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Heavy meson decays into light resonances

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We analyze the Lorentz structures of weak decay matrix elements between meson states of arbitrary spin. Simplifications arise in the transition amplitudes for a heavy meson decaying into the light one via a Bethe-Salpeter approach which incorporates heavy quark symmetry. The phenomenological consequences of our results on several semileptonic, nonleptonic, and flavor-changing neutral-current-induced decays of heavy flavored mesons are derived and discussed.

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I. INTRODUCTION

Decays of the b quark into light u, d, s quarks offer ways of testing the standard model and probing new physics. Thus the $b \to u$ decays give a direct determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$ while the flavor-changing neutral-current-(FCNC-)induced transition $b \to s$ allows one to extract knowledge of the yet to be discovered t quark, and is sensitive to new physics beyond the standard model. Although the charmed channels $b \to c$ dominate the total decay rate of the B meson, the much rarer decays of the b quark into light flavored quarks provide important information about the parameters of the CKM matrix; in those rare decays there is so much available phase space that it is possible to produce many light meson resonances (p, d, f,... wave) in the final state, not just the ground state mesons. Thus those rare processes tell us something about the hadronic structure of light meson resonances, apart from giving us information about weak interaction elements; hadronic matrix elements in weak decays have certainly attracted much theoretical attention since it is hard to calculate them directly from the first QCD principles. The main purpose of this paper is to explore and understand the Lorentz structures of transition matrix elements of the weak current between meson states of arbitrary spin. Most of the recent progress in the heavy quark effective theory has been concentrated in the area of heavy hadron to heavy hadron transitions [1–7], but a few results have appeared in the literature about heavy to light hadron transitions in their ground states [2,8]. It is our aim here to extend the arguments to the heavy to light meson resonances of arbitrary spin.

We will begin by outlining a general rule for counting the number of form factors representing independent Lorentz structures in Sec. II, and we present explicit forms of those elements for processes $0 \to J'$ and $1 \to J'$ before detailing the general case $J \to J'$. It is widely accepted that the heavy quark limit is a reliable approximation for treating mesons and baryons containing a b quark, as long as the momentum of the light degrees of freedom is small compared to the mass of the heavy quark. Therefore in Sec. III we take that limit for the initial B mesons and show how the number of form factors is reduced. The expressions for decay rates (in terms of those form factors) are worked out in Sec. IV and our conclusions are stated in Sec. V. An Appendix contains some useful technicalities about sums over polarization tensors.

II. HADRONIC MATRIX ELEMENTS

We begin with considering the matrix elements of a vector (or axial vector) current between a spin $J = 0$ meson state of momentum p and a meson resonance of momentum p' with spin J'. For the simple case when $J' = 0$, it is well known that there are just two Lorentz-invariant form factors parametrizing the matrix element:

$$\begin{aligned}\langle p', 0|\mathcal{J}_\mu|p, 0\rangle &\equiv f_1(p\cdot p')p_\mu + f_2(p\cdot p')p'_\mu \\ &\equiv a_+(q^2)(p+p')_\mu + a_-(q^2)(p-p')_\mu, \quad (1)\end{aligned}$$

where $q = p - p'$, and we have made no assumptions about current conservation at this stage. Before we present the results for any J', let us take the $J' = 1$ case as an example of the proliferation of form factors when J' and J grow. Here, in addition to the vectors p_μ and p'_μ, we have available the final vector meson polarization vector $\phi'^*_\mu(p')$, satisfying $\phi'_\mu p'^\mu = 0$. Allowing also for the Levi-Civita tensor [9] (since we have made no assumptions about parity as yet), we arrive at four form factors:

$$\begin{aligned}\langle p', 1|\mathcal{J}_\mu|p, 0\rangle \equiv {} & [a_+(q^2)(p+p')_\mu + a_-(q^2)(p-p')_\mu]p^\nu\phi'^*_\nu \\ & + f(q^2)\phi'^*_\mu + ig(q^2)\epsilon_{\mu\alpha\beta\gamma}\phi'^{*\alpha}p^\beta p'^\gamma .\end{aligned}$$

Naturally, if we impose parity conservation, then either one or three of the above structures disappear.

For higher spin J', we may represent the final meson by a Lorentz tensor of rank J', namely, $\phi'^*_{\{\mu_1\cdots\mu_{J'}\}}(p')$; it is of course transverse to p', symmetric, and traceless. Because the final result must be a Lorentz vector, the indices of the polarization tensor may either be completely saturated with p to form the scalar $\phi'^*_{\{\mu_1\cdots\mu_{J'}\}}p^{\mu_1}\cdots p^{\mu_{J'}}$, or we may leave one index free, $\phi'^*_{\{\mu\mu_2\cdots\mu_{J'}\}}p^{\mu_2}\cdots p^{\mu_{J'}}$. As well we should allow for a Levi-Civita tensor coupling to the polarization tensor in the form

$$\epsilon_{\mu\mu_1\alpha\beta}(\phi'^{*\{\mu_1\mu_2\cdots\mu_{J'}\}}p_{\mu_2}\cdots p_{\mu_{J'}})p^\alpha p'^\beta .$$

Altogether then we can construct the vectorial matrix element

$$\langle p', J' \geq 1|\mathcal{J}_\mu|p, 0\rangle = a_+^{(J')}(q^2)(\phi'^*_{\{\mu_1\cdots\mu_{J'}\}}p^{\mu_1}\cdots p^{\mu_{J'}})(p+p')_\mu + a_-^{(J')}(q^2)(\phi'^*_{\{\mu_1\cdots\mu_{J'}\}}p^{\mu_1}\cdots p^{\mu_{J'}})(p-p')_\mu$$
$$+f^{(J')}(q^2)(\phi'^*_{\{\mu\mu_2\cdots\mu_{J'}\}}p^{\mu_2}\cdots p^{\mu_{J'}}) + ig^{(J')}(q^2)\epsilon_{\mu\mu_1\alpha\beta}(\phi'^{*\{\mu_1\mu_2\cdots\mu_{J'}\}}p_{\mu_2}\cdots p_{\mu_{J'}})p^\alpha p'^\beta. \qquad (2)$$

Some remarks are in order at this stage.

(1) Except for the $J' = 0$ case where the number of independent form factors is two, there are at most *four* form factors for arbitrary higher spin J' when $J = 0$.

(2) The number of form factors depends of course on the angular momentum which the current carries. The transverse part of a vector (or axial vector) current will carry spin 1, while the longitudinal part corresponds to spin 0 and is not relevant for conserved currents. Thus we may associate three pieces of (2) with orbital angular momentum $J'+1$, J', and $J'-1$ for the transverse current, and one orbital piece J' for the longitudinal current. In the special case $J' = 0$, one transverse form factor and one longitudinal form factor survive.

(3) Keeping these points in mind and following the authors of Ref. [10], it is appropriate to express the vectorial matrix element for general J' in the form

$$\begin{aligned}\langle p', J'|\mathcal{J}_\mu|p, 0\rangle = {} & F_0^{(J')}(q^2)(\phi'^*_{\{\mu_1\cdots\mu_{J'}\}}p^{\mu_1}\cdots p^{\mu_{J'}})\frac{M^2-M'^2}{q^2}q_\mu \\ & +F_1^{(J')}(q^2)(\phi'^*_{\{\mu_1\cdots\mu_{J'}\}}p^{\mu_1}\cdots p^{\mu_{J'}})\left[(p+p')_\mu - \frac{M^2-M'^2}{q^2}q_\mu\right] \\ & +F_2^{(J')}(q^2)\left[\phi'^*_{\{\mu\mu_2\cdots\mu_{J'}\}}p^{\mu_2}\cdots p^{\mu_{J'}} - \frac{q_\mu}{q^2}\phi'^*_{\{\mu_1\cdots\mu_{J'}\}}p^{\mu_1}\cdots p^{\mu_{J'}}\right] \\ & +F_3^{(J')}(q^2)\epsilon_{\mu\alpha\beta\gamma}\phi'^{*\{\alpha\mu_2\cdots\mu_{J'}\}}p_{\mu_2}\cdots p_{\mu_{J'}}p^\beta p'^\gamma, \qquad (3)\end{aligned}$$

where the last two terms are not present when $J' = 0$.

With the experience gained from the work presented in the previous paragraph, we may now analyze matrix elements between initial spin-1 state and a final state of arbitrary spin. For the $1 \to 0$ case, all results for $0 \to 1$ discussed previously are retained except that one should replace the final polarization vector by the initial one and exchange momenta (crossing).

When both final and initial spins are 1, we have two polarization vectors: the initial ϕ_μ and the final ϕ'^*_ν; the matrix element is a bilinear of them. From them we can first construct scalar invariants such as

$$(\phi_\mu p'^\mu)(\phi'^*_\nu p^\nu), \quad (\phi_\mu \phi'^{*\mu}), \quad \epsilon^{\alpha\beta\gamma\delta}\phi_\alpha\phi'^*_\beta p_\gamma p'_\delta.$$

In combination with either p_μ or p'_μ, each of the above three scalars generates two form factors; so at the moment we have six in all. Next, allowing the polarization vectors to carry the Lorentz index of the current, we gain two more form factors:

$$\phi_\mu(\phi'^*_\nu p^\nu), \quad (\phi_\nu p'^\nu)\phi'^*_\mu.$$

Finally, using the Levi-Civita tensor we get two more structures,

$$\epsilon_{\mu\alpha\beta\gamma}\phi^\alpha\phi'^{*\beta}p^\gamma \quad \text{and} \quad \epsilon_{\mu\alpha\beta\gamma}\phi^\alpha\phi'^{*\beta}p'^\gamma.$$

In all there are therefore *ten* form factors in the case $J = J' = 1$. Although one can contemplate structures like

$$(\phi'_\nu p^\nu)\epsilon^{\mu\alpha\beta\gamma}\phi_\alpha p_\beta p'_\gamma, \quad (\phi_\nu p'^\nu)\epsilon^{\mu\alpha\beta\gamma}\phi'_\alpha p_\beta p'_\gamma,$$

the identity

$$P_\mu\epsilon_{\alpha\beta\gamma\lambda} = P_\alpha\epsilon_{\mu\beta\gamma\lambda} + P_\beta\epsilon_{\alpha\mu\gamma\lambda} + P_\gamma\epsilon_{\alpha\beta\mu\lambda} + P_\lambda\epsilon_{\alpha\beta\gamma\mu}$$

can be massaged to show that these new terms are not independent of the previous ones.

