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Incompatibility of multipole predictions for the nucleon spin-polarizability and Drell-Hearn-Gerasimov sum rules

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Energy-weighted integrals of the difference in helicity-dependent photoreaction cross sections ($\sigma_{1/2}$-$\sigma_{3/2}$) provide information on the nucleon's spin-dependent polarizability (γ), and on the spin-dependent part of the asymptotic forward Compton amplitude through the Drell-Hearn-Gerasimov (DHG) sum rule. Estimates from current π-photoproduction multipole analyses, particularly for the *proton-neutron difference*, are in good agreement with relativistic-one-loop chiral calculations for γ but predict large deviations from the DHG sum rule. Either (a) *both* the two-loop corrections to the spin-polarizability are large *and* the existing multipoles are wrong, *or* (b) modifications to the Drell-Hearn-Gerasimov sum rule are required to fully describe the isospin structure of the nucleon.

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The recent experiments on deep-inelastic scattering of polarized leptons from polarized protons and neutrons have raised interesting questions on the spin structure of the nucleon [1–3]. Because nonperturbative QCD corrections can potentially influence the interpretation of these measurements, considerable attention has recently been paid to the Q^2 evolution of the spin observables [4–9]. The $Q^2=0$ limit is determined by the total spin-dependent photoabsorption cross sections measured with the photon and nucleon polarizations parallel, $\sigma_{3/2}$, and antiparallel, $\sigma_{1/2}$.

A variety of sum rules have been derived for the integrals of these photoreaction cross sections [10,11]. Two that are quite sensitive to the nucleon spin structure are the spin-dependent polarizability [12] (or "spin polarizability" γ), and the Drell-Hearn-Gerasimov (DHG) integrals [13,14]. For the sake of the subsequent discussion, we recall briefly their origins. Both are derived from considerations of the forward Compton scattering amplitude of Gell-Mann, Goldberger, and Thirring (GGT) [15] which, using crossing symmetry, takes the form

$$A(\omega)=f(\omega^2)\varepsilon'\cdot\varepsilon+i\omega g(\omega^2)\sigma\cdot(\varepsilon'\times\varepsilon). \qquad (1)$$

Here ε and ε' are the incident and final photon polarization vectors and σ is the target spinor. For small values of the photon energy (ω), the functions f and g can be written as

$$f(\omega^2)=f(0)+f'(0)\omega^2+O(\omega^4),$$

$$g(\omega^2)=g(0)+g'(0)\omega^2+O(\omega^4), \qquad (2)$$

where the prime indicates differentiation with respect to ω^2. $f'(0)$ is identified with the sum of the electric and magnetic polarizabilities of the target ($\alpha+\beta$) [16], and by analogy $g'(0)$ is referred to as the spin polarizability γ [12]. The

GGT dispersion relations provide sum rules for these quantities [15], and for the spin polarizability we have

$$\gamma=g'(0)=\frac{1}{4\pi^2}\int_{\omega_0}^{\infty}\frac{\sigma_{1/2}-\sigma_{3/2}}{\omega^3}\,d\omega, \tag{3}$$

where the integration, weighted by the third power of the photon energy, runs from the π threshold, ω_0, to infinity. The polarizabilities are particularly interesting quantities since they can also be calculated with chiral perturbation theory (χPT).

The DHG sum rule [13,14] provides another evaluation of the same difference of spin-dependent cross sections by combining a GGT dispersion relation

$$g(0)-g(\infty)=\frac{1}{4\pi^2}\int_{\omega_0}^{\infty}\frac{\sigma_{1/2}-\sigma_{3/2}}{\omega}\,d\omega, \tag{4a}$$

with the low-energy theorem of Low, Gell-Mann, and Goldberger (LGG) [17],

$$g(0)=-\alpha\kappa^2/2m^2. \tag{4b}$$

The additional assumption that $g(\infty)=0$ in Eq. (4a) results in an integral of the cross sections, weighted by a single power of the photon energy, in terms of the anomalous magnetic moment (κ) of the target,

$$\mathrm{DHG}=\int_{\omega_0}^{\infty}\frac{\sigma_{1/2}-\sigma_{3/2}}{\omega}\,d\omega=-\frac{2\pi^2\alpha}{m^2}\kappa^2. \tag{5}$$

