# Signals for the formation of the singlet $D$ and $P$ states of charmonium in $\bar{p}p$ collisions

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We calculate the angular distribution of the photons produced in the decays of the singlet $D$ and singlet $P$ states in charmonium directly produced by unpolarized proton-antiproton ($p\bar{p}$) collisions. The reactions we study are (1) $p\bar{p} \to {}^1D_2 \to {}^1P_1 + \gamma$, (2) $p\bar{p} \to {}^1D_2 \to 1\,{}^3S_1 + \gamma$, and (3) $p\bar{p} \to {}^1P_1 \to {}^1S_0 + \gamma$. We also study the angular distribution of the electron ($e^-$) in the cascade process (4) $p\bar{p} \to {}^1P_1 \to \psi + \pi^0 \to (e^+e^-) + (\gamma_1\gamma_2)$. Simple and distinct expressions are found for each case. The observation of the photon or the electron (as the case may be) with the appropriate energy and the distinct angular distribution may be taken as a signal for the formation of the singlet $D$ or $P$ state resonance in $p\bar{p}$ collisions.

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The singlet $D$ and $P$ states of charmonium are interesting in several respects. Even though the mass of the singlet $D$ state $1\,{}^1D_2$ is supposed [1,2] to be above the charm threshold, it is expected to have a narrow width since the strong transition ${}^1D_2 \to D + \bar{D}^*$ or $D^* + \bar{D}$ is energetically forbidden. This is the case as long as the $1\,{}^1D_2$ mass is below 3875 MeV. Both ${}^1D_2$ and the singlet $P$ state ${}^1P_1$ cannot be seen in $e^+e^-$ collisions either as a resonance or through radiative transitions from $e^+e^-$ resonances. The best hope for producing these states is through $\bar{p}p$ collisions now being studied by the E760 group [3] at Fermilab. In fact, the E760 group recently reported [3] the discovery of the ${}^1P_1$ state in $\bar{p}p$ collisions. The hyperfine splitting between the singlet $P$ state and the center of gravity (c.o.g.) of the triplet $P$ state was successfully predicted in several potential models [4]. The experiments [3] have not yet established the $J^{PC}$ quantum numbers of the postulated ${}^1P_1$ state with a mass around 3526.2 MeV. In order to establish the $J^{PC}$ quantum numbers of the postulated ${}^1D_2$ and ${}^1P_1$ states, one must calculate the angular distribution of the decay products from general model-independent methods and compare it with experiment. In this paper we discuss the angular distributions of the photons and the electron in the following processes: (1) $\bar{p}p \to {}^1D_2 \to {}^1P_1 + \gamma$; (2) $\bar{p}p \to {}^1D_2 \to 1\,{}^3S_1 + \gamma$; (3) $\bar{p}p \to {}^1P_1 \to {}^1S_0 + \gamma$; and (4) $\bar{p}p \to {}^1P_1 \to \psi + \pi^0 \to (e^+e^-) + (2\gamma)$.

We chose to discuss these cases because the angular distribution in each case has a strikingly simple and distinct form, which should serve as a signal for the formation of these states.

Both processes $\bar{p}p \to {}^1D_2$ and $\bar{p}p \to {}^1P_1$ are forbidden in perturbative QCD under some questionable approximations. By $C$ and $P$ invariance, the ${}^1D_2$ and ${}^1P_1$ states can be formed only if $p$ and $\bar{p}$ have the same helicities as $p$ and $\bar{p}$. Now if we assume that the valence quarks and antiquarks in $p$ and $\bar{p}$ have the same helicities as $p$ and $\bar{p}$ and that the $u, d$ quarks and $\bar{u}, \bar{d}$ antiquarks are massless, the formation of these singlet states is forbidden in perturbative QCD since $\bar{u}_{\lambda 1}\gamma^\mu v_{\lambda 2}$ vanishes for massless spinors when the helicity indices $\lambda_1$ and $\lambda_2$ are equal. But this helicity selection rule is strongly violated as seen in the observed processes $\eta_c \to p\bar{p}$ and $p\bar{p} \to {}^1P_1$, etc. So we expect a $p\bar{p} \to {}^1D_2$ cross section comparable to the production cross sections of other charmonium states.

