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Low energy measurements of the strong coupling constant and the question of a light gluino

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The hadronic width of the J/ψ is anomalously small when compared with that of the Υ in standard QCD in the nonrelativistic approximation. We discuss the extent to which current ideas concerning relativistic corrections and/or supplementing QCD with a light gluino can reconcile these measurements with each other and with other low energy data including that from lattice calculations and from τ decay.

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I. INTRODUCTION

Although precision measurements at the Z pole have yielded information on the electroweak couplings with an error of the order of 0.1%, the experimental value of $\alpha_3(M_Z)$ from the CERN e^+e^- collider LEP has an error of the order of 10%. For instance, we know that

$$\alpha^{-1}(M_Z) = 127.9 \pm 0.2 \ , \tag{1}$$

$$\sin^2\theta_W(M_Z) = 0.2324 - 0.005 \times [m_t^2/(138 \text{ GeV})^2 - 1] \pm 0.0003 \ , \tag{2}$$

whereas from LEP studies the current published value [1] of the strong-coupling constant is

$$\alpha_3(M_Z) = 0.123 \pm 0.006 \ . \tag{3a}$$

An analysis of more recent data from LEP may have the effect of bringing this value down significantly, without, however, reducing the error [2]:

$$\alpha_3(M_Z) = 0.108 \pm 0.012 \ . \tag{3b}$$

Current jet studies at LEP are still plagued by hadronization effects and other systematic errors. The substantial uncertainty in α_3 has had the effect of severely limiting the predictive power of supersymmetric (SUSY) unification schemes. On the other hand, electron annihilation studies over a range of lower energies allow for a test of the presence of strong nonperturbative effects since these will typically fall off with a power of $\sqrt{s}$ which can then be easily distinguished from the predictions of perturbative QCD which fall off only logarithmically. Two jet measures which were predicted to have negligible nonperturbative effects are the jet mass difference and the integrated energy-energy asymmetry. We discuss current data on these measures and compare the resulting values of $\alpha_3(M_Z)$ with those from quarkonia decays and lattice calculations. Relativistic effects in quarkonia decays and the effect of a possible light gluino are also considered.

In Sec. II of this work we discuss the implications of low-energy jet data on this coupling at the scale of the Z. Section III is concerned with quarkonia branching ratios and lattice analyses. Section IV deals with the effect of the τ decay data. Conclusions together with a discussion of other analyses are given in Sec. V.

II. α_3 FROM JET PROPERTIES

The jet (squared) mass difference scaled by the (squared) center-of-mass energy s is an infrared insensitive quantity whose perturbation theory prediction [3] as a function of s is

$$\langle M_h^2 - M_l^2\rangle/s = 1.05\frac{\alpha_3(\sqrt{s})}{\pi}\left(1 + 2.76\frac{\alpha_3(\sqrt{s})}{\pi} + \cdots\right) . \tag{4}$$

M_h and M_l are, respectively, the heavier and lighter jet masses where, by definition, the hadrons are associated with two jets of minimum mass. For this quantity the second-order corrections are small and the nonperturbative effects are expected to approximately cancel between the two jet masses. This can be experimentally tested by comparing the experimental results with the QCD prediction and looking for additional power-law terms in $\sqrt{s}$. In fact it is found that the energy dependence of this quantity is consistent with perturbative QCD alone; there is no evidence for the presence of power-law corrections to Eq. (4). It would be theoretically discouraging if the nonperturbative contributions to an infrared insensitive quantity had the same logarithmic behavior as the perturbative contributions, thus frustrating any separation. On general grounds (they are higher twist terms) the nonperturbative contributions should fall off as a power of the QCD Λ parameter divided by $\sqrt{s}$. There is very little room in the data for such contributions. We feel therefore that these data give a reasonable estimate of the strong-coupling constant without appreciable uncertainties due to nonperturbative effects. As we shall see the results interpolate well through the quarkonium region and are in reasonable agreement with the lattice results, which at present are the only quasirigorous treatment of nonperturbative effects.

A similarly infrared insensitive quantity is the integrated energy-energy asymmetry

$$\int_{30^\circ}^{90^\circ} d\chi A_{EE}(\chi) = 0.766\frac{\alpha_3(\sqrt{s})}{\pi}\left(1.0 + 3.59\frac{\alpha_3(\sqrt{s})}{\pi}\right) \tag{5}$$

where $A_{EE}(\chi) = E(180^\circ - \chi) - E(\chi)$ and $E(\chi)$ is the distribution of the energy-weighted angle χ between the directions of any two particles in the event. Any nonperturbative corrections to this quantity must also be power behaved in $\sqrt{s}$ and no such corrections are found. The CELLO data [4] on these two quantities are plotted in Fig. 1 versus $\sqrt{s}$. Each result for $\alpha_3(\sqrt{s})$ is extrapolated to M_Z and averaged there assuming standard QCD running with five flavors. These can be compared with a recent precision lattice result [5] extrapolated to the Z:

$$\alpha_3(M_Z) = 0.102^{+0.0029}_{-0.0100} \text{ (lattice)} . \tag{6}$$

This is consistent with the previous published lattice result [6]. The asymmetric errors quantify the preference of lattice calculations for small values of α_3. The band of α_3 values shown in Fig. 1 corresponds to the weighted average of this lattice value with that from the CELLO data:

$$\alpha_3(M_Z) = 0.0965^{+0.002}_{-0.003} \text{ (weighted average)} . \tag{7}$$

