CP-violating polarizations in semileptonic heavy meson decays

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We study the T-violating lepton transverse polarization ($P_l^\perp$) in three body semileptonic heavy meson decays to pseudoscalar mesons and to vector mesons. We calculate these polarizations in the heavy quark effective limit, which simplifies the expressions considerably. After examining constraints from CP-conserving (including $b \to s\gamma$) and CP-violating processes, we find that in B decays $P^\perp$ of the muon in multi-Higgs-doublet models can be of order 13%, while $P^\perp$ of the τ can even approach unity. In contrast, $P_\mu^\perp$ in D decays is at most 1.5%. We discuss possibilities for detection of $P_l^\perp$ at current and future B factories. We also show that $P_l^\perp$ in decays to vector mesons, unlike in decays to pseudoscalars, can get contributions from left-right models. Unfortunately, $P_l^\perp$ in that case is proportional to W_L-W_R mixing, and is thus small.

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I. INTRODUCTION

The standard model (SM) has thus far met with incredible experimental success. Nevertheless, many hypothetical extensions to the SM remain phenomenologically viable. Since new physics often provides new sources of CP violation (CPV), one good way to search for such extensions is to consider CP-violating observables which are negligible in the SM, but which can have large contributions from other sources of CPV.

A major barrier to any candidate for such an observable is the upper bound on the electric dipole moment of the neutron, d_n, which is now around $10^{-25}e\,\mathrm{cm}$ [1]. The SM explanation for CPV, the Cabbibo-Kobayashi-Maskawa (CKM) mechanism [2], has come to be accepted by many as the source of CP violation in the neutral K sector not only because it predicts ϵ to be in the right range, but also because it predicts d_n to be negligible [3]. As the upper bound on d_n has plummeted, many potential explanations for ϵ from other sources have run aground, and thus it is more difficult to find observables which have good prospects of detecting CPV beyond the SM.

One such observable is the transverse polarization of the lepton in semileptonic $K_{\mu 3}$ decays [4], $P_l^\perp$, which is the T-violating projection of the lepton spin onto the normal of the decay plane, i.e., $P_l^\perp \sim \mathbf{s}_l \cdot (\mathbf{k} \times \mathbf{p})$ [5], where $\mathbf{k}$ and $\mathbf{p}$ are decay product momenta. It arises from the interference between two amplitudes with nonzero relative phase. In practice, one measures the asymmetry between the number of particles parallel and antiparallel to the normal of the decay plane:

$$P_l^\perp = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}. \tag{1}$$

There are several advantages to using such a semileptonic CP-violating observable. First, semileptonic decays occur through a single SM diagram at the tree level, so that CP-violating contributions to $P_l^\perp$ are negligible in the SM [6]. Second, final state interactions in charged semileptonic meson decays are negligible. Thus a nonzero $P_l^\perp$ in such decays is a signal for new physics. Third, theoretical uncertainties in semileptonic decays are much smaller than in purely hadronic decays. Finally, $P_l^\perp$ in semileptonic decays comes from both the quark and lepton sectors, so that purely hadronic or purely leptonic CP-violating observables, such as d_n or d_e, do not necessarily strongly constrain $P_l^\perp$ [7]. In fact, there exist reasonable models for which $P_l^\perp$ in $K_{\mu 3}$ decays can be of the order of 10^{-2}–10^{-3}, consistent with all other constraints [8, 9]. Such values are well within reach of experiments. The last measurements of $P_l^\perp$ were done at Brookhaven National Laboratory on $K^+ \to \pi^0\mu^+\nu_\mu$ decays. Their combined result was [10]

$$P_\mu^\perp\left(K^+ \to \pi^0\mu^+\nu_\mu\right) = -1.85 \pm 3.60 \times 10^{-3}, \tag{2}$$

which implies a 95% confidence upper bound of about 0.9%. There is also an experiment currently under construction at KEK [11] which hopes to push this bound down by a factor of 10 [12].

In this paper we consider $P_l^\perp$ in heavy meson decays of the type $M \to m\,l\,\nu_l$ and $M \to m^*\,l\,\nu_l$, where M and m are pseudoscalar mesons, and m^* is a vector meson. $P_l^\perp$ has been studied in decays to pseudoscalars [13, 14], but not in decays to vector mesons. We derive expressions for $P_l^\perp$ in $M \to m^*\,l\,\nu_l$ decays, as well as in $M \to m\,l\,\nu_l$ decays, in the heavy quark effective limit. This greatly simplifies our results. One can even obtain analytic expressions for $\overline{P_l^\perp}$, the polarization averaged over all kinematical variables.

In decays to pseudoscalars ($M \to m\,l\,\nu_l$), $P_l^\perp$ is sensitive only to spin-0 effective Lagrangians [4, 15], which makes it a good tool for probing non-SM Higgs physics [8]. We find that this holds for $P_l^\perp$ in decays to *longitudinally polarized* vector mesons ($M \to m_L^*\,l\,\nu_l$), but that $P_l^\perp$ in decays to *transversely polarized* vector mesons

($M \to m^*_{T1}\, l\, \nu_l$ and $M \to m^*_{T2}\, l\, \nu_l$) is sensitive only to new V and A physics, such as left-right models. Unfortunately, $P_l^\perp$ in that case is proportional to W_L-W_R mixing, which is constrained to be small. However, in the former case, multi-Higgs-doublet models yield encouraging results, even after imposing CP-conserving and CP-violating constraints. There are reasonable models in which $P_\tau^\perp$ in $B \to D\tau\nu$ decays can even approach unity.

Section II lists the form factors needed for our calculation. We consider contributions to $P_l^\perp$ from multi-Higgs-doublet models in Sec. III, and from left-right models in Sec. IV. Possibilities for detecting $P_l^\perp$ in various decay modes are discussed in Sec. V.

II. FORM FACTORS

From Lorentz invariance and basic symmetry considerations, we can write the hadron matrix elements (HME's) for decays of pseudoscalar mesons (M) to pseudoscalar (m) and vector mesons (m^*) as

$$\begin{aligned}
\langle m(k)|\, V_\mu\, |M(K)\rangle &= f_+\,(K+k)_\mu + f_-\,(K-k)_\mu\,,\\
\langle m(k)|\, A_\mu\, |M(K)\rangle &= 0,\\
\langle m^*(k,\varepsilon^*_\lambda)|\, V_\mu\, |M(K)\rangle &= \frac{iV_1}{M}\left(\epsilon_{\mu\alpha\beta\gamma}\, \varepsilon^{*\,\alpha}_\lambda K^\beta k^\gamma\right),\\
\langle m^*(k,\varepsilon^*_\lambda)|\, A_\mu\, |M(K)\rangle &= A_1 M \varepsilon^*_{\lambda\mu} + \frac{A_2}{M}(\varepsilon^*_\lambda\cdot K)(K+k)_\mu\\
&\quad + \frac{A_3}{M}(\varepsilon^*_\lambda\cdot K)(K-k)_\mu, \qquad (3)
\end{aligned}$$

where, for M^+ decay, $V_\mu = \bar{D}\gamma_\mu U$ and $A_\mu = \bar{D}\gamma_\mu\gamma^5 U$ (U and D are the appropriate up- and down-type quarks for M and m). The axial vector HME for $M \to m$ is zero because there is no way to form an axial vector with just K^α and k^β. We have used M and m to represent both a meson and its mass. The form factors $f_\pm$, V_1, and A_{1-3} are functions of $(K\cdot k)$ and $r \equiv m/M$. Here λ is the polarization index. We will refer to the m^* longitudinal polarization by the label $\lambda = L$, and the two transverse polarizations by the label $\lambda = T_1,\ T_2$, for $\boldsymbol{\varepsilon}_\lambda$ in the decay plane and perpendicular to the decay plane, respectively.