## I. DECAYS OF THE ${}^1D_2$ STATE FORMED IN $\bar{P}P$ COLLISIONS

We previously [5] examined the angular distributions in the following decay schemes of ${}^1D_2$ : (1) $\bar{p}p \to {}^1D_2 \to {}^1P_1+\gamma_1 \to (1\,{}^1S_0+\gamma_2)+\gamma_1$, (2) $\bar{p}p \to {}^1D_2 \to 1\,{}^3S_1+\gamma \to (e^+e^-) + \gamma$. We have derived [5] the angular distributions of $\gamma_1$ and $\gamma_2$ in process (1) and of $\gamma$ and $e^-$ in process (2). The angular distributions of the photon in the processes (1) $\bar{p}p \to 1\,{}^1D_2 \to 1{}^1P_1 + \gamma_1$ and (2) $\bar{p}p \to 1\,{}^1D_2 \to 1\,{}^3S_1 + \gamma$ can be obtained from the results of Ref. [5] by integrating over the directions of $\gamma_2$ and $e^-$ in the cascade processes mentioned above. After integrating over $(\theta', \phi')$ the results of Eqs. (29) and (41) of Ref. [5], we obtain for either processes (1) or (2) the following angular distribution function for $\gamma_1$ or $\gamma$:

$$W(\theta) = \frac{1}{4\pi}\left[1 + \frac{\sqrt{5}}{7}\sqrt{4\pi}(2|A_0|^2 + |A_1|^2 - 2|A_2|^2)Y_{20}(\theta) + \frac{1}{7}\sqrt{4\pi}(6|A_0|^2 - 4|A_1|^2 + |A_2|^2)Y_{40}(\theta)\right], \quad (1)$$

where $A_0$, $A_1$, and $A_2$ are the angular momentum helicity amplitudes as defined in Ref. [5], and $\theta$ is the angle $\bar{p}$ makes with $\gamma$. These helicity amplitudes $A_\nu (\nu = 0, 1, 2)$ are related to the $E1$, $M2$, and the $E3$ multipole amplitudes $a_1, a_2$, and $a_3$ by the orthogonal transformation:

 

$$A_0 = \frac{1}{\sqrt{10}}a_1 + \frac{1}{\sqrt{2}}a_2 + \sqrt{\frac{2}{5}}a_3 \; , A_1 = \sqrt{\frac{3}{10}}a_1 + \frac{1}{\sqrt{6}}a_2 - \sqrt{\frac{8}{15}}a_3 \; , A_2 = \sqrt{\frac{3}{5}}a_1 - \frac{1}{\sqrt{3}}a_2 + \frac{1}{\sqrt{15}}a_3 \; . \tag{2}$$

Substituting Eqs. (2) into Eq. (1) we obtain

$$W(\theta) = \frac{1}{4\pi}\Bigg[1 + \frac{\sqrt{5}}{7}\sqrt{4\pi}\left(1 - \frac{17}{10}|a_1|^2 - \frac{1}{2}|a_2|^2 + \frac{1}{5}|a_3|^2 + \frac{7}{\sqrt{5}}\text{Re}(a_1a_2^*) - \frac{4}{5}\text{Re}(a_1a_3^*) - \frac{4}{\sqrt{5}}\text{Re}(a_2a_3^*)\right)Y_{20}(\theta)$$

$$+\frac{1}{7}\sqrt{4\pi}\left(\frac{8}{3}|a_2|^2 + \frac{1}{3}|a_3|^2 + 6\ \text{Re}(a_1a_3^*) + \frac{10\sqrt{5}}{3}\text{Re}(a_2a_3^*)\right)Y_{40}(\theta)\Bigg] \; . \tag{3}$$

We used the normalizations

$$|A_0|^2 + |A_1|^2 + |A_2|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \; . \tag{4}$$

In process (1) $[\bar{p}p \to {}^1D_2 \to {}^1P_1 + \gamma], a_1, a_2$, and $a_3$ are the $E1$, $M2$, and $E3$ amplitudes. On the other hand, in process (2) $[p\bar{p} \to {}^1D_2 \to 1\,{}^3S_1+\gamma], a_1, a_2$, and $a_3$ refer to the $M1$, $E2$, and $M3$ amplitudes. In process (1), $a_2$ and $a_3$ are of order $v^2/c^2$ compared to $a_1$, whereas in process (2) they are all of order $v^2/c^2$ since $1\,{}^1D_2 \to 1\,{}^3S_1+\gamma$ does not take place in the nonrelativistic limit because the nonrelativistic $M1$ transition operator does not connect the two states.