We also plot in Fig. 1 the lattice result in Eq. (6) extrapolated to 5 GeV. Equation (7) is consistent with that obtained in 1984 from earlier experiments on the same jet measures. It is known that the presence of light gluinos would raise the value of α_3 required to fit jet measures

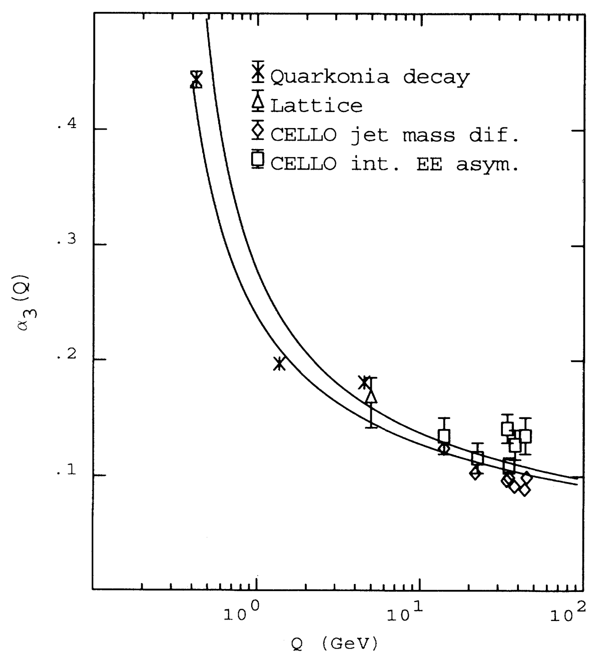

FIG. 1. The running of the strong-coupling constant in standard QCD compared to data on decays of vector strangeonium, charmonium, and bottomonium into dissimilar quarks, jet data, and the lattice result. The solid lines correspond to $\alpha_3(M_Z) = 0.0985$ and 0.0935.

[7] by about 10%. The actual value of $\alpha_3(M_Z)$ deduced from the CELLO data in the presence of light gluinos would then be about 0.11.

III. α_3 FROM QUARKONIA BRANCHING RATIOS AND LATTICE STUDIES

The most accurate data from the three-gluon decays of the $^3S_1$ vector quarkonia in the nonrelativistic approximation are also plotted in Fig. 1 for comparison. The other quarkonia measurements are consistent with these but have larger errors. The results are taken from [8] but are updated to include the results of the 1994 Particle Data Group averages. In the nonrelativistic quark model one finds

$$\alpha_3^0(\mu) = \left(\frac{\Gamma(^3S_1 \to \text{dissimilar hadrons})81\alpha^2\pi e_q^2}{\Gamma(^3S_1 \to e^+e^-)10(\pi^2 - 9)}\right)^{1/3} . \tag{8}$$

The scale μ is chosen so that the next-to-leading order QCD corrections vanish. It takes the natural form of approximately the quark mass. Such a scale choice is usually very close to that defined by the principle of minimum sensitivity. The superscript 0 indicates that α_3^0 is the result of pure perturbative QCD in the nonrelativistic heavy quark approximation with no attempt to allow for model-dependent relativistic corrections. We regard it as quite interesting that one can predict the nonstrange decay of the Φ meson from perturbative QCD if the strong coupling is as low as Eq. (7). This was first pointed out in [9]. On the other hand, a strong coupling this low is inconsistent with minimal SUSY unification. In [8] it was noted that a gluino of mass 5 GeV or less could restore consistency with unification; this was the first positive indication that the gluino might be light. Other positive indications have since been noted [10,11,7].

It is now generally agreed that there are three windows at low energy for the gluino mass [12] and vigorous proposals are being made to close these windows definitively or to discover a light gluino [13]. The current status of these windows is shown in Fig. 2 which updates the figure of [12] to show the effect of LEP measurements and the Helios result [14]. The latter experiment is a missing energy experiment which complements the beam dump results. It, however, is not sensitive to gluinos of mass or lifetime significantly greater than charm. The sloping boundaries of the regions ruled out by the stable particle searches, the Helios experiment, and the UA1 experiment are defined, respectively, by gluino lifetimes of 10^{-8}, 10^{-9}, and 10^{-10} s. These are related to the squark masses via the assumption of independent decay of the gluino into $q\bar{q}\tilde{\gamma}$ with an effective mass equal to half the $g\tilde{g}$ bound-state mass. The upper boundary of the ultra low-mass gluino window is given by the CUSB [15] result $m_{\tilde{g}} < 0.6$ GeV or $m_{\tilde{g}} > 2.6$ GeV. The CUSB experiment searched for monoenergetic photons from Υ decay which could signal the presence of gluinoballs. The theoretical prediction for this rate is governed by the gluinoball wave function at the origin which is somewhat model de-

