Role of scalar meson resonances in $K_L^0 \to \pi^0\gamma\gamma$ decay

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Corrections to $K_L^0 \to \pi^0\gamma\gamma$ decay induced by scalar meson exchange are studied within chiral perturbation theory. In spite of the poor knowledge of the scalar meson parameters, the calculated branching ratio can be changed only up to a few percent.

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I. INTRODUCTION

The $K \to \pi\gamma\gamma$ decays have been the subject of intensive theoretical studies during the last few years [1–9]. The experimentally measured branching ratio [10, 11] of $K_L^0 \to \pi^0\gamma\gamma$ is not so well explained theoretically as it seems from previous calculations. In this decay, $O(p^4)$ terms in the chiral Lagrangian give the leading order contribution of chiral perturbation theory (CHPT), resulting in the branching ratio $B \simeq 0.7 \times 10^{-6}$. The amplitude $K_L^0 \to \pi\gamma\gamma$ is finite at the one-loop level in CHPT.

The experimentally observed values are $(1.70 \pm 0.3) \times 10^{-6}$ (NA31 result) [10] and $(1.86 \pm 0.60 \pm 0.60) \times 10^{-6}$ (E731 result) [11]. At the same time, the observed invariant-mass distribution of the final photons is in good agreement with the theoretical predictions [1, 3–5, 8].

The vector meson exchange, resulting in the $O(p^6)$ contribution of CHPT, was studied by the authors of [3, 6] and it was found that this contribution is very important. In addition to vector meson exchange present at next-to-leading order, $O(p^6)$ of CHPT, the two-pion intermediate state was taken into account [8, 12]. It was found that these corrections raise the rate by 20%. At the $O(p^4)$ order in the chiral Lagrangian, both vector and scalar resonance exchange helps to explain $K \to \pi\pi\pi$ and $K \to \pi\pi$ amplitudes [13–18].

Motivated by this effect, we investigate the role of the scalar resonances in the $K_L^0 \to \pi^0\gamma\gamma$ decay. We notice that scalar mesons also induce a contribution of the $O(p^6)$ in CHPT. The masses of the $a_0(983)$ and $f_0(975)$ scalar mesons are close to the scale characterizing the CHPT [19, 20] expansion $\Lambda \simeq 1$ GeV, and therefore they should be taken into account.

The outline of the work is the following. In Sec. II we derive the $O(p^6)$ effective Lagrangian for $K_L^0 \to \pi^0\gamma\gamma$ decay. In Sec. III we discuss and comment on our results.

II. SCALAR MESONS AND $O(P^6)$ EFFECTIVE LAGRANGIANS

There have been many attempts to understand the nature of scalar mesons [21–28]. In the chiral Lagrangian we deal only with the quantum numbers of scalar mesons, and we apply the approach of Refs. [1⁵,18,19].

Very nice descriptions of CHPT up to $O(p^4)$ can be found in [19,20,29,30]. We follow their notation. Here we describe only the part of the chiral Lagrangian necessary for our purpose.

The kinetic term of the Lagrangian describing scalar mesons is given by

$$\mathcal{L}_k(S) = \frac{1}{2}\mathrm{tr}(\nabla_\mu S \nabla^\mu S - M_S^2 S^2), \tag{1}$$

where S is the scalar octet and M_S corresponds to the scalar masses in the chiral limit. For the scalar singlet the kinetic term of the Lagrangian is

$$\mathcal{L}_k(S_1) = \frac{1}{2}(\partial^\mu S_1 \partial_\mu S_1 - M_{S_1}^2 S_1^2). \tag{2}$$

The scalar meson resonance $f_0(983)$ can be described as a linear combination of octet and singlet states of SU(3), while the $a_0(975)$ belongs completely to its octet.

