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$SU(3)_L \otimes U(1)_N$ and $SU(4)_L \otimes U(1)_N$ gauge models with right-handed neutrinos

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Pisano and Pleitez have introduced an interesting $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ gauge model which has the property that gauge anomaly cancellation requires the number of generations to be a multiple of 3. We consider generalizing that model to incorporate right-handed neutrinos. We find that there exists a nontrivial generalization of the Pisano-Pleitez model with right-handed neutrinos which is actually simpler than the original model in that symmetry breaking can be achieved with just three $SU(3)_L$ triplets [rather than three $SU(3)_L$ triplets and a sextet]. We also consider a gauge model based on $SU(3)_C \otimes SU(4)_L \otimes U(1)_N$ symmetry. Both of these new models also have the feature that the anomalies cancel only when the number of generations is divisible by 3.

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Pisano and Pleitez [1] have proposed an interesting model based on the gauge group

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_N \tag{1}$$

(for further work on this model see Refs. [2,3]). This model has the interesting feature that each generation of fermions is anomalous, but that with three generations the anomalies canceled.

In this paper we point out that if right-handed neutrinos are included then there is an alternative $SU(3)_L \otimes U(1)_N$ gauge model. This alternative model is actually simpler than the Pisano-Pleitez model because it turns out that less Higgs mulitplets are needed in order to allow the fermions to gain masses and to break the gauge symmetry. This alternative model also has the interesting property that anomalies only cancel when all three generations are included. This alternative model allows for Dirac neutrino masses. We will also discuss a $SU(3)_C \otimes SU(4)_L \otimes U(1)_N$ gauge model which also includes right-handed neutrinos and also requires three generations to cancel the anomalies.

We start by briefly reviewing the Pisano-Pleitez model. In that model the three lepton generations transform under the gauge symmetry, Eq. (1), as

$$f_L^a = \begin{pmatrix} \nu_L^a \\ e_L^a \\ (e_R^c)^a \end{pmatrix} \sim (1,3,0), \tag{2}$$

where $a=1,2,3$ is the generation index.

Two of the three quark generations transform identically and one generation transforms in a different representation of $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ (although note that all of the right-handed u- and d-type quarks have the same gauge quantum numbers):

$$\begin{gathered} Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ J_1 \end{pmatrix}_L \sim (3,3,2/3), \\ u_{1R} \sim (3,1,2/3), \quad d_{1R} \sim (3,1,-1/3), \\ J_{1R} \sim (3,1,5/3), \\ Q_{iL} = \begin{pmatrix} d_i \\ u_i \\ J_i \end{pmatrix}_L \sim (3,\bar{3},-1/3), \\ u_{iR} \sim (3,1,2/3), \quad d_{iR} \sim (3,1,-1/3), \\ J_{iR} \sim (3,1,-4/3), \end{gathered} \tag{3}$$

where $i=2,3$.

Symmetry breaking and fermion mass generation can be achieved by three scalar $SU(3)_L$ triplets and a sextet. For these details the reader can see Ref. [1].

If right-handed neutrinos are included then we can add either three $SU(3)_L \otimes U(1)_N$ singlets $\nu_R^a \sim (1,1,0)$ (which is a trivial generalization of the Pisano-Pleitez model), or we can try to modify the quantum numbers of the fermions such that ν_R replaces e_R as the third component of the lepton triplets. We will show that this latter case is possible and it leads to an $SU(3)_L$ model which is simpler than the Pisano-Pleitez model with the same nice features.

The gauge quantum numbers of the fermions are as follows: The leptons consist of

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$$f_L^a = \begin{pmatrix} \nu_L^a \\ e_L^a \\ (\nu_R^c)^a \end{pmatrix} \sim (1,3,-1/3), \quad e_R^a \sim (1,1,-1), \tag{4}$$

where $a=1,\ldots,3$. As in the case of the Pisano-Pleitez model, two of the quark generations transform identically, and one generation transforms differently (again note that the right-handed u- and d-type quarks of each generation transform the same way):

$$Q_{1L} = \begin{pmatrix} u_{1L} \\ d_{1L} \\ u'_{1L} \end{pmatrix} \sim (3,3,1/3),$$

$$u_{1R} \sim (3,1,2/3), \quad d_{1R} \sim (3,1,-1/3),$$

$$u'_{1R} \sim (3,1,2/3),$$

$$Q_{iL} = \begin{pmatrix} d_{iL} \\ u_{iL} \\ d'_{iL} \end{pmatrix} \sim (3,\bar{3},0),$$

