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General effective actions

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We investigate the structure of the most general actions with the symmetry group G, spontaneously broken down to a subgroup H. We show that the only possible terms in the Lagrangian density that, although not G invariant, yield G-invariant terms in the action, are in one to one correspondence with the generators of the fifth cohomology classes. For the special case of $G=\mathrm{SU}(N)_L\times\mathrm{SU}(N)_R$ broken down to the diagonal subgroup $H=\mathrm{SU}(N)_V$, there is just one such term for $N\geq 3$, which for $N=3$ is the original Wess-Zumino-Witten term.

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Effective field theories are increasingly used to understand the dynamics of the Goldstone bosons that result from spontaneous breaking of continuous symmetries. If the action of a theory is invariant under a (compact) Lie group G of global symmetries, spontaneously broken to a subgroup H, then the Goldstone fields $\pi^a(x)$ in the effective action parametrize the coset space G/H with $a=1,\ldots,\dim G/H$, and accordingly transform under linear representations of H, but under nonlinear realizations of the broken symmetries of G. The power of effective field theories arises largely from the fact that the nonlinearly realized broken symmetry allows only a finite number of terms in the action, up to any given order in an expansion in powers of derivatives or momenta.

A general method for constructing invariant nonlinear effective actions was given in Ref. [1] for $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ and was extended to the case of arbitrary G and H in Ref. [2]. But although this method yields the most general G-invariant term in the effective Lagrangian, its results are not quite complete. Wess and Zumino [3] showed that fermion loops produce a four-derivative term in the effective Lagrangian for the strong-interaction Goldstone octet that is not invariant under SU(3)×SU(3), but rather changes under SU(3)×SU(3) transformations by a total derivative, so that the action *is* SU(3)× SU(3) invariant. Subsequently Witten [4] was able to reexpress this term as the integral over an invariant Lagrangian density in five dimensions. The Wess-Zumino-Witten (WZW) action has since then been generalized in Ref. [5] to G/H models with arbitrary G and H.

It is natural to ask whether there are any more possible terms in the action (not necessarily related to anomalies in the underlying theory), that, although invariant under a nonlinearly realized symmetry G, are not the four-dimensional integrals of G-invariant Lagrangian densities. This question seems to us important, as the effective field theory approach is based on our ability to catalog *all* invariant terms in the action with a given number of derivatives.

The first step is to show that even where the action is not the integral of a G-invariant Lagrangian density, its variation with respect to the Goldstone boson fields is an invariant density. The Goldstone boson fields $\pi^a(x)$ enter the action as a parametrization of a general spacetime-dependent G transformation $U(\pi(x))$, so the variation of the action under an arbitrary change in π may be written as

$$\delta S[\pi]=\int d^4x\ \mathrm{Tr}\{(U^{-1}\delta U)_{\mathscr{X}}J\}\ , \tag{1}$$

where a subscript $\mathscr{X}$ or $\mathscr{H}$ will denote the terms proportional to the broken and unbroken symmetry generators x_a and t_i, respectively, and the coefficient J is a local function of the Goldstone boson fields and their derivatives. Let us work out how J transforms. According to the general formalism of [2], under a global transformation $g\in G$, the Goldstone boson fields undergo the transformation $\pi\rightarrow\pi'$, with

$$gU(\pi)=U(\pi')h(\pi,g)\ , \tag{2}$$

where $h(\pi,g)$ is some element of the unbroken subgroup H. Since $S[\pi]=S[\pi']$ for all π, the variational derivatives are also equal:

$$\frac{\delta S[\pi']}{\delta\pi^a}=\frac{\delta S[\pi]}{\delta\pi^a}\ .$$

(Note that the derivative is with respect to π, not π', on both sides of the equation.) Using Eq. (1), this is

$$\mathrm{Tr}\left\{\left[U^{-1}(\pi')\frac{\partial U(\pi')}{\partial\pi^a}\right]_{\mathscr{X}}J(\pi')\right\}=\mathrm{Tr}\left\{\left[U^{-1}(\pi)\frac{\partial U(\pi)}{\partial\pi^a}\right]_{\mathscr{X}}J(\pi)\right\}\ . \tag{3}$$