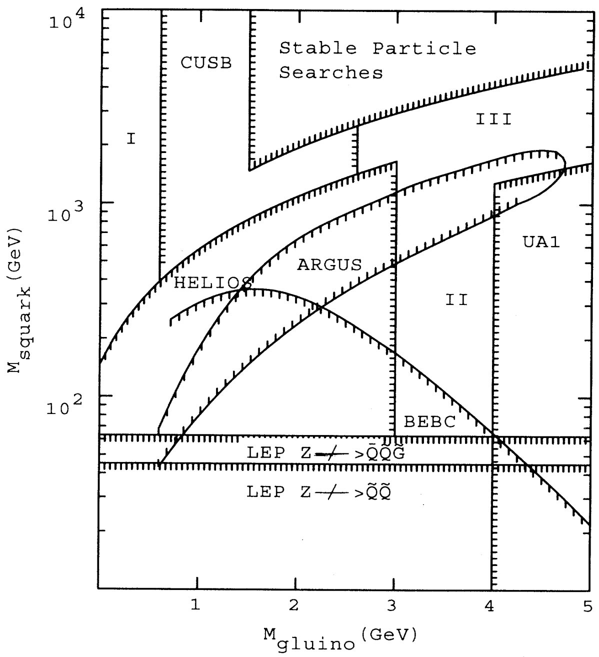

FIG. 2. Gluino windows in the space of gluino and squark masses. Hatched areas are disfavored by the indicated experiments. The currently allowed windows are labeled I, II, and III. The quarkonium data prefer a gluino in the lowest-energy window.

pendent. The CUSB constraint will become more compelling when the monoenergetic photons associated with the expected glueballs or with the known pseudoscalar mesons are discovered in Υ decay. However, even accepting this result, a very light gluino which should lead to an approximately supersymmetric multiplet of glueballs, gluinoballs, and glueballinos could exist.

This above-noted consistency of light gluinos with SUSY unification, however, required that only the gluino and photino be light. If, as is the case in the minimal supergravity-inspired model, the W-inos, Z-ino, and Higgsinos are also below the Z when the gluino is light, the consistency is destroyed [16] and solutions are only found for $0.122 < \alpha_3(M_Z) < 0.132$. Although this is not in disagreement with the LEP data of Eq. (3a), it causes serious problems for low-energy QCD as is obvious from Fig. 1. Independent of this problem, it was pointed out in [10] that the discrepancy in Fig. 1 between the J/ψ and Υ data can be greatly alleviated if the gluino is in the lowest-energy window around 0.4 GeV. The possibility that the vector quarkonia can decay into gluino-containing hadrons modifies the quarkonium measurement of $\alpha_3(M_q)$ according to the relation

$$\alpha_3^0(M_q) = \alpha_3(M_q)\left(1 + \frac{3}{\pi}\alpha_3(M_q)R(m_{\tilde{G}}/M_q)\right)^{1/3} , \quad (9)$$

where R is a steeply falling function of its argument given by [17]. The result then is that $\alpha_3(M_Z)$ is 0.113 ± 0.003, significantly higher than Eq. (7). Another approach to the J/ψ-Υ discrepancy of Fig. 1 is to make a linear parametrization of the relativistic corrections of the form [18]

$$\alpha_3^0(M_q) = \alpha_3(M_q)(1 - C\langle v^2/c^2\rangle)^{1/3} . \quad (10)$$

This modifies the three-gluon decay rate of the $q\bar{q}$ bound state by a factor

$$f_{r,q} = 1 - C\langle v^2/c^2\rangle .$$

Here v is the quark velocity in the vector bound state. In a typical potential model

$$\langle v^2/c^2\rangle = \begin{cases} 0.23 & \text{for } J/\psi , \\ 0.077 & \text{for } \Upsilon . \end{cases} \quad (11)$$

This is a dangerous procedure for several reasons apart from the model dependence of Eq. (11). First of all, the fit values of C are greater than 3 implying that the J/ψ width is reduced by some 70% by "relativistic corrections." This would strongly suggest that quadratic effects are non-negligible and should go in the opposite direction. It is possible to do a quadratic fit which would reconcile the J/ψ and Υ data without such large corrections. Such a fit could easily bring the quarkonium data into agreement with the jet mass and energy asymmetry data without requiring such large relativistic corrections although then all predictive power from quarkonia is lost. Second, if one makes a phenomenological linear parametrization of the relativistic corrections, one must give up hope of understanding the Φ decay, and the Zweig rule at low energies, since the simple linear fit in $\langle v^2/c^2\rangle$ is not likely to be meaningful. Third, the agreement of the quarkonia data with the lattice results and with the infrared insensitive jet measures is destroyed. In addition the large increase in α_3 at both the bottom quark and charm quark scales in the linear parametrization is not supported by the data from $GGG/GG\gamma$ branching ratios of vector quarkonia nor by that from the $GG/\gamma\gamma$ branching ratio of the η_c as we will now discuss. Theoretically these ratios are

$$\frac{\Gamma(V \to GGG)}{\Gamma(V \to GG\gamma)} = \frac{5}{36\alpha e_q^2}\alpha_3(\mu)\left\{1 + \frac{\alpha_3(\mu)}{\pi}\left[-2.3 + 0.93\beta_0 + \beta_0/2\ln(\mu/M_V)\right]\right\} , \quad (12)$$

$$\frac{\Gamma(\eta_c \to GG)}{\Gamma(\eta_c \to \gamma\gamma)} = \frac{2}{9\alpha^2 e_q^4}\alpha_3^2(\mu)\left\{1 + \frac{\alpha_3(\mu)}{\pi}\left[-0.9 + \frac{20-\pi^2}{3} + \beta_0[4/3 + \ln(\mu/M_{\eta_c})]\right]\right\} , \quad (13)$$

TABLE I. Five quarkonia branching ratios and the resulting values of the strong coupling in the nonrelativistic quark model. The scale μ is chosen to eliminate first-order QCD corrections.