From these vector and axial vector HME's, one can derive scalar and pseudoscalar HME's using the Dirac equation [13]

$$\begin{aligned}
\langle m(k)|\, S\, |M(K)\rangle &= \frac{-M^2}{m_D - m_U}\left[f_+(1-r^2) + f_- t\right],\\
\langle m(k)|\, P\, |M(K)\rangle &= 0,\\
\langle m^*(k,\varepsilon^*_\lambda)|\, S\, |M(K)\rangle &= 0,\\
\langle m^*(k,\varepsilon^*_\lambda)|\, P\, |M(K)\rangle &= \frac{-M}{m_D + m_U}(\varepsilon^*_\lambda\cdot K)[A_1\\
&\quad + A_2(1-r^2) + A_3 t], \qquad (4)
\end{aligned}$$

where for M^+ decays, $S = \bar{D}U$, and $P = \bar{D}\gamma^5 U$, and $t \equiv (K-k)^2/M^2$. The masses (m_D, m_U) are (m_b, m_c) in B decays and (m_s, m_c) in D decays. The middle two parity-odd matrix elements in (4) are zero because there is no way to form a pseudoscalar using only K^α, k^β, and $\varepsilon_\lambda{}^\gamma$. Note that the factor $(\varepsilon^*_\lambda\cdot K)$ implies that $\langle m^*(k,\varepsilon^*_\lambda)|\, P\, |M(K)\rangle$ is nonzero only for longitudinally polarized vector mesons.

Recently there has been a lot of interest in *heavy quark effective theory* (HQET), which considers the limit M, $m \to \infty$. The principal tenet of HQET is that v_μ (v'_μ), the four-velocity of M ($m^{(*)}$), is unchanged by QCD corrections [16]. Thus it makes sense to write the HME's in terms of velocity [17]:

$$\begin{aligned}
\langle m(v')|\, V_\mu\, |M(v)\rangle &= \sqrt{Mm}\,[\xi_+(v+v')^\mu\\
&\quad + \xi_-(v-v')^\mu]\,,\\
\langle m(v')|\, A_\mu\, |M(v)\rangle &= 0,\\
\langle m^*(v',\varepsilon^*_\lambda)|\, V_\mu\, |M(v)\rangle &= i\sqrt{Mm}\,\xi_{V_1}\left(\epsilon_{\mu\alpha\beta\gamma}\,\varepsilon^{*\,\alpha}_\lambda v'^\beta v^\gamma\right),\\
\langle m^*(v',\varepsilon^*_\lambda)|\, A_\mu\, |M(v)\rangle &= \sqrt{Mm}[\xi_{A_1}(1+v\cdot v')\varepsilon^*_{\lambda\mu}\\
&\quad - \xi_{A_2}(\varepsilon^*_\lambda\cdot v)v_\mu\\
&\quad - \xi_{A_3}(\varepsilon^*_\lambda\cdot v)v'_\mu]. \qquad (5)
\end{aligned}$$

From (3) and (5), one can derive relations between the form factors [18]:

$$\begin{aligned}
f_\pm &= \pm\frac{1}{2\sqrt{r}}\left((1\pm r)\xi_+ - (1\mp r)\xi_-\right) \;\to \pm\frac{1\pm r}{2\sqrt{r}}\xi,\\
V_1 &= -\frac{1}{\sqrt{r}}\xi_{V_1} \;\to -\frac{1}{\sqrt{r}}\xi,\\
A_1 &= \frac{x+r}{\sqrt{r}}\xi_{A_1} \;\to \frac{x+r}{\sqrt{r}}\xi,\\
A_2 &= -\frac{1}{2\sqrt{r}}\left(\xi_{A_3} + r\xi_{A_2}\right) \;\to -\frac{1}{2\sqrt{r}}\xi,\\
A_3 &= +\frac{1}{2\sqrt{r}}\left(\xi_{A_3} - r\xi_{A_2}\right) \to +\frac{1}{2\sqrt{r}}\xi, \qquad (6)
\end{aligned}$$

where $x \equiv (K\cdot k)/M^2 = r\, v\cdot v'$. In the M, $m \to \infty$ limit, $\xi_+ = \xi_{V_1} = \xi_{A_1} = \xi_{A_3} = \xi$ and $\xi_- = \xi_{A_2} = 0$, so that all the form factors can be written in terms of the Isgur and Wise function, $\xi(x)$ [19]. Note that the HME's are normalized, and so $\xi(x)$ is equal to 1 at zero recoil ($x = r$ or $v\cdot v' = 1$) [20]. Specific forms for $\xi(x)$ are listed in the Appendix.

III. HIGGS MODELS

A. Transverse polarization

As we said, semileptonic pseudoscalar decays to pseudoscalar mesons, $M \to m\, l\, \nu$, and to longitudinally polarized vector mesons, $M \to m^*_L\, l\, \nu$, can arise only from the interference of new scalar physics with the SM. In this section, we consider contributions to $P^\perp(M \to m\, l\, \nu_l)$ and $P^\perp(M \to m^*_L\, l\, \nu_l)$ from models with charged Higgs scalars. Other types of contributions are possible, such as from scalar leptoquarks [4, 8, 9].

Lee first proposed that CP could be violated via phases in a model with two Higgs doublets [21]. This idea was refined by Weinberg with the elimination of flavor-changing neutral currents (FCNC's) in a model with three Higgs doublets, using a symmetry to ensure that only one Higgs doublet couples to each right-handed fermion field, what is commonly referred to as natural flavor conservation

(NFC) [22]. There are various other ways to avoid the FCNC problem [23,24], but for simplicity, we concentrate on models where NFC is either exact or partially broken [25]. We will assume that the CKM phase is nonzero, and so we do *not* impose strong constraints on CPV in the Higgs sector from ϵ. Even if CP is broken only spontaneously, a nonzero CKM phase can arise after integrating out superheavy fields, and so we see no reason to take it zero.

We are interested in the interference of a charged Higgs boson with the SM W boson, and so one need only parametrize an effective Lagrangian for the charged Higgs coupling to fermions. In a model with N charged scalar fields, one obtains a Lagrangian in terms of the $N-1$ physical charged Higgs bosons [13]:

$$-\mathcal{L}_{H^+} = \frac{1}{v}\sum_{i=1}^{N-1}\Big[\alpha_i \bar{U}_L V_L M_D D_R H_i^+ + \beta_i \bar{U}_R M_U V_L D_L H_i^+ + \gamma_i \bar{N}_L M_E E_R H_i^+\Big] + \text{H.c.} \tag{7}$$

Here v is the SM Higgs vacuum expectation value (VEV), $v = (4G_F/\sqrt{2})^{-1/2} \simeq 174\text{GeV}$; U, D, N, and E are fields for the up quarks $[U^T = (u\ c\ t)]$, down quarks, neutrinos, and charged leptons; M_D, M_U, and M_E are the diagonal mass matrices; and V_L is the CKM matrix.