To put this in a useful form, take the derivative of Eq. (2) with respect to π^a, and multiply on the left with $U(\pi')^{-1}$ and on the right with $h^{-1}(\pi,g)$:

$$U^{-1}(\pi')\frac{\partial U(\pi')}{\partial\pi^a}=h(\pi,g)U^{-1}(\pi)\frac{\partial U(\pi)}{\partial\pi^a}h^{-1}(\pi,g)-\frac{\partial h(\pi,g)}{\partial\pi^a}h^{-1}(\pi,g)$$

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and so

$$\left[U^{-1}(\pi')\frac{\partial U(\pi')}{\partial\pi^a}\right]_{\mathscr{X}} = h(\pi,g)\left[U^{-1}(\pi)\frac{\partial U(\pi)}{\partial\pi^a}\right]_{\mathscr{X}} h^{-1}(\pi,g) \ . \quad (4)$$

Equation (3) then becomes

$$\mathrm{Tr}\left\{\left[U^{-1}(\pi)\frac{\partial U(\pi)}{\partial\pi^a}\right]_{\mathscr{X}}[h^{-1}(\pi,g)J(\pi')h(\pi,g)]\right\} = \mathrm{Tr}\left\{\left[U^{-1}(\pi)\frac{\partial U(\pi)}{\partial\pi^a}\right]_{\mathscr{X}}J(\pi)\right\} \ . \quad (5)$$

From linear combinations of the quantities $[U^{-1}(\pi)\partial U(\pi)/\partial\pi^a]_{\mathscr{X}}$ we can form arbitrary linear combinations [6] of the broken symmetry generators x_a, so (5) yields the transformation rule for J:

$$J(\pi')=h(\pi,g)J(\pi)h^{-1}(\pi,g) \ . \quad (6)$$

Following the same arguments that led to (4), we easily see that also

$$[U^{-1}(\pi')\delta U(\pi')]_{\mathscr{X}} = h(\pi,g)[U^{-1}(\pi)\delta U(\pi)]_{\mathscr{X}}h^{-1}(\pi,g) \ , \quad (7)$$

so $\mathrm{Tr}\{(U^{-1}\delta U)_{\mathscr{X}}J\}$ is invariant under G.

This result leads to a natural five-dimensional formulation of the theory. As usual, we compactify spacetime to a four-sphere M_4 by requiring that all fields approach definite limits as $x^\mu\to\infty$. The operator $U(\pi(x))$ therefore traces out a four-sphere in the manifold of G/H as x^μ varies over M_4. If the homotopy group $\pi_4(G/H)$ is trivial [as is the case for SU(N)×SU(N) spontaneously broken to SU(N) with $N\geqslant 3$], or if $U(\pi(x))$ belongs to the trivial element of $\pi_4(G/H)$, then we may introduce a smooth function $\tilde\pi^a(x,t_1)$, such that $\tilde\pi^a(x,1)=\pi^a(x)$, and $\tilde\pi^a(x,0)=0$. In this way spacetime is extended to a five-ball B_5 with boundary M_4 and coordinates x^μ and t_1. The action may then be written in the five-dimensional form

$$S[\pi]=\int_{B_5} d^4x dt_1 \ \mathscr{L}_1 , \quad (8)$$

where $\mathscr{L}_1$ is the G-invariant density $\mathrm{Tr}\{(U^{-1}\partial U/\partial t_1)_{\mathscr{X}}J\}$. [When $\pi_4(G/H)\neq 0$, we may interpolate between $\pi^a(x)$ and a fixed representative $\pi_0^a(x)$ of the homotopy class of $\pi^a(x)$. The difference $S[\pi]-S[\pi_0]$ is given by the integral over the cylinder $M_4\times[0,1]$ of the same density $\mathscr{L}_1$ as in (8) and the arguments to be presented below still hold. In some cases, G/H may be naturally embedded into a larger space with vanishing fourth homotopy group, as is the case for SU(2) embedded in SU(3), considered in [4].]