This means that we treat the a_0 and f_0 like ρ and ω vector mesons: $a_0(975)$ is identified with S_3 and

$$f_0(975) = \frac{1}{\sqrt{3}}S_8 + \frac{2}{\sqrt{6}}S_1. \tag{3}$$

Their interactions with Goldstone pseudoscalars can be described by writing the most general $SU(3)_L \times SU(3)_R$ Lagrangian taking into account the C and P properties of pseudoscalars and scalars [19,20,29]:

$$\mathcal{L}_{SPP} = c_d \mathrm{Tr}(S u_\mu u^\mu) + c_m \mathrm{Tr}(S\chi_+) + \bar{c}_d S_1 \mathrm{Tr}(u_\mu u^\mu) + \bar{c}_m S_1 \mathrm{Tr}(\chi_+), \tag{4}$$

where

$$u_\mu = i u^\dagger D_\mu U u^\dagger, \tag{5}$$

$$D_\mu U = \partial_\mu U + ie(A_\mu U - U A_\mu), \tag{6}$$

$$\chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u, \tag{7}$$

and $U = u^2$ is a unitary 3×3 matrix, with $u = \exp(-\frac{i}{\sqrt{2}}\frac{\Phi}{f})$, $\Phi = \frac{1}{\sqrt{2}}\sum_{i=1}^{8}\lambda_i\varphi^i$, $\mathrm{Tr}(\lambda_i\lambda_j) = \delta_{ij}$, Φ is the matrix of the pseudoscalar fields, A_μ is the electromagnetic field, and χ_+ is related to the quark mass matrix as in [19,29]. The experimental values of the decay widths of f_0, a_0 are given by the Particle Data Group [31]:

$$\Gamma(f_0 \to \pi\pi) = 36 \text{ MeV}, \tag{8}$$

$$\Gamma(a_0 \to \eta\pi) = 59 \text{ MeV}. \tag{9}$$

Assuming that $M_S \simeq M_{S_1}$ and fitting the experimental data for decay widths (8) and (9) we derive

$$c_d = \pm 0.022 \text{ GeV}, \quad c_m = \pm 0.029 \text{ GeV} \tag{10}$$

and

$$\bar{c}_d = 0.019 \text{ GeV}, \quad \bar{c}_m = -0.024 \text{ GeV} \tag{11}$$

or

$$\bar{c}_d = -0.019 \text{ GeV}, \quad \bar{c}_m = 0.011 \text{ GeV}. \tag{12}$$

These fits are obtained from a simultaneous fit of l_i, $i = 1, 2, ..., 6$, defined in [19] and the a_0, f_0 decay widths (11) and (12), taking $\bar{c}_d$ positive (11), and negative (12). We take into account η-η' mixing through their mixing angle θ. The results of our fit to the l_i are slightly different from those obtained in Ref. [19], where the a_0 decay rate (9) and the large N_c limit were used. These authors [3, 19] emphasized that the widely used nonet assumption for η, η' mesons: in the lowest-order Lagrangian $\mathcal{L}_2$ is by no means unique. In our proceeding calculations we shall also use their fit for $c_d, c_m, \bar{c}_m, \bar{c}_d$:

$$c_d = \pm 0.032 \text{ GeV}, \quad c_m = \pm 0.042 \text{ GeV} \tag{13}$$

and

$$\bar{c}_d = \pm 0.019 \text{ GeV}, \quad \bar{c}_m = \pm 0.024 \text{ GeV}. \tag{14}$$

We support the idea of scalar meson dominance in the counterterm's couplings, introduced in [19]. In Table I, we present both sets of the parameters.

In order to have two-photon-scalar couplings, we add to the Lagrangian the two-photon interaction with scalar mesons:

$$\mathcal{L}_{S\gamma\gamma} = ge^2 \text{Tr}(Q^2 S) F_{\mu\nu} F^{\mu\nu} + g' e^2 S_1 F_{\mu\nu} F^{\mu\nu}, \tag{15}$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor and Q is the quark charge matrix.