Branching ratio	Experiment	Ref.	μ (GeV)	α_3^0 (μ)	α_3^0 (M_Z)
$\Gamma(\Upsilon \to GGG)/\Gamma(\Upsilon \to \mu^+\mu^-)$	32.5 ± 1.3	[19]	4.54	0.181 ± 0.002	0.1045 ± 0.0007
$\Gamma(J/\psi \to GGG)/\Gamma(J/\psi \to e^+e^-)$	10.7 ± 0.7	[19]	1.37	0.197 ± 0.004	0.0909 ± 0.0008
$\Gamma(\Upsilon \to GG\gamma)/\Gamma(\Upsilon \to GGG)$	0.0279 ± 0.0015	[20]	2.55	0.216 ± 0.011	0.1043 ± 0.0024
$\Gamma(J/\psi \to GG\gamma)/\Gamma(J/\psi \to GGG)$	0.10 ± 0.04	[21]	0.80	$0.23^{+0.16}_{-0.06}$	$0.089^{+0.014}_{-0.010}$
$\Gamma(\eta_c \to GG)/\Gamma(\eta_c \to \gamma\gamma)$	1470 ± 650	[19]	0.60	0.27 ± 0.06	$0.0895^{+0.0049}_{-0.0101}$

where $\beta_0 = 11 - 2n_f/3$ for n_f active quarks. In each case a separate μ can be chosen so that the large square brackets in Eqs. (12) and (13) vanish. The results are summarized in Table I. Since the structure of the interactions in the numerators and denominators of Eqs. (12) and (13) are identical, these ratios should be insensitive to relativistic corrections. The corresponding values of α_3 are shown, on an expanded scale, in Fig. 3 in comparison with the values from the quarkonia and lattice data of Fig. 1. The dashed line in Fig. 3 corresponds to the value $\alpha_3(M_Z) = 0.108$ which is the 1σ lower limit in the fit of Kobel [18]. In that fit the α_3 values from J/ψ and Υ are moved above the dashed line by relativistic corrections ($C \approx 3$). If the radiative-to-gluonic-branching ratios are indeed insensitive to relativistic corrections, it is clear from Fig. 3 that the value of α_3 from the GGG decay of the Υ cannot rise by as much as would be suggested by a value $C = 3$ in Eq. (10). In other words, the value of $C \sim 3$ required to reconcile the J/ψ with the Υ hadronic decay data is disfavored at the 1σ level by the Υ radiative decay. Similarly the data from the J/ψ and η_c radiative decays, the fourth and fifth rows in Table I, do not support as large an increase in $\alpha_3(m_c)$ as would be implied by the linear fit. The problem, in fact, goes beyond the linear fit. As one can see from Fig. 3, the discrepancies cannot be resolved by any relativistic corrections to the three-gluon decays of J/ψ and Υ which, as expected, do not affect the radiative branching ratios in Table I. The best one could hope for in this approach would be for the relativistic corrections to increase the α_3 value from the hadronic J/ψ decays while decreasing that from the hadronic Υ decays. This could be done by adding a quadratic term to the phenomenological relativistic correction of Eq. (10). The resulting value of $\alpha_3(M_Z)$ would then be close to Eq. (7) although the minimum χ^2 per degree of freedom in such a fit including the radiative decays would still be unacceptably large. As a final note of caution concerning the linear parametrization one should note that, since in such a fit one can trivially reconcile the J/ψ data with the Υ data (ignoring the radiative data), it begs the question of whether there is some nonstandard physics in the low-energy region.

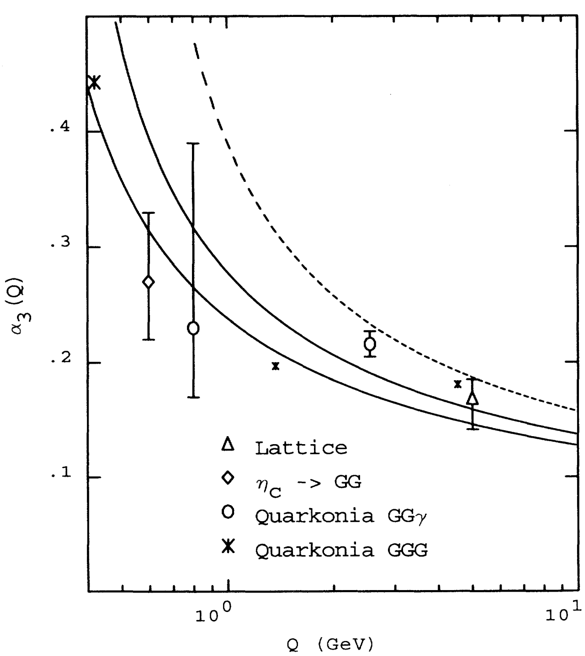

FIG. 3. Lattice and quarkonium data shown on an expanded scale. The radiative quarkonia decays are expected to be insensitive to relativistic corrections. The quarkonia values shown correspond to the α_3^0. See text.