If the coefficients α_i, β_i, and γ_i are complex, the interference between the charged Higgs and W boson amplitudes in Fig. 1 produces a T-violating transverse polarization of the lepton. Since the H^+ amplitude is proportional to the matrix elements in (4), one gets contributions to $P_l^\perp(M \to m^* l\nu)$ only for decays in which the m^* is longitudinally polarized. This means that if one can veto decays with transversely polarized m^*'s, the denominator in (1) will be reduced while the numerator will remain unchanged, leading to a larger polarization.

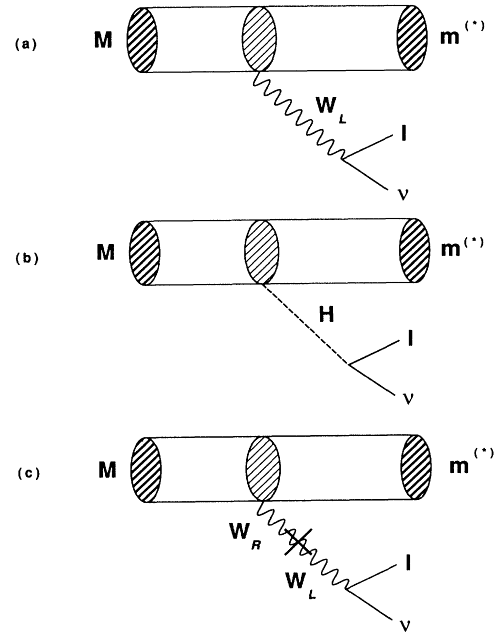

FIG. 1. Diagrams which contribute to $M \to m^{(*)} l\nu$ from (a) the SM W exchange, (b) charged Higgs boson exchange, (c) W_L-W_R mixing.

Let us evaluate $P_l^\perp$ in terms of the velocity-dependent form factors. Then we can take the heavy quark effective limit, which allows us to write $P_l^\perp$ with only one form factor $\xi(x)$. We calculate $P_l^\perp$ for semileptonic pseudoscalar decays to pseudoscalar mesons, to longitudinally polarized vector mesons, and to unpolarized vector mesons in this limit:

$$P_l^\perp(x)\left(M \overset{H^+}{\to} ml\nu\right) = C_{H^+}\frac{3\pi}{4}\frac{(1-r^2)(x+r)(x^2-r^2)\sqrt{t}}{(1+r)^2 x_1^3}\frac{\xi(x)^2}{\xi(x)^2}, \tag{8}$$

$$P_l^\perp(x)\left(M \overset{H^+}{\to} m_L^* l\nu\right) = C_{H^+}\frac{3\pi}{4}\frac{(1-r^2)(x+r)(x^2-r^2)\sqrt{t}}{(1-r)^2(x+r)^2 x_1}\frac{\xi(x)^2}{\xi(x)^2}, \tag{9}$$

$$P_l^\perp(x)\left(M \overset{H^+}{\to} m^* l\nu\right) = C_{H^+}\frac{3\pi}{4}\frac{(1-r^2)(x+r)(x^2-r^2)\sqrt{t}}{(1-r)^2(x+r)^2 x_1 + 4t(x+r)x x_1}\frac{\xi(x)^2}{\xi(x)^2}. \tag{10}$$

We list the full expressions with general form factors in the Appendix. Note that we have already integrated $P_l^\perp$ over one kinematical variable $[(K\cdot p)/M^2]$ so that $P_l^\perp$ is only a function of the remaining kinematical variable x [where $x = (K\cdot k)/M^2$ and $x_1 \equiv \sqrt{x^2-r^2}$]. This integration gives the factor $3\pi/4$ in (8)–(10). For $M^+ \to \bar{m}^{0\,(*)} l^+ \nu_l$ and $M^0 \to m^{-\,(*)} l^+ \nu_l$ decays, the new physics coefficient is given by

$$C_{H^+} = \frac{M m_l}{M_W^2}\sum_i^{N-1}\frac{M_W^2}{M_{H_i}^2}\left(\frac{m_D}{m_D \mp m_U}\text{Im}\alpha_i\gamma_i^* \pm \frac{m_U}{m_D \mp m_U}\text{Im}\beta_i\gamma_i^*\right), \tag{11}$$

while C_{H^+} for the CP conjugate decays has the opposite sign [26]. Here m_l, m_U, and m_D are the lepton and current quark masses specific to each decay, and M_{H_i} and the coefficients α_i, β_i, and γ_i come from the effective Lagrangian (7). The upper (lower) signs apply to $M \to ml\nu$ ($M \to m^* l\nu$) decays. Since $m_U > m_D$ in D decays, it follows that $P_l^{\perp}(D^+ \to \bar{K}^0 l^+ \nu)$ has the opposite sign as $P_l^{\perp}(K^+ \to \pi^0 l^+ \nu)$ in three-Higgs-doublet models. It also means that C_{H^+}'s in the decays to m^* are somewhat suppressed over those to m when m_D and m_U are of the same order, as in B decays.

We have neglected all lepton mass effects in the denominator of (8)–(10). For $l = \mu$, this is always a very small effect. In $l = \tau$ decays, it changes our results only qualitatively when $P_\tau^{\perp} \sim 1$, i.e., when H^+ effects are important in the denominator of (1). In that case, it might be possible to see new physics effects in changes to the branching ratio of $B \to D^{(*)} \tau\nu$.

To estimate the size of $P_l^{\perp}$ in various models, we must integrate over the remaining kinematical variable x. In an experiment, one generally measures the overall asymmetry in (1), rather than measuring $P_l^{\perp}(x)$ for each x and then averaging. So we must integrate the numerator and denominator of (8)–(10) separately:

$$
\begin{aligned}
\overline{P_l^{\perp}}\left(M \overset{H^+}{\to} ml\nu\right) &= C_{H^+}\frac{3\pi}{4}\frac{I_\perp}{I_S},\\
\overline{P_l^{\perp}}\left(M \overset{H^+}{\to} m_L^* l\nu\right) &= C_{H^+}\frac{3\pi}{4}\frac{I_\perp}{I_L},\\
\overline{P_l^{\perp}}\left(M \overset{H^+}{\to} m^* l\nu\right) &= C_{H^+}\frac{3\pi}{4}\frac{I_\perp}{I_L + I_T},
\end{aligned}
\tag{12}
$$

where $I_\perp$, I_S, I_L, and I_T are integrals of the kinematics in (8)–(10). Unfortunately, this means we must know something about the overall form factor $\xi(x)$. In the Appendix, we list two possible parametrizations for $\xi(x)$: a relativistic oscillator model and a monopole approximation. $\overline{P_l^{\perp}}$ in decays to pseudoscalars in these models differs by at most 15% for r in the region of interest ($r > 0.25$ for all the decays we study), and considerably less for decays to vector mesons. If we set the monopole

