_D$  by  $M_{K^*}$  in Eq. (11), taking  $x, y \approx 1$ , and noting that  $z \approx 0$  for the  $p$ -wave decay  $K^* \rightarrow K\pi$ ]. Such a contribution would show up quite dramatically in  $\rho_{K/B}^{\text{tot}}(\tilde{u})$  for a penguin decay rate of order 10% and indeed would allow ready determination of  $\epsilon$  in Eq. (17) to values as small as  $\epsilon = 0.02$  by integrating over the total kaon yield above  $u = 0.7$ . In an ideal case of infinite  $B$  mass the  $s$  quark from  $b \rightarrow sg$  or  $b \rightarrow s\gamma$  would have identical fragmentation independent of the identity of the recoiling system (gluon jet or single photon). However, the use of this inclusive data for the actual  $B$  mass,  $M_B = 5.3$  GeV, is somewhat dubious. For  $b \rightarrow s\gamma$ , the relevant total energy of the hadronizing system of the  $s$  plus spectator  $\bar{q}$  is rather low, being roughly only

$$\begin{aligned} W_{X_s} &= \sqrt{2E_s "m_{\bar{q}}"} \\ &\approx (5.3 \text{ GeV} \times 0.3 \text{ GeV})^{1/2} = 1.3 \text{ GeV}, \end{aligned} \quad (18)$$

as indeed is manifest by the fact that the  $K^*$  and only a few other kaon resonances dominate the data. Barring precocious scaling, extraction of accurate information on inclusive functions such as  $D_{K^{+,0}/s}(u)$  in such kinematics is questionable.

The kinematics in the case of  $b \rightarrow sg$  is, on the other hand, different. Assuming the  $s$  quark to recoil against an oppositely moving  $g\bar{q} = \bar{3}$  source, the total energy available for hadronization is now  $M_B = 5.3$  GeV and we expect the "leading"  $s \rightarrow K$  fragmentation to be similar to that in an  $e^+e^- \rightarrow s\bar{s}$  process at these energies. Indeed comparing even just the expected standard model rate, Eq. (4), with the upper bound given last year by CLEO for  $B \rightarrow K\pi$  ( $< 2.6 \times 10^{-5}$ ) indicates that the inclusive rate will not be dominated by just a few resonances.

Unfortunately  $s \rightarrow K$  fragmentation contributions in  $e^+e^-$  is charged suppressed by a factor  $q_s^2 / \sum q_i^2$  and are hence difficult to extract from the known data on inclusive kaon production [17]. Likewise,  $s$  quark jet production in deep inelastic  $\nu$  scattering is Cabibbo suppressed

by  $|V_{us}|^2$  and hence not particularly useful. We are thus forced to work by analogy, guided by general symmetry principles.

Starting first from SU(3) flavor symmetry, the inclusive  $\pi^+$  spectrum from  $e^+e^-$  annihilation has been recently [18] nicely fitted using a primary, direct  $u \rightarrow \pi^+$  term  $D_p$  and a secondary distribution term  $D_s$ :

$$D_p(z, t_0) = \frac{5}{6}(1-z)^2 \quad D_s(z, t_0) = \frac{5(1-z)^4}{6z} \quad (19)$$

where  $t_0 \equiv \ln(Q_0^2/\Lambda^2)$  reflects the general fact that in QCD these distributions run with  $Q^2$ . The motivation of the authors of [18] for departing from other, perhaps more common forms in which  $D(z)$  is given jointly as one smooth function [e.g., as in [19] where  $D(z) = (1-z)^2/4z$ ] is that the physics dictating the two end point regions is rather different. The logarithmic rise in total kaon number produced for  $z \rightarrow 0$  is driven by mesons produced out of secondary quarks formed in the fragmentation chain of the outgoing quark jet. The region  $z \rightarrow 1$  is dominated by mesons made out of the original quark in the jet. A separation of these two phenomena was crucial for the study in [18] in which (as is the case here) the authors were particularly interested in the  $z \rightarrow 1$  regime of the fragmentation function in order to compare with competing processes.