A mundane resolution of the puzzle might be to assume that the relativistic corrections also affect the radiative branching ratios in Table I, but a sufficiently large effect would be extremely surprising. At present therefore one should not rule out a "new physics" resolution. On the other hand, the fit with a light gluino by itself did not succeed in totally reconciling the J/ψ $1S$ data with the Υ. An acceptable fit including the J/ψ $1S$ data was found in [10] only by adding in quadrature to the experimental error an extra 10% as a concession to the seeming necessity for some relativistic corrections. We therefore consider a combination of gluino and relativistic corrections writing for the effective coupling for the Υ and J/ψ gluonic decays:

$$\alpha_3^0(\mu) = \alpha_3(\mu)\left(1 + \frac{3}{\pi}\alpha_3(\mu)R(m_{\tilde{G}}/M_q)\right)^{1/3} f_{r,q}^{1/3} .$$

One could write a quadratic form for the relativistic correction factors:

$$f_{r,q} = 1 - C_1\langle v^2/c^2\rangle - C_2\langle v^2/c^2\rangle^2 . \tag{14}$$

But since we have two quark systems and two parameters, this is equivalent to allowing arbitrary separate relativistic corrections to the J/ψ and Υ gluonic decay rates. In the following, therefore, we will allow $f_{r,c}$ and $f_{r,b}$ to float separately. For the Υ radiative decay we will assume no relativistic correction and write an effective coupling

$$\alpha_3^0(\mu) \equiv \frac{36\alpha e_q^2}{5}\frac{\Gamma(V \to GGG)}{\Gamma(V \to GG\gamma)} = \alpha_3(\mu)\left(1 + \frac{1}{\pi}\alpha_3(\mu)R(m_{\tilde{G}}/M_q)\right) . \tag{15}$$

For the η_c hadronic to photonic branching ratio we write

$$\alpha_3^0(\mu) \equiv \left(\frac{9\alpha^2 e_q^4}{2}\frac{\Gamma(\eta_c \to GG)}{\Gamma(\eta_c \to \gamma\gamma)}\right)^{1/2}$$
$$= \alpha_3(\mu)\left(1 + \frac{2}{\pi}\alpha_3(\mu)R(m_{\tilde{G}}/M_q)\right)^{1/2} . \quad (16)$$

The coefficient of the gluino correction is taken, as an approximation, to be proportional to the difference in the numerator and denominator processes of the number of zeroth-order gluons each of which can split into a gluino pair. In these equations α_3^0 is the uncorrected result of Table I while α_3 is the resulting value of the strong-coupling constant in the presence of gluino and relativistic corrections. For each value of $m_{\tilde{G}}$ there is a narrow band of $f_{r,c}$ and $f_{r,b}$ values that will bring the data into reasonable agreement. We have scanned the space of $m_{\tilde{G}}$, $f_{r,c}$, and $f_{r,b}$ in a Monte Carlo simulation to find these allowed solutions. We require consistency between the J/ψ and Υ GGG decay data as well as consistency with the $\Upsilon \to GGG/GG\gamma$ and $\eta_c \to GG/\gamma\gamma$ branching ratios which are taken to be independent of relativistic corrections. We also require that the relativistic correction to the Υ decay be no larger than that to J/ψ decay since this correction goes to zero for sufficiently large quark mass. Because of its large experimental uncertainty we do not use in the fit the J/ψ radiative branching ratio, the fourth row in Table I, although one can see from Fig. 3 that its inclusion would not qualitatively change our results. Instead we use a constraint from the lattice result.

We turn therefore to consider the effect of light gluinos on the lattice analysis [22]. In this work calculations are made in the zero-flavor (quenched) approximation and a correction factor $1 + 2n_f/33 \approx 1.24$ is applied to the resulting α_3 to account for the effect of quark loops. Actually we feel this is an overestimate since it is unlikely that the strange and charm quarks contribute equally with the light flavors. We would find it preferable to replace n_f by the mass-dependent β-function contribution

$$n_f = \sum_{i=1}^{4} n(m_i, m_b) \; , \quad (17)$$

where [23]

$$n(m_i, Q) = 1 - 6m_i^2/Q^2 + 24(m_i^4/Q^4)\tfrac{1}{w}\mathrm{arccoth}(w) \quad (18)$$

with

$$w = \sqrt{1 + 4m_i^2/Q^2} \; . \quad (19)$$

This has the effect of reducing the lattice value of $\alpha_3(m_b)$ by about 2%. Similarly the effect of light gluinos can be incorporated by writing

$$\alpha_3(5\text{ GeV}) = (0.169^{+0.017}_{-0.027})$$
$$\times\{1 + 2[n_f + 3n(m_{\tilde{G}}, m_b)]/33\}/1.24 \quad (20)$$

where we have supplied the color-statistics factor of 3 to relate the effect of an octet of color gluinos to that of a single quark flavor. The lattice value of α_3 increases with decreasing gluino mass while the Υ value decreases. The result is that they are consistent only for a very restricted range of gluino masses $m_{\tilde{G}} < 2$ GeV.

Thus, in our Monte Carlo simulation, we vary four parameters $\alpha_3(M_Z)$, $m_{\tilde{G}}$, $f_{r,c}$, and $f_{r,b}$. For each set of these four parameters we calculate the χ^2 per degree of freedom for the five data points represented by the lattice result and the processes of Table I excluding the J/ψ radiative decay. In the running of the coupling constant the contribution of the gluino is decoupled for values of $Q < m_{\tilde{G}}$. Figure 4 shows a scatter plot of $m_{\tilde{G}}$ versus $\chi^2/5$ with the $\alpha_3(M_Z)$ values indicated by shape coding. Solutions are found with $0.1047 < \alpha_3(M_Z) < 0.1231$. Values of $\alpha_3(M_Z)$ in the lower or higher half of this range are indicated in Fig. 4 by squares or $\times$'s, respectively. The plotted points are a random nonoverlapping subset from a random sample of 5000 solutions. The figure represents all possible values of $f_{r,b}$ and $f_{r,c}$ with $0.1 < m_{\tilde{G}} < 5$ GeV and a resulting $\chi^2/5 < 3$. Values of $m_{\tilde{G}}$ above 5 GeV have $\chi^2/5$ above 2.2 and lower values of $\alpha_3(M_Z)$. For $m_{\tilde{G}} > M_Z$ this minimum $\chi^2/5$ corresponds to $\alpha_3(M_Z) = 0.0985 \pm 0.0015$ which overlaps well with the result of Eq. (7). As can be seen from Fig. 4, a $\chi^2/5 < 1$ requires a gluino mass below 0.7 GeV. In Fig. 5, we show the values of α_3 (1.36 GeV) as a function of the gluino mass with the $\chi^2/5$ values indicated in the shape coding. Solutions are found with $\chi^2/5$ down to 0.54 and the range from this value to 3.0 is divided into a lower and upper half represented in Fig. 5 by squares or $\times$'s, respectively. The solutions shown are a random nonoverlapping subset of the 5000 events used also in Fig. 4. The best solutions ($\chi^2/5 < 1.0$) correspond to relativistic correction factors $0.45 < f_{r,c} < 0.78$ for charmonium and $0.83 < f_{r,b} < 1.17$ for bottomonium. These cor-