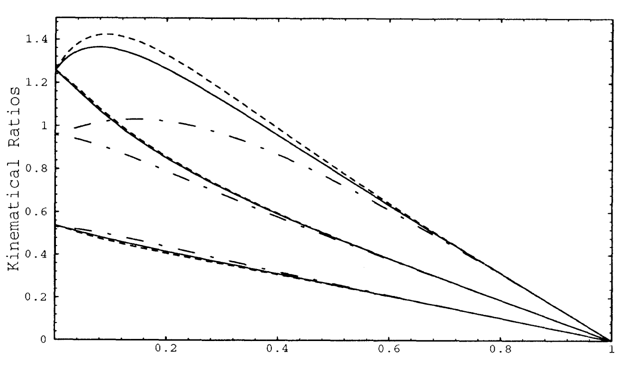

FIG. 2. $\overline{P_l^{\perp}}/C_{H^+}$ as a function of $r \equiv m/M$ for $\xi(x)$ given by the monopole approximation (A7) with $\rho = 1$ (solid lines), $\rho = 1.2$ (dashed lines), and where $\xi(x)^2$ is naively divided out (dash-dotted lines). The top, middle, and bottom sets of curves correspond to $M \to ml\nu$, $M \to m_L^* l\nu$, and $M \to m^* l\nu$ decays, respectively.

parameter ρ equal to 1 in (A7), we can obtain analytic expressions for $\overline{P_l^{\perp}}$ in terms of r. We list the corresponding I's in the Appendix. From Fig. 2, we see that choosing $\rho = 1$ instead of 1.2 (in order to obtain analytic expressions) changes $\overline{P_l^{\perp}}$ by only a few percent (for $r > 0.25$). Even naively dividing out $\xi(x)^2$ from the numerator and denominator of (8)–(10) gives results which (for $r > 0.25$) differ by 30%, or much less, from the other parametrizations of $\xi(x)$.

B. Constraints

For the purposes of placing constraints on $P_l^{\perp}$, we make two simplifying assumptions. First, we take the α_i, β_i, and γ_i to be flavor diagonal. This strictly holds only in models with NFC, and so Higgs models without NFC may have somewhat weaker, more model-dependent bounds [27]. Second, we will assume that the lightest charged Higgs boson mass eigenstate h^+ gives the dominant contribution, so that we can drop the subscript i on the coefficients α, β, and γ. In three-Higgs-doublet models (3HDM's), $\mathrm{Im}\alpha_1\gamma_1^* = -\mathrm{Im}\alpha_2\gamma_2^*$ and $\mathrm{Im}\beta_1\gamma_1^* = -\mathrm{Im}\beta_2\gamma_2^*$, and so in that case we are simply making the replacement $M_{H_1^+}^{-2} - M_{H_2^+}^{-2} \to M_{h^+}^{-2}$. This has virtually no effect on CP-violating constraints, because they have the same behavior, and the CP-conserving constraints tend to require a large splitting between $M_{H_1^+}$ and $M_{H_2^+}$ anyway.

We want to constrain C_{H^+}, which now depends upon $\mathrm{Im}\alpha\gamma^*$, $\mathrm{Im}\beta\gamma^*$, $M_W^2/M_{h^+}^2$, and the masses involved with M and $m^{(*)}$. In the general case (given our two assumptions), we can bound $\mathrm{Im}\alpha\gamma^* \, M_w^2/M_{h^+}^2$ directly from the experimental upper bound on $P_\mu^{\perp}(K^+ \to \pi^0 \mu^+ \nu_\mu)$ of 0.9% [10] to obtain [28]

$$
|\mathrm{Im}\alpha\gamma^*| \, \frac{M_W^2}{M_{h^+}^2} < 940. \tag{13}
$$

Since m_U is small, $P_\mu^{\perp}(K^+ \to \pi^0 \mu^+ \nu_\mu)$ is insensitive to $\mathrm{Im}\beta\gamma^*$. The best we can do is to use $|\mathrm{Im}\beta\gamma^*| < |\beta| \cdot |\gamma|$. From the bounds placed upon $|\beta|$ and $|\gamma|$ by [29], we obtain

$$
|\mathrm{Im}\beta\gamma^*| \, \frac{M_W^2}{M_{h^+}^2} < 160\frac{M_W}{M_{h^+}} < 285. \tag{14}
$$

From (14), one sees that the upper bound on $\mathrm{Im}\beta\gamma^* \, M_W^2/M_{h^+}^2$ decreases with increasing M_{h^+} and is at its maximum when M_{h^+} is at the model-independent lower bound of $M_Z/2$. We can use (13) and (14) in (12) to obtain upper bounds on $\overline{P_l^{\perp}}$ for various decays. Our results are summarized in the first column of Table I.

Let us now specialize to the case of 3HDM's. The CP-violating coefficients can be written [8]

$$
\begin{aligned}
\mathrm{Im}\alpha\gamma^* &= \tfrac{1}{2}\sin 2\theta_3 \, \sin\delta \, \tfrac{v v_u}{v_d v_e},\\
\mathrm{Im}\beta\gamma^* &= \tfrac{1}{2}\sin 2\theta_3 \, \sin\delta \, \tfrac{v v_d}{v_u v_e},\\
\mathrm{Im}\alpha\beta^* &= \tfrac{1}{2}\sin 2\theta_3 \, \sin\delta \, \tfrac{v v_e}{v_u v_d},
\end{aligned}
\tag{15}
$$

where v_u, v_d, and v_e give mass to the up quarks, down

TABLE I. Maximum values of the transverse polarization ($\overline{P_l^\perp}$) for various decay modes due to SM interference with charged Higgs bosons in the general case and in two specific models. K_L^{0*} (D_L^{0*}) refers to longitudinally polarized K^{0*}'s (D^{0*}'s). For the VE (VH) model, we use $\kappa < 1.2$ ($\kappa < 0.54$) as derived in the text. Numbers of order 1 are approximate since we neglect H^+ effects in the denominator of (1).