Ignoring completely secondary kaon production, and assuming for the kaon the same primary distribution  $D_p(z, t)$  as in the pion (evolved down from the fitted  $Q_0 = 29$  GeV data, to  $Q = M_B$  using the analytic expression provided in [18]), the result for  $\rho_{K/B}^{\text{tot}}(\tilde{u})$ , Eq. (17), using a value for  $\epsilon = 0.2$  is shown in Fig. 2. For comparison the result of Fig. 1 is included (i.e.  $\epsilon = 0$ ), and as is appropriate, we have focused only on the end-point region  $\tilde{u} \rightarrow 1$ . We see that for this "maximal" value of  $\epsilon$ , a significant difference has developed in the expected kaon distributions once  $\tilde{u} \geq 0.7$ . Accurate data at larger  $\tilde{u}$  allow one to probe significantly smaller  $\epsilon$ , so that for  $\epsilon = 0.05$  comparable differences appear at  $\tilde{u} = 0.8$  and at  $\tilde{u} = 0.9$ , a value as small as  $\epsilon = 0.02$  could be discerned.

Having demonstrated what should be an experimen-

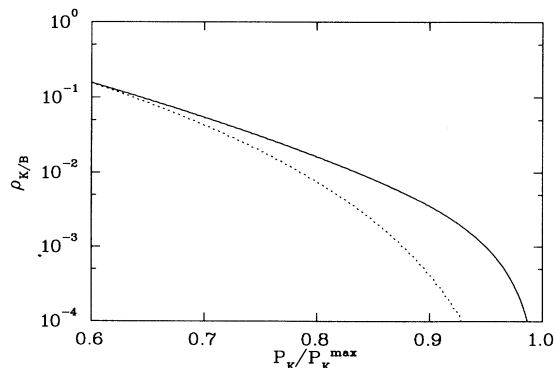


FIG. 2. The expected distribution of kaons if 20% of the decays arose via gluonic penguins and the  $s \rightarrow K$  fragmentation function was SU(3) symmetric with  $u \rightarrow \pi^+$ . In dots, the tail of Fig. 1 has also been included for comparison.

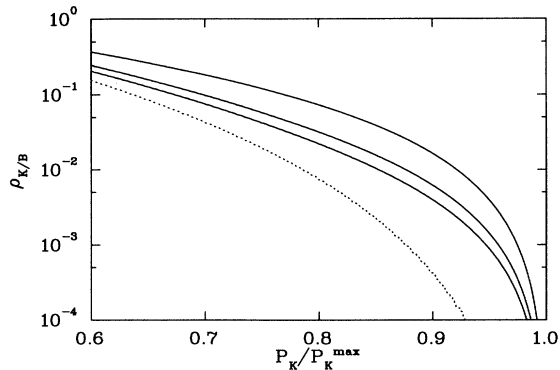


FIG. 3. Same as Fig. 2 except a heavy quark fragmentation function has been used for  $s \rightarrow K$ . The various plots, from top to bottom, are for  $\epsilon_s = 1, 4, 10$ , respectively, in Eq. (20).

tally testable effect, it is likely that these estimates are, however, conservative. SU(3) flavor symmetry is in fact broken and it is likely that the  $s$  quark fragmentation function is harder than that of a  $u$  quark. As an extreme alternative, we show the results in Fig. 3 of using the heavy quark fragmentation function of Peterson *et al.* [20],

$$D_{K^{+},0/s}(z) = \frac{N}{z[1 - 1/z - \epsilon_Q/(1-z)]^2} \quad (20)$$

in which  $N$  is determined by fixing the normalization to integrate to one and  $\epsilon_Q$  is qualitatively  $m_q^2/m_Q^2$ , the ratio of effective light to heavy quark masses. Note that the  $\epsilon \rightarrow \infty$  limit smoothly matches up to the parametrization of Baier *et al.* [19] for a light-quark fragmentation function. In the case of charm a good fit to the data [21] is found using  $\epsilon_C = 0.15$ . Since in the case of the  $s$  quark the choice of  $\epsilon_s$  is more ambiguous (if at all correct), Fig. 3 contains plots for a few possible  $\epsilon_s$  values. As expected, the resulting distributions yield a significantly greater departure from the cascade scenario of kaon production than the one indicated by Fig. 2 where perfect SU(3) flavor symmetry was assumed.