$$u_{iR} \sim (3,1,2/3), \quad d_{iR} \sim (3,1,-1/3),$$

$$d'_{iR} \sim (3,1,-1/3), \tag{5}$$

where $i=2,3$. It is straightforward to check that all gauge anomalies cancel with the above choice of gauge quantum numbers. As in the Pisano-Pleitez model, each generation is anomalous but with all three generations the anomalies cancel. Symmetry breaking and fermion mass generation can be achieved with just three $SU(3)_L$ Higgs triplets. We define them by their Yukawa Lagrangians as[1]

$$\mathscr{L}_{\rm Yuk}^{\chi} = \lambda_1 \bar{Q}_{1L} u'_{1R} \chi + \lambda_{2ij} \bar{Q}_{iL} d'_{jR} \chi^* + \text{H.c.}, \tag{6}$$

where $\chi \sim (1,3,-1/3)$ and, if χ gets the vacuum expectation value (VEV),

$$\langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix}, \tag{7}$$

then the exotic 2/3 and $-1/3$ quarks gain masses and the gauge symmetry is broken to the standard model gauge symmetry:

$$\begin{gathered} SU(3)_c \otimes SU(3)_L \otimes U(1)_N \\ \downarrow \langle \chi \rangle \\ SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \end{gathered} \tag{8}$$

[1]Note that we have implicitly defined the d'_{jR} and u'_{1R} fields as the linear combination of right-handed u and d quarks which couples to the χ field. It should be clear that this can be done without loss of generality.

where $Y = 2N - \sqrt{3}\lambda_8/3$ [$\lambda_8 = \text{diag}(1,1,-2)/\sqrt{3}$] is the combination of N and λ_8 which annihilates the VEV (i.e., $Y\langle\chi\rangle = 0$). Note that Y is identical to the standard hypercharge of the standard model. Electroweak symmetry breaking and ordinary fermion mass generation are achieved with two $SU(3)_L$ triplets ρ, η which we define through their Yukawa Lagrangians as

$$\begin{aligned} \mathscr{L}_{\rm Yuk}^{\rho} &= \lambda_{1a} \bar{Q}_{1L} d_{aR} \rho + \lambda_{2ia} \bar{Q}_{iL} u_{aR} \rho^* + G_{ab} \bar{f}_L^a (f_L^b)^c \rho^* \\ &\quad + G'_{ab} \bar{f}_L^a e_R^b \rho + \text{H.c.}, \\ \mathscr{L}_{\rm Yuk}^{\eta} &= \lambda_{3a} \bar{Q}_{1L} u_{aR} \eta + \lambda_{4ia} \bar{Q}_{iL} d_{aR} \eta^* + \text{H.c.}, \end{aligned} \tag{9}$$

where $a,b = 1,\ldots,3$, $i=2,3$ and $\rho \sim (1,3,2/3)$, $\eta \sim (1,3,-1/3)$. Note that we require the vacuum structure

$$\langle \rho \rangle = \begin{pmatrix} 0 \\ u \\ 0 \end{pmatrix}, \quad \langle \eta \rangle = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}. \tag{10}$$

The VEV $\langle\rho\rangle$ will generate masses for the three charged leptons and two of the neutrinos will gain degenerate Dirac masses (with one necessarily massless) and two up-type quarks and one down-type quark will also gain masses while the VEV $\langle\eta\rangle$ will generate masses for the remaining quarks. Observe that, while the model accommodates neutrino masses it does not seem to give any indication as to why they are smaller than the masses of the charged fermions. In the model as we have presented it, one must assume that this is due to rather small values of the Yukawa couplings G_{ab}. The VEV's $\langle\rho\rangle,\langle\eta\rangle$ also give the electroweak gauge bosons masses and results in the symmetry breaking:

$$\begin{gathered} SU(3)_c \otimes SU(3)_L \otimes U(1)_N \\ \downarrow \langle \chi \rangle \\ SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \\ \downarrow \langle \rho \rangle, \langle \eta \rangle \\ SU(3)_c \otimes U(1)_Q . \end{gathered} \tag{11}$$

Note that because $\langle\rho\rangle,\langle\eta\rangle$ transform as part of a $Y=1$, $SU(2)_L$ doublet [under the $SU(2)_L \otimes U(1)_Y$ subgroup of $SU(3)_L \otimes U(1)_N$] the correct W, Z mass relation ensues, and the model essentially reduces to the standard model provided $\langle\chi\rangle \gg \langle\eta\rangle,\langle\rho\rangle$.