We determine the constants g and g' by fitting the experimental data [31]. We are, however, aware that these fits should be regarded with special caution:

$$\Gamma(f_0 \to 2\gamma) = 0.56 \times 10^{-6} \text{ GeV}, \tag{16}$$

$$\Gamma(a_0 \to 2\gamma) = 0.24 \times 10^{-6} \text{ GeV}. \tag{17}$$

We find the following possible combinations for g and g':

$$(1) \quad g = 0.07 \text{ GeV}^{-1}, \quad g' = 0.03 \text{ GeV}^{-1}, \tag{18}$$

$$(2) \quad g = 0.07 \text{ GeV}^{-1}, \quad g' = -0.07 \text{ GeV}^{-1}, \tag{19}$$

$$(3) \quad g = -0.07 \text{ GeV}^{-1}, \quad g' = 0.07 \text{ GeV}^{-1}, \tag{20}$$

$$(4) \quad g = -0.07 \text{ GeV}^{-1}, \quad g' = -0.03 \text{ GeV}^{-1}. \tag{21}$$

The authors of Ref. [32] assumed nonet symmetry for scalar mesons which requires $c_m = \bar{c}_m = 0$. They use the measured cross section $\sigma(\gamma\gamma \to \pi^0\eta) \simeq 30$ nb at the $a_0(980)$ [33] to determine the relevant parameters.

The Lagrangian (4) and (15) can be used [14,19] to construct an effective Lagrangian describing two-pseudoscalar–two-photon couplings dominated by scalar meson exchange. Eliminating scalar mesons, as in [19], we derive

$$\mathcal{L}^s_{PP\gamma\gamma} = ge^2 \frac{1}{M_S^2} F_{\mu\nu} F^{\mu\nu} [c_d \text{Tr}(Q^2 u^\alpha u_\alpha) - \tfrac{1}{3} c_d \text{Tr}(Q^2) \text{Tr}(u^\alpha u_\alpha) + c_m \text{Th}(\chi_+ Q^2) - c_m \tfrac{1}{3} \text{Tr}(\chi_+) \text{Tr}(Q^2)]$$
$$+ g' e^2 \frac{1}{M_S^2} F_{\mu\nu} F^{\mu\nu} [\bar{c}_d \text{Tr}(u^\alpha u_\alpha) + \bar{c}_m \text{Tr}(\chi_+)]. \tag{22}$$

The superscript S of $\mathcal{L}_{PP\gamma\gamma}$ is to show the presence of one strong vertex. Adding to this Lagrangian the lowest-order weak Lagrangian [14] describing $K^0 \to \pi^0, \eta, \eta'$ transitions

$$\mathcal{L}_w = c_2 \text{Tr}(\lambda_6 u_\mu u^\mu), \tag{23}$$

we easily obtain the amplitude, in which π^0, η, η' are poles, as presented in Fig. 1(a). We take, as in [3],

TABLE I. V, A, S, S_1 contributions to the coupling constants l_i^r, $i = 1, 2, 3, 4, 5, 6, 8$ (l_7 is explained by the higher pseudoscalar meson state) in units of 10^{-3} and compared with the values from [29]. The values are calculated for $c_d = \pm 0.022$ GeV, $c_m = \pm 0.022$ GeV, $\bar{c}_d = 0.19$ GeV, and $\bar{c}_m = -0.024$ GeV while in the parentheses are values obtained for $\bar{c}_d = -0.19$ GeV and $\bar{c}_m = 0.011$ GeV. In the last column there are the l_i^r from [12,13], at the scale M_ρ.

l_i	V	A	S	S_1	Total	Total [29]	$l_i^r(M_\rho)$
l_1	0.6	0	−0.09	0.19	0.7	0.6	0.7 ± 0.4
l_2	1.2	0	0	0	1.2	1.2	1.3 ± 0.7
l_3	−3.6	0	0.27	0	−3.33	−3.0	-4.4 ± 2.5
l_4	0	0	−0.22	−0.48(−0.22)	−0.7(−0.44)	0.0	0.3 ± 0.5
l_5	0	0	0.66	0	0.66	1.4	1.3 ± 0.5
l_6	0	0	−0.15	0.30(0.14)	−0.15(−0.01)	0.0	-0.2 ± 0.3
l_8	0	0	0.45	0	0	0.9	0.9 ± 0.3