### III. EXCLUSIVE DECAYS

The observation last year [16] by the CLEO group of the two-body exclusive decay  $B \rightarrow K^* \gamma$  was the first unambiguous experimental evidence of penguin processes. One likewise expects that rare two-body hadronic decays of the  $B$  meson to be a particularly useful means of measuring gluon mediated,  $b \rightarrow s$  transitions. For concreteness, we will focus on the mode  $B \rightarrow K\pi$  which, as mentioned earlier, was likely seen last year at CLEO [10] (the ambiguity involves insufficient experimental resolution to separate candidate  $\pi\pi$  from  $K\pi$  decay channels).

For the two-body hadronic decays, penguin processes compete with another rare decay,  $b \rightarrow u$ , which occurs at tree-level in the standard model but is proportional to the Cabibbo-Kobayashi-Mashawa (CKM) matrix el-

ements  $|V_{ub}V_{us}|^2$ . Judicious comparison with analogous processes proportional to  $|V_{ub}V_{ud}|^2$  allows one to infer the relative importance of the gluonic penguin contribution. In the case of  $B \rightarrow K\pi$  the corresponding decay mode is  $B \rightarrow \pi\pi$ . (Hence the further importance that CLEO resolve these two modes, the sum of which were reported with a total branching rate of  $2.4 \pm_{0.7}^{0.8} \pm 0.2 \times 10^{-5}$  [10].) An observed ratio of branching rates much above (or below) the naive  $|V_{us}/V_{ud}|^2 \approx 1/20$  must be due to penguins.

Using perturbative QCD methods recently seen [22] to give a good description of the two-body hadronic decays of the  $B$  meson, we estimate in the standard model that

$$\mathcal{B}(B \rightarrow K\pi) \approx 0.5 \times 10^{-5}, \quad (21)$$

not far from the CLEO data of last year [10]. Such a result clearly does not allow much room for enhancement. However, the importance of bound-state effects must be emphasized. We will therefore sketch how our estimate Eq. (21) was obtained, in which some particularly simplifying approximations were used. A full analysis of the decay is the subject of a forthcoming work [23].

Since the two-body exclusive decays of the  $B$  involve large momentum transfers, they are short distance events. A twist expansion in perturbative QCD suggests [24] that only the contribution from the lowest order Fock component expansions of the  $B$  and of the outgoing mesons are relevant. The decay rate of the  $B$  then involves a perturbatively calculable hard amplitude convoluted with a soft physics wave function,  $\psi_m$ , from each of the mesons. These wave functions, although as yet uncalculable from first principles, are universal for each meson; i.e., they factorize from the hard amplitude and hence are independent of the process involved. Thus as was employed in Ref. [22], ideally one can phenomenologically parametrize these wave functions using a (few) measured cross-sections and/or decay rates. For simplicity, we use the factorization scheme advocated by Brodsky and Lepage [24] and take the momenta of the quarks as some fraction  $x$  of the total momentum of the parent meson weighted by a soft physics distribution amplitude  $\phi(x)$  [ $\phi(x)$  being then simply the quark's wave function  $\psi(x, k_\perp)$  integrated over transverse momentum,  $k_\perp$ ].

An important ingredient in the perturbative QCD approach to the two-body exclusive decays of the  $B$  is that the decay amplitude can acquire an imaginary part because some heavy quark propagators in various Feynman graphs can go on-shell in the integration over the mesonic distribution amplitudes. Here as elsewhere [25], [26], picking up such poles is legitimate in Perturbative QCD (PQCD) as they are not pinched singularities and hence by the Coleman-Norton theorem [27], are not associated with a long distance event. These poles arise (in part) because of the factorization scheme we employ and because of what one believes to be the correct relation between quark and meson masses:

$$M_B = m_b + \Lambda, \quad (22)$$

where  $\Lambda \sim 500$  MeV. Since it has been found in practice [22] that these imaginary parts tend to dominate the

decay amplitude when they occur, we will here focus on them to obtain our estimate Eq. (21) with the more complete study to be presented elsewhere [23].