Now we proceed to matrix elements for $1 \to J' \geq 2$. Besides the polarization vector ϕ_μ of the initial meson, we have the Lorenz tensor of rank J', $\phi'_{\{\mu_1\cdots\mu_{J'}\}}$ for the final meson. When this polarization tensor occurs in the contracted vector form

$$\varphi'_\mu = \phi'_{\{\mu\mu_2\cdots\mu_{J'}\}}p^{\mu_2}\cdots p^{\mu_{J'}},$$

we can repeat the earlier analysis $(1 \to 1)$ and obtain ten form factors. In addition we should consider the possibility that two indices remain uncontracted:

$$\varphi'_{\mu\nu} = \phi'_{\{\mu\nu\mu_3\cdots\mu_{J'}\}}p^{\mu_3}\cdots p^{\mu_{J'}},$$

from which we can build two more vectorial covariants, $\phi^\nu\varphi'_{\mu\nu}$ and $\epsilon^{\mu\alpha\beta\gamma}\phi^\nu\varphi'_{\alpha\nu}p_\beta p'_\gamma$. Hence the number of form factors rises to *twelve.* As far as the counting is concerned, this ties in very nicely with the classical analysis based on angular momentum addition.

(1) Letting $S = 1$ correspond to the transverse current and coupling it to $J = 1$, we obtain total spin $2, 1, 0$. To these we may add orbital angular momenta $L = J'+2, J'+1, J', J'-1, J'-2$, $L = J'+1, J', J'-1$, and $L = J'$, respectively. Consequently, there are $5 + 3 + 1 = 9$ form factors when $J' \geq 2$.

(2) Setting $S = 0$ for the longitudinal part, there is only total spin 1 (of the initial meson). So here we get three form factors, associated with $L = J'+1, J', J'-1$, provided that $J' \geq 1$.

Adding (1) and (2), the total (maximum) number of form factors is 12 — but is of course reduced to a smaller number when $J' \leq 1$.

We are now in a position to outline the general rule for counting how many independent form factors are needed to describe a vector (or axial vector) current between spin-J and spin-J' meson states. The analysis is best carried out in the channel of the current. First, we decompose the current itself into a transverse part ($S = 1$) and a longitudinal part ($S = 0$). Second, we compose the spins of the two mesons into the set

$$J + J',\ J + J' - 1, \ldots, |J - J'|$$

and ask what angular momentum values L are needed to give total spin S. For $S = 0$, L necessarily equals the total mesons' spin, while for $S = 1$ there is a threefold possibility for L (assuming the total mesons' spin exceeds 0). Hence the total number of L values, and thus form factors, equals the sum N of N_0 and N_1 where

$$N_0 = 1 + 2\mathrm{Min}(J, J'), \quad S = 0,$$

$$N_1 = 1 + \sum_{k=0}^{2\mathrm{Min}(J,J')} [1 + 2\mathrm{Min}(1, |J - J'| + k) + 1], \qquad S = 1,$$

leading to

$$N = 4(2J' + 1) \ \text{ for } J' < J,$$

$$N = 4(2J + 1) - 2 \ \text{ for } J' = J,$$

$$N = 4(2J + 1) \ \text{ for } J' > J.$$

These structures may be given a Lorentz covariant form. We shall not write them all out as they are not needed in the present investigation. We will just content ourselves by stating what they reduce to at a special kinematical point, zero recoil, where $Mp' = M'p$ and M, M' stand for the masses of the mesons in the initial and final states. In this limit only three structures survive for the element $\langle J'|\mathcal{J}_\lambda|J\rangle$: namely,

$$\phi'^{*\{\mu_1\cdots\mu_{J'}\}}\phi_{\{\lambda\mu_1\cdots\mu_{J'}\}} \ \text{ for } J' = J - 1,$$

$$\phi'^{*\{\mu_1\cdots\mu_J\}}\phi_{\{\mu_1\cdots\mu_J\}}(p + p')_\lambda \ \text{ for } J' = J,$$

$$\phi'^{*}_{\{\lambda\mu_1\cdots\mu_J\}}\phi^{\{\mu_1\cdots\mu_J\}} \ \text{ for } J' = J + 1.$$

III. HEAVY TO LIGHT TRANSITIONS

It has been demonstrated that Bethe-Salpeter approach is as useful as the so-called tensor method of the heavy quark effective theory for treating hadronic matrix elements [2,5,7]. We shall use this approach, incorporating heavy quark symmetry, to investigate weak decays of a heavy meson into light resonances in this section. In analogy with the interpolating field method, we shall consider the Bethe-Salpeter amplitude $\phi^\beta_\alpha = \langle 0|TQ_\alpha\bar{q}^\beta|p\rangle$ for the meson state in momentum-space,

$$\phi^\beta_\alpha = [\chi_{(\mu_1\cdots\mu_L)}(p)]^\sigma_\rho\ (A^{\mu_1\cdots\mu_L})^{\rho\beta}_{\sigma\alpha}, \tag{4}$$

where the spin-parity projector χ represents the Lorentz-covariant wave function of the external meson. The structure of spin-parity projectors for resonances of higher spin have been worked out by us [6,7], and here we just list the results:

$$\chi^{(^1L_L)}_{(\mu_1\cdots\mu_L)}(p) = \gamma_5 P_{\{\mu_1\cdots\mu_L\}}(p), \tag{5}$$

$$\chi^{(^3L_{L+1})}_{(\mu_1\cdots\mu_L)}(p) = \gamma^\mu V^{L+1}_{\{\mu\mu_1\cdots\mu_L\}}(p), \tag{6}$$

$$\chi^{(^3L_L)}_{(\mu_1\cdots\mu_L)}(p) = -i\gamma^\mu \sum_k \frac{p^\lambda}{m}\epsilon_{\lambda\mu\mu_k\nu}d^{\nu\nu'}(p)V^L_{\{\mu_1\cdots\bar{k}\cdots\mu_L\nu'\}}(p), \tag{7}$$

and

$$\chi^{(^3L_{L-1})}_{(\mu_1\cdots\mu_L)}(p) = \gamma^\mu\left[\sum_k d_{\mu\mu_k}(p)V^{L-1}_{\{\mu_1\cdots\bar{k}\cdots\mu_L\}}(p) - \frac{2}{2L-1}\sum_{kl} d_{\mu_k\mu_l}(p)V^{L-1}_{\{\mu\mu_1\cdots\bar{k}\bar{l}\cdots\mu_L\}}(p)\right], \tag{8}$$

where we have adopted the standard notation $^{2S+1}L_J$, and $d_{\mu\nu}(p)$ is given in the Appendix. The parity of the meson resonances is given by $(-1)^{L+1}$ and $CP = -1$ for the singlet and $+1$ for the triplet. When a heavy meson contains an on-shell heavy quark, one has further

$$\phi^\beta_\alpha = \left[\frac{1+\not{v}}{2}\chi_{(\mu_1\cdots\mu_L)}(v)\right]^\sigma_\alpha\ (A^{\mu_1\cdots\mu_L})^\beta_\sigma. \tag{9}$$

For heavy mesons, it is conventional to organize the terms as eigenfunctions of projectors corresponding to the total angular momentum of the light degrees of freedom. Even though we really have no detailed knowledge of the configuration of the light degrees of freedom, the decoupling of the heavy quark spin tells us the two components in a doublet generated by the heavy quark spin operator tie in with those of the light degrees of freedom. Using this line of argument the spin-parity operators have been presented by Falk [4] and are related to ours through Clebsch-Gordan coefficients. We will return to this issue soon. The matrix element for the heavy to light transition takes the form [11]

$$\langle X_L(p)|\bar{q}\Gamma h_v|X(v)\rangle = \mathrm{Tr}\left[M^{\nu_1\cdots\nu_L}(v,p)\Gamma\frac{1+\not{v}}{2}\chi_{(\nu_1\cdots\nu_L)}(v)\right]. \tag{10}$$

The overlap integral M involves the light degrees of freedom in both heavy and light hadrons and embodies the spin and parity of the light meson:

$$(M^{\nu_1\cdots\nu_L})^\beta_\alpha = [\bar{\chi}_{(\mu_1\cdots\mu_{L'})}(p)]^\sigma_\rho [\mathcal{M}^{\mu_1\cdots\mu_{L'};\nu_1\cdots\nu_L}(p,v)]^{\rho\beta}_{\sigma\alpha}. \tag{11}$$

Here $\mathcal{M}^{\mu_1\cdots\mu_L;\nu_1\cdots\nu_L}$ is a Lorentz tensor with parity $(-1)^{L+L'}$, which contains all the nonperturbative physics of the matrix element, carrying also the symmetry properties of the external current Γ. In particular, this symmetry has been used in phenomenological studies of rare B decays [12,13], where loop diagrams of the standard model result in a current of $\sigma_{\mu\nu}p^\nu$ structure. However, as far as the multispinor is concerned, we can always decompose $\mathcal{M}$ into $\mathcal{M} = \mathcal{D} \otimes D$, with D being one of $\Gamma = I, \gamma_5, \gamma_\lambda, \gamma_\lambda\gamma_5, \sigma_{\lambda\gamma}$, and attribute all momentum dependence to $\mathcal{D}$. Thus we can always rewrite the overlap integral in the form

$$M^\beta_\alpha = \mathcal{D}^\beta_\alpha \mathrm{Tr}[D\bar{\chi}(p)]. \tag{12}$$

Evidently for $S = 0$ resonances, only the contribution corresponding to $D = \gamma_5$ to the overlap integral M survives, while the part $D = \gamma_\lambda$ is associated with $S = 1$. Based on this, we shall construct the most general form of M in terms of the tensors P and V, which are symmetric, transverse, and traceless.