We next show that this is the integral of a G-invariant five-form on G/H. Consider a general deformation $\pi(x)\to\tilde\pi(x;t)$, where t^i are a set of $\dim(G/H)-4$ free parameters, that along with the x^μ provide a set of coordinates for G/H. The coordinate t_1 in (8) can be chosen to be any one of these parameters. We have shown that

$$\frac{\partial S[\tilde\pi]}{\partial t_i}=\int_{M_4} d^4x \ \mathscr{L}_i , \quad (9)$$

where $\mathscr{L}_i\equiv\mathrm{Tr}\{(\tilde U^{-1}\partial\tilde U/\partial t^i)_{\mathscr{X}}J\}$ are G-invariant functions of $\tilde\pi^a$ and its derivatives. The general rules of [2] would allow a wide variety of terms in $\mathscr{L}_i$, but these are limited by integrability conditions. From (9) we see that

$$\int_{M_4} d^4x\left(\frac{\partial\mathscr{L}_i}{\partial t^j}-\frac{\partial\mathscr{L}_j}{\partial t^i}\right)=0 \ .$$

Since this integral vanishes for all $\tilde\pi(x)$, its integrand must be an x derivative [7]:

$$\frac{\partial\mathscr{L}_i}{\partial t^j}-\frac{\partial\mathscr{L}_j}{\partial t^i}=-\partial_\mu\mathscr{L}^\mu_{ij} .$$

This can be written in the language of differential forms, as $d_tF_1=-d_xF_2$, where

$$d_t\equiv dt^i\partial_i , \quad d_x\equiv dx^\mu\partial_\mu$$

and F_1 and F_2 are the five-forms

$$F_1\equiv\tfrac{1}{24}\epsilon_{\mu\nu\rho\sigma}\,\mathscr{L}_i dt^i dx^\mu dx^\nu dx^\rho dx^\sigma ,$$

$$F_2\equiv\tfrac{1}{12}\epsilon_{\mu\nu\rho\sigma}\,\mathscr{L}^\mu_{ij} dt^i dt^j dx^\nu dx^\rho dx^\sigma \ .$$

It follows that $0=d_t^2F_1=d_x(d_tF_2)$, so by an extension of Poincaré's lemma, in any simply connected patch we will have $d_tF_2=-d_xF_3$, where F_3 is a five-form $\epsilon_{\mu\nu\rho\sigma}\mathscr{L}^{\mu\nu}_{ijk}dt^i dt^j dt^k dx^\mu dx^\nu$. Continuing in this way, we can construct five-forms F_4 and F_5 proportional, respectively, to four and five dt factors, with $d_tF_3=-d_xF_4$, $d_tF_4=-d_xF_5$, and $d_tF_5=0$. Hence $F\equiv\Sigma_{N=1}^5F_N$ is a closed five-form on G/H:

$$dF=0, \quad d\equiv d_x+d_t . \quad (10)$$

Also, because B_5 has t_2,t_3, etc., all constant, Eq. (8) may be written

$$S[\pi]=\int_{B_5} F \ . \quad (11)$$

So far, only the term F_1 has been shown to be G invariant. The group G acts transitively on the manifold G/H, so a G transform of a form is always continuously connected to the original form. Thus the two-forms are homotopic and define the same de Rham cohomology class. One can construct a G-invariant form in this cohomology class by integrating the form over the group G with the invariant Haar measure [8,9]. This has no effect on (11), since the integral depends only on F_1, which is already invariant. Also, one can similarly show that any two invariant p-forms in the same cohomology class differ not only by an exterior derivative, but specifically by the exterior derivative of an *invariant* $(p-1)$-form. Such an exterior derivative term in the five-form F would yield a term in (8) that can be written as the four-dimensional integral of a G-invariant density, so the classification of terms in $S[\pi]$ that *cannot* be so written is now reduced to the problem of finding the fifth de Rham cohomology group $H^5(G/H;\mathbf{R})$ of the manifold G/H [10].