$c_2/f^4 = 9\times10^{-6}$ GeV^{-2} and $f \simeq f_\pi = 0.093$ GeV. From the direct weak kaon transition to pion and scalar meson, it is possible to derive a new contribution to $K^0 \to \pi^0\gamma\gamma$. In this contribution a scalar meson decays into 2γ.

There are two procedures in the literature used to determine the effective weak Largangian: the "factorization model" [14] and the "weak deformation model" [1, 3–5, 14]. It seems that the "weak deformation model" has a realistic chance of describing a rather large number of processes involving the higher-order weak Lagrangian. This model has obtained more support after the successful application of the $O(p^4)$ terms in $K^+ \to \pi^+\gamma^*$ decay [3], where weak counterterms satisfy scale-independent relations. The weak Lagrangian containing vectors for $K^0 \to \pi^0\gamma\gamma$ was derived using this method [3]. In order to maintain a consistent calculation of the vector and scalar resonance exchange, we apply this procedure too. We find, knowing (22),

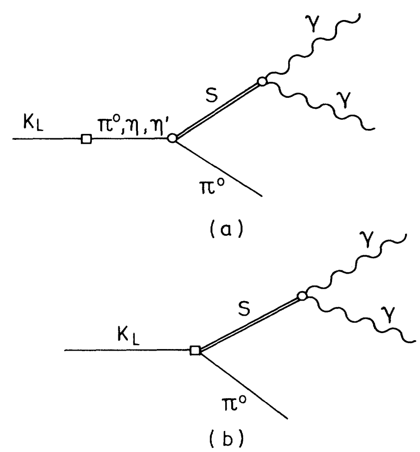

FIG. 1. Scalar meson exchange diagram for $K_L^0 \to \pi^0\gamma\gamma$, with π, η, η' poles (a) and with direct weak transition (b).

$$\mathcal{L}^w_{PP\gamma\gamma} = ge^2\frac{c_2c_d}{M_S^2f^2}F_{\mu\nu}F^{\mu\nu}[\mathrm{Tr}(\lambda_6u^\alpha u_\alpha Q^2) - \tfrac{4}{3}\mathrm{Tr}(\lambda_6u^\alpha u_\alpha)\mathrm{Tr}(Q^2) + \mathrm{Tr}(\lambda_6Q^2u^\alpha u_\alpha) + \mathrm{Tr}(\lambda_6u^\alpha Q^2u_\alpha)]$$
$$+g'e^2\frac{c_2\bar{c}_d}{M_S^2f^2}4F_{\mu\nu}F^{\mu\nu}[\mathrm{Tr}(\lambda_6u^\alpha u_\alpha Q^2)], \qquad (24)$$

where the superscript w denotes the direct weak vertex [see Fig. 1(b)].

III. EFFECTIVE SCALAR MESON CONTRIBUTION TO THE DECOMPOSED $K_L^0 \to \pi^0\gamma\gamma$ AMPLITUDE

The general decomposition for the $K^0 \to \pi^0\gamma\gamma$ amplitude is given by

$$M(K^0(k)\to\pi^0(p)\gamma(q_1)\gamma(q_2)) = \epsilon_\mu(q_1)\epsilon_\nu(q_2)\Big(\frac{A(y,z)}{m_K^2}(q_1^\nu q_2^\mu - q_1\cdot q_2g^{\mu\nu})$$
$$+2\frac{B(y,z)}{m_K^4}(p\cdot q_1p\cdot q_2g^{\mu\nu} + q_1\cdot q_2p^\mu p^\nu - p\cdot q_1q_2^\mu p^\nu - p\cdot q_2q_1^\nu p^\mu)\Big) \qquad (25)$$