In process (1), $a_2$ and $a_3$ are of order $k/m$ compared to $a_1$, where $k$ is the photon energy ($\sim$ 300 MeV) and $m$ is the $c$ quark mass. So $|a_2|^2$, $|a_3|^2$, and $\text{Re}(a_2a_3^*)$ can be neglected compared to $|a_1|^2$, and $|a_1|^2$ is nearly 1. Furthermore, in potential models [6] $a_2$ and $a_3$ are purely imaginary if $a_1$ is real. Therefore, $\text{Re}(a_1a_2^*)$ and $\text{Re}(a_1a_3^*)$ vanish. As a result, the angular distribution of the $\gamma$ photon produced in the process $\bar{p}p \to {}^1D_2 \to {}^1P_1 + \gamma$ will be

$$W(\theta) \simeq \frac{1}{4\pi}\left[1 - \frac{1}{2}\sqrt{\frac{4\pi}{5}}Y_{20}(\theta)\right] = \frac{1}{16\pi}(5 - 3\cos^2\theta) \; , \tag{5}$$

where $\theta$ is the angle between $\gamma_1$ and the direction of $\bar{p}$ in the $p\bar{p}$ c.m. frame. This pure $E_1$ distribution of $\gamma_1$ with an energy of about 300 MeV can be thought of as a signal for the ${}^1D_2$ formation in $\bar{p}p$ collisions.

In process (2), $a_1$, $a_2$, and $a_3$ are all of order $v^2/c^2$ or $k/m$. In fact, in potential models [6] $a_2$ and $a_3$ are numerically much bigger than $a_1$. Therefore, in Eq. (3) we cannot neglect the coefficient of $Y_{40}(\theta)$. In fact, it may be bigger than the coefficient of $Y_{20}(\theta)$. The angular distribution of $\gamma$ in $\bar{p}p \to {}^1D_2 \to 1\,{}^3S_1+\gamma$ takes the form

$$W(\theta) = \frac{1}{4\pi}[1 + a'Y_{20}(\theta) + b'Y_{40}(\theta)] \; , \tag{6}$$

where the expressions for the coefficients $a'$ and $b'$ can be obtained from Eq. (3). If we take the results of the potential model calculation of Ref. [6] we have

$$\begin{aligned} a_1 &\simeq i(0.083), \\ a_2 &\simeq -i(0.970), \\ a_3 &\simeq -i(0.229) \; . \end{aligned} \tag{7}$$

Therefore, the numerical values of $a'$ and $b'$ will be

$$a' \simeq 0.78 \; , \tag{8}$$

$$b' \simeq 2.06 \; . \tag{9}$$

So

$$W(\theta) \simeq \frac{1}{4\pi}[1 + 0.78Y_{20}(\theta) + 2.06Y_{40}(\theta)] \; . \tag{10}$$

We expect the coefficients of $Y_{20}$ and $Y_{40}$ to be very sensitive to the potential models. So by measuring the angular distribution of $\gamma$ in $\bar{p}p \to {}^1D_2 \to \psi + \gamma$ we can test the different potential models.

## II. DECAYS OF THE SINGLET $P$ STATE (${}^1P_1$) FORMED IN $\bar{p}p$ COLLISIONS

Here we consider processes (1) $\bar{p}p \to {}^1P_1 \to 1\,{}^1S_0 + \gamma$ and (2) $\bar{p}p \to {}^1P_1 \to \psi + \pi^0 \to (e^+e^-) + (\gamma_1\gamma_2)$.

*Process* (1): $\bar{p}p \to {}^1P_1 \to 1{}^1S_0 + \gamma$. In this process, the photon is pure $E1$ by angular momentum and parity conservation and a calculation of the type described in Ref. [5] will give the angular distribution of $\gamma$ by the function

$$W(\theta) = \frac{3}{8\pi}(1 - \cos^2\theta) \; . \tag{11}$$

*Process* (2): $\bar{p}p \to {}^1P_1 \to \psi + \pi^0 \to (e^+e^-) + (2\gamma)$.