A. s wave $\to J'$

First, let us examine decays of a heavy meson of the $(0^-,\ 1^-)$ doublet into a light resonance of higher spin. The spin-parity projector for the heavy meson is spin 0, i.e., $\chi = \gamma_5$ and $\gamma \cdot V$, so we need only find the general form for the overlap integral with no Lorentz index. For a final resonance which is a spin singlet, we remain with

$$\begin{aligned} M^{({}^1L'_{L'})}(v,p) = {} & \gamma_5 \left[G_1^{({}^1L'_{L'})}(v\cdot p) + G_2^{({}^1L'_{L'})}(v\cdot p)\ \not{p}\right] v^{\mu_1}\cdots v^{\mu_{L'}} P_{\{\mu_1\cdots\mu_{L'}\}}(p) \\ & + \gamma_5 \left[G_3^{({}^1L'_{L'})}(v\cdot p) + G_4^{({}^1L'_{L'})}(v\cdot p)\ \not{p}\right] \gamma^{\mu_1} v^{\mu_2}\cdots v^{\mu_{L'}} P_{\{\mu_1\cdots\mu_{L'}\}}(p). \end{aligned} \tag{13}$$

To explain why that is all, we note that $P_{\{\mu_1\cdots\mu_{L'}\}}$ is transverse to p and traceless so that only products of v^μ and γ^μ may be contracted with it. Hence a product involving v^μ purely gives the first two form factors in Eq. (13). Furthermore, since $\gamma^\mu\gamma^\nu = g^{\mu\nu} - i\sigma^{\mu\nu}$, and bearing in mind the symmetry property, two γ matrices are not permitted; only one γ^μ is allowed, its position being irrelevant. Hence we have the second two terms (which are absent when $L' = 0$).

We turn now to the spin triplet. For $J' = L' + 1$ resonances, we shall build up the Dirac bispinor M using $V^{L'+1}_{\{\mu\mu_1\cdots\mu_{L'}\}}(p)$ and v and Dirac matrices. Notice that $V^{L'+1}$ has the same properties as P in Eq. (13), except for the rank which does not matter. This leads us to

$$\begin{aligned} M^{({}^3L'_{L'+1})}(v,p) = {} & \left[G_1^{({}^3L'_{L'+1})}(v\cdot p) + G_2^{({}^3L'_{L'+1})}(v\cdot p)\ \not{p}\right] v^\mu v^{\mu_1}\cdots v^{\mu_{L'}} V^{L'+1}_{\{\mu\mu_1\cdots\mu_{L'}\}}(p) \\ & + \left[G_3^{({}^3L'_{L'+1})}(v\cdot p) + G_4^{({}^3L'_{L'+1})}(v\cdot p)\ \not{p}\right] \gamma^\mu v^{\mu_1}\cdots v^{\mu_{L'}} V^{L'+1}_{\{\mu\mu_1\cdots\mu_{L'}\}}(p). \end{aligned} \tag{14}$$

The analysis for $J' = L'$ resonances of the spin triplet instead goes as follows. On the face of it we can construct the Lorentz scalars

$$\gamma^\mu \sum_k \frac{p^\lambda}{m} v^{\mu_k} \epsilon_{\lambda\mu\mu_k\nu} d^{\nu\nu'} V^{L'}_{\{\mu_1\cdots\bar{k}\cdots\mu_{L'}\nu'\}}(p) \underbrace{v^{\mu_1}\cdots v^{\mu_{L'}}}_{\text{without } v^{\mu_k}},$$

$$\gamma^\mu \sum_k \frac{p^\lambda}{m} \gamma^{\mu_k} \epsilon_{\lambda\mu\mu_k\nu} d^{\nu\nu'} V^{L'}_{\{\mu_1\cdots\bar{k}\cdots\mu_{L'}\nu'\}}(p) \underbrace{v^{\mu_1}\cdots v^{\mu_{L'}}}_{\text{without } v^{\mu_k}},$$

$$\gamma^\mu \sum_k \frac{p^\lambda}{m} v^{\mu_k} \epsilon_{\lambda\mu\mu_k\nu} d^{\nu\nu'} V^{L'}_{\{\mu_1\cdots\bar{k}\cdots\mu_{L'}\nu'\}}(p) \gamma^{\mu_1} \underbrace{v^{\mu_2}\cdots v^{\mu_{L'}}}_{\text{without } v^{\mu_k}},$$

$$\gamma^\mu \sum_k \frac{p^\lambda}{m} \gamma^{\mu_k} \epsilon_{\lambda\mu\mu_k\nu} d^{\nu\nu'} V^{L'}_{\{\mu_1\cdots\bar{k}\cdots\mu_{L'}\nu'\}}(p) \gamma^{\mu_1} \underbrace{v^{\mu_2}\cdots v^{\mu_{L'}}}_{\text{without } v^{\mu_k}},$$

and terms with an additional $\not{p}$. However, using identities

$$i\gamma^\mu \epsilon_{\mu\nu\lambda\sigma} = \gamma_5(g_{\nu\lambda}\gamma_\sigma + g_{\lambda\sigma}\gamma_\nu - g_{\sigma\nu}\gamma_\lambda - \gamma_\nu\gamma_\lambda\gamma_\sigma)$$

and

$$i\sigma^{\mu\nu}\epsilon_{\mu\nu\lambda\sigma} = 2\gamma_5\sigma_{\lambda\sigma},$$

all of these structures can be reduced into four independent forms: namely,

$$M^{(^3L'_{L'})}(v,p) = \gamma_5\left[G_1^{(^3L'_{L'})}(v\cdot p) + G_2^{(^3L'_{L'})}(v\cdot p)\ \not{p}\right] V^{L'}_{\{\mu_2\cdots\mu_{L'}\nu'\}}(p)v^{\mu_2}\cdots v^{\mu_{L'}}v^{\nu'}$$
$$+\gamma_5\left[G_3^{(^3L'_{L'})}(v\cdot p) + G_4^{(^3L'_{L'})}(v\cdot p)\ \not{p}\right] V^{L'}_{\{\mu_2\cdots\mu_{L'}\nu'\}}(p)v^{\mu_2}\cdots v^{\mu_{L'}}\gamma^{\nu'}. \quad (15)$$

Apart from the change in the value of S the structure of $M(v,p)$ is exactly the same as that for resonances of spin singlet in Eq. (13). Following a similar procedure it is straightforward to work out the results for spin triplet of $J' = L' - 1$; these read

$$M^{(^3L'_{L'-1})}(v,p) = \left[G_1^{(^3L'_{L'-1})}(v\cdot p) + G_2^{(^3L'_{L'-1})}(v\cdot p)\ \not{p}\right] v^{\mu_2}\cdots v^{\mu_{L'}} V^{L'-1}_{\{\mu_2\cdots\mu_{L'}\}}(p)$$
$$+\left[G_3^{(^3L'_{L'-1})}(v\cdot p) + G_4^{(^3L'_{L'-1})}(v\cdot p)\ \not{p}\right] \gamma^{\mu_2}v^{\mu_3}\cdots v^{\mu_{L'}} V^{L'-1}_{\{\mu_2\mu_3\cdots\mu_{L'}\}}(p), \quad (16)$$

in which only the first two form factors contribute when $L' = 1$.

Compared with the general problem, discussed in Sec. II, the simplification resulting from the heavy quark approximation is twofold: in the first place the decaying 1^- meson shares the same complexity as a 0^- meson; given the state of the light degrees of freedom of the $(0^-, 1^-)$ doublet, the overlap integral is actually determined by the state of the light resonance; in the second place a set of four "universal" form factors are sufficient to parametrize all matrix elements of bilinear operators $\bar{q}\Gamma h_v$ for each $^{2S+1}L_J$ configuration. However, given a particular current, it is possible for some of them to be absent.

B. p wave $\to J'$

Here it will prove convenient to mix the $^1P_1$ and $^3P_1$ states of the heavy decaying meson to track the spin of the constituent light degrees of freedom. The way to do this has been delineated in Ref. [4], and in our case we state the decomposition much more explicitly. Given our spin-parity projectors for the p wave,

$$\chi_\nu^{(^1P_1)} = \gamma_5\phi^5_\nu,$$
$$\chi_\nu^{(^3P_2)} = \phi_{\nu\lambda}\gamma^\lambda,$$
$$\chi_\nu^{(^3P_1)} = -\gamma_5[\phi_\nu - (\phi_\lambda\gamma^\lambda)(\gamma_\nu - v_\nu)],$$
$$\chi_\nu^{(^3P_0)} = -(\gamma_\nu - v_\nu),$$

we arrange them into a pair of doublets corresponding with two distinct states of the total light angular momentum. Thus the doublet of higher spin $(1^+, 2^+)$ is

$$\chi_\nu^{(\Uparrow)} = \begin{pmatrix} -\phi_{\nu\lambda}\gamma^\lambda \\ \gamma_5\left[\varphi_\nu - \frac{1}{3}(\varphi_\lambda\gamma^\lambda)(\gamma_\nu - v_\nu)\right] \end{pmatrix},$$

and the lower spin doublet $(0^+, 1^+)$ is

$$\chi_\nu^{(\Downarrow)} = \begin{pmatrix} \gamma_5(\frac{1}{3}\varphi_\lambda - \frac{1}{\sqrt{2}}\phi_\lambda)\gamma^\lambda \\ \frac{1}{\sqrt{3}} \end{pmatrix}(\gamma_\nu - v_\nu),$$

where $\varphi_\nu \equiv \phi^5_\nu + \frac{1}{\sqrt{2}}\phi_\nu$ and the constraint, $\gamma^\nu\chi_\nu^{(\Uparrow)} = 0$ applies to the components of $(1^+, 2^+)$ doublets. In fact when we take the trace according to Eq. (10) with the spin-parity projector of $(0^+, 1^+)$, the vector factor $(\gamma_\nu - v_\nu)$ in the $\chi_\nu^{(\Downarrow)}$ can be absorbed into $M^\nu(v,p)$, leaving us a scalar matrix in Dirac space. The ensuing analysis is thus exactly the same as the $(0^-, 1^-)$ doublet, and we do not repeat it here. With respect to the projector for the $\chi_\nu^{(\Uparrow)}$ doublet, we construct