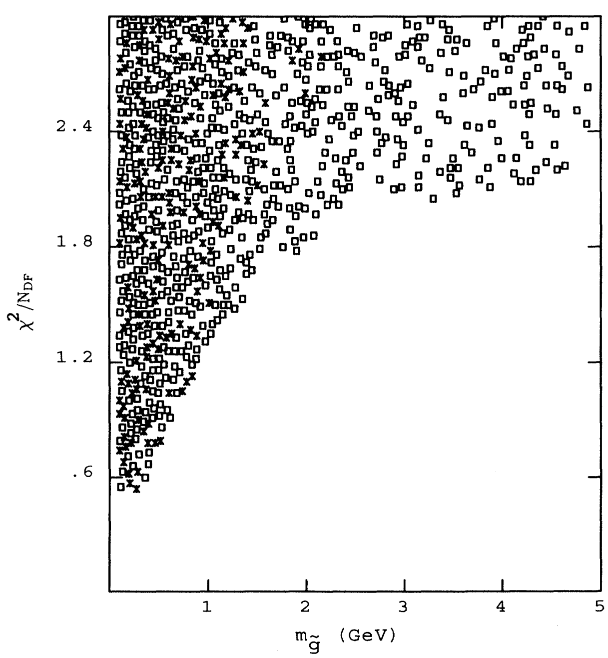

FIG. 4. Correlation between the gluino mass and the $\chi^2/5$ for the data of Fig. 3 excluding the Φ hadronic decay data and the J/ψ radiative decay. The shape coding indicates the resulting value of $\alpha_3(M_Z)$ in each solution. See text.

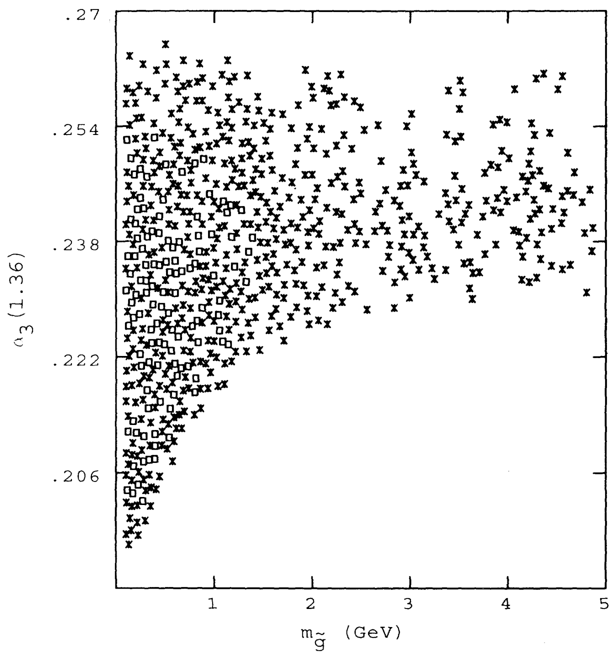

FIG. 5. Correlation between the gluino mass and the resulting α_3 (1.36 GeV) allowing arbitrary relativistic corrections to the J/ψ and Υ hadronic decays. The shape coding indicates the $\chi^2/N_{\rm DF}$. Solutions with $\chi^2/5 < 1.77$ are indicated by squares. See text.

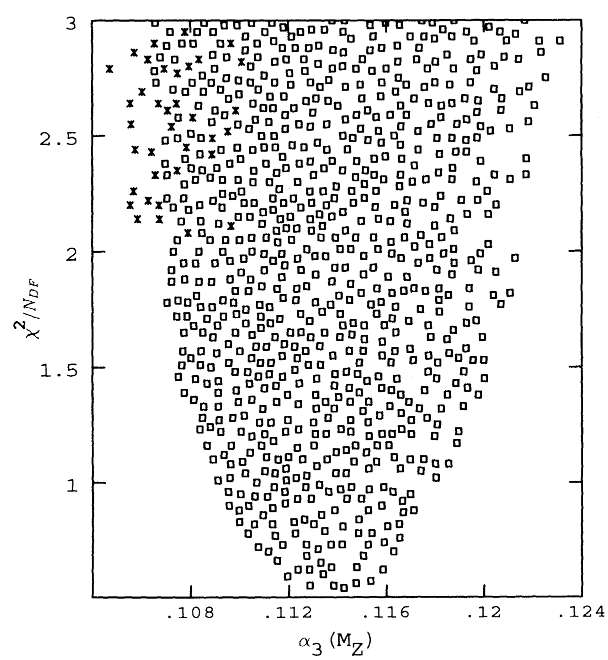

FIG. 6. Correlation between $\alpha_3(M_Z)$ and the $\chi^2/N_{\rm DF}$ values for $m_{\tilde G} < 5$ GeV. The solutions are plotted as squares or $\times$'s if the gluino mass is in the lower or upper half, respectively, of the range from 0.1 to 5 GeV. Values of $m_{\tilde G}$ above this range have $\chi^2/5 > 2.2$ and lower values of $\alpha_3(M_Z)$ (see text).