with dimensionless invariant amplitudes A, B which are functions of the Dalitz variables

$$y = |k\cdot(q_1-q_2)|/m_K^2, \qquad (26)$$

$$z = (q_1+q_2)^2/m_K^2. \qquad (27)$$

Including loop effects at $\mathcal{O}(p^4)$, vector mesons [1, 3–7, 12] and the exchange of scalar mesons calculated in this approach, we can write

$$A = \frac{c_2}{f^4}\frac{m_K^2\alpha}{\pi}\Big[F(z/r_\pi^2)\left(1-\frac{r_\pi^2}{2}\right) + F(z)\left(\frac{1+r_\pi^2}{z}-1\right)$$
$$+(a_V+a_s^1)(3-z+r_\pi^2)+a_s^0\Big], \qquad (28)$$

where

$$B = -2a_V\frac{c_2}{f^4}\frac{m_K^2\alpha}{\pi}, \quad r_\pi = \frac{m_\pi}{m_K}, \qquad (29)$$

$$a_V = \frac{512\pi^2h_V^2m_K^2}{9m_V^2}. \qquad (30)$$

In [3] it was calculated that $a_V = -0.32$ without η-η' mixing, and $a_V \simeq -0.19$ when this mixing was included. We find

$$a_s^1 = \frac{16\pi^2m_K^2}{M_S^2}\left(\frac{2c_dg}{3}+\frac{2}{9}c_dg\beta(\theta)\right) \qquad (31)$$

with

$$\beta(\theta) = \Big(-\frac{m_K^2}{m_\eta^2-m_K^2}[(\cos\theta+2\sqrt{2}\sin\theta)(\cos\theta-\sqrt{2}\sin\theta)] - \frac{m_K^2}{m_\eta'^2-m_K^2}(\sin\theta-2\sqrt{2}\cos\theta)(\sin\theta+\sqrt{2}\cos\theta)\Big), \qquad (32)$$

TABLE II. The branching ratio B for $K_L^0 \to \pi^0\gamma\gamma$ decay. For this calculation all possible combinations of the parameters g, g', c_d, c_m, $\bar{c}_d$, $\bar{c}_m$ are used. The g, g' are in GeV^{-1} and c_d, c_m, $\bar{c}_d$, $\bar{c}_m$ are given in GeV. In the last column the relative contribution of the scalar meson exchange is presented in percent. $\Delta B = (B - B_0)/B_0$ with $B_0 = 8.87 \times 10^{-7}$ denotes the contribution without scalar mesons; $a_V = -0.32$ was taken.

g	g'	c_d	c_m	$\bar{c}_d$	$\bar{c}_m$	$10^7 B$	ΔB (%)
0.07	0.03	0.022	0.029	0.019	−0.024	8.54	−3.7
0.07	0.03	−0.022	−0.029	0.019	−0.024	9.06	2.1
0.07	0.03	0.022	0.029	−0.019	0.011	8.65	−2.5
0.07	0.03	−0.022	−0.029	−0.019	0.011	9.18	3.5
0.07	−0.07	0.022	0.029	0.019	−0.024	8.80	−0.8
0.07	−0.07	−0.022	−0.029	0.019	−0.024	9.33	5.2
0.07	−0.07	0.022	0.029	−0.019	0.011	8.54	−3.7
0.07	−0.07	−0.022	−0.029	−0.019	0.011	9.05	2.0
−0.07	0.07	0.022	0.029	0.019	−0.024	8.95	0.1
−0.07	0.07	−0.022	−0.029	0.019	−0.024	8.44	−4.8
−0.07	0.07	0.022	0.029	−0.019	0.011	9.23	4.1
−0.07	0.07	−0.022	−0.029	−0.019	0.011	8.70	−0.8
−0.07	−0.03	0.022	0.029	0.019	−0.024	9.22	4.0
−0.07	−0.03	−0.022	−0.029	0.019	−0.024	8.70	−2.0
−0.07	−0.03	0.022	0.029	−0.019	0.011	9.10	2.6
−0.07	−0.03	−0.022	−0.029	−0.019	0.011	8.58	−3.3