Decay	$\lvert\overline{P_l^\perp}\rvert$ General case	$\lvert\overline{P_l^\perp}\rvert$ VE model	$\lvert\overline{P_l^\perp}\rvert$ VH model
$K^+ \to \pi^0\,\mu^+\,\nu_\mu$	0.9%	1×10^{-5}	0.9%
$D^+ \to \bar{K}^0\,\mu^+\,\nu_\mu$	1.5%	4×10^{-5}	0.4%
$D^+ \to \bar{K}_L^{0*}\,\mu^+\,\nu_\mu$	0.54%	2×10^{-5}	0.13%
$D^+ \to \bar{K}^{0*}\,\mu^+\,\nu_\mu$	0.29%	1×10^{-5}	0.07%
$B^+ \to \bar{D}^0\,\mu^+\,\nu_\mu$	13%	2×10^{-4}	12%
$B^+ \to \bar{D}_L^{0*}\,\mu^+\,\nu_\mu$	4.1%	6×10^{-5}	3.7%
$B^+ \to \bar{D}^{0*}\,\mu^+\,\nu_\mu$	2.1%	3×10^{-5}	1.9%
$B^+ \to \bar{D}^0\,\tau^+\,\nu_\tau$	~ 1	3×10^{-3}	~ 1
$B^+ \to \bar{D}_L^{0*}\,\tau^+\,\nu_\tau$	$\sim 68\%$	1×10^{-3}	$\sim 62\%$
$B^+ \to \bar{D}^{0*}\,\tau^+\,\nu_\tau$	$\sim 35\%$	0.5×10^{-3}	$\sim 32\%$

quarks, and charged leptons, respectively. θ_3 (δ) is a free, CP-conserving (CP-violating) parameter of the model. For convenience, let us define

$$\kappa \equiv |\sin 2\theta_3\,\sin\delta|\,\frac{M_W^2}{M_{h^+}^2}, \tag{16}$$

so that $\mathrm{Im}\alpha\gamma^*\,M_W^2/M_{h^+}^2$ and $\mathrm{Im}\beta\gamma^*\,M_W^2/M_{h^+}^2$ are just given in terms of κ and the VEV's. The relations in (15) are not enough by themselves to better the constraints given by (13) and (14), and so we consider specific models.

A common assumption is that the three VEV's are all of the same order, i.e., $v_u \simeq v_d \simeq v_e$. We refer to this as the VEV equality (VE) model. In the VE model, all three CP-violating coefficients are of order 1, and $\overline{P_\mu^\perp}$ will be quite small. But with one VEV for each type of massive fermion, this need not be the case. Since fermion masses are proportional to the VEV's as well as the Yukawa couplings, it is quite reasonable to suppose that the hierarchy in the fermion masses lies in the VEV's, and not the Yukawa couplings [8]. Suppose the third family Yukawa couplings are of the same order. Then one has $v_u : v_d : v_e \sim m_t : m_b : m_\tau$, which implies that

$$|\mathrm{Im}\alpha\gamma^*|\,\frac{M_W^2}{M_{h^+}^2} \sim \frac{m_t^2}{m_b m_\tau}\kappa, \tag{17}$$

so that $\overline{P_l^\perp}$ need not be small [8]. We will refer to this as the VEV hierarchy (VH) model. While the VH model provides a reasonable justification for considering large ratios of VEV's, it does not solve all the mass hierarchy problems. We view the VE and VH models as two reasonable extremes, much in the same way that the range 1 to m_t/m_b is considered for "$\tan\beta$" in 2HDM's.

For simplicity, we define the VEV's in the VE model to be identically equal, and in the VH model to have the ratio $m_t : m_b : m_\tau$ exactly. Since (16) implies $\kappa < 3.2$ or so, $\overline{P_\mu^\perp}(K^+ \to \pi^0\mu^+\nu_\mu)$ does not put any further constraints on κ in the VE model. However, the VH model can reach the upper bound on $\overline{P_\mu^\perp}(K^+ \to \pi^0\mu^+\nu_\mu)$, and one needs $\kappa < 0.54$. We now must consider if there are any other constraints on κ which would force $\overline{P_l^\perp}$ to be small.

As we said in the Introduction, the most stringent constraint on CPV often comes from the electric dipole moment of the neutron, d_n. The purely hadronic coefficient $\mathrm{Im}\alpha\beta^*$ is very constrained by d_n [8], and in the VE model we find that we need $\kappa < 1.2$. However, in the VH model, $\mathrm{Im}\alpha\gamma^*/\mathrm{Im}\alpha\beta^*$ is large, and the upper bound on κ is only about 5, which is 10 times weaker than the $K_{\mu 3}$ bound. This is a consequence of the semileptonic decay—only quark-lepton CPV is enhanced in the VH model.

CP-conserving processes may also constrain $\overline{P_l^\perp}$. Consider the inclusive decay $b \to s\gamma$, whose branching ratio has been measured by the CLEO Collaboration to be $(2.3 \pm 0.7) \times 10^{-4}$ [30], which corresponds to a 95% C.L. experimental bound of $0.9 \times 10^{-4} < B(b \to s\gamma) < 3.7 \times 10^{-4}$. On the theoretical side, there is much uncertainty in the SM prediction, due to the choice of renormalization scale [31], and to possible large next-to-leading-order QCD effects [32], though it is important to note that the calculation in [32] is incomplete and therefore could be very misleading. We take $(1.5\text{–}4) \times 10^{-4}$ as a reasonable range for the SM prediction. In 2HDM's, the charged Higgs contribution adds constructively with the SM contribution, and one can put a lower bound the charged Higgs boson mass [33]. One would like to generalize this result to 3HDM's. The amplitude for $b \to s\gamma$ (at the W mass scale) can be written [34]

$$\begin{aligned} A = {} & F_1\left(\frac{m_t^2}{M_W^2}\right) + \frac{1}{3}\,|\beta_i|^2\,F_1\left(\frac{m_t^2}{M_{H_i^+}^2}\right) \\ & +\mathrm{Re}\alpha_i\beta_i^*\,F_2\left(\frac{m_t^2}{M_{H_i^+}^2}\right) \\ & +i\mathrm{Im}\alpha_i\beta_i^*\,F_2\left(\frac{m_t^2}{M_{H_i^+}^2}\right), \end{aligned} \tag{18}$$

where the sum over i runs from 1 to $N-1$, and one can show that $\beta_i\beta_i^* = (v^2 - v_u^2)/v_u^2$, $\mathrm{Re}\alpha_i\beta_i^* = 1$, and $\mathrm{Im}\alpha_i\beta_i^* = 0$. In the SM, only the first term is nonzero. For $N = 2$, we recover the 2HDM limit, i.e., $(|\beta|^2, \mathrm{Re}\alpha\beta^*, \mathrm{Im}\alpha\beta^*) \to (v_d^2/v_u^2, 1, 0)$. In 3HDM's, one can have cancellations between the pieces as long as H_1^+ and H_2^+ are not degenerate in mass. It turns out that for both the VE and VH models, $\mathrm{Re}\alpha_1\beta_1^*$ can be less than zero, so that for sufficiently large $M_{H_2^+}$, there is no bound from $B(b \to s\gamma)$ on $M_{H_1^+}$. For $M_{h^+} \sim M_W$ (or smaller), $\sin 2\theta_3 \sin\delta$ must be somewhat smaller than one [35], but this is not enough to better the constraints on κ we have derived thus far.

There are also constraints from $B(b \to s\gamma)$ on $\text{Im}\alpha\beta^*$ [34,37], which in turn constrains $P_l^\perp$ via (15). Since the last term in (18) is purely imaginary, it does not destructively interfere with the other terms, so that the contribution from $\text{Im}\alpha\beta^*$ to $B(b \to s\gamma)$ is always positive. However, even for $M_{h^+} \sim M_Z/2$, one can only bound $\text{Im}\alpha\beta^* < 1.4$, which is satisfied in both the VE and VH models. Since the CLEO has set a *lower* limit on $B(b \to s\gamma)$ of about 10^{-4}, the constraint on $\text{Im}\alpha\beta^*$ from $b \to s\gamma$ will never be able to strongly constrain $P_l^\perp$ in these models.