rection factors are significantly more moderate (closer to unity) than those required in the fit of [18] ($f_{r,c} < 0.3$). However, if one makes a quadratic fit in the quark velocities to these correction factors one still finds fairly large coefficients C_1 and C_2. This may suggest that the relativistic corrections are not such simple functions of the quark velocities. For $m_{\tilde G} < 5$ GeV, Fig. 6 shows the resulting $\chi^2/5$ values as a function of $\alpha_3(M_Z)$. The preferred solutions have $\alpha_3(M_Z) = 0.114 \pm 0.005$ with $m_{\tilde G} < 0.7$ GeV. Solutions with $m_{\tilde G} > 2.55$, plotted in Fig. 6 as $\times$'s illustrate the tendency of a heavier gluino to worsen the χ^2 and shift the $\alpha_3(M_Z)$ values to a lower range.

IV. α_3 FROM τ DECAY

The constraints on α_3 from τ decay are often quoted in support of an $\alpha_3(M_\tau) \approx 0.33$ which would suggest standard QCD running between M_τ and M_Z [24,25] and hence the absence of a light gluino. Such a large value of $\alpha_3(M_\tau)$ would be in conflict with the large body of low-energy data shown in Figs. 1 and 3. In fact, there are at least three ways to measure the strong coupling in τ decay and the current anomalies in the world average data can be interpreted in terms of an inconsistency in the α_3 values deduced from these various measurements [26]. Thus, on very general grounds, the electronic branching ratio $B_\tau(e)$, the τ lifetime τ_τ, and the τ single hadron branching ratio $B_\tau(h)$ are all related by

$$\tau_\tau = B_\tau(e)\tau_\mu\left(\frac{M_\mu}{M_\tau}\right)^5 , \tag{21}$$

$$B_\tau(h) = \frac{\tau_\tau}{2M_\mu^2 M_\tau}\left(\frac{M_\pi^3 B_\pi(\mu)(M_\tau^2 - M_\pi^2)}{\tau_\pi(M_\pi^2 - M_\mu^2)^2} + \frac{M_K^3 B_K(\mu)(M_\tau^2 - M_K^2)^2}{\tau_K(M_K^2 - M_\mu^2)^2}\right) . \tag{22}$$

Each of these three quantities can be used as a measure of the strong-coupling constant through the theoretical expression

$$B_\tau(e) = \{1.973 + 3.0582[1 + \delta_{\rm pert} + \delta_{\rm np}]\}^{-1} . \tag{23}$$

The perturbative contribution is usually written

$$\delta_{\rm pert} = \left(\frac{\alpha_s}{\pi}\right) + 5.2023\left(\frac{\alpha_s}{\pi}\right)^2 + 26.366\left(\frac{\alpha_s}{\pi}\right)^3 + \delta_{\rm HO} \tag{24}$$

and usually $\alpha_s(M_\tau)$ is evaluated from the electronic branching ratio of the τ with $\delta_{\rm HO}$ taken to be zero or negative and $\delta_{\rm np}$ (the nonperturbative contribution) estimated to be ≈ -0.007 [24]. This leads to the quoted result for $\alpha_3(M_\tau)$ above 0.3. However, in view of the large quadratic and cubic terms in Eq. (24), it seems

TABLE II. τ decay data and the suggested values of the strong-coupling constant assuming a gluino above the Z.

	World ave.	ALEPH
$B_\tau(e)$	0.1776 ± 0.0015	0.1820 ± 0.0035
$\alpha_3(M_\tau)$	0.305 ± 0.013	0.264 ± 0.034
τ_τ	295.7 ± 3.2 ps	$294.7 \pm 5.4 \pm 3.0$ ps
$\alpha_3(M_\tau)$	0.274 ± 0.018	0.277 ± 0.035
$B_\tau(h)$	0.1279 ± 0.0029	0.1281 ± 0.0034
$\alpha_3(M_\tau)$	0.243 ± 0.041	0.231 ± 0.059
ave. $\alpha_3(M_\tau)$	0.285 ± 0.006	0.262 ± 0.013
ave. $\alpha_3(M_Z)$	0.1095 ± 0.0008	0.1063 ± 0.0020

more reasonable to us to write

$$\delta_{\rm pert} = \left(\frac{\alpha_s}{\pi}\right)\left[1 - 5.202\left(\frac{\alpha_s}{\pi}\right)\right]^{-1} - 0.69\left(\frac{\alpha_s}{\pi}\right)^3 + \delta_{\rm HO} \tag{25}$$

with the residual $\delta_{\rm HO}$ assumed negligible. This amounts to assuming that the higher-order corrections fall off approximately as a geometric series as suggested by the first two known corrections. Equations (25) and (24) are equivalent up to the known order. In addition, we neglect $\delta_{\rm np}$ since its current estimate is comparable to unknown fourth-order perturbative contributions and the theory of nonperturbative corrections is still highly model dependent. In Table II we show the values of the three related τ decay parameters in both the world average data where discrepancies persist and in the ALEPH data where consistency is obtained although the errors are larger. In each case we calculate α_3 using Eq. (25) neglecting $\delta_{\rm np}$ and $\delta_{\rm HO}$. The weighted means are also given. The data in Table II are taken from reviews at the Second Workshop on τ Lepton Physics [27,28].