Consider the cascade process

$$\bar{p}(\lambda_1)p(\lambda_2) \to {}^1P_1(\nu) \to \psi(\sigma)+\pi^0 \to e^-(\kappa_1)+e^+(\kappa_2)+\pi^0,$$

where the Greek symbols in parentheses after the particle symbols represent their helicity indices. We will work in the $\bar{p}p$ c.m. frame where ${}^1P_1$ is produced at rest. So the symbol $\nu$ represents the value of the $Z$ component of the total angular momentum of ${}^1P_1$. The probability amplitude for this process can be written as

$$\begin{aligned} T^{\kappa_1\kappa_2}_{\lambda_1\lambda_2} = \sum_{\nu,\sigma} {}_{{}^1P_1}\langle e^-(\kappa_1)e^+(\kappa_2)|C|\psi(\sigma)\rangle_{{}^1P_1} \\ \times {}_{{}^1P_1}\langle\psi(\sigma)\pi^0|A|{}^1P_1(\nu)\rangle_{{}^1P_1} \\ \times {}_{{}^1P_1}\langle{}^1P_1(\nu)|B|\bar{p}(\lambda_1)p(\lambda_2)\rangle_{{}^1P_1} \ , \end{aligned} \tag{12a}$$

where each matrix element is evaluated in the rest frame of ${}^1P_1$ or the c.m. frame of the $\bar{p}p$ system. In Eq. (12a), $B$, $A$, and $C$ are the appropriate transition operators:

$$\nu,\sigma = -1,0,+1 \ . \tag{12b}$$

The two-particle helicity state $|\bar{p}(\lambda_1)p(\lambda_2)\rangle$ in the c.m. frame of $\bar{p}p$ can be written as [7]

$$\begin{aligned} |p(\theta,\phi);\lambda_1\lambda_2\rangle = \sum_{JM}\sqrt{\frac{2J+1}{4\pi}}D^J_{M\lambda}(\phi,\theta,-\phi) \\ \times|pJM;\lambda_1\lambda_2\rangle \ , \end{aligned} \tag{13}$$

where

$$\lambda = \lambda_1 - \lambda_2 \ , \tag{14}$$

and $(\theta,\phi)$ represent the spherical polar angles giving the direction of $\bar{p}$ momentum whose magnitude is $p$. We will choose the $Z$ axis along the direction of motion of $\psi$. So

$$\begin{aligned} {}_{{}^1P_1}\langle{}^1P_1(\nu)|B|\bar{p}(\lambda_1)p(\lambda_2)\rangle_{{}^1P_1} \\ = \sqrt{\frac{3}{4\pi}}D^1_{\nu\lambda}(\phi,\theta,-\phi)\langle 1\nu|B|1\nu;\lambda_1\lambda_2\rangle \\ = \sqrt{\frac{3}{4\pi}}D^1_{\nu\lambda}(\phi,\theta,-\phi)B_{\lambda_1\lambda_2} \ , \end{aligned} \tag{15}$$

where $B_{\lambda_1\lambda_2}$ are the angular momentum helicity amplitudes which do not depend on the angles or the index $\nu$. By $C$ variance [7],

$$B_{\lambda_1\lambda_2} = B_{\lambda_2\lambda_1} \ . \tag{16}$$

By $P$ variance [7],

$$B_{\lambda_1\lambda_2} = -B_{-\lambda_1-\lambda_2} \ . \tag{17}$$

From Eqs. (16) and (17),

$$\begin{aligned} B_{+-} = -B_{-+} = -B_{+-} = B_1 = 0, \\ B_{++} = -B_{--} = B_0 \ . \end{aligned} \tag{18}$$

There is only one independent $B$ helicity amplitude and we call it $B_0$. Since the momentum $p'$ of $\psi$ in the ${}^1P_1$ rest frame is along the positive $Z$ axis,

$$\begin{aligned} {}_{{}^1P_1}\langle\psi(\sigma)\pi^0|A|{}^1P_1(\nu)\rangle_{{}^1P_1} \\ = \langle p'(0,0);\sigma 0|A|1\nu\rangle \\ = \sum_{JM}\sqrt{\frac{2J+1}{4\pi}}D^{J*}_{M\sigma}(0,0,0)\langle p'JM;\sigma 0|A|1\nu\rangle \ . \end{aligned} \tag{19}$$