$$(M^\nu)^\beta_\alpha = p^\nu[\bar{\chi}(p)]^\sigma_\rho\ [\mathcal{M}_0(p,v)]^{\rho\beta}_{\sigma\alpha},$$

for $L' = 0$ and like before, the scalar $\mathcal{M}_0$ can be expressed in terms of two unknown functions. When $L' = 1$, we have

$$(M^\nu)^\beta_\alpha = p^\nu[\bar{\chi}_\mu(p)]^\sigma_\rho\ [\mathcal{M}_1^\mu(p,v)]^{\rho\beta}_{\sigma\alpha},$$

which formally has the the same structure as the M in Eq. (13). In addition to those four form factors, we need two more to describe

$$(M^\nu)^\beta_\alpha = [\bar{\chi}^\nu(p)]^\sigma_\rho\ [\mathcal{M}_1(p,v)]^{\rho\beta}_{\sigma\alpha},$$

making a total of six. A similar analysis for $L' \geq 2$ produces the forms

$$(M^\nu)^\beta_\alpha = p^\nu[\bar{\chi}_{(\mu_1\cdots\mu_{L'})}(p)]^\sigma_\rho\ [\mathcal{M}^{\mu_1\cdots\mu_{L'}}(p,v)]^{\rho\beta}_{\sigma\alpha},$$

$$(M^\nu)^\beta_\alpha = g^{\mu_1\nu}[\bar{\chi}_{(\mu_1\mu_2\cdots\mu_{L'})}(p)]^\sigma_\rho\ [\mathcal{M}^{\mu_2\cdots\mu_{L'}}(p,v)]^{\rho\beta}_{\sigma\alpha},$$

to each of which belong four form factors. Therefore we finish up with eight form factors. (As discussed before, there are four unknown functions for the first M^ν above and another four for the second when $L' > 1$, but only two when $L' = 1$. However, if $L' = 0$, only the two form factors from the first M^ν contribute.)

In summary, we have *two* universal form factors for the transition from heavy p wave to light s wave, *six* form factors to light p wave, and *eight* to light d wave or states of higher spin.

C. L wave $\to J'$

We are now in a position to complete our analysis for the general case. We rearrange the four states ${}^3L_{L+1}$, ${}^3L_L$, ${}^3L_{L-1}$, and ${}^1L_L$ into a pair of doublets:

$$\chi^{(\Uparrow)}_{(\nu_1\cdots\nu_L)} = \begin{pmatrix} -\phi^{L+1}_{\{\nu_1\cdots\nu_L\lambda\}}\gamma^\lambda \\ \gamma_5\left[\varphi^L_{\{\nu_1\cdots\nu_L\}} - \frac{1}{2L+1}\sum_k \varphi^L_{\{\nu_1\cdots\bar{k}\cdots\nu_L\lambda\}}\gamma^\lambda(\gamma_{\nu_k}-v_{\nu_k})\right] \end{pmatrix},$$

and

$$\chi^{(\Downarrow)}_{(\nu_1\cdots\nu_L)} = \sum_k \begin{pmatrix} \gamma_5\left(\frac{1}{2L+1}\varphi^L_{\{\nu_1\cdots\bar{k}\cdots\nu_L\lambda\}} - \frac{1}{\sqrt{2L}}\phi^L_{\{\nu_1\cdots\bar{k}\cdots\nu_L\lambda\}}\right)\gamma^\lambda \\ \frac{1}{L}\frac{\sqrt{2L-1}}{\sqrt{2L+1}}\left[\phi^{L-1}_{\{\nu_1\cdots\bar{k}\cdots\nu_L\}} - \frac{1}{2L-1}\sum_l \phi^{L-1}_{\{\nu_1\cdots\bar{k}\bar{l}\cdots\nu_L\lambda\}}\gamma^\lambda(\gamma_{\nu_l}+v_{\nu_l})\right] \end{pmatrix}(\gamma_{\nu_k}-v_{\nu_k}),$$

with

$$\varphi_{\{\nu_1\cdots\nu_L\}} = \phi^5_{\{\nu_1\cdots\nu_L\}} + \frac{1}{\sqrt{2}}\phi_{\{\nu_1\cdots\nu_L\}}.$$

(The "mixing angle" is uniform for all L.) For the higher spin doublet $(L, L+1)$ we have the following $(L'+1)$-fold structures, when $L' \le L$:

$$(M^{\nu_1\cdots\nu_L})^\beta_\alpha = p^{\nu_1}\cdots p^{\nu_L}[\bar{\chi}_{(\mu_1\cdots\mu_{L'})}(p)]^\sigma_\rho\,[\mathcal{M}^{\mu_1\cdots\mu_{L'}}(p,v)]^{\rho\beta}_{\sigma\alpha},$$

$$(M^{\nu_1\cdots\nu_L})^\beta_\alpha = g^{\nu_1\mu_1}p^{\nu_2}\cdots p^{\nu_L}[\bar{\chi}_{(\mu_1\cdots\mu_{L'})}(p)]^\sigma_\rho\,[\mathcal{M}^{\mu_2\cdots\mu_{L'}}(p,v)]^{\rho\beta}_{\sigma\alpha}r,$$

and

$$(M^{\nu_1\cdots\nu_L})^\beta_\alpha = g^{\nu_1\mu_1}\cdots g^{\nu_{L'}\mu_{L'}}p^{\nu_{L'+1}}\cdots p^{\nu_L}[\bar{\chi}_{(\mu_1\cdots\mu_{L'})}(p)]^\sigma_\rho\,[\mathcal{M}(p,v)]^{\rho\beta}_{\sigma\alpha}.$$

As before, each of these objects corresponds to four form factors, except the last one which just has two form factors. Altogether then there are $4L'+2$ form factors. On the other hand, if $L' > L$ it is easy to work out that the number of the form factors is $4(L+1)$. Turning next to the doublet of lower spin $(L-1, L)$, we can surely absorb the factors such as $(\gamma_{\nu_k} - v_{\nu_k})$ into the overlap integral and thereby consider the case of a Lorentz tensor of rank $L-1$. This leads us to the conclusion that the number of independent form factors is $4L'+2$ if $L' \le L-1$, but $4L$ if $L' > L-1$.

The situation for heavy to heavy transitions is simpler. Since the spin-parity projector for the final resonance is also a Rarita-Schwinger object ($\gamma^\mu P_{\{\mu\cdots\}} = 0$), form factors $G_3^{({}^{2S+1}L'_{J'})}$ and $G_4^{({}^{2S+1}L'_{J'})}$ do not contribute, and $G_1^{({}^{2S+1}L'_{J'})}$ and $G_2^{({}^{2S+1}L'_{J'})}$ collapse into one form factor for an on-shell heavy quark ($\not{p} = M'$).

IV. EXCLUSIVE DECAY RATES

Before applying our formalism to specific heavy to light matrix elements, we must describe to what extent the formal heavy quark symmetry relations between form factors lead to simplifications phenomenologically in weak decays. The applicable region of heavy quark symmetries in decays is determined by the average momentum transfer Q_l of the light degrees of freedom. It is estimated heuristically by [14] that

$$Q_l^2 \simeq \frac{\Lambda}{m}\frac{\Lambda}{M}(q^2_{\max} - q^2); \qquad q^2_{\max} = (M-m)^2.$$

Here q is the momentum transfer of mesons, M and m are the masses of initial heavy and final light mesons, respectively, and $\Lambda \sim \Lambda_{\rm QCD}$ sets the scale for the light degrees of freedom. The relations developed in Sec. III will certainly hold in the region where $q^2 \simeq q^2_{\max}$ or $Q_l \ll M$, but actually, the suppression factor Λ/M allows us to continue q away from the maximum by about 1.0 GeV. In semileptonic and rare dileptonic B decays, this region may correspond to high energy lepton pairs. From a phenomenological point of view, leptons of large energy in B decays are particularly useful for extracting information about the underlying physics. By looking at the $l\bar{\nu}_l$ of large q in semileptonic decays, we can separate of the $b \to u$ decays from the $b \to c$ decays [15] and likewise by measuring $l\bar{l}$ pairs of q larger than the masses of J/ψ and ψ' in rare B decays ($b \to s$) we obtain signals of short-distance physics dominated by top quark contributions [16]. We shall present the invariant mass spectrum of lepton pairs in the first part of this section.

In the second part, we shall examine two-body heavy meson decays such as those into K resonance plus charmonium or photon, where q^2 is fixed kinematically in each process. Heavy quark symmetries only make approximate sense in such decays where the value of $v\cdot p$ (v the velocity of the M meson and p the momentum of the m meson) does not change much between processes and is not too large compared with m. However, it has been argued that phenomenological models for soft processes still work when Q_l is about 1.0 GeV [14,17]. In this case, relations between form factors may remain roughly applicable. The attempt of applying them to rare radiative B decays and nonleptonic decays with the assumption of factorization is also made below. But we ought to remember that rare radiative decays experience a relatively large recoil and factorization suffers from uncertainties due to nonperturbative QCD. Therefore we do not expect those decays are entirely suitable cases for applying heavy quark symmetry.

We shall now evaluate rates for various exclusive processes including semileptonic, rare dilepton, rare radiative, and nonleptonic decays. We shall restrict ourselves to pseudoscalar decays into the light resonances of spin J. Thus we make the substitutions $J\to 0$ and $J'\to J$ in the earlier formulas. As expected, all of these exclusive rates are written formally in terms of four form factors. Here are the results and relevant discussions case by case.