The inconsistencies in the world average data can be gauged by the discrepancies in the values of α_3 deduced from the three measurements. Until these discrepancies are resolved the small error in the average α_3 may not be reliable. The ALEPH data is more self-consistent and therefore its weighted average α_3 may be more meaningful. Extrapolated to 1.36 GeV the resulting value of α_3 is 0.300 ± 0.018 in the case of a heavy gluino and 0.289 ± 0.016 in the case of a light gluino. These are two or three standard deviations from the value predicted by good fits to the quarkonium and lattice data as can be seen from Fig. 5. The τ data could be brought into one standard deviation agreement if the nonperturbative contribution were $\delta_{\rm np} \approx +0.007$ which is small compared to the perturbative contribution.

V. CONCLUSIONS

The current analysis suggests two consistent values of $\alpha_3(M_Z)$ depending on whether or not the gluino is light. These are

$$\text{standard QCD}: \quad \alpha_3(M_Z) = 0.097 \pm 0.003\ , \tag{26}$$

$$\text{light gluino}: \quad \alpha_3(M_Z) = 0.114 \pm 0.005\ . \tag{27}$$

The first result is that of Eq. (7) and Fig. 1. As discussed above it does not have a good χ^2. The result in the case of a light gluino corresponds to the minimum χ^2 of Fig. 6. Both of these are inconsistent with minimal supersymmetric unification predictions (with supergravity inspired soft SUSY breaking) [8,16] which prefers significantly higher values. We will come back to this in a subsequent publication. An important question, of course, is what is the likelihood that the actual value of the strong-coupling constant in standard QCD is significantly greater than Eq. (26). Many higher values of α_3 have in fact been reported from jet studies including some quoted in [10] in support of a light gluino but all of these depend on large model-dependent hadronization corrections. Recent LEP values from the Z width are consistent at the 1σ level with both Eqs. (26) and (27) [2]. It is of course possible that, in the future, better understanding of the bound-state corrections to the quarkonia widths and refinements in the lattice calculations may lead to somewhat larger values of $\alpha_3(M_Z)$ but it is unlikely that such effects could lead to consistency between low-energy physics and minimal SUSY unification.

We should mention at this point other analyses of the strong-coupling constant. A somewhat more physical model for the bound-state corrections [29] taking into account one of three possible effects has recently obtained limited upward corrections to α_3. Their results are that, in standard QCD,

$$\text{from the } J/\psi \text{ width}: \quad \alpha_3(M_Z) = 0.1056 \pm 0.0013\ , \tag{28}$$

$$\text{from the } \Upsilon \text{ width}: \quad \alpha_3(M_Z) = 0.110 \pm 0.002\ . \tag{29}$$

The fact that the value of $\alpha_3(M_Z)$ from the J/ψ lies below that from the Υ continues to support the possibility of a light gluino.

Comparable numbers have also come from analysis of deep-inelastic scattering [30]: namely,

$$\text{deep inelastic}: \quad \alpha_3(M_Z) = 0.108 \pm 0.002\ . \tag{30}$$

These authors find that the deep-inelastic data in the presence of a light Majorana gluino requires $\alpha_3(M_Z) = 0.124 \pm 0.001$. This figure, although in agreement with the published LEP analyses leading to Eq. (3), is inconsistent with the jet data and lattice results. The deep-inelastic data, if they are currently correctly interpreted, may therefore rule out a gluino in the 5-GeV mass region. It is not clear whether the effects of an ultralight gluino (below 0.7 GeV) on the deep-inelastic data have been thoroughly analyzed.

A number similar to Eq. (30) comes, in the heavy gluino case, from the Brodsky-Lepage-Mackenzie (BLM) [31] proposal for choice of scale:

$$\text{BLM scale choice}: \quad \alpha_3(M_Z) = 0.107 \pm 0.003\ . \tag{31}$$

Similarly, recent lattice work [32] might point to an increase in α_3, the new preliminary values being about $\alpha_3(M_Z) = 0.110 \pm 0.004$, two standard deviations from the result of [5]. It is clear from [18] that further analysis of the dependence on potential models in the lattice calculations may be needed to resolve a possible discrepancy between the lattice values and the values from the jet mass difference and the energy-energy asymmetry. For the present we regard Eqs. (26) and (27) as the best estimates from low-energy data of the value of $\alpha_3(M_Z)$ although the current situation is such that an ultimate two to three standard deviation departure from those values would not be totally surprising. This would not affect our major conclusions which are as follows.

(1) Current ideas about relativistic corrections in quarkonium decays in the standard model still leave a $2\sigma\sim 3\sigma$ discrepancy between values of the strong-coupling constant from J/ψ and Υ decay. This discrepancy could be resolved by further progress in understanding relativistic corrections or by new physics such as a light gluino.

(2) If there is no new physics in the low-energy region, the large body of low-energy data that is free of obvious, large, model-dependent effects when taken as a whole prefers values of $\alpha_3(M_Z)$ below 0.11 and perhaps below 0.10.

It is to be hoped that the various analyses will soon come to the point where the data will be clearly inconsistent with either a heavy or a light gluino.

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