Since the transition operator $A$ is rotationally invariant, $M$ is equal to $\nu$ and $J$ is equal to 1. So

$$\begin{aligned} {}_{{}^1P_1}\langle\psi(\sigma)\pi^0|A|{}^1P_1(\nu)\rangle_{{}^1P_1} = \sqrt{\frac{3}{4\pi}}D^{1*}_{\nu\sigma}(0,0,0)A^1_{\sigma 0} \\ = \sqrt{\frac{3}{4\pi}}\delta_{\nu\sigma}A_{\sigma 0} \ . \end{aligned} \tag{20}$$

$C$ invariance is trivially satisfied in this decay. By $P$ invariance [7],

$$A_{\sigma 0} = A_{-\sigma 0} \ . \tag{21}$$

So

$$\begin{aligned} A_{10} = A_{-10} = A_1, \\ A_{00} = A_0 \ , \end{aligned} \tag{22}$$

$$\begin{aligned} {}_{{}^1P_1}\langle e^-(\kappa_1)e^+(\kappa_2)|C|\psi(\sigma)\rangle_{{}^1P_1} &= {}_{\psi}\langle e^-(\kappa_1)e^+(\kappa_2)|U^\dagger_\Lambda({}^1P_1,\psi)CU_\Lambda({}^1P_1,\psi)|\psi(\sigma)\rangle_\psi \\ &= {}_{\psi}\langle e^-(\kappa_1)e^+(\kappa_2)|C|\psi(\sigma)\rangle_\psi \ , \end{aligned} \tag{23}$$

where $U_\Lambda(A,B)$ is the unitary operator corresponding to a Lorentz transformation which takes us from the $B$ rest frame to the $A$ rest frame. In Eq. (23) we made use of the fact that

$$U^\dagger_\Lambda({}^1P_1,\psi)CU_\Lambda({}^1P_1,\psi) = C \ . \tag{24}$$

Now we notice

$$\begin{aligned}\psi\langle e^-(\kappa_1)e^+(\kappa_2)|C|\psi(\sigma)\rangle_\psi &= \langle p''(\theta',\phi');\kappa_1\kappa_2|C|1\sigma\rangle \\ &= \sum_{JM}\sqrt{\frac{2J+1}{4\pi}}D^{J^*}_{MK}(\phi',\theta',-\phi')\langle JM;\kappa_1\kappa_2|C|1\sigma\rangle \\ &= \sqrt{\frac{3}{4\pi}}D^{1^*}_{\sigma\kappa}(\phi',\theta',-\phi')\langle 1\sigma;\kappa_1\kappa_2|C|1\sigma\rangle \\ &= \sqrt{\frac{3}{4\pi}}D^{1^*}_{\sigma\kappa}(\phi',\theta',-\phi')C_{\kappa_1\kappa_2}\ , \end{aligned} \tag{25}$$

where

$$\kappa = \kappa_1 - \kappa_2\ . \tag{26}$$

By $C$ invariance [7],

$$C_{\kappa_1\kappa_2} = C_{\kappa_2\kappa_1}\ . \tag{27}$$

By $P$ invariance [7],

$$C_{\kappa_1\kappa_2} = C_{-\kappa_1-\kappa_2}\ . \tag{28}$$

So we get

$$\begin{aligned} C_{++} &= C_{--} = C_0\ , \\ C_{+-} &= C_{-+} + C_1\ . \end{aligned} \tag{29}$$

If $\psi$ decays to $e^+e^-$ through a virtual photon, $C_0 \simeq \frac{m}{E}C_1$, where $m$ and $E$ are mass and energy of the electron.

Later we will neglect $C_0$ compared to $C_1$. So using Eqs. (12a), (15), (20), (23), and (25), we obtain

$$T^{\kappa_1\kappa_2}_{\lambda_1\lambda_2} = \frac{3}{4\pi}B_{\lambda_1\lambda_2}C_{\kappa_1\kappa_2}\sum_{\sigma}^{-1,0,+1} A_\sigma D^{1^*}_{\sigma\kappa}(\Omega')D^1_{\sigma\lambda}(\Omega)\ , \tag{30}$$

where $\Omega = (\theta,\phi)$ and $\Omega' = (\theta',\phi')$.