The fifth de Rham cohomology group is well known where G/H is itself a simple Lie group. For $G=\mathrm{SU}(N)$ with $N\geqslant 3$ [including the case $\mathrm{SO}(6)\sim\mathrm{SU}(4)$], $H^5(G;\mathbf{R})$ has a single generator

$$\Omega_5=\frac{-i}{240\pi^2}\mathrm{Tr}(U^{-1}dU)^5 . \tag{12}$$

(Here and henceforth, we suppress wedges in the exterior product of differential forms, reserving them for the products of cohomology groups.) This is in particular the case for $\mathrm{SU}(N)\times\mathrm{SU}(N)$ spontaneously broken to $\mathrm{SU}(N)$ with $N\geqslant 3$, where G/H is itself just $\mathrm{SU}(N)$. Equation (12) is the original Wess-Zumino-Witten term, which we now see is indeed unique. All other simple [or U(1)] Lie groups have trivial fifth cohomology groups. For the original case [1] of $\mathrm{SU}(2)\times\mathrm{SU}(2)$ spontaneously broken to SU(2) the cohomology is trivial, so all invariant actions are the integrals of invariant Lagrangian densities.

Where G/H is a product space, we use the Künneth formula [8,9]

$$H^k(K_1\times K_2;\mathbf{R})=\sum_{k_1+k_2=k} H^{k_1}(K_1;\mathbf{R})\wedge H^{k_2}(K_2;\mathbf{R}) , \tag{13}$$

which gives $H^5(G/H;\mathbf{R})$ in terms of the cohomologies of its factors up to degree 5. For this purpose, we need to know that [11–13] for all simple Lie groups G, $H^k(G;\mathbf{R})$ vanishes for $k=1,2,4$ while $H^3(G;\mathbf{R})$ has a single generator (corresponding to the Goldstone-Wilczek topologically conserved current [14])

$$\Omega_3=\frac{i}{12\pi}\mathrm{Tr}(U^{-1}dU)^3 . \tag{14}$$

Also $H^k(U(1);\mathbf{R})$ vanishes for $k>1$, while for $k=1$ it has a single generator

$$\Omega_1=-i\mathrm{Tr}(U^{-1}dU) . \tag{15}$$

Finally, $H^0(K;\mathbf{R})=\mathbf{R}^c$, where c is the number of connected components of K; for our purposes this just means that if $H^5(K;\mathbf{R})$ for some space K has a generator Ω_5, then $H^5(K\times K';\mathbf{R})$ has the same generator for any K'. To each generator of $H^5(G/H;\mathbf{R})$, there corresponds a WZW-like term in the five-dimensional Lagrangian, and an independent coupling constant. In particular, if G is semisimple, with precisely p factors $\mathrm{SU}(N_i)$ with $N_i\geqslant 3$ and all other simple factors with $H^5=0$, then we have p different terms of the Wess-Zumino-Witten type, each of which has an independent coupling constant in the action. This result is of course expected for a product of groups, and is known to appear explicitly in the low energy effective action when massive fermions are integrated out of the path integral [15].

When G/H is not itself a Lie group, the fifth cohomology group of G/H may still be obtained from that of G. For any simple group G and subgroup H, we may construct a "projected" five-form on G/H that is invariant under local H transformations [5,15–17], and is given by

$$\Omega_5(U;V)=\frac{-i}{240\pi^2}\{\mathrm{Tr}(U^{-1}DU)^5-5\mathrm{Tr}W(U^{-1}DU)^3 + 10\mathrm{Tr}W^2U^{-1}DU\} , \tag{16}$$

where V is the H connection $V=(U^{-1}dU)_{\mathscr{H}}$, DU is the H-covariant derivative $DU=dU-UV$, and the trace is evaluated in any convenient representation of G, usually taken as the defining representation. In general, $\Omega_5(U;V)$ is neither closed nor simply related to the generator $\Omega_5(U;0)$ of $H^5(G;\mathbf{R})$. Rather,