For our present purposes then, the most relevant graph contributing to the decay rate  $B \rightarrow K\pi$  is shown in Fig. 4. The square indicates a gluonic penguin diagram, the operator structure of which will be presently discussed. The cross indicates the heavy-quark propagator that can go on shell. We note that graphs at this order in  $\alpha_s$  other than that shown have imaginary parts, but they are suppressed by factors of  $\epsilon = \Lambda/M_B$ . Unlike the case of the inclusive cross section considered in the previous section, the restriction to explicit hadronic modes in the final state means that the virtuality of the quark and gluon legs entering the penguin decay  $b \rightarrow sg$  cannot be discounted. Hence more than merely the simple chromomagnetic penguin operator  $O_g^M$ , Eq. (3), contributes and in particular, it is found that chromoelectric penguin diagrams,

$$O_g^E = \frac{g_s}{16\pi^2} \bar{s}\tau^a \gamma_\nu \frac{1}{2} (1 - \gamma_5) b (D_\mu G_{\mu\nu})^a, \quad (23)$$

are highly relevant (indeed they in fact dominate our es-

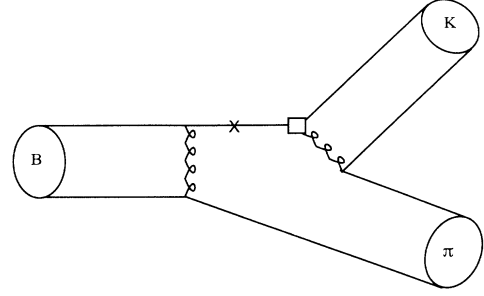


FIG. 4. The diagram with largest imaginary phase contributing to  $B \rightarrow K\pi$  in a perturbative QCD analysis. The cross indicates the heavy quark propagator that can go on shell. The square represents gluonic penguin operators from  $H_{\text{eff}}$ .

timate for the  $B \rightarrow K\pi$  decay rate). Including then both operators  $O_g^M$  and  $O_g^E$ , with Wilson coefficients  $C_M(\mu)$  and  $C_E(\mu)$ , respectively, the contribution (I) of the diagram in Fig 1. to the decay amplitude is given by the expression

$$(I) = \frac{8A}{\epsilon_B} C_M(\mu) \int dx \frac{\phi_\pi(x)(1-x)}{x - 2\epsilon_B - i\eta} \int dy \phi_k(y)y + \frac{4A}{\epsilon_B} C_E(\mu) \int dx \frac{\phi_\pi(x)(1-x)(1+x-2\epsilon_B)}{x - 2\epsilon_B - i\eta} \int dy \phi_k(y)y(1-y), \quad (24)$$

where  $A$  is given by

$$A = \frac{8}{9} \alpha_s^2 f_B f_k f_\pi G_F U_{bt} U_{ts}^*, \quad (25)$$

and a peaking approximation has been used for  $\phi_B(z)$ :

$$\phi_B(z) = \frac{1}{2\sqrt{3}} f_B \delta(z - \epsilon). \quad (26)$$

$f_B$  is the decay constant of the  $B$ .

Using the distribution amplitude of Chernyak and Zhitnitsky [28] for the pion and, for simplicity, the asymptotic distribution amplitude [29] for the kaon,

$$\phi_\pi(x) = 5(1 - 2x)^2, \quad (27)$$

$$\phi_k(y) = 1,$$

the imaginary piece of Eq. (24) becomes

$$\text{Im}(I) = \frac{5A\pi}{\epsilon_B} \left[ \frac{2}{3} C_E(\mu) + 4C_M(\mu) \right] (1 - 2\epsilon)(1 - 4\epsilon)^2. \quad (28)$$

We use  $\text{Im}(I)$  to obtain our estimate of the decay rate:

$$\Gamma^{\text{est}} = \frac{[\text{Im}(I)]^2}{16\pi M_B}. \quad (29)$$

As parameters we use

$$\begin{aligned} V_{ts} &= 0.045, \\ f_B &= \sqrt{2} f_\pi, \\ \epsilon &= \frac{0.5}{M_B}, \\ \Lambda_{\text{QCD}} &= 0.2 \text{ GeV}. \end{aligned} \quad (30)$$

For the scale  $\mu$  we take the virtuality of the softest gluon exchanged,  $\mu^2 \approx 0.5(\text{GeV})^2$ . For the evolution of  $C_E(\mu)$ , only Cella *et al.* of Ref. [5] have calculated the anomalous dimension mixing matrix relevant for the chromo-electric penguins, and thus we use their results. This might be thought imprudent, considering the controversy that was associated with the chromomagnetic penguins [4–6]. We note, however, that in the case of the chromomagnetic penguin diagrams, the final results of Ciuchini *et al.* [4] differ only slightly from that of Cella *et al.* for those operators mutually calculated, and hence one might expect that any errors due to scheme dependence would be kept at a minimum. Such is our hope in the present work, although we acknowledge and stress the importance that these expectations be confirmed.