A. Semileptonic B decays into light mesonic resonances

The amplitude for $\bar{B}\to X_q l\bar{\nu}_l$ contains hadronic and leptonic currents

$$\frac{G_F}{\sqrt{2}}V_{qb}L_\mu\langle X_q(p)|\bar{q}\gamma^\mu(1-\gamma_5)b|\bar{B}(v)\rangle, \tag{17}$$

in which $L_\mu=\bar{u}_l\gamma_\mu(1-\gamma_5)v_{\nu_l}$. The Lorentz-invariant form factors in Eq. (2) for $V-A$ currents are related to our G_i by

$$\begin{aligned} a_+^{(J)} &= \frac{1}{M^{J-\frac{1}{2}}}\left(\frac{G_1^{(^{2S+1}L_J)}}{M}\pm G_2^{(^{2S+1}L_J)}-G_4^{(^{2S+1}L_J)}\right),\\ a_-^{(J)} &= \frac{1}{M^{J-\frac{1}{2}}}\left(\frac{G_1^{(^{2S+1}L_J)}}{M}\mp G_2^{(^{2S+1}L_J)}+G_4^{(^{2S+1}L_J)}\right),\\ f^{(J)} &= \frac{2}{M^{J-\frac{3}{2}}}[\pm G_3^{(^{2S+1}L_J)}+(v\cdot p)G_4^{(^{2S+1}L_J)}],\\ g^{(J)} &= -\frac{2}{M^{J-\frac{1}{2}}}G_4^{(^{2S+1}L_J)}, \end{aligned} \tag{18}$$

where the upper sign applies to $J=L$ and the lower one to $J=L\pm 1$. As our G functions do *not* scale as the heavy mass when $v\cdot p$ is close to the mass of light mesons, we can easily read off the scale of form factors $a_\pm^{(J)}$, $f^{(J)}$, and $g^{(J)}$. With ground-state mesons in the final state for example, one deduces [8]

$$\begin{aligned} a_+^{(0^-)}+a_-^{(0^-)} &= \frac{2}{\sqrt{M}}G_1^{(^1S_1)},\\ a_+^{(0^-)}-a_-^{(0^-)} &= 2\sqrt{M}G_2^{(^1S_1)}, \end{aligned} \tag{19}$$

$$\begin{aligned} a_+^{(1^-)}+a_-^{(1^-)} &= \frac{2}{\sqrt{M^3}}G_1^{(^3S_1)},\\ a_+^{(1^-)}-a_-^{(1^-)} &= -\frac{2}{\sqrt{M}}[G_2^{(^3S_1)}+G_4^{(^3S_1)}], \end{aligned} \tag{20}$$

$$\begin{aligned} f^{(1^-)} &= 2\sqrt{M}[-G_3^{(^3S_1)}+(v\cdot p)G_4^{(^3S_1)}],\\ g^{(1^-)} &= -\frac{2}{\sqrt{M}}G_4^{(^3S_1)}. \end{aligned} \tag{21}$$

(Above, a logarithmic dependence of the form factor on the heavy mass, arising from the anomalous scaling of quark currents in the effective theory, is expected. Note also that ratios of these combinations have scaling properties which are independent of J.) However, the relative contribution of these form factors to the decay rates will not necessarily follow such scaling behavior when kinematical factors are taken into account. Making use of the general formula for polarization sums in the Appendix, we arrive at the differential distribution of the decay rate:

$$\begin{aligned} \frac{d\Gamma}{dq^2d\Omega_l d\Omega'} &= \frac{2^J(J!)^2}{(4\pi)^5(2J)!}\left(\frac{G_F}{\sqrt{2}}\right)^2|V_{qb}|^2\left(\frac{M_i}{M}\right)\left(\frac{q^2}{M^2}\right)\left(\frac{\Delta}{M_i}\right)^{2J-1}\\ &\times\left\{\frac{8}{q^2M_i^2}[\Delta^2-(k\cdot p)^2]\left|\Delta\, a_+^{(J)}+\frac{p\cdot q}{2\Delta}f^{(J)}\right|^2\right.\\ &\left.+\frac{J+1}{J}\left[[\Delta^2+(k\cdot p)^2]\left(\left|\frac{f^{(J)}}{\Delta}\right|^2+|g^{(J)}|^2\right)+4(k\cdot p)\mathrm{Re}(f^{(J)}g^{(J)*})\right]\right\}, \end{aligned} \tag{22}$$

where q^2 is the squared invariant mass of the $(l\bar{\nu}_l)$ pair, Ω_l the solid angle of the charged lepton in the $l\bar{\nu}_l$ frame in which $\vec{q}=0$, Ω' the angle of the final meson in the rest frame of the initial meson, and $\Delta^2=M^2[(v\cdot p)^2-M_i^2]$. k is the relative momentum of the $(l\bar{\nu}_l)$. The ratio $k\cdot p/\Delta$ turns out to be $\cos\theta_l$ in the $(l\bar{\nu}_l)$ frame in which $\pi-\theta_l$ is the polar angle of the charged lepton with respect to the direction of motion of the decaying meson. The form factor $a_-^{(J)}$ in Eq. (2) makes no contribution in the limit of massless leptons. We notice that the contribution of $a_+^{(J)}$ is suppressed near the zero recoil point, $v\cdot p=M_i$, where the heavy quark approximation is supposed to work well; the differential distribution is then dominated by two form factors $f^{(J)}$ and $g^{(J)}$, or equivalently by $G_3^{(^{2S+1}L_J)}$ and $G_4^{(^{2S+1}L_J)}$.

As these form factors depend only on q^2, we integrate over all angles to obtain the invariant mass distribution of the $l\bar{\nu}_l$:

$$\frac{d\Gamma}{dq^2} = \frac{2^J(J!)^2}{48\pi^3(2J)!}\left(\frac{G_F}{\sqrt{2}}\right)^2 |V_{qb}|^2 \left(\frac{M_i}{M}\right)^3 q^2 \left(\frac{\Delta}{M_i}\right)^{2J+1} \times\left[\frac{4}{q^2M_i^2}\left|\Delta\, a_+^{(J)} + \frac{p\cdot q}{2\Delta} f^{(J)}\right|^2 + \frac{J+1}{J}\left(\left|\frac{f^{(J)}}{\Delta}\right|^2 + \left|g^{(J)}\right|^2\right)\right]. \tag{23}$$

It is worthwhile presenting the differential forward-backward asymmetry in θ_l of the charged lepton, which is defined by

$$\mathcal{A}^{\rm FB}(q^2) = \frac{\int_0^1 \, d\Gamma(\cos\theta_l) - \int_{-1}^0 \, d\Gamma(\cos\theta_l)}{\int_0^1 \, d\Gamma(\cos\theta_l) + \int_{-1}^0 \, d\Gamma(\cos\theta_l)}.$$

The angular integral in the numerator picks out the $k\cdot p$ term in Eq. (19) and $\Delta a_+^{(J)}$ in the normalization can be ignored when $\Delta \to 0$; this leaves

$$\mathcal{A}^{\rm FB}|_{v\cdot p\to M_i} = \frac{3}{2}\frac{{\rm Re}\left[\frac{f^{(J)}}{\Delta}g^{(J)*}\right]}{\left(1+\frac{J}{J+1}\frac{(p\cdot q)^2}{q^2M_i^2}\right)\left|\frac{f^{(J)}}{\Delta}\right|^2 + |g^{(J)}|^2}. \tag{24}$$

Supposing the charmed quark is very heavy compared to the momentum of light degrees of freedom, we can apply all of the above results to semileptonic D decays. In this case we may make use of exclusive process of the Cabibbo favored $c \to sl\bar{\nu}$ decay to determine the values of the desired form factors. First of all, we notice that a subset of our $G_3^{(^{2S+1}L_J)}$ and $G_4^{(^{2S+1}L_J)}$ can be expressed in terms of $f^{(J)}$ and $g^{(J)}$, which may be found by fitting the angular and q^2 distribution of the data. Further, a linear combination of $G_1^{(^{2S+1}L_J)}$ and $G_2^{(^{2S+1}L_J)}$ can be extracted if $a_+^{(J)}$ is accessed experimentally. Now in order to determine $a_-^{(J)}$ experimentally one must include the lepton masses, whose effect is unfortunately suppressed by a factor of m_l^2/q^2 and is thus difficult to measure in the e and μ channels, especially at relatively large q^2. The contribution of the μ mass, for instance, has been found to be less than 5% in semileptonic $D \to K(K^*)$ decays [18]. On the other hand, it is impossible to measure $a_-^{(J)}$ in the τ channel of D decays because of the phase space. Evidently, this missing form factor makes it hard for us to separate $G_1^{(^{2S+1}L_J)}$ from $G_2^{(^{2S+1}L_J)}$ as far as semileptonic D decays are concerned. (But we are optimistic that these form factors may be separated in the τ channel of B meson decays [19].)

Experimentally, individual form factors have been studied extensively during the past few years for semileptonic $D \to K^*$, $D_s \to \phi$, $D \to K$, and $D_s \to (\eta+\eta')$ decays in the e and μ channels [20–22]. The form factor $a_+^{(0^-)}$ has been measured in $D \to K$ decays and the average value of CLEO, E687, and E691 is $a_+^{DK}(0) = 0.76 \pm 0.02$ [23]. Here we shall examine $D \to K^*$ decays in some detail and extract the corresponding values of $G_3^{(^3S_1)}$ and $G_4^{(^3S_1)}$. At fixed target experiments [21], three form factors A_1, A_2, and V (proportional to $f^{(1^-)}$, $a_+^{(1^-)}$, and $g^{(1^-)}$, respectively) have been determined by fitting to the angular and q^2 distribution of the data. As far as A_2 is concerned, there appears to be considerable disagreement among experimental results, but as this form factor is kinematically suppressed in the regime close to zero point, it is not vital in the following discussion. Using the measured form factors A_1 and V at $q^2 = 0$ of Ref. [21], for which E691, E687, and CLEO groups are in agreement and the assumption of the nearest pole dominance for the q^2 dependence of form factors, which is used by all of these groups, we evaluate the averaged form factors at $q^2_{\rm max}$, $A_1 = 0.61 \pm 0.04$ and $V = 1.22 \pm 0.20$, corresponding to $f_D^{(1^-)} = -(1.69\pm0.11)$ GeV and $g_D^{(1^-)} = 0.88 \pm 0.15$ GeV^{-1}. Since the range of q^2 in this decay is only about 1 GeV^2, small compared to heavy pole masses, $M(D_s^*) = 2.11$ GeV and $M(D_{s1}) = 2.54$ GeV, the resulting form factors are not sensitive to the assumed q^2 dependence. (This variation in q^2 becomes more prominent in the case of higher K resonances.) These numerical results tell us that $G_{3D}^{(^3S_1)} = 0.081 \pm 0.098$ GeV$^{1/2}$ and $G_{4D}^{(^3S_1)} = -(0.60 \pm 0.10)$ GeV$^{-1/2}$ at $v\cdot p = M_{K^*}$. Translating them into form factors of B decays by multiplying the logarithmic dependence on the heavy quark mass $[\alpha_s(m_b)/\alpha_s(m_c)]^{-6/25}$, we have that $G_{3B}^{(^3S_1)} = 0.089 \pm 0.108$ GeV$^{1/2}$ and $G_{4B}^{(^3S_1)} = -(0.66 \pm 0.11)$ GeV$^{-1/2}$, where $\alpha_s(m_b) = 0.19$ and $\alpha_s(m_c) = 0.29$, [24] have been used. Correspondingly, we find $f_B^{(1^-)} = -(3.13 \pm 0.18)$ GeV and $g_B^{(1^-)} = 0.58 \pm 0.10$ GeV^{-1}. (The masses of mesons are $M_{K^*} = 0.892$ GeV, $M_D = 1.87$ GeV, and $M_B = 5.28$ GeV.) Here the magnitude of $G_3^{(^3S_1)}$ turns out to be small compared to $G_4^{(^3S_1)}$. We may rewrite Eq. (21) as