In writing Eqs. (3) and (5), both integrals are implicitly assumed to converge. This is a reasonably safe assumption for γ, because of the $1/\omega^3$ weighting in the integrand. However, the lower power in the energy weighting of the DHG integrand requires that the cross-section difference, which is just the imaginary part of $g(\omega)$ from the optical theorem, falls off with energy faster than $1/\ln\omega$. This, and the explicit requirement that $g(\infty)=0$, make the DHG sum rule critically dependent upon high-energy behavior.

As yet there are no direct measurements of $\sigma_{1/2}$ or $\sigma_{3/2}$. Nonetheless, it is possible to estimate their difference using photoproduction amplitudes constructed from measurements of the different charge channels in pion production. This has been carried out by a number of authors who have reported varying levels of agreement [5,10,15,16]. The purpose of this report is to draw attention to a *significant inconsistency* between multipole predictions for the γ and DHG integrals of Eqs. (3) and (5).

Predictions of helicity-dependent reaction cross sections for both the proton and the neutron can be constructed from an isospin decomposition of multipoles into isovector (VV), isoscalar (SS), and mixed (VS) terms, so that $\sigma_{p,n}=(\sigma^{VV}+\sigma^{SS})\pm\sigma^{VS}$. This has been done using the recent FA93 multipole analysis of single-π production from VPI [20], which extends up to 1.7 GeV. Following Karliner's prescription [18], the known $\pi\pi N/\pi N$ branching ratios of N^* resonances have been used to estimate 2π photoproduction. In Fig. 1 the results for the γ and DHG proton integrals are plotted against E_γ, the upper limit of integration, and are shown as the fraction accumulated up to the 1.7 GeV limit of

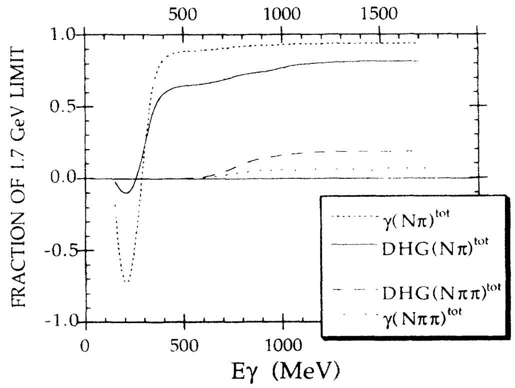

FIG. 1. The fraction of the spin-dependent polarizability and DHG integrals, as a function of the upper limit of integration, compared with the values computed up to 1.7 GeV from the FA93 amplitudes of Workman and Arndt [20]. The "total" value, the sum of the three isospin contributions (VV, SS, and VS), represent the expected results for the proton.

the VPI data base. The large variations below 500 MeV come from the Δ resonance, through the VV component. The flattening of all of the curves above 1 GeV supports convergence of the integrals. The 2π contribution is never large, and is particularly small for γ since the Δ has no $\pi\pi N$ branch and the effect of the higher-lying N^* resonances is drastically reduced by the $1/\omega^3$ weighting of the integral in Eq. (3). For the DHG integral of Eq. (5), these results are similar to those previously reported [19] using an earlier multipole solution (SP92).

In Table I we compare the FA93 predictions for γ to the relativistic χPT calculation of Bernard *et al.* [12]. Although the one-loop results differ considerably from those of the FA93 multipoles, the effect of the Δ has been estimated by Bernard *et al.* and found to be significant and negative ($\gamma^{\Delta}_{p,n}=-366\times10^{-6}$ fm^4), thereby bringing the χPT proton and neutron values much closer to those of the multipole analysis. This is very encouraging, although complete verification must await a calculation including all Δ effects to the same order. Nevertheless, since to this order the Δ contribution is the same for the proton and neutron, the vector-scalar component, $\gamma_{VS}=\frac{1}{2}(\gamma_p-\gamma_n)$, will be unaffected by the Δ, and for γ_{VS}, relativistic one-loop χPT agrees very well with FA93.