$$d\Omega_5(U;V)=\frac{i}{24\pi^2}d_{rst}W^rW^sW^t \tag{17}$$

and

$$\Omega_5(U;V)=\Omega_5(U;0)+\Omega_5(1;V)+d\gamma(U;V) , \tag{18}$$

where W is the field strength $W=dV+V^2$, and d_{rst} is the trace of the symmetrized product of generators ρ^r of H, $2d_{rst}=\mathrm{Tr}\rho^r\{\rho^s,\rho^t\}$, which plays a key role in the study of the chiral anomaly in four dimensions [18]. But if $d_{rst}=0$, then the five-form $\Omega_5(U;V)$ is closed, and also each term in the five-dimensional Chern-Simons term $\Omega_5(1;V)$ for the $\mathscr{H}$-valued gauge field V vanishes. The form $\Omega_5(U;V)$ then belongs to the same cohomology class as $\Omega_5(U;0)$ and it can be shown that there is a one to one correspondence between the fifth cohomology generators of G/H and those of G. On the other hand, if $d_{rst}\neq 0$, then the projected form of (16) is not closed and it can be shown that the fifth cohomology is trivial in this case. For example, any coset space of the type $\mathrm{SU}(n)/H$ with $n\geqslant 3$, where H is embedded in G in such a way that $d_{rst}=0$, has one cohomology generator, given in (16).

It is noteworthy that the simple groups SU(2), Sp($2N$), SO(N), $N\geqslant 7$; E_6, E_7, E_8, F_4, G_2 that have zero fifth cohomology are also those that have vanishing d symbols. We now see that for such groups, the coset spaces G/H have $H^5(G/H;\mathbf{R})=0$ for all subgroups H. These properties are easily verified for the special case of compact symmetric spaces [12,13]. Also, when $\mathrm{rank}(G)=\mathrm{rank}(H)$, a classic theorem [13] states that all odd cohomology classes vanish. An example of a general class of manifolds G/H with $\mathrm{rank}(H)\leqslant\mathrm{rank}(G)$ for which $H^5(G;\mathbf{R})\neq 0$ and $H^5(G/H;\mathbf{R})=0$ is provided [12] by

$$\mathrm{SU}(n)/S(U(k_1)\times\cdots\times U(k_q)),\quad k=\sum_{\alpha=1}^{q}k_\alpha\geqslant 3 ,$$

with the $U(k_\alpha)$ embedded in SU(n) in such a way that the defining representation of SU(n) transforms also as the defining representation of $S(U(k_1)\times\cdots\times U(k_q))$.

Finally, if G is not simple, and H is a nontrivial subgroup, the cohomology problem can be solved by analyzing the restriction of the d symbols of G to the subgroup H [19]. If G is semisimple (and H is connected), two types of cohomology generators arise [20]. First, the projected form of

(16) is now obtained as a linear combination of $\Omega_5(U;V)$ on each simple component of G with nonvanishing fifth cohomology. Linear combinations for which d_{rst} on the subgroup H vanishes yield generators of $H^5(G/H;R)$. Second, there may be generators that are linear combinations of products of cohomology generators on G/H of degrees 2 and 3. Generators of degree 2 correspond to the field strength associated with generators of invariant Abelian subgroups of H [i.e., $U(1)$ factors]. Generators of degree 3 correspond to the Goldstone-Wilczek current of (14), projected to G/H. When G is not semisimple and contains extra $U(1)$ factors, there are also linear combinations of products of generators of degree 1 with generators of degrees 1, 2, 3, and 4.

We conclude with a brief discussion of global quantization conditions. Different interpolating maps are generally topologically inequivalent [their equivalence classes being given by $\pi_5(G/H)$], and there is no natural way of choosing one interpolation above another. Witten has argued that the quantum action can be allowed to be multiple valued, provided the action changes additively by integer multiples of 2π [4]. The dependence of interpolation becomes invisible in the quantum theory provided the coupling constants multiplying Ω_5 as normalized in (12) are integers. In the present case, this quantization condition must be enforced on every independent coupling constant multiplying each nontrivial WZW term normalized as in (12).

A slight refinement of this quantization condition is required when $\pi_4(G)=0$ and $\pi_4(H)\neq 0$. For all simple groups H we have $\pi_4(H)=0$, except when H is a symplectic group, for which $\pi_4(Sp(2n))=\mathbf{Z}_2$. Whenever $\pi_4(H)=\mathbf{Z}_2$, H has a discrete anomaly [21], even though its d symbols vanish identically, and it can be shown that the coupling constant of the corresponding term of $H^5(G/H;\mathbf{R})$ must be quantized in terms of *even* integers to obtain a single-valued path integral [22].

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