Finally, we note that the VE and VH models give specific predictions for $\text{Im}\beta\gamma^*$ [see (15)], and in both cases it must be less than 2. In general 3HDM's, one cannot improve upon the bound in (14), though large $\text{Im}\beta\gamma^*$ would require small v_u/v_d as well as very large v/v_e, which is not as appealing theoretically as either the VE or VH models.

In Table I, we summarize the maximum values for $\overline{P_l^\perp}$ allowed in the VE (VH) model, with a bound of $\kappa < 1.2$ ($\kappa < 0.54$) coming from the upper bound on d_n ($P_l^\perp$ in K decays).

IV. LEFT-RIGHT MODELS

Decays to vector mesons, $M \to m^* l\nu$, have one more four-vector than $M \to ml\nu$ decays with which to construct hadronic matrix elements. The m^* polarization vector lets us construct both a vector and an axial vector current [see (3)], allowing a nonzero V and A interference term. The upshot is that $P_l^\perp(M \to m^* l\nu)$ gets contributions from spin-1 effective CP-violating Lagrangians as well as those of spin 0.

Let us therefore consider left-right models [38], whose charged gauge boson couplings to fermions can be parametrized by the effective Lagrangian

$$-\mathcal{L}_{W^+} = \frac{g_L}{\sqrt{2}} \left[\bar{U}_L \gamma_\mu V_L D_L + \bar{N}_L \gamma_\mu E_L\right] W_L^{+\,\mu} + \frac{g_R}{\sqrt{2}} \left[\bar{U}_R \gamma_\mu V_R D_R\right] W_R^{+\,\mu} + \text{H.c.}, \tag{19}$$

where V_R is the right-handed CKM matrix. We neglect right-handed currents coupled to leptons because they yield polarizations proportional to m_ν. This means that $P_l^\perp$ must arise from the interference of the SM W_L diagram and a diagram containing W_L-W_R mixing (see Fig. 1) [4]. We define the mixing angle ζ by

$$\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix} \begin{pmatrix} W_L \\ W_R \end{pmatrix}, \tag{20}$$

where W_1 and W_2 are the two mass eigenstates. The interference between V and A HME's vanishes for longitudinally polarized m^*'s, and so $P_l^\perp$ is only nonzero for transversely polarized m^*'s. Further, the numerator of (1) has the same magnitude, but opposite sign, for m^*'s with $T1$ and $T2$ polarizations. Therefore, the polarization in the sum of decays to both transversely polarized m^* states, $P_l^\perp(M \to m_T^* l\nu)$, is identically zero, and we must consider $P_l^\perp$ for either $T1$ or $T2$. We again write $P_l^\perp$ in the heavy quark effective limit,

$$\begin{aligned} P_l^\perp(x)\left(M \overset{W_R^+}{\to} m_{T1}^* l\nu\right) &= C_{W_R^+} \frac{3\pi}{4} \frac{(x+r)(x^2-r^2)\sqrt{t}}{2t(x+r)x_1(x-r/2)} \frac{\xi(x)^2}{\xi(x)^2}, \\ P_l^\perp(x)\left(M \overset{W_R^+}{\to} m_{T2}^* l\nu\right) &= -C_{W_R^+} \frac{3\pi}{4} \frac{(x+r)(x^2-r^2)\sqrt{t}}{2t(x+r)x_1(x+r/2)} \frac{\xi(x)^2}{\xi(x)^2}, \end{aligned} \tag{21}$$

and list the full expressions in the Appendix. The coefficient

$$C_{W_R^+} = 2\,\frac{m_l}{M}\,\tan\zeta\ \text{Im}\left(\frac{g_R V_R{}^{UD}}{g_L V_L{}^{UD}}\right) \tag{22}$$

depends upon the W_L-W_R mixing angle ζ, the left and right CKM elements V_L^{ij} and V_R^{ij} ($i,j = U, D$), and gauge coupling constants g_L and g_R.

We can find an averaged polarization by integrating the numerator and denominator of (21) over x:

$$\begin{aligned} \overline{P_l^\perp}\left(M \overset{W_R^+}{\to} m_{T1}^* l\nu\right) &= C_{W_R^+} \frac{3\pi}{4} \frac{I_\perp/(1-r^2)}{I_{T1}}, \\ \overline{P_l^\perp}\left(M \overset{W_R^+}{\to} m_{T2}^* l\nu\right) &= -C_{W_R^+} \frac{3\pi}{4} \frac{I_\perp/(1-r^2)}{I_{T2}}. \end{aligned} \tag{23}$$

We again use the $\rho = 1$ monopole expression for $\xi(x)$, which results in the I_{T1} and I_{T2} listed in the Appendix.

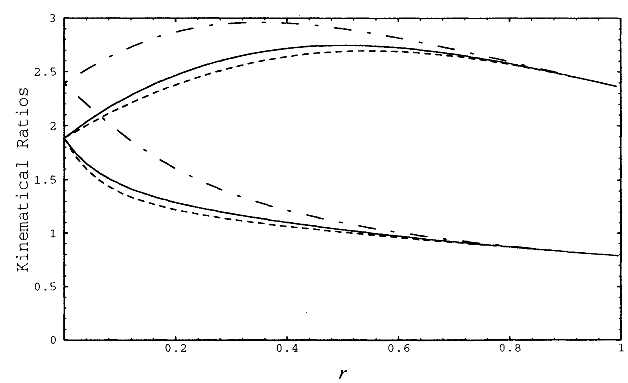

FIG. 3. $\overline{P_l^\perp}/C_{W_R^+}$ as a function of r. Notation is the same as in Fig. 2, with the top and bottom sets of curves corresponding to $M \to m_{T1}^* l\nu$ and $M \to m_{T2}^* l\nu$ decays, respectively.

We have normalized the I's so that $I_{T1}+I_{T2}=I_T$. Figure 3 shows that using $\rho=1$ (to obtain an analytic expression) instead of 1.2 is a good approximation since $\xi(x)^2$ appears in both the numerator and denominator in (21).

Let us consider constraints on $P_\tau^\perp$ in $B_{\tau 3}$ decays. Our Lagrangian in (7) gives a *tree level* contribution to ϵ' [39], and we can relate $P_l^\perp$ and ϵ'. If $\mathrm{Im}(V_R^{UD}/V_L^{UD})$ is roughly the same order for all UD, then $\overline{P_\tau^\perp}\sim 10^{-2}(\epsilon'/\epsilon)$, which is tiny. It is in principle possible that $\mathrm{Im}(V_R^{ud}/V_L^{ud})\simeq \mathrm{Im}(V_R^{us}/V_L^{us})\simeq 0$ while $\mathrm{Im}(V_R^{cb}/V_L^{cb})\sim 1$, which gives $\overline{P_\tau^\perp}\sim 2\zeta$. Nevertheless, $|\zeta|$ is constrained to be less than about 6% from μ decays [40], and less than about 2% from $b\to s\gamma$ [41], so that we can bound $\overline{P_\tau^\perp}$ to be less than about 4%.