An important phenomenological difference between this model and the Pisano-Pleitez model is that the exotic quarks have electric charges 2/3 and $-1/3$ (in the Pisano-Pleitez model the exotic quarks had electric charges 5/3 and $-4/3$). A consequence of this is that the exotic quarks can mix with the ordinary ones. Indeed, in Eq. (9), we can have extra terms obtained by replacing d_{aR}, u_{aR} with d'_{iR}, u'_{1R}. One important consequence of this type of mixing is that small flavor-changing neutral currents (FCNC's) will be induced due to the breakdown of the Glashow-Iliopoulos-Maiani (GIM) mechanism (one can easily check however that as $\langle\chi\rangle$ goes to infinity these induced FCNC's go to zero). This type of situation has been discussed previously and bounds on the mixing strengths can be obtained from the experimental nonobservation of FCNC's beyond those predicted by the standard model [4].

The required vacuum structure Eqs. (7) and (10) can be obtained from a Higgs potential. The most general Higgs potential (i.e., the one including all terms consistent with gauge invariance and renormalizability) is very complicated. However, it turns out that under the assumption of the discrete symmetry,[2] $\chi \to -\chi$, then the potential can be written in such a way so that the required vacuum is manifest. The most general potential consistent with the discrete symmetry $\chi \to -\chi$ contains 13 parameters, which can be written in terms of three VEV's and 10 dimensionless coupling constants in the following way:

$$\begin{aligned}V(\chi,\rho,\eta)=&\lambda_1(\chi^\dagger\chi-w^2)^2+\lambda_2(\rho^\dagger\rho-u^2)^2+\lambda_3(\eta^\dagger\eta-v^2)^2+\lambda_4[(\chi^\dagger\chi-w^2)+(\rho^\dagger\rho-u^2)]^2\\&+\lambda_5[(\chi^\dagger\chi-w^2)+(\eta^\dagger\eta-v^2)]^2+\lambda_6[(\rho^\dagger\rho-u^2)+(\eta^\dagger\eta-v^2)]^2\\&+\lambda_7(\chi^\dagger\eta+\eta^\dagger\chi)^2+\lambda_8(\chi^\dagger\rho)(\rho^\dagger\chi)+\lambda_9(\chi^\dagger\eta)(\eta^\dagger\chi)+\lambda_{10}(\eta^\dagger\rho)(\rho^\dagger\eta),\end{aligned} \tag{12}$$

where $\lambda_1,\ldots,\lambda_{10}$ are required to be real in order for the potential to be Hermitian. Having written the 13 parameters of the Higgs potential in this way, then for the range of parameters in which $\lambda_1,\ldots,\lambda_{10}\geqslant 0$, then $V(\chi,\rho,\eta)\geqslant 0$. This is because each term in the potential is then either positive or zero. If values of χ, ρ, η are found such that each term of the potential is zero, then it will be the vacuum of the theory. Clearly the vacuum given by Eqs. (7) and (10) is the vacuum of this potential (provided $\lambda_1,\ldots,\lambda_{10}\geqslant 0$) since one can check that $V(\langle\chi\rangle,\langle\rho\rangle,\langle\eta\rangle)=0$. Note that the first six terms (i.e., with couplings $\lambda_1,\ldots,\lambda_6$) imply that χ, ρ, and η have nonzero VEV's, but do not give any information about how they are aligned, whereas the last three terms (i.e., the terms with couplings $\lambda_8,\ldots,\lambda_{10}$) imply that the VEV's are orthogonal to each other. In other words, the correct vacuum alignment results.

We now turn to another possibility beyond the standard model. This extension of the standard model is based on $SU(3)_C\otimes SU(4)_L\otimes U(1)_N$ gauge group [3]. In this $SU(3)_C\otimes SU(4)_L\otimes U(1)_N$ model, the three lepton generations transform under the gauge symmetry as

$$f_L^a=\begin{pmatrix}\nu_L^a\\ e_L^a\\ (\nu_R^c)^a\\ (e_R^c)^a\end{pmatrix}_L \sim(1,4,0), \tag{13}$$

where $a=1,2,3$ is the generation index. In the quark sector, two of the three quark generations transform identically and one generation transforms in a different representation of $SU(4)_L\otimes U(1)_N$. The quarks have the following representation under the $SU(3)_C\otimes SU(4)_L\otimes U(1)_N$ gauge group:

$$Q_{1L}=\begin{pmatrix}u_1\\ d_1\\ u_1'\\ J_1\end{pmatrix}_L \sim(3,4,2/3),$$

$$u_{1R}\sim(3,1,2/3),\quad d_{1R}\sim(3,1,-1/3),$$

$$u'_{1R}\sim(3,1,2/3),\quad J_{1R}\sim(3,1,5/3),$$

$$Q_{iL}=\begin{pmatrix}d_i\\ u_i\\ d_i'\\ J_i\end{pmatrix}_L \sim(3,\bar{4},-1/3),$$

$$u_{iR}\sim(3,1,2/3),\quad d_{iR}\sim(3,1,-1/3),$$

$$d'_{iR}\sim(3,1,2/3),\quad J_{iR}\sim(3,1,-4/3), \tag{14}$$

where $i=2,3$. All gauge anomalies cancel in this theory. As discussed in Ref. [3] this type of construction is only anomaly-free when the number of generations is divisible by 3. In the fermion representations we have added right-handed neutrinos and the exotic quarks $u_1', d_{2,3}', J_{1,2,3}$.

We now discuss symmetry breaking in this model. We introduce the Higgs field

$$\chi_1\sim(1,4,-1), \tag{15}$$

which couples via the Yukawa Lagrangian

$$\mathscr{L}_{\mathrm{Yuk}}^{\chi_1}=\lambda_1\bar{Q}_{1L}J_{1R}\chi_1+\lambda_{1ij}\bar{Q}_{iL}J_{jR}\chi_1{}^*+\text{H.c.}, \tag{16}$$

where $i,j=2,3$. If χ_1 gets the VEV,

$$\langle\chi_1\rangle=\begin{pmatrix}0\\0\\0\\w_1\end{pmatrix}, \tag{17}$$

then the exotic charged 5/3 and $-4/3$ quarks ($J_{1,2,3}$) gain masses. In order for u_1' and $d_{2,3}'$ to gain masses, we introduce the Higgs field

$$\chi_2\sim(1,4,0), \tag{18}$$

which couples via the Yukawa Lagrangian

[2]This can be made into a symmetry of the full Lagrangian, by assuming that $u'_{1R}\to -u'_{1R}, d'_{jR}\to -d'_{jR}$; $j=1,2$.

$$\mathcal{L}^{\chi_2}_{\text{Yuk}}=\lambda_2\bar{Q}_{1L}u'_{1R}\chi_2+\lambda_{2ij}\bar{Q}_{iL}d'_{jR}\chi_2^*+\text{H.c.}, \tag{19}$$

where $i,j=2,3$ and χ_2 gets the VEV:

$$\langle\chi_2\rangle=\begin{pmatrix}0\\0\\w_2\\0\end{pmatrix}. \tag{20}$$

With the two Higgs fields $\chi_{1,2}$ the gauge symmetry is broken to the standard model, as indicated below:

$$\begin{gathered}\text{SU}(3)_C\otimes\text{SU}(4)_L\otimes\text{U}(1)_N\\ \downarrow\langle\chi_1\rangle,\langle\chi_2\rangle\\ \text{SU}(3)_C\otimes\text{SU}(2)_L\otimes\text{U}(1)_Y,\end{gathered} \tag{21}$$

where Y is the linear combination of λ_8,λ_{15} and N which annihilates $\langle\chi_1\rangle$ and $\langle\chi_2\rangle$ and one can easily check that it is given by

$$Y=2N-\frac{1}{\sqrt{3}}\lambda_8-\frac{2\sqrt{6}}{3}\lambda_{15}, \tag{22}$$

where λ_8 and λ_{15} are diagonal SU(4) generators with $\lambda_8=\text{diag}(1,1,-2,0)/\sqrt{3}$ and $\lambda_{15}=\text{diag}(1,1,1,-3)/\sqrt{6}$. One can easily check that Y is numerically identical to the standard model hypercharge.