where we take as usual $\theta \simeq 20°$. We define

$$a_s^0 = -2a_s^1 - \frac{16\pi^2 m_K^2}{M_S^2}\{\tfrac{4}{9}c_m g[1+\beta(\theta)] + 4g'\bar{c}_m\}. \quad (33)$$

The scalar meson exchange does not influence the B invariant amplitude. An interesting implication of this result is that the CP-conserving amplitude of $K_L^0 \to \pi^0 e^+e^-|_{\gamma\gamma}$ decay, proceeding through $\gamma\gamma$ states [1–3, 34, 35], is not influenced by scalars.

As we have mentioned already, the choice of parameters describing scalar mesons is the most troublesome part of this work. We make all possible allowed combinations of the parameters $c_d, c_m, \bar{c}_d, \bar{c}_m, g$, and g' and we present the numerical results in Tables II–IV. Without vector and scalar mesons the branching ratio was found to be [1]

$$B_0(K_L^0 \to \pi^0\gamma\gamma) = 6.67 \times 10^{-7}. \quad (34)$$

TABLE III. The branching ratio B for $K_L^0 \to \pi^0\gamma\gamma$ decay. For this calculation all possible combinations of the parameters g, g', c_d, c_m, $\bar{c}_d$, $\bar{c}_m$ are used, as described in the paper. The g, g' are in GeV^{-1} and c_d, c_m, $\bar{c}_d$, $\bar{c}_m$ are given in GeV. In the last column the relative contribution of the scalar meson exchange is presented in percent. $\Delta B = (B - B_0)/B_0$ with $B_0 = 7.80 \times 10^{-7}$ denotes the contribution without scalar mesons. η-η' mixing was taken into account and $a_V = -0.19$ was used.

g	g'	c_d	c_m	$\bar{c}_d$	$\bar{c}_m$	$10^7 B$	ΔB(%)
0.07	0.03	0.022	0.029	0.019	−0.024	7.44	−4.6
0.07	0.03	−0.022	−0.029	0.019	−0.024	8.04	3.1
0.07	0.03	0.022	0.029	−0.019	0.011	7.53	−3.5
0.07	0.03	−0.022	−0.029	−0.019	0.011	8.15	4.5
0.07	−0.07	0.022	0.029	0.019	−0.024	7.65	−1.9
0.07	−0.07	−0.022	−0.029	0.019	−0.024	8.29	6.3
0.07	−0.07	0.022	0.029	−0.019	0.011	7.43	−4.7
0.07	−0.07	−0.022	−0.029	−0.019	0.011	8.04	3.1
−0.07	0.07	0.022	0.029	0.019	−0.024	7.94	1.8
−0.07	0.07	−0.022	−0.029	0.019	−0.024	7.35	−5.8
−0.07	0.07	0.022	0.029	−0.019	0.011	8.19	5.0
−0.07	0.07	−0.022	−0.029	−0.019	0.011	7.57	−3.0
−0.07	−0.03	0.022	0.029	0.019	−0.024	8.19	5.0
−0.07	−0.03	−0.022	−0.029	0.019	−0.024	7.57	−3.0
−0.07	−0.03	0.022	0.029	−0.019	0.011	8.08	3.4
−0.07	−0.03	−0.022	−0.029	−0.019	0.011	7.47	−4.2