V. DISCUSSION

Let us consider the various decay modes. In particular, we discuss whether one should study charged or neutral decays, of B or D mesons, to pseudoscalar or vector mesons, with $l=\mu$ or $l=\tau$.

Technically, the transverse polarization $P_l^\perp$ is *motion reversal violating*, which is equivalent to T violation only in the absence of final state interaction (FSI) effects [42]. This is irrelevant in charged decays, e.g., $M^+\to\bar{m}^0 l^+\nu_l$, because they have only one charged decay product, and FSI's are negligible. In neutral decays, e.g., $M^0\to m^- l^+\nu_l$, there are two charged particles in the final state, and so one can expect FSI effects of order $\alpha_{\rm EM}/\pi$ [43]. For this reason, measurements of $P_l^\perp$ in $K_{\mu 3}$ decays are done on the $K^+\to\pi^0\mu^+\nu_\mu$ mode. But if the experimental sensitivity to $P_l^\perp$ in a given decay is only at the percent level, one can study decays of neutral mesons as well. Actually, since both B and D mesons are produced in pairs, one must be able to determine the charge of the lepton (because $P_l^\perp$ flips sign for the CP conjugate decay) so that one effectively measures the asymmetry

$$A_{\rm CPV}\equiv\frac{1}{2}\left[P_l^\perp(M\to\bar{m}l^+\nu_l)-P_l^\perp(\bar{M}\to ml^-\bar{\nu}_l)\right], \tag{24}$$

which is a true CP-violating observable. Since FSI effects cancel in $A_{\rm CPV}$, charged decays are in principle not preferable to neutral decays.

From Table I, it is clear that B decays give larger $\overline{P_l^\perp}$ than D decays. One can see from (11) that this has two causes: M_D is smaller than M_B, and the heavier quark mass in D decays, m_c, is proportional to $\mathrm{Im}\beta\gamma^*$ instead of $\mathrm{Im}\alpha\gamma^*$. The former coefficient is more constrained than the latter, and models in which $\mathrm{Im}\beta\gamma^*$ is large tend to be less theoretically appealing. For example, in 3HDM's, one would need v_u/v_d to be small while v/v_e is very large.

Let us estimate the number of decays necessary to see a 5σ signal of $\overline{P_l^\perp}$ with the maximum allowed values in the general case (column 1 of Table I). We use $N=25k/\overline{P_l^\perp}^2$, and take $k\sim 10$. One needs about 1.5×10^4 $B\to D\mu\nu$ decays, and about 4.4×10^5 $B\to D^*\mu\nu$ decays (or about 2.2×10^5 $B\to D^*\mu\nu$ decays, if one can veto all decays to transversely polarized D^*'s) to see a 5σ signal. It is not clear which mode is preferable. Naively, one needs only about 10^6 B's (including $B^\pm$, B^0, and $\bar{B}^0$) to get enough $B\to D\mu\nu$ decays, versus about 10^7 B's for enough $B\to D^*\mu\nu$ decays, but reconstructing $B\to D\mu\nu$ decays is in practice rather difficult [44]. But $P_l^\perp$ is almost certainly easier to observe in B decays than in D decays. One needs about 1.1×10^6 (1.7×10^7) $D\to K\mu\nu$ decays to observe $\overline{P_l^\perp}$ for the maximum value in the general case (VH model), which naively requires 3×10^7 (6×10^8) D's.

To observe $P_l^\perp$ of a muon, one needs to stop the muon so it can decay. At a *symmetric* B factory, such as CESR or DORIS II, the muon in $B\to D\mu\nu$ will have momentum of up to 2.3 GeV, which would require perhaps 1.3 kg/cm^2 of material (e.g., $\sim$ 4.5 m of Al) to stop it [45]. Stopping muons would be more difficult at the asymmetric SLAC B factory, since the muon momenta will be higher in the laboratory frame, but if it could be accomplished, the luminosity should be sufficient to see a 13% polarization. One could consider measuring $P_\mu^\perp$ at a hadron collider, where the number of $B_{\mu 3}$ and $D_{\mu 3}$ decays would be much greater, but the hurdle of stopping the muon would need to be overcome.

A better possibility may be $B_{\tau 3}$ decays, because one can have $\overline{P_\tau^\perp}\sim 1$. One needs perhaps 250 $B\to D\tau\nu$ decays and 2500 $B\to D^*\tau\nu$ decays to see a 5σ signal. Both of these may prove difficult for CLEO or ARGUS, but should be no problem at the SLAC B factory. Unlike muons, τ's do not need to be stopped, and one can measure the polarization of the τ from its decay spectrum [46]. In $\tau^\pm\to\pi^\pm\nu_\tau$ decays, for example, the decay width has the behavior $d\Gamma\sim 1\mp\mathbf{P}_{\tau^\pm}\cdot\hat{p}_\pi\sim 1-P_{\tau^+}^\perp\cos\theta$ [47], where $\mathbf{P}_{\tau^\pm}$ is the polarization vector of the $\tau^\pm$, $\hat{p}_\pi$ is a unit vector in the pion direction, and θ is the angle of $\hat{p}_\pi$ from the normal of the B decay plane. The main problem with $B_{\tau 3}$ decays at the SLAC B factory (or any source of B's which are not at rest in the laboratory frame) may lie in defining the decay plane, since the B's do not decay at rest, in which case we may have underestimated k [10, 11].

Finally, we note that $P_l^\perp$ from left-right models is probably unobservable at the SLAC B factory. In addition to the small values for $\overline{P_l^\perp}$ required by the bounds on W_L-W_R mixing, one needs to measure the polarization of m^* as well as of l, so that our k is perhaps 100 or more. For $\overline{P_\tau^\perp}\sim 4\%$, one needs more than 10^6 $B\to D^*\tau\nu$ decays.

We have derived expressions for the transverse polarization of the lepton in semileptonic meson decays, in the heavy quark effective limit. Reasonable multi-Higgs-doublet models can give a muon polarization in B decays of order 13% and a τ polarization of order unity. Both of these should be within the luminosity reach of the SLAC B factory, though the τ polarization has the advantage of not requiring a stopper. Should a nonzero signal be observed, implying the existence of physics beyond the standard model, the best place to study $P_l^\perp$ would be at a high luminosity *symmetric* B factory.