$$f^{(1^-)} = 2M_{K^*}\sqrt{M}(1+\delta_G)G_4^{(^3S_1)}, \quad g^{(1^-)} = -\frac{2}{\sqrt{M}}G_4^{(^3S_1)}, \tag{25}$$

in which we define a dimensionless parameter,

$$\delta_G = -\left.\frac{G_3^{(^3S_1)}}{M_{K^*}G_4^{(^3S_1)}}\right|_{v\cdot p\to M_{K^*}}, \tag{26}$$

which takes a value of $\delta_G = 0.14 \pm 0.18$, being effectively negligible at the present level of the measurement. Therefore the data of form factors in semileptonic $D \to K^*$ decays imply that the heavy-flavor-independent δ_G is potentially zero. This suggests that form factors of the end point for the K^* channel may be well approximated by

$$\frac{f^{(1^-)}}{M\ g^{(1^-)}} = -M_{K^*}, \tag{27}$$

upon setting $\delta_G = 0$. The preliminary measurements, but with fairly big errors [22], of $D_s^+ \to \phi\mu^+\nu_\mu$ also show evidence of this relation. In the quark model of Altomari and Wolfenstein (also Gilman and Singleton) [25,26], we find

$$\delta_G^{\text{AW-GS}} = \frac{2m_s}{M_{K^*}} - 1,$$

which vanishes when $m_s = M_{K^*}/2$, a good approximation of $m_s = 0.45$ GeV used in Ref. [26]. On the other hand, we should point out that the δ_G parameter for $B \to D^*$ decays equals one in the on-shell limit of heavy quark effective theory [27].

As we shall soon see, such form factors extracted from D decays can be used directly in rare dilepton $\bar{B} \to K^*$ decays. Indeed since ρ and K^* are in the same SU(3) octet, the above form factors will help us in determining V_{ub} in $\bar{B} \to \rho$ when corrections of SU(3) breaking are taken into account. In the same manner, but more straightforwardly, form factors at $v \cdot p = m_\rho$ determined in Cabibbo suppressed $D \to \rho$ decays will help to fix V_{ub}.

B. Rare dilepton *B* decays into light meson resonances

The effective Hamiltonian relevant to flavor-changing one-loop processes $b \to s l\bar{l}$ is given by [29]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}}\left(\frac{\alpha}{4\pi s_W^2}\right)[\bar{s}\Gamma_\mu^A b\ \bar{l}\gamma^\mu(1-\gamma_5)l + \bar{s}\Gamma_\mu^B b\ \bar{l}\gamma^\mu(1+\gamma_5)l], \tag{28}$$

with effective vertices

$$\Gamma_\mu^{A(B)} = A(B)\gamma_\mu(1-\gamma_5) - im_b s_W^2 F_2 \sigma_{\mu\nu}q^\nu(1+\gamma_5)/q^2.$$

In this Hamiltonian, heavy particles, $W^\pm$ bosons, and the top quark are integrated out and their masses together with QCD corrections are absorbed into coefficient functions,

$$A(B,F_2) = \sum_{q=u,c,t} V_{qs}^* V_{qb} A_q(B_q, F_2^q), \tag{29}$$

which are dominated in the standard model by the top quark contributions except for the long distance effect which proceeds mainly through CKM favored $c\bar{c}$ intermediate vector meson states. Here V_{qs} and V_{qb} are elements of CKM matrix and $A_q(B_q, F_2^q)$ are given in Ref. [29]. The process $b \to s\bar{\nu}\bar{\nu}$ is induced by pure $V-A$ currents and only the A term in the above Hamiltonian survives. The effective quark current $\bar{s}\Gamma_\mu^{A(B)}b$ in question has two different Dirac structures, but is still a spin-1 object; so its hadronic matrix element must assume the form of Eq. (2). We shall reserve form factors $a_\pm, f$, and g for $V-A$ currents and express the additional tensor current in terms of extra $\tilde{a}_\pm, \tilde{f}$, and $\tilde{g}$. According to Eq. (10) and Eq. (14), the trace with $-i\sigma_{\mu\nu}q^\nu(1+\gamma_5)$ gives

$$\begin{aligned}
\tilde{a}_+^{(J)} &= \frac{1}{M^{J-\frac{1}{2}}}\left(\pm\frac{G_2^{(^{2S+1}L_J)}}{M} \mp \frac{G_3^{(^{2S+1}L_J)}}{q^2} - \frac{M}{q^2}G_4^{(^{2S+1}L_J)}\right),\\
\tilde{a}_-^{(J)} &= \frac{1}{M^{J-\frac{1}{2}}}\left(\mp\frac{M^2-M_i^2}{q^2}\frac{G_2^{(^{2S+1}L_J)}}{M} \mp \frac{G_3^{(^{2S+1}L_J)}}{q^2} - \frac{M}{q^2}G_4^{(^{2S+1}L_J)}\right),\\
\tilde{f}^{(J)} &= \frac{1}{M^{J-\frac{3}{2}}}\left(\pm\frac{M^2-M_i^2+q^2}{q^2}\frac{G_3^{(^{2S+1}L_J)}}{M} + \frac{M^2-M_i^2-q^2}{q^2}G_4^{(^{2S+1}L_J)}\right),\\
\tilde{g}^{(J)} &= \frac{2}{M^{J-\frac{3}{2}}}\left(\mp\frac{G_3^{(^{2S+1}L_J)}}{M} - G_4^{(^{2S+1}L_J)}\right)\frac{1}{q^2}.
\end{aligned} \tag{30}$$

Conservation of the current produces the constraint

$$(M^2 - M_i^2)\tilde{a}_+^{(J)} + q^2\tilde{a}_-^{(J)} + \tilde{f}^{(J)} = 0.$$

Therefore there is a freedom to eliminate one of the four form factors, and this is reflected by disappearance of G_1 in Eq. (30). To be consistent with the semileptonic decays, we shall use $\tilde{a}_+, \tilde{f}$, and $\tilde{g}$ in this section. From Eq. (30), we can easily read off their scaling behavior in the heavy mass near the zero recoil region, remembering that $G^{(^{2S+1}L_J)}$ scales only with the light mass. For the case of B decays to the ground state K meson in the final state, we find

$$s = \tilde{a}_+^{(0^-)} = \frac{1}{\sqrt{M}}G_2^{(^1S_1)}, \tag{31}$$

$$\begin{aligned}
h &= \frac{q^2(\tilde{a}_+^{(1^-)} + \tilde{a}_-^{(1^-)})}{M^2 - M_{K^*}^2 + q^2} + \frac{\tilde{f}^{(1^-)}}{M^2 - M_{K^*}^2 + q^2} + \frac{1}{2}\tilde{g}^{(1^-)}\\
&= -\frac{1}{\sqrt{M^3}}G_2^{(^3S_1)},
\end{aligned} \tag{32}$$

$$\begin{aligned}
g_+ + g_- &= -\tilde{f}^{(1^-)} - \frac{1}{2}(M^2 - M_{K^*}^2 - q^2)\tilde{g}^{(1^-)} = \frac{2}{\sqrt{M}}G_3^{(^3S_1)},\\
g_+ - g_- &= \tilde{f}^{(1^-)} + \frac{1}{2}(M^2 - M_{K^*}^2 + q^2)\tilde{g}^{(1^-)}\\
&= -2\sqrt{M}G_4^{(^3S_1)},
\end{aligned} \tag{33}$$

agreeing with that of Ref. [8](where form factors s, h, and $g_\pm$ were used [30]).

As we have pointed out Eqs. (18) and (30) imply a connection between exclusive semileptonic and rare dileptonic decays of the heavy meson, through flavor independent $G^{(^{2S+1}L_J)}$ functions. At the endpoint, using the δ_G parameter of Eq. (26), we rewrite form factors of the $B \to K^*$ decay as

$$\tilde{f}^{(1^-)} = \frac{2M_{K^*}\sqrt{M}}{M - M_{K^*}}(1+\delta_G)G_4^{(^3S_1)},$$
$$\tilde{g}^{(1^-)} = -\frac{2\sqrt{M}}{(M-M_{K^*})^2}\left(1+\frac{M_{K^*}}{M}\delta_G\right)G_4^{(^3S_1)}. \tag{34}$$

Recalling similar relations for $V - A$ form factors in Eq. (25), we may eliminate $G_4^{(^3S_1)}$ and obtain

$$\tilde{f}^{(1^-)} = \frac{1}{M - M_{K^*}}\, f^{(1^-)},$$
$$\tilde{g}^{(1^-)} = \frac{M}{(M-M_{K^*})^2}(1+r\delta_G)\, g^{(1^-)}. \tag{35}$$

Effectively they decouple as the product of $r = M_{K^*}/M$, and δ_G is quite small.