With these inputs we obtain that

$$B^I(B \rightarrow K\pi) \approx 0.3 \times 10^{-5}, \quad (31)$$

where the superscript  $I$  reminds the reader this estimate is based only on picking up the (leading) imaginary part of the decay amplitude. Roughly assuming that the real part (which adds incoherently but involves many more graphs and also in principle more off-shell operators) is of the same order, we obtain Eq. (21).

We note that similar such bound-state effects have been previously considered [30] in the case of  $B \rightarrow K^* \gamma$  decays, however, there the physical on-shell photon eliminates the electromagnetic version of Eq. (23). The purely hadronic two-body decay  $B \rightarrow K \pi$  does not, however, enjoy such simplifying features. Chromoelectric penguins are found to be highly relevant because the restriction to a  $K \pi$  final state allows for the gluons to be significantly off shell and because the Wilson coefficient of the chromoelectric operator is appreciably larger than that of the chromomagnetic operator (at either the  $M_W$  scale or when ran to lower scales appropriate for the decay of the  $B$ ).

The complications of bound-state effects and in particular the introduction of additional operators thus allows, at least in principle, significant differences between the inclusive rates discussed in Sec. II and that of any particular decay mode as discussed here. One tenable although perhaps contrived scenario is that *only*  $O_g^M$  is significantly enhanced. Although somewhat bizarre, especially since for the gluon penguin diagrams it is at a fundamental level the same Feynman graphs that determine the Wilson coefficients of both the chromomagnetic as well as chromoelectric operators, such a scenario cannot nevertheless be precluded. Indeed the fact that QCD corrections play an important role and produce significant changes in both the absolute magnitudes and relative sizes of the various coefficients, means that such a scenario might yet be feasible. However, one should recall that this preferential treatment would also have to extend into the electromagnetic sector where the data [8,16] also do not allow significant departure from standard model predictions.

#### IV. CONCLUSIONS

It is well known that deviation of  $b \rightarrow s \gamma$  and  $b \rightarrow sg$  amplitudes from standard model estimates could directly indicate new physics. The study of potential  $b \rightarrow sg$  enhancement is further motivated by its possible contribution to resolving the  $B$  semileptonic problem. We have

here discussed the possibility of detecting such enhancement and estimating the overall strength of  $b \rightarrow sg$  transitions by focusing on the resulting inclusive kaon distribution in the region near  $P_K = P_K^{\max}$  and alternatively looking at the extreme case of  $P_K = P_K^{\max}$  corresponding to the (penguin generated)  $B \rightarrow K \pi$  exclusive final state.

Clearly future studies could make use of the much richer topologies in an effort to have a more sensitive extraction of  $b \rightarrow sg$ .<sup>1</sup> Remarkably, however, mere use of the inclusive kaon distribution will be sufficiently sensitive to an enhanced  $b \rightarrow sg$  total rate of even just a few percent as the significantly harder kaons near  $P_K = P_K^{\max}$  that result from the penguin decays would dominate over the softer kaons from the ordinary cascade  $B \rightarrow D(+X) \rightarrow K(+X')$ .

The exclusive  $B \rightarrow K \pi$  decay mode is sensitive to the chromoelectric penguin term. Present data and standard model estimates already would appear to exclude enhancement of this operator by more than a factor of two or so, although potential uncertainties exist concerning the evolution of such "off-shell" operators that may yet modify these estimates. Even barring such complications, these conclusions while suggestive nevertheless do not preclude a significantly large enhancement in the total, inclusive production of kaons as the chromoelectric operator vanishes for on-shell gluons and hence does not contribute to the perturbatively calculated, inclusive  $b \rightarrow sg$  decay rate (obtained using chromomagnetic penguin transitions).

Given these initial results and in view of the fact that enhanced  $b \rightarrow sg$  decays are indications of new physics, the simple first step of looking for it via the inclusive kaon spectrum, complemented by better data on the exclusive two-body decays, seems an undoubtedly worthwhile enterprise.

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