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APPENDIX

So far, we have used the heavy quark effective limit, in which $\xi_+(x) = \xi_{V_1}(x) = \xi_{A_1}(x) = \xi_{A_3}(x) = \xi(x)$, and $\xi_-(x) = \xi_{A_2}(x) = 0$. For completeness, we list the expressions for the polarization without that simplification:

$$P_l^\perp(x)\left(M \overset{H^+}{\to} ml\nu\right) = C_{H^+}\frac{3\pi}{4}\frac{x_1^2\sqrt{t}}{x_1^3}\frac{[(1+r)\xi_+(x)-(1-r)\xi_-(x)]}{[(1+r)\xi_+(x)-(1-r)\xi_-(x)]^2} \times [(1-r)(x+r)\xi_+(x)-(1+r)(x-r)\xi_-(x)]\,, \tag{A1}$$

$$P_l^\perp(x)\left(M \overset{H^+}{\to} m_L^* l\nu\right) = C_{H^+}\frac{3\pi}{4}\frac{x_1^2\sqrt{t}}{(x+r)^2x_1}\frac{\{\xi_{A_1}(x)(x+r)(x-r^2)-[\xi_{A_3}(x)+r\xi_{A_2}(x)]x_1^2\}}{\{\xi_{A_1}(x)(x-r^2)-[\xi_{A_3}(x)+r\xi_{A_2}(x)](x-r)\}^2} \times \left\{\xi_{A_1}(x)(x+r)-\tfrac{1}{2}[\xi_{A_3}(x)+r\xi_{A_2}(x)](1-r^2)+\tfrac{1}{2}[\xi_{A_3}(x)-r\xi_{A_2}(x)]t\right\}, \tag{A2}$$

$$\begin{aligned} P_l^\perp(x)\left(M \overset{H^+}{\to} m^* l\nu\right) = {}& C_{H^+}\frac{3\pi}{4}\frac{x_1^2\sqrt{t}}{(x+r)x_1}\{\xi_{A_1}(x)(x+r)(x-r^2)-[\xi_{A_3}(x)+r\xi_{A_2}(x)]x_1^2\} \\ &\times\left\{\xi_{A_1}(x)(x+r)-\tfrac{1}{2}[\xi_{A_3}(x)+r\xi_{A_2}(x)](1-r^2)+\tfrac{1}{2}[\xi_{A_3}(x)-r\xi_{A_2}(x)]t\right\} \\ &\times\Big((x+r)\{\xi_{A_1}(x)(x-r^2)-[\xi_{A_3}(x)+r\xi_{A_2}(x)](x-r)\}^2 \\ &+2r^2t\left[\xi_{V_1}(x)^2(x-r)+\xi_{A_1}(x)^2(x+r)\right]\Big)^{-1}, \end{aligned} \tag{A3}$$

where C_{H^+} is given in (11). The corresponding expressions for LR contributions are

$$P_l^\perp(x)\left(M \overset{W_R^+}{\to} m_{T1}^* l\nu\right) = C_{W_R^+}\frac{3\pi}{4}\frac{(x+r)x_1^2\sqrt{t}}{2t(x+r)x_1}\frac{\xi_{A_1}(x)\xi_{V_1}(x)}{\left[\frac{3}{4}\xi_{V_1}(x)^2(x-r)+\frac{1}{4}\xi_{A_1}(x)^2(x+r)\right]}, \tag{A4}$$

$$P_l^\perp(x)\left(M \overset{W_R^+}{\to} m_{T2}^* l\nu\right) = -C_{W_R^+}\frac{3\pi}{4}\frac{(x+r)x_1^2\sqrt{t}}{2t(x+r)x_1}\frac{\xi_{A_1}(x)\xi_{V_1}(x)}{\left[\frac{1}{4}\xi_{V_1}(x)^2(x-r)+\frac{3}{4}\xi_{A_1}(x)^2(x+r)\right]}, \tag{A5}$$

where $C_{W_R^+}$ is given by (22).

To find the average polarization, we must integrate both numerator and denominator over x. For this, one must know $\xi(x)$. One possible choice comes from a relativistic oscillator model [18]:

$$\xi(x) = \frac{2r}{(x+r)}e^{-\beta\frac{(x-r)}{(x+r)}}, \tag{A6}$$

where $\beta \simeq 1.85$ [18]. Another possibility is a monopole approximation:

$$\xi(x) = \frac{1}{1+\rho^2(x-r)/r}, \tag{A7}$$

where $\rho \simeq 1.2 \pm 0.25$ [48]. For most choices of $\xi(x)$, the integration over x must be done numerically, but for the monopole approximation with $\rho = 1$, one can obtain reasonably simple analytic expressions (see below). Since $\xi(x)^2$ appears both in the numerator and denominator of (1), $P_l^\perp$ is fairly insensitive to the choice of $\xi(x)$. We find that for decays with the lowest value of r (~ 0.25), the difference between $P_l^\perp$ using $\xi(x)$ from (A7) for $\rho = 1$ (analytic case) and $\rho = 1.2$, and (A6) is no more than 15%, and considerably less in most cases (see Figs. 2 and 3). Thus we use (A7) with $\rho = 1$ to obtain the following analytic expressions for the integrals in (12) and (23):

$$\begin{aligned} I_\perp &= \int_r^{(1+r^2)/2} dx\,(1-r^2)(x+r)x_1^2\sqrt{t}\,\xi(x)^2 \\ &= \frac{1}{15}r^2(1-r)(1+6r-6r^5-r^6) \\ &\quad - \frac{2r(1-r+r^3-r^4)}{\sqrt{1+r^2}}\arctan\left(\frac{1-r}{\sqrt{1+r^2}}\right), \end{aligned} \tag{A8}$$

$$\begin{aligned} I_S &= \int_r^{(1+r^2)/2} dx\,(1+r)^2x_1^3\,\xi(x)^2 \\ &= \frac{1}{8}\frac{r^2(1-r)(1+r)^3(1+10r^2+r^4)}{1+r^2} \\ &\quad + \frac{3}{2}r^4(1+r)^2\ln r, \end{aligned} \tag{A9}$$

$$I_L = \int_r^{(1+r^2)/2} dx\,(1-r)^2(x+r)^2 x_1\,\xi(x)^2 = \frac{1}{8}\frac{r^2(1-r)^3(1+r)(1+8r-6r^2+8r^3+r^4)}{1+r^2} + 2r^4(1-r)^2\left[\arctan\left(\frac{2r}{1-r^2}\right)-\frac{\pi}{2}-\frac{1}{4}\ln r\right], \tag{A10}$$

$$I_T = \int_r^{(1+r^2)/2} dx\,4t(x+r)xx_1\,\xi(x)^2 = \frac{1}{6}r^2(1+r)^3(1+3r-3r^2-r^3) + 4r^4(1+r^2)\left[\arctan\left(\frac{2r}{1-r^2}\right)-\frac{\pi}{2}\right] + 2r^4(1-r)^2\ln r, \tag{A11}$$

$$I_{T1} = \int_r^{(1+r^2)/2} dx\,2t(x+r)x_1(x-r/2)\,\xi(x)^2 = \frac{1}{12}r^2(1-r^2)(1+3r+34r^2+3r^3+r^4) + r^4(1+r)^2\left[\arctan\left(\frac{2r}{1-r^2}\right)-\frac{\pi}{2}\right] + r^4(2-r+2r^2)\ln r, \tag{A12}$$

$$I_{T2} = \int_r^{(1+r^2)/2} dx\,2t(x+r)x_1(x+r/2)\,\xi(x)^2 = \frac{1}{12}r^2(1-r^2)(1+9r-14r^2+9r^3+r^4) + r^4\left[3(1+r)^2-8r\right]\left[\arctan\left(\frac{2r}{1-r^2}\right)-\frac{\pi}{2}\right] - 3r^5\ln r. \tag{A13}$$

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