TABLE IV. The branching ratio B for $K_L^0 \to \pi^0\gamma\gamma$ decay. For this calculation all possible combinations of the parameters g, g', c_d, c_m, $\bar{c}_d$, $\bar{c}_m$ are used. The g, g' are in GeV^{-1}, while c_d, c_m, $\bar{c}_d$, $\bar{c}_m$ are given in GeV and their values are taken from [29]. In the last column the relative contribution of the scalar meson exchange is presented in percent. $\Delta B = (B - B_0)/B_0$ with $B_0 = 8.87 \times 10^{-7}$ denotes the contribution without scalar mesons; $a_V = -0.32$ was taken.

g	g'	c_d	c_m	$\bar{c}_d$	$\bar{c}_m$	$10^7 B$	$\Delta B[\%]$
0.07	0.03	0.032	0.042	0.019	0.024	9.18	3.5
0.07	0.03	−0.032	−0.042	0.019	0.024	8.43	−5.0
0.07	0.03	−0.032	−0.029	−0.019	−0.024	8.58	3.2
0.07	0.03	0.032	0.042	−0.019	−0.024	9.35	5.4
0.07	−0.07	0.032	0.042	0.019	0.024	8.33	−6.1
0.07	−0.07	−0.032	−0.042	0.019	0.024	9.07	2.2
0.07	−0.07	−0.032	−0.042	−0.019	−0.024	9.46	6.7
0.07	−0.07	0.032	0.042	−0.019	−0.024	8.68	−2.1
−0.07	0.07	0.032	0.042	0.019	0.024	9.46	6.7
−0.07	0.07	−0.032	−0.042	0.019	0.024	8.68	−2.1
−0.07	0.07	−0.032	−0.042	−0.019	−0.024	8.33	−6.1
−0.07	0.07	0.032	0.042	−0.019	−0.024	9.07	2.2
−0.07	−0.03	0.032	0.042	0.019	0.024	8.58	−3.2
−0.07	−0.03	−0.032	−0.042	0.019	0.024	9.35	5.4
−0.07	−0.03	−0.032	−0.042	−0.019	−0.024	9.18	3.5
−0.07	−0.03	0.032	0.042	−0.019	−0.024	8.43	5.0

When vector mesons and loops are included, the branching ratio is [3, 6]

$$B(K_L^0 \to \pi^0\gamma\gamma) = 8.87 \times 10^{-7} \tag{35}$$

for $a_V = -0.32$, and for $a_V = -0.19$, when mixing with η-η' was accounted for, is

$$B(K_L^0 \to \pi^0\gamma\gamma) = 7.80 \times 10^{-7}. \tag{36}$$

As can be seen from Tables II–IV, the largest contribution to the branching ratio is obtained for the following choice of parameters: $g = \mp 0.07\ \text{GeV}^{-1}$, $g' = \pm 0.07\ \text{GeV}^{-1}$, $c_d = \mp 0.032$ GeV, $c_m = \mp 0.042$ GeV, $\bar{c}_d = \mp 0.019$ GeV, $\bar{c}_m = \mp 0.024$ GeV giving $a_s^1 = -0.06$ and $a_s^0 = 0.14$, where either upper or lower signs are taken correspondingly. They give

$$B(K_L^0 \to \pi^0\gamma\gamma) = 9.46 \times 10^{-7}. \tag{37}$$

Taking values derived in [32] where $gc_d = g'\bar{c}_d \simeq \pm 0.16 \times 10^{-3}$, $a_V = -0.32$ we get $B(K_L^0 \to \pi^0\gamma\gamma) = 8.83 \times 10^{-7}$ for the plus sign, while for the minus sign it appears according [32], $B(K_L^0 \to \pi^0\gamma\gamma) = 8.92 \times 10^{-7}$. In the calculation [8, 9], where the exchange of two charged pions was taken into account, the physical amplitude $K^0 \to \pi^0\pi^+\pi^-$ was used to compute the absorptive part of the $K_L^0 \to \pi^0\gamma\gamma$ amplitude and then subtracted dispersion relations were applied to obtain the full amplitude. These corrections increase the branching ratio by about 20% in comparison with the leading order term $O(p^4)$, created by pion and kaon loops. In our analysis we do not add these corrections, since it was found [13–15] that the amplitude $K^0 \to \pi^0\pi^+\pi^-$ at $O(p^4)$ is already explained by the resonance exchange.