In Table II we list the predictions of FA93 for the three isospin components of the DHG integral, together with an earlier analysis by Karliner [18]. The right-most column

TABLE I. Estimates of the nucleon spin-dependent polarizability, using the VPI-FA93 multipole analysis [20], compared with the χPT prediction of Bernard *et al.* [12]. Tabulated values are in units of 10^{-6} fm^4.

	Multipole estimate	χPT	
	FA93	Relativistic one-loop	One-loop $+\Delta$
γ_p	-134	$+216$	-150
γ_n	-38	$+320$	-46
γ_{VS}	-48	-52	-52

TABLE II. The isospin decomposition of the DHG integral, as estimated from photoproduction multipoles, in units of 10^{-4} fm^2, or μb.

	Karliner Ref. [18]			FA93 Ref. [20]	Karliner Ref. [18]		DHG Eq. (5)
	$N\pi$	$N\pi\pi$	Total	$N\pi$	$N\pi\pi$	Total	
VV	-170	-49	-219	-178	-49	-227	-218.5
SS	-2	-1	-3	$+3$	-1	$+2$	-0.3
VS	-24	-15	-39	-50	-15	-65	$+14.7$
Proton			-261			-289	-204.1
Neutron			-183			-160	-233.5

gives the results expected from the magnetic moments in the right-hand side of Eq. (5). The large isovector (VV) component, obtained in these analyses from the $N\pi$ multipoles, appears to be quite stable. Combining this with the estimate for the $N\pi\pi$ contribution gives quite reasonable agreement with the sum rule value. Similarly, the total isoscalar (SS) contribution ($N\pi+N\pi\pi$) is consistently small, as is the corresponding DHG value.

The total multipole estimates from the recent FA93 solution, for both the proton and the neutron, are 40% different from the full DHG sum rule predictions, and this discrepancy is almost entirely due to the vector-scalar (VS) contribution which differs in magnitude from the magnetic-moment value by a factor of 4, and is of the opposite sign. (Here the VS component from FA93 is somewhat larger than that of the SP92 solution reported in Ref. [19].) As seen Fig. 1, almost $\frac{2}{3}$ of the DHG value is saturated in integrating up to 500 MeV, largely because of the $1/\omega$ energy weighting. This is the energy region containing the greatest concentration of published measurements and, thus, it is precisely the region where multipole analyses would be expected to be the most reliable.

The earlier analysis of Karliner also predicted a large negative DHG$_{VS}$ contribution, -39 μb (Table II). Here, the disagreement with the $+15$ μb value from the sum rule, although still appreciable and of opposite sign, is less than that of FA93. The values for $\Delta\sigma^{VS}=[\sigma_{3/2}-\sigma_{1/2}]^{VS}$ predicted by these multipoles are shown in Fig. 2. The main differences between the Karliner [18] and FA93 results lie in the region between 400 and 600 MeV, and occur mostly in the contributions from charged-π production. In the absence of direct measurements, an unambiguous prediction of the $\Delta\sigma=(\sigma_{3/2}-\sigma_{1/2})$ difference requires accurate knowledge, in both isospin channels, of seven quantities: the unpolarized cross section, the three single-polarization observables, and a minimum of 3 out of 12 possible double-polarization observables [21]. There have been many measurements of spin-observables in recent years, although a complete set is still lacking. However, there was almost no information on polarization degrees of freedom at the time of the 1973 Karliner analysis, and this is the main limitation of that work. Figure 2 also shows the predictions of a new VPI solution, SP94. The chief difference between FA93 and SP94 is the recent inclusion in the VPI data base of large sets of high precision single-polarization data from the BNL Laser Electron Gamma Source (LEGS) and Bonn facilities [22]. Nonetheless, the results for $\Delta\sigma^{VS}=[\sigma_{3/2}-\sigma_{1/2}]^{VS}$ are almost unchanged. Further refinements will require double-polarization data.