The normalized angular distribution function for unpolarized $\bar{p}p$ collisions is given by

$$\begin{aligned} W(\Omega;\Omega') &= \frac{N}{4}\sum_{\substack{\lambda_1\lambda_2\\ \kappa_1\kappa_2}} T^{\kappa_1\kappa_2}_{\lambda_1\lambda_2}T^{\kappa_1\kappa_2^*}_{\lambda_1\lambda_2} \\ &= \frac{N}{4}\left(\frac{3}{4\pi}\right)^2\sum_{\substack{\lambda_1\lambda_2\\ \kappa_1\kappa_2}}|C_{\kappa_1\kappa_2}|^2|B_{\lambda_1\lambda_2}|^2 \\ &\quad\times\sum_{\sigma,\sigma'}^{-1,0,1} A_\sigma A^*_{\sigma'}D^1_{\sigma\kappa}(\Omega')D^{1^*}_{\sigma'\kappa}(\Omega') \\ &\quad\times D^1_{\sigma\lambda}(\Omega)D^{1^*}_{\sigma'\lambda}(\Omega). \end{aligned} \tag{31}$$

Now we make use of the fact that

$$D^{1^*}_{\sigma'\lambda} = (-1)^{\sigma'-\lambda}D^1_{-\sigma',-\lambda'} \tag{32}$$

and

$$D^{j_1}_{m_1'm_1}D^{j_2}_{m_2'm_2} = \sum_J\langle j_1j_2;m_1m_2|JM\rangle\langle j_1j_2;m_1m_2;|JM'\rangle \times D^J_{M'M}\ . \tag{33}$$

We also use the relabeling

$$d = \sigma - \sigma'$$

and

$$s = \sigma + \sigma'\ . \tag{34}$$

We then finally obtain

$$\begin{aligned} W(\Omega;\Omega') &= \frac{N}{4}\left(\frac{3}{4\pi}\right)^2\sum_{L_1}^{0,2}\beta_{L_1}\sum_{L_2}^{0,2}\epsilon_{L_2} \\ &\quad\times\sum_{d}^{\mathrm{Min}(L_1,L_2)}\left(1-\frac{\delta_{d0}}{2}\right)\alpha_d^{L_1L_2}\mathcal{Y}_d^{L_1L_2}(\Omega;\Omega')\ , \end{aligned} \tag{35}$$

where the coefficients are

$$\beta_{L_1} = |B_0|^2\langle 1100|L_{10}\rangle\ , \tag{36}$$

$$\begin{aligned}\epsilon_{L_2} &= \sum_\alpha^{0,1}(-1)^\alpha|C_\alpha|^2\langle 11;\alpha-\alpha|L_20\rangle \\ &\simeq -|C_1|^2\langle 11;1-1|L_20\rangle\ , \end{aligned} \tag{37}$$

$$\begin{aligned}\alpha_d^{L_1L_2} &= \sum_{s(d)}^{\geq 0}\left(1-\frac{\delta_{s0}}{2}\left(A_{\frac{s+d}{2}}A^*_{\frac{s-d}{2}} + A^*_{\frac{s+d}{2}}A_{\frac{s-d}{2}}\right)\right. \\ &\quad\times\left\langle 11;\frac{s+d}{2},-\frac{s-d}{2}\right| L_1d\Big\rangle \\ &\quad\times\left\langle 11;\frac{s+d}{2},-\frac{s-d}{2}\right| L_2d\Big\rangle\ , \end{aligned} \tag{38}$$

where

$$s(d) = \tfrac{1}{2}[1-(-1)^d],\ \tfrac{1}{2}[1-(-1)^d]+2,\ldots,(2-d)\ . \tag{39}$$

Finally the angular function $\mathcal{Y}_d^{L_1L_2}(\Omega;\Omega')$ is given by

$$\mathcal{Y}_d^{L_1L_2} = D_{d0}^{L_1}(\Omega)D_{d0}^{L_2^*}(\Omega') + D_{d0}^{L_1^*}(\Omega)D_{d0}^{L_2}(\Omega')$$
$$= \sqrt{\frac{4\pi}{2L_1+1}}\sqrt{\frac{4\pi}{2L_2+1}} \times[Y_{L_1d}^*(\Omega)Y_{L_2d}(\Omega') + Y_{L_1d}(\Omega)Y_{L_2d}^*(\Omega')] \ . \quad (40)$$