In the last subsection, we determined form factors $f_B^{(1^-)}$ and $g_B^{(1^-)}$, as well as $G_{3B}^{(^3S_1)}$ and $G_{4B}^{(^3S_1)}$. Now we are able to evaluate additional ones via Eq. (35) at $v \cdot p = M_{K^*}$; the results turn out to be $\tilde{f}_B^{(1^-)} = -(0.72 \pm 0.14)$ and $\tilde{g}_B^{(1^-)} = 0.163 \pm 0.024$ GeV^{-2}, respectively. As explained previously, we are not able to determine $G_{2B}^{(^{2S+1}L_J)}$, or $\tilde{a}_+^{(J)}$, in this way because of the missing form factor $a_-^{(J)}$. But the two kinds of form factors entering semileptonic D decays will allow us to predict rare decay distributions close to the zero recoil point. To see this, let us examine the formula of the mass spectrum and differential forward-backward charge asymmetry.

The dilepton invariant mass spectrum is given by

$$\frac{d\Gamma}{dq^2} = \frac{2^J(J!)^2}{48\pi^3(2J)!}\left(\frac{G_F}{\sqrt{2}}\right)^2\left(\frac{\alpha}{4\pi s_W^2}\right)^2\left(\frac{M_i}{M}\right)^3 q^2\left(\frac{\Delta}{M_i}\right)^{2J+1}$$
$$\times\left[\frac{4}{q^2M_i^2}\left|\Delta\left(Aa_+^{(J)} + m_b s_W^2 F_2 \tilde{a}_+^{(J)}\right) + \frac{p\cdot q}{2\Delta}\left(Af^{(J)} + m_b s_W^2 F_2 \tilde{f}^{(J)}\right)\right|^2\right.$$
$$\left.+\frac{J+1}{J}\left(\left|\frac{1}{\Delta}(Af^{(J)} + m_b s_W^2 F_2 \tilde{f}^{(J)})\right|^2 + \left|Ag^{(J)} + m_b s_W^2 F_2 \tilde{g}^{(J)}\right|^2\right) + A \longleftrightarrow B\right]. \tag{36}$$

Just as with semileptonic processes the contributions of $a_+^{(J)}$ and $\tilde{a}_+^{(J)}$ are suppressed near the zero recoil point; consequently, the *same* two form factors $G_3^{(^{2S+1}L_J)}$ and $G_4^{(^{2S+1}L_J)}$ via $f^{(J)}, g^{(J)}, \tilde{f}^{(J)}$, and $\tilde{g}^{(J)}$ determine the end point spectrum. Likewise, the forward-backward charge asymmetry of dilepton production is

$$\mathcal{A}^{\rm FB}|_{v\cdot p\to M_i} = \frac{3}{2}\,\frac{\mathrm{Re}\left[\frac{f^{(J)}}{\Delta}g^{(J)*}|A|^2 + m_b s_W^2 F_2\left(\frac{f^{(J)*}}{\Delta}\tilde{g}^{(J)} + \frac{\tilde{f}^{(J)}}{\Delta}g^{(J)*}\right)A^* - A \longleftrightarrow B\right]}{\left(1+\frac{J}{J+1}\frac{(p\cdot q)^2}{q^2M_i^2}\right)\left|\frac{Af^{(J)} + m_b s_W^2 F_2 \tilde{f}^{(J)}}{\Delta}\right|^2 + |Ag^{(J)} + m_b s_W^2 F_2 \tilde{g}^{(J)})|^2 + A \longleftrightarrow B}, \tag{37}$$

and has the potential to be fairly large in the standard model, following the argument of the authors of Ref. [31]. For $m_t/M_W \geq 2$, as suggested by the Collider Detector at Fermilab (CDF) value of m_t, the contribution of the Z-exchange diagrams becomes important and the coefficient of the left-handed leptonic current grows as m_t^2, leading to a substantial asymmetry. (The asymmetry in inclusive $B \to X_s l\bar{l}$ processes has also been investigated in great detail in Ref. [32].) Our scheme discussed here permits the asymmetry in *exclusive* B decays into K resonances to be studied in a model independent way.

As far as the $B \to K^* l^+ l^-$ channel is concerned, the δ_G parameter defined in Eq. (26) contains the whole dependence on hadronic form factors of the forward-backward asymmetry at the end point,

$$\left.\frac{\mathcal{A}^{\rm FB}(B\to K^*\, l^+l^-)}{\sqrt{\left(\frac{v\cdot p}{M_{K^*}}\right)^2 - 1}}\right|_{v\cdot p\to M_{K^*}} = -\frac{\frac{1}{1+\delta_G}\left[|\,(1-r)A + s_W^2F_2\,|^2 - |\,(1-r)B + s_W^2F_2\,|^2\right] + r\mathrm{Re}[s_W^2F_2\,(A-B)^*]}{|\,(1-r)A + s_W^2F_2\,|^2 + |\,(1-r)B + s_W^2F_2\,|^2}. \tag{38}$$

Actually there is a contribution independent of δ_G, but it is suppressed by $r = M_{K^*}/M$ (where we use the limit of $m_b = M$); as a result, the asymmetry is mainly proportional to $\frac{1}{1+\delta_G}$. Thus a small δ_G parameter will enhance the forward-backward asymmetry. Considering that δ_G seems negligible according to present measurements, we expect there will be a considerable asymmetry at the end point of the $B \to K^*\, l^+l^-$ channel. Given the input of the δ_G determined in $D \to K^*$ decays and coefficients $A_t - B_t$, B_t, and F_2^t evaluated at $m_t = 150$, 174 (of the CDF group), and 200 GeV, and using the formulas in Ref. [29] including QCD corrections, we obtain the results

$$\left.\frac{\mathcal{A}^{\rm FB}(B \to K^*\, l^+l^-)}{\sqrt{\left(\frac{v\cdot p}{M_{K^*}}\right)^2 - 1}}\right|_{v\cdot p \to M_{K^*}} = -(87 \pm 13,\ 89 \pm 12,\ 88 \pm 14)\%,$$

respectively. They are not sensitive to the variation of the top mass in this range. We anticipate that planned experiments of rare dileptonic B decays will provide detailed measurements enabling us to test these predictions.

We conclude the subsection with some comments.

(1) Sometimes the spatial components of the hadronic current are parametrized alternatively in the literature in terms of one longitudinal and two transverse helicity amplitudes in the massless lepton limit [18,26]. The transverse helicity amplitudes are linear combinations of our functions G_3 and G_4. The numerator of the formulas for the forward-backward asymmetry in both semileptonic and rare dilepton processes depends only on transverse amplitudes, or G_3 and G_4.

(2) It is well known that there is helicity suppression of the longitudinal helicity component in $D \to K$ and $B \to D$ decays. For the differential decay rate into higher spin K resonances close to the end point an analogous effect occurs insofar as the contributions $a_+^{(J)}$ and $\tilde{a}_+^{(J)}$ become part of the longitudinal amplitude. As a result, two form factors dominate the invariant mass distribution and normalize the forward-backward charge asymmetry.

(3) Combining these kinematical factors with the heavy quark limit allows us to employ the transverse amplitudes extracted from semileptonic decays in order to reduce uncertainties of form factors for exclusive rare dilepton processes. Hence we can make predictions that largely avoid model dependence of hadronic form factors, as we have done for $B \to K^*$ decay. Conversely when experiments for rare decays are carried out at ongoing and planned B meson facilities, they will substantially impact upon the semileptonic processes.

(4) The authors of Ref. [33] reported lattice QCD data for one of the form factors in $B \to K^*$ decays (proportional to $\tilde{f}^{(1^-)}$) at the end point. Their results support the scaling law of the heavy quark limit and agree within 25% with our numerical value $\tilde{f}^{(1^-)} = 0.72 \pm 0.14$ in the leading order of the heavy quark effective theory.

C. Rare radiative B decays into K meson resonances

The rare radiative decays induced by the penguin diagrams of the standard model are mediated by an effective Hamiltonian [34]

$$H_{\rm eff} = C_\gamma m_b \epsilon_\mu^* \bar{s}\sigma^{\mu\nu} q_\nu (1+\gamma_5) b, \qquad (39)$$

where m_b is the mass of the bottom quark, ϵ_μ^* the photon polarization vector,

$$C_\gamma = -\frac{G_F}{\sqrt{2}}\left(\frac{e}{4\pi^2}\right) C_7 V_{ts}^* V_{tb},$$

and C_7 is a Wilson coefficient. The exclusive decay rate here reads

$$\Gamma(\bar{B} \to K^i\gamma) = \frac{2^J (J!)^2}{8\pi(2J)!}\frac{J+1}{J}|C_\gamma|^2 (m_b M)^2 \times \left[\frac{(v\cdot p)^2}{M_i^2} - 1\right]^J |H^{(J)}(v\cdot p)|^2, \qquad (40)$$

with $v\cdot p = (M^2 + M_i^2)/2M$. There is but a single form factor $H^{(J)}$, and this is related to our form factors G_i :

$$H^{(J)} = \pm \frac{G_3^{(^{2S+1}L_J)}}{M} + G_4^{(^{2S+1}L_J)},$$

with the upper (lower) sign for $J = L$ $(J = L \pm 1)$. The ground state version of such a relation was obtained in Ref. [8]. It is interesting and important to check out the average momentum transfer of light degrees of freedom: For the lower lying K resonances considered in Ref. [13], it is easy to estimate Q_l at about 600 MeV with $\Lambda = 330$ MeV. This heuristic argument suggests that the above relation probably holds in radiative decays.