Electroweak symmetry breaking and the fermion masses are assumed to be due to the VEV's of the Higgs bosons:

$$\rho\sim(1,4,1),\quad \eta\sim(1,4,0),\quad S\sim(1,10,0). \tag{23}$$

These Higgs bosons couple to the fermions through the Yukawa Lagrangian:

$$\begin{aligned}\mathcal{L}^{\rho}_{\text{Yuk}}&=\lambda_{1a}\bar{Q}_{1L}d_{aR}\rho+\lambda_{ia}\bar{Q}_{iL}u_{aR}\rho^*+\text{H.c.},\\ \mathcal{L}^{\eta}_{\text{Yuk}}&=\lambda'_{1a}\bar{Q}_{1L}u_{aR}\eta+\lambda'_{ia}\bar{Q}_{iL}d_{aR}\eta^*+\text{H.c.},\\ \mathcal{L}^{S}_{\text{Yuk}}&=G_{ab}\bar{f}_{aL}(f_{bR})^cS+\text{H.c.},\end{aligned} \tag{24}$$

where $a,b=1,\ldots,3$ and $i=2,3$. If ρ gets the VEV,

$$\langle\rho\rangle=\begin{pmatrix}0\\u\\0\\0\end{pmatrix}, \tag{25}$$

two up and one down quarks gain mass. If η gets the VEV,

$$\langle\eta\rangle=\begin{pmatrix}v\\0\\0\\0\end{pmatrix}, \tag{26}$$

then the remaining quarks get masses. If S gets the VEV [note that the 10 representation of SU(4) can be represented as a 4×4 symmetric matrix],

$$\langle S\rangle=\begin{pmatrix}0&0&0&0\\0&0&0&\frac{v'}{\sqrt{2}}\\0&0&0&0\\0&\frac{v'}{\sqrt{2}}&0&0\end{pmatrix}, \tag{27}$$

then all of the leptons get masses. In this model the Higgs sector is more complicated due to the presence of the S field and it is not clear that the required vacuum can be obtained as the minimum of a Higgs potential. With the VEV's $\langle\rho\rangle,\langle\eta\rangle,\langle S\rangle$ the intermediate electroweak gauge symmetry is spontaneously broken as follows:

$$\begin{gathered}\text{SU}(3)_C\otimes\text{SU}(2)_L\otimes\text{U}(1)_Y\\ \downarrow\langle\rho\rangle,\langle\eta\rangle,\langle S\rangle\\ \text{SU}(3)_C\otimes\text{U}(1)_Q.\end{gathered} \tag{28}$$

The electric charge operator has been identified as $Q=I_3+Y/2$ where I_3 is given by the SU(4) generator $\text{diag}(1,-1,0,0)/2$ and Y is given in Eq. (22).

Note that the VEV's of ρ, η and S transform as $Y=1$,SU(2)$_L$ doublets [under the SU(2)$_L\otimes$U(1)$_Y$ subgroup of SU(3)$_L\otimes$U(1)$_N$ which is left unbroken by $\langle\chi_{1,2}\rangle$]. For this reason and the fact that Y [Eq. (22)] is identical to the standard model hypercharge, it is clear that the model reduces to the standard model with the correct low energy phenomenology provided $\langle\chi_{1,2}\rangle\gg\langle\rho\rangle,\langle\eta\rangle,\langle S\rangle$.

Note that in the limit $\langle\chi_2\rangle\gg\langle\chi_1\rangle,\langle\rho\rangle,\langle\eta\rangle,\langle S\rangle$, the model reduces to the Pisano-Pleitez model (with right-handed singlet neutrinos) at an energy scale much less than $\langle\chi_2\rangle$. On the other hand, if $\langle\chi_1\rangle\gg\langle\chi_2\rangle,\langle\rho\rangle,\langle\eta\rangle,\langle S\rangle$, the model reduces to the $\text{SU}(3)_C\otimes\text{SU}(3)_L\otimes\text{U}(1)_N$ model discussed before (in the present paper), which has ν_R^a in the SU(3)$_L$ triplets.

Note added. After we completed this paper we became aware of a paper by Pisano and Pleitez (Sao-Paulo Report No. IFT-P.003/94; hep-ph/9401272) where they consider a $\text{SU}(4)_L\otimes\text{U}(1)_N$ model which is essentially identical to the $\text{SU}(4)_L\otimes\text{U}(1)_N$ model (which is the second model in this paper) discussed here.

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[1] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992); R. Foot, O. F. Hernandez, F. Pisano, and V. Pleitez, *ibid.* **47**, 4158 (1993); P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).

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[3] This gauge group and fermion representations were already discussed by F. Pisano and Tran Anh Tuan, in Proceedings of the XIVV Encontro National de Física de Partícular e Campos, Caxambu, 1993 (unpublished). However, they did not consider symmetry breaking, which is done in the present paper.

[4] See, for example, P. Langacker and D. London, Phys. Rev. D **38**, 886 (1988).