The FA93 calculations for γ_{VS} and DHG$_{VS}$ are plotted in Fig. 2 as a function of the upper integration bound. Their apparent convergence leaves little room for reconciling the predictions for these quantities. The multipole calculations for the DHG$_{VS}$ integral (Table II) consistently predict the opposite sign (negative) and a significantly larger magnitude than the sum rule of Eq. (5). But, this is in sharp contrast to the FA93 prediction for γ_{VS} (Table I) which is within 8% of the relativistic one-loop χPT value. In principal, the different energy weighings of the γ_{VS} and DHG$_{VS}$ integrals admit the possibility that contributions above the 1.7 GeV limit of the VPI database could bring the DHG$_{VS}$ value up to that expected by the sum rule, without appreciably affecting γ_{VS}. However, 1.7 GeV is already so large that such $\Delta\sigma^{VS}=\frac{1}{2}(\Delta\sigma_p-\Delta\sigma_n)$ differences would have to be huge in order to overcome the $1/\omega$ DHG energy weighting. For example, $\Delta\sigma^{VS}=-200$ μb between 2 and 3 GeV, which would require a prominent but as yet unidentified resonance, or a constant level of $\Delta\sigma^{VS}=-20$ μb extending up to 100 GeV, which would be much larger than the contributions of the resonance region of Fig. 2 in which the isospin structure of the Δ and N^* states can be expected to enhance the proton-

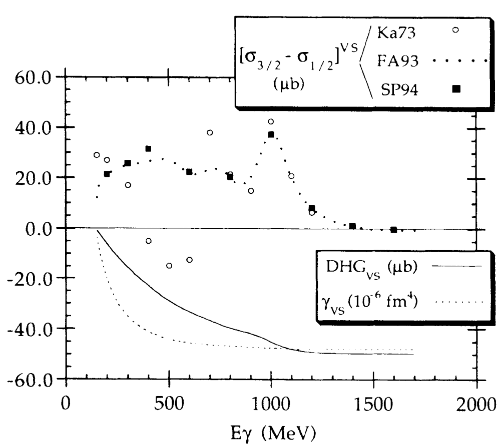

FIG. 2. The predictions for $\Delta\sigma=(\sigma_{3/2}-\sigma_{1/2})$, for the proton-neutron difference $\Delta\sigma^{VS}=\frac{1}{2}(\Delta\sigma_p-\Delta\sigma_n)$, in μb, as computed by Karliner [18], Ka73, and with the FA93 and SP94 multipoles from VPI [20]. The VS contributions to the γ and DHG integrals, computed with FA93, are shown in the lower curves as a function of the upper energy limit of integration.

neutron difference. Furthermore, in either case, the $\frac{1}{2}(\Delta\sigma_p+\Delta\sigma_n)$ sum would have to remain unaffected so as not to destroy the agreement with the isovector and isoscalar components in Table II. Apart from such scenarios, which seem highly unlikely, there are only two other possibilities. Either (a) *both* the two-loop corrections to the spin-polarizability are large *and* the existing multipoles are wrong, *or* (b) modifications to the Drell-Hearn-Gerasimov sum rule are needed to fully describe the isospin structure of the nucleon.

It is possible that the two-loop corrections to the χPT calculation of γ are large. Although this is not usually the case for χPT expansions, it would not be without precedent. Since the $N\pi\pi$ contributions do not appear to be large, a significant two-loop component to γ would then imply that the $N\pi$ multipoles require modifications. Although some modifications could alter the predictions for several other observables, the effects might also be quite subtle. To recover the DHG values in Table II, DHG$_p$ would have to increase by ~ 80 μb while DHG$_n$ decreased by the same amount. If this were achieved by changing $\sigma_{1/2}$ and $\sigma_{3/2}$ by amounts of equal magnitude but of opposite sign, for both the proton and the neutron, then the unpolarized cross sections, and related sum rules such as

$$\alpha+\beta=f'(0)=\frac{1}{4\pi^2}\int_{\omega_0}^{\infty}\frac{\sigma_{1/2}+\sigma_{3/2}}{\omega^2}\,d\omega \tag{6}$$

for the nucleon polarizabilities, would remain in agreement with experiment. Nonetheless, even in such complicated scenarios other double-polarization observables would certainly be affected, notably the beam-recoil asymmetry C_Z and the target-recoil asymmetry L_Z, which, like $\Delta\sigma$ and the unpolarized cross section, are constructed from different combinations of the squares of the helicity amplitudes [21]. Unfortunately, there are as yet no measurements of these quantities.