D. Nonleptonic B decays into K meson resonances plus charmonia

Now we consider the two-body hadronic decays $\bar{B} \to K^i(\bar{c}c)$, in which charmonia $(\bar{c}c)$ could be $J/\psi, \psi', \chi_{1c}, \eta_c$, etc. The effective Hamiltonian relevant to processes $b \to s\bar{c}c$ is given by [35]

$$H_{\rm eff} = C\bar{s}\gamma^\mu(1-\gamma_5)b\ \bar{c}\gamma_\mu(1-\gamma_5)c, \qquad (41)$$

with

$$C = \frac{G_F}{\sqrt{2}}(c_1 + c_2/3)V_{cs}^* V_{cb}.$$

Here V_{cs} and V_{cb} are elements of CKM matrix and c_1 and c_2 are Wilson coefficients. Assuming factorization and using decay constants $f_P q_\mu = \langle 0|\bar{c}\gamma_\mu\gamma_5 c|(\bar{c}c)_P\rangle$ for the pseudoscalar charmonium such as η_c, and $f_V\phi_\mu = \langle 0|\bar{c}\gamma_\mu c|(\bar{c}c)_V\rangle$ for the vector charmonium like J/ψ, results in

$$H_{\rm eff}(b \to s(\bar{c}c)_P) = C f_P q_\mu \bar{s}\gamma^\mu(1-\gamma_5)b, \tag{42}$$

and

$$H_{\rm eff}(b \to s(\bar{c}c)_V) = C f_V \phi_\mu \bar{s}\gamma^\mu(1-\gamma_5)b. \tag{43}$$

When $(\bar{c}c)$ is a vector, the form factor $a_-^{(J)}$ does not contribute to the amplitude because $\phi_\mu q^\mu = 0$. It is straightforward, though tedious, to calculate the decay rate using Eq. (2); the result reads

$$\Gamma(\bar{B} \to K^i(\bar{c}c)_V) = \frac{2^J (J!)^2 f_V^2}{16\pi(2J)!}|C|^2 \left(\frac{M_i^3}{M^2}\right)\left[\frac{(v\cdot p)^2}{M_i^2} - 1\right]^{J-\frac{1}{2}} \times \left\{\frac{8M^2}{M_V^2}\left|\left[\frac{(v\cdot p)^2}{M_i^2}-1\right]M^J a_+^{(J)} + \frac{1}{2}\left(\frac{v\cdot p}{M_i} - \frac{M_i}{M}\right)\frac{M^{J-1}f^{(J)}}{M_i}\right|^2 + \frac{J+1}{J}\left(\left|\frac{M^{J-1}f^{(J)}}{M_i}\right|^2 + \left[\frac{(v\cdot p)^2}{M_i^2}-1\right]\left|M^J g^{(J)}\right|^2\right)\right\}, \tag{44}$$

where all form factors are *fixed* at $v \cdot p = (M^2 + M_i^2 - M_V^2)/2M$. Some decays into lower lying K resonances plus $J/\psi(\psi', \chi_{1c})$ are investigated in Ref. [36]. The value of $v \cdot p$ is in a range of 1.4 $\sim$ 2.0 GeV, which, unfortunately, does not overlap the range of 2.6–3.0 GeV in radiative decays.

For the sake of completeness, we present the exclusive decay rate for $\bar{B} \to K^i(\bar{c}c)_P$ which depends only upon the longitudinal part of the matrix element, namely the F_0-term of the parametrization in Eq. (3). Making use of the general formula for polarization sums in the Appendix, we arrive at the decay rate

$$\Gamma(\bar{B} \to K^i(\bar{c}c)_P) = \frac{2^J (J!)^2 f_P^2}{8\pi(2J)!}|C|^2\left(\frac{M_i}{M^2}\right)(M^2 - M_i^2)^2 \times \left[\frac{(v\cdot p)^2}{M_i^2} - 1\right]^{J+\frac{1}{2}} |M^J F_0^{(J)}(v\cdot p)|^2, \tag{45}$$

with $v \cdot p = (M^2 + M_i^2 - M_P^2)/2M$. This longitudinal form factor F_0 may be related our to our set G_i via

$$F_0^{(J)} = \frac{2}{M^{J-\frac{1}{2}}(M^2 - M_i^2)}[(M - v\cdot p)G_1^{(^{2S+1}L_J)} \mp (M_i^2 - Mv\cdot p)G_2^{(^{2S+1}L_J)} \pm MG_3^{(^{2S+1}L_J)} - M_i^2 G_4^{(^{2S+1}L_J)}],$$

where the upper sign applies to $J = L$ and the lower one to $J = L \pm 1$. This form does not arise of course in the light leptonic processes.

V. CONCLUSION

In this paper we have outlined the general Lorentz structure for matrix elements of current operators between meson states with arbitrary spins with particular focus on a pseudoscalar (or scalar) meson decaying into resonances of higher spin. The matrix element for these processes resembles very closely the extensively studied $0^- \to 1^-$ decays. Without reference to parity, *three* form factors pertain to the transverse (conserved) part of the current. One extra form factor is needed to describe the longitudinal part. If the full angular distribution of the exclusive rate can be determined experimentally then it is possible in principle to extract each of the 4 form factors.

Using the heavy quark approximation for the decaying heavy flavored meson, we may achieve a great simplification in the matrix elements, reflected in a decrease of the number of form factors; such matrix elements may be expressed in terms of a set of universal form factors which are independent of the mass and spin of the heavy quark inside the decaying heavy meson, as well as the Dirac structure of the current operator. Four of these form factors, for instance, are sufficient to parametrize any matrix elements between 0^- and spin-J states. Importantly, this allows us to link various decay processes induced by different currents: for example, semileptonic decays via $V-A$ and rare dilepton decays by an effective current (arising from one-loop diagrams). This procedure enables us to make use of the end-point spectrum to determine certain CKM matrix elements and to test FCNC sector of B decays. At the phenomenological level, we have formulated rates for various exclusive $\bar{B}$ decays into light resonances of higher spin and expressed them in terms of certain universal form factors. As more experimental measurements of $\bar{B}$ decays become available in the near future, we may hope to determine these form factors through different decay modes and thereby test the heavy quark approximation. To be sure, all of these results can be applied to D-meson decays too, as long as we assume the charm quark is heavy enough compared to the QCD scale. In this way one may use the heavy flavor symmetry to relate universal form factors between B-meson and D-meson decays.

A parallel analysis for baryonic states is currently being undertaken.

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APPENDIX: POLARIZATION SUMS

We first carry out the polarization sums in the rest frame of the spin J meson $\vec{p}'=0$. Representing such a polarization tensor by a spacelike, symmetric, traceless vector with J indices, $\phi^{(\lambda)}_{i_1\cdots i_J}$, the fundamental formula is

$$\sum_{(\lambda)} q^{i_1}\cdots q^{i_J}\phi^{(\lambda)}_{i_1\cdots i_J}\phi^{*(\lambda)}_{j_1\cdots j_J}q'^{j_1}\cdots q'^{j_J} = \frac{2^J(J!)^2}{(2J)!}(|\vec{q}||\vec{q}'|)^J P_J(\vec{n}\cdot\vec{n}'),$$

where n and n' are unit vectors along arbitrary vectors $\vec{q}$ and $\vec{q}'$. Setting $q=q'=p$ we obtain the elementary result

$$\sum_\lambda |p^{i_1}\cdots p^{i_J}\phi^{(\lambda)}_{i_1\cdots i_J}|^2 = \frac{2^J(J!)^2}{(2J)!}|\vec{p}|^{2J}.$$

By differentiating the first formula with respect to q and q' we may peel off indices, one at a time. Doing this just once for q and q', we get

$$\sum_\lambda q^{i_2}\cdots q^{i_J}\phi^{(\lambda)}_{ii_2\cdots i_J}q'^{j_2}\cdots q'^{j_J}\phi^{(\lambda)*}_{jj_2\cdots j_J} = \frac{2^J(J!)^2}{J^2(2J)!}(|\vec{q}||\vec{q}'|)^{J-1}\Big[\delta_{ij}P'_J-(n_in_j+n'_in'_j)P''_{J-1} + n_in'_j(P''_{J-2}-2P''_{J-1})+n'_in_jP''_J\Big].$$

Then setting $q=q'=p$, one obtains

$$\sum_\lambda p^{i_2}\cdots p^{i_J}\phi^{(\lambda)}_{ii_2\cdots i_J}\phi^{(\lambda)*}_{jj_2\cdots j_J}p'^{j_2}\cdots p'^{j_J} = \frac{2^J(J!)^2}{(2J)!}|\vec{p}|^{2(J-1)}\left[\frac{J+1}{2J}\delta_{ij}+\frac{J-1}{2J}\frac{p_ip_j}{|\vec{p}|^2}\right].$$

This trick can be continued to free up all the indices, but fortunately this will not be required in what follows.

Boosting the above results to an arbitrary frame is easy: one simply makes the replacements

$$\delta_{ij}\to -g_{\mu\nu}+\frac{p'_\mu p'_\nu}{M'^2}\equiv d_{\mu\nu}(p'),\quad p_i\to p_\mu-\frac{p\cdot p'}{M'^2}p'_\mu,$$
$$\phi^{(\lambda)}_{i_1\cdots i_J}\to\phi^{(\lambda)}_{\mu_1\cdots\mu_J},$$

leading to

$$\sum_\lambda |p^{\mu_1}\cdots p^{\mu_J}\phi^{(\lambda)}_{\mu_1\cdots\mu_J}(p')|^2 = \frac{2^J(J!)^2}{(2J)!}\left(\frac{\Delta}{M'}\right)^{2J},$$

and

$$\sum_\lambda p^{\mu_2}\cdots p^{\mu_J}\phi^{(\lambda)}_{\mu\mu_2\cdots\mu_J}(p')p^{\nu_2}\cdots p^{\nu_J}\phi^{(\lambda)*}_{\nu\nu_2\cdots\nu_J}(p') = \frac{2^J(J!)^2}{(2J)!}\left(\frac{\Delta}{M'}\right)^{2(J-1)}\left[\frac{(J+1)}{2J}d_{\mu\nu}(p')+\frac{(J-1)}{2J}\frac{(p_\mu-\frac{p\cdot p'}{M'^2}p'_\mu)(p_\nu-\frac{p\cdot p'}{M'^2}p'_\nu)}{(\frac{\Delta}{M'})^2}\right],$$

where $\Delta^2\equiv p^4+p'^4+q^4-2p^2q^2-2p'^2q^2-2p^2p'^2$ and $q=p-p'$.

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