If we use the normalizations,

$$|A_0|^2 + 2|A_1|^2 = |B_0|^2 = C_0|^2 + |C_1|^2 = 1 \ , \quad (41)$$

the explicit expressions for the nonzero coefficients take the form

$$\begin{aligned}
\beta_0 &= -\frac{1}{\sqrt{3}}, \\
\beta_2 &= \sqrt{\frac{2}{3}} \ , \\
\epsilon_0 &= -\frac{1}{\sqrt{3}} \ , \\
\epsilon_2 &= -\frac{1}{\sqrt{6}}(1-3|C_0|^2) \simeq -\frac{1}{\sqrt{6}} \ , \\
\alpha_0^{00} &= \tfrac{1}{3} \ , \qquad (42) \\
\alpha_0^{20} = \alpha_0^{02} &= -\frac{\sqrt{2}}{3}(|A_0|^2 - |A_1|^2) \ , \\
\alpha_0^{22} &= \tfrac{1}{3}(2|A_0|^2 + |A_1|^2) \ , \\
\alpha_1^{22} &= \mathrm{Re}(A_1A_0^*) \ , \\
\alpha_2^{22} &= -|A_1|^2 \ .
\end{aligned}$$

The normalization constant $N/4$ is now 1. If the experimentalists can measure the angular distribution of the electron and of $\pi^0$ or, equivalently, that of the total momentum of the two $\gamma$ photons into which $\pi^0$ decays, one can use Eqs. (35)–(42) and get the relative magnitudes of the angular momentum helicity amplitudes $A_0$ and $A_1$ in the process ${}^1P_1 \to \psi + \pi^0$ as well as the cosine of the relative phase between the two helicity amplitudes.

If we do not measure the angular distribution of the total momentum of the two $\gamma$'s we will integrate over $\Omega$ and then get the angular distribution of the electron alone as

$$W_e = \int d\Omega\, W(\Omega;\Omega')$$
$$= \frac{3}{8\pi}\left[|A_0|^2 + |A_1|^2 + (|A_1|^2 - |A_0|^2)\cos^2\theta'\right] . \quad (43)$$

The angle $\theta'$ is the angle between the positive $z$ axis and the electron as measured in the $\psi$ rest frame. Since $\pi^0$ moves along the negative $z$ axis and since

$$\cos^2(\pi - \theta') = \cos^2\theta' \ , \quad (44)$$

we may also take $\theta'$ to be the angle between $\pi^0$ and the electron as measured in the $\psi$ rest frame. The angle $\theta'$ is also the angle between the total momentum of the two $\gamma$ photons and the $e^-$ in the $\psi$ rest frame, since $\pi^0$ decays into $2\gamma$ and the total momentum is conserved. The experimentalists will probably measure the angles in the $p\bar{p}$ c.m. frame. If $\tilde{\theta}'$ represents the angle between the total momentum of the two photons and $e^-$ in the $p\bar{p}$ c.m. frame, it is related to $\theta'$ by the relations

$$\begin{aligned}
\phi' &= \tilde{\phi}' \ , \\
\cos\theta' &= \frac{\cos\tilde{\theta}' - \beta}{1 - \beta\cos\tilde{\theta}'} \ , \\
\sin\theta' &= \gamma\sin\tilde{\theta}'\left[1 + \beta\frac{(\cos\tilde{\theta}' - \beta)}{(1-\beta\cos\tilde{\theta}')}\right] \ , \\
\gamma &= \frac{1}{\sqrt{1-\beta^2}} \ , \qquad (45)
\end{aligned}$$

where

$$\beta = \frac{v}{c} = \frac{\sqrt{[M_{^1p_1}^2 - (M_\psi + M_\pi 0)^2][M_{^1p_1}^2 - (M_\psi - M_\pi 0)^2]}}{[M_{^1p_1}^2 + (M_\psi^2 - M_\pi^2 0)]}$$

By measuring the angular distribution of the electron, namely $W_e(\theta')$, one can estimate the ratio of the magnitudes of the two helicity amplitudes $|A_0|$ and $|A_1|$. By confirming the angular distribution given by Eq. (43) we are also confirming the $J^{PC}$ quantum number of the postulated ${}^1p_1$ state of charmonium.

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