Alternatively, if the multipoles are basically correct, then the DHG sum rule requires a modification of the form

$$\begin{aligned}\mathrm{DHG}_{VS}&=\frac{1}{2}\int_{\omega_0}^{\infty}\frac{[\sigma_{1/2}-\sigma_{3/2}]_p-[\sigma_{1/2}-\sigma_{3/2}]_n}{\omega}\,d\omega\\&=-\frac{1}{2}(\kappa_p^2-\kappa_n^2)\frac{2\pi^2\alpha}{m^2}-C.\end{aligned} \tag{7}$$

The simplest choice for the correction factor needed to bring the *VS* sum rule down to the FA93 values of Table II would be $C=2\pi^2[g_p(\infty)-g_n(\infty)]$, with $g_p(\infty)\approx -g_n(\infty)\approx 2$ μb so as to preserve the existing agreement in the *VV* and *SS* components. In other words, contrary to the original DHG assumptions, g_p and g_n would tend to nearly equal but opposite constants at high energy. The physical origin of such constants would be quite interesting. Abarbanel and Goldberger [10] have shown that such a situation can result from a $J=1$ fixed pole in the angular momentum plane, but there may be other explanations. In addition, the DHG$_{VS}$ integral is just the $Q^2=0$ limit of the Bjorken sum rule integral [23], and $C=0$ has been assumed in modeling its Q^2 evolution [4–6,8]. The Q^2 dependence of a possible nonzero $g(\infty)$ remains to be considered.

On the other hand, changes in the DHG sum rule due to QCD-current algebra effects, arising from possible corrections to the commutator of the charge densities generated by the quark fields, have also been proposed [24,25]. Estimating such a modification involves a number of assumptions, which are minimized in the proton-neutron difference. In a recent work, Chang, Liang, and Workman have proposed a modified DHG$_{VS}$ sum rule with $C=\alpha g_A/6F_\pi^2$ in Eq. (7). Here, $g_A=1.25$ is the axial-vector coupling constant and $F_\pi=93$ MeV is the pion β-decay constant. This term would bring the expected DHG$_{VS}$ value reasonably close (-54 μb) to the multipole predictions of Table II. However, as pointed out by Kawarabayshi and Suzuki [24], such current-algebra modifications could potentially change the LGG low-energy theorem and it remains to be checked that Eq. (4b) is preserved in such schemes.

In summary, there is a *significant incompatibility* between the χPT calculation for the nucleon spin polarizability, the evaluation of the conventional DHG sum rule, and the predictions for these quantities using recent multipole analyses. If the two-loop corrections to γ are indeed large, it will be quite important to provide an experimental constraint since such calculations are quite demanding. Alternatively, if the corrections to the DHG sum rule are large, a determination of the DHG$_{VS}$ integral could provide a unique constraint on the high-energy spin-dependent Compton amplitude, and on possible components of the quark currents that have previously remained elusive. Ultimately, this situation can only be resolved with direct measurements of the $\sigma_{1/2}$ and $\sigma_{3/2}$ cross sections on both the proton and the neutron. Experiments below 1 GeV are in preparation at the LEGS and Mainz (MAMI) accelerator facilities, and higher energy extensions are planned at the Continuous Electron Beam Accelerator Facility (CEBAF), the Bonn Electron Stretcher Accelerator (ELSA), and the Grenoble Anneau Accélérateur Laser (GRAAL) facility. Since the key physics issues with the least model dependence are in the proton-neutron difference, each of which involve cross section differences themselves, considerable care must be taken to minimize systematic uncertainties.

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