It was pointed out in [3] that there are many $\mathcal{O}(p^8)$ contributions related to vector mesons exchange, as well as some $\mathcal{O}(p^5)$ terms induced by $VP\gamma$ couplings which are not considered yet. We do not take these effects into account.

At $O(p^6)$ of the weak Lagrangian there are terms proportional to l_i^2, induced by the $O(p^4)$ part of the chiral Lagrangian, which could contribute, but their overall couplings are an order of magnitude smaller than the vector and scalar meson exchange considered in the present paper.

Motivated by the CHPT study in [1], NA31 [10] has extracted the bound on a_V from the Dalitz plot distribution of the two photon $-0.32 < a_V < 0.19$. From our result it is obvious that a_s^1 can increase or decrease

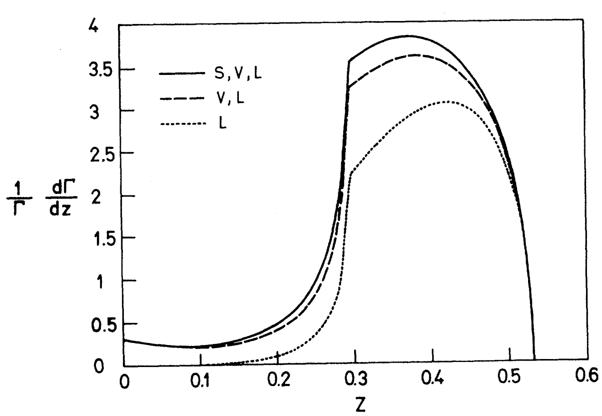

FIG. 2. Normalized spectra in the $\gamma\gamma$ invariant mass $z = (q_1 + q_2)^2/m_K^2$ for $a_V = 0$ (L, dotted curve), $a_V = -0.32$ (V, L, dashed curve), and $a_s^1 = -0.6$, $a_s^0 = 0.14$ (S, V, L, full curve).

a_V depending on the choice of the parameters from 20% to 30%. On the other hand, only a_V contributes to the $B(y,z)$ invariant amplitude of $K_L^0 \to \pi^0\gamma\gamma$ decay. In Tables II–IV we present the possible combinations of the parameters $c_d, c_m, \bar{c}_d, \bar{c}_m, g, g'$ and their influence on B.

We see from the results presented in Tables II and III that the model used for η-η' mixing is important, since if the nonet assumption for η's is not used, the branching ratio is increased by 15%. In Fig. 2, $\frac{1}{\Gamma}\frac{d\Gamma}{dz}$ is presented as a function of the $\gamma\gamma$ invariant mass for $a_V = a_s^1 = a_s^0 = 0$, for $a_V = 0.32$, $a_s^1 = a_s^0 = 0$ (V, L), and $a_V = 0.32$, $a_s^1 = -0.06$, and $a_s^0 = 0.14$ (S, V, L).

Finally we can summarize the following:

(i) The corrections coming from scalar meson exchange are rather small, but not negligible. They might increase the branching ratio up to 6.7%.

(ii) The corrections strongly depend on the parameters determined by the scalar meson data.

(iii) The CP-conserving $K_L^0 \to \pi^0 e^+e^-|_{\gamma\gamma}$ decay rate is not influenced by scalar meson exchange.

(iv) The interference of vector and scalar mesons in the study of the A part of the invariant amplitude in $K_L^0 \to \pi^0\gamma\gamma$ decay is not negligible.

(v) It seems that the large experimental value for $B(K_L^0 \to \pi^0\gamma\gamma)$ can be explained theoretically when all possible contributions coming from loops at $O(p^4)$ and the accumulation of the smaller effects of $O(p^6)$, or even $O(p^8)$, are taken into account consistently.

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