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Weak scale radiative lepton mass

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We construct a Z_8 model for leptons where all Yukawa couplings are of order unity, but known lepton masses are generated radiatively, *order by order*. The seed is provided by fourth generation leptons E and N, and two additional Higgs doublets are introduced to give nearest-neighbor Yukawa couplings. Loop masses are generated when Z_8 is *softly* broken down to Z_2, while m_e and m_μ generation require Higgs bosons to be weak scale. The Z_2 symmetry forbids $\mu\to e\gamma$. The most stringent bound comes from $\tau\to\mu\mu^\pm e^\mp$. The model has implications for $\tau\to e\gamma$, $\mu\bar{e}\to\bar{\mu}e$ conversion, $\mu\to e\nu_e\bar{\nu}_\mu$, and FCNC decays of E and N (such as $E\to\tau e^\pm\mu^\mp$).

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A major mystery regarding fermion flavor is the very wide range of its mass spectrum. Neutrinos appear to be massless, while known masses range from the electron's 0.511 MeV, to over 120 GeV [1] for the top quark. In the standard $SU(2)_L\times U(1)$ electroweak model, a single Higgs doublet is responsible for generating all masses. The natural scale is $v=(\sqrt{2}G_F)^{-1/2}\approx 246$ GeV, where G_F is the Fermi coupling. The puzzle is then twofold. On one hand, the dimensionless Yukawa couplings f are scattered over a wide range, and there is an empirical family hierarchy, e.g., $f_e\ll f_\mu\ll f_\tau$, for each type of fermion. On the other hand, we have $f\ll 1$ for all known fermions, except for the top quark where $f_t\sim 1$. This is in contrast with the gauge couplings (at W scale) $g_1\cong 0.36$, $g_2\cong 0.65$, and $g_3\simeq 1.2$ for strong SU(3). It was suggested a long time ago [2] that the observed fermion mass hierarchy may be due to radiative mechanisms. It is desirable to have Yukawa couplings $f\sim 1$, just like gauge couplings. Given that $\sqrt{2}m_\tau/v\cong 0.01$ is itself rather small, we would naturally need *new* leptons to provide the "seed" for mass generation in the lepton sector. Neutrino counting $N_\nu=2.99\pm 0.04$ and direct search limits m_E, $m_N>M_Z/2$ [3] imply that new sequential leptons E and N are at scale v, deviating sharply from earlier patterns [4]. In this Rapid Communication, we construct a simple model where E and N receive mass at the tree level, but all lower generation lepton masses are generated by loop processes.

We start with the lepton sector of minimal "3+1" generations [4], where there is only one right-handed neutral lepton N_R. Consider a discrete Z_8 symmetry ($\omega^8=1$). We assign both $\bar{l}_{iL}=(\bar{\nu}_{iL},\bar{e}_{iL})$ and e_{iR} to transform as ω^3, ω^2, ω^1, ω^4 for $i=1$–4, respectively, while N_R transforms as ω^4. The scalar sector consists of three doublets Φ_0, Φ_3, and Φ_5, transforming as 1, ω^3, and ω^5, respectively. Thus, aside from $E\simeq e_4$ and N, only nearest-neighbor Yukawa couplings are allowed,

$$
\begin{aligned}
-\mathcal{L}_Y={}&f_{44}\bar{l}_{4L}e_{4R}\Phi_0+\tilde{f}_{44}\bar{l}_{4L}N_R\tilde{\Phi}_0+f_{43}\bar{l}_{4L}e_{3R}\Phi_3\\
&+f_{34}\bar{l}_{3L}e_{4R}\Phi_3+\tilde{f}_{34}\bar{l}_{3L}N_R\tilde{\Phi}_5+f_{32}\bar{l}_{3L}e_{2R}\Phi_5\\
&+f_{23}\bar{l}_{2L}e_{3R}\Phi_5+f_{21}\bar{l}_{2L}e_{1R}\Phi_3\\
&+f_{12}\bar{l}_{1L}e_{2R}\Phi_3+\text{H.c.},
\end{aligned}
\tag{1}
$$

where $\tilde{\Phi}\equiv i\sigma_2\Phi^*=(\phi^{0*},-\phi^-)$ as usual. We assume CP invariance for simplicity.

If only $\langle\phi_0^0\rangle=v/\sqrt{2}$, E and N become massive and are naturally at v scale if f_{44}, $\tilde{f}_{44}\sim 1$. The lower generation leptons remain massless at this stage, protected by the Z_8 symmetry. To allow for radiative mass generation, the Z_8 symmetry is *softly* broken down to Z_2 in the Higgs potential by Φ_3-Φ_5 mixing. Explicitly,

$$
\begin{aligned}
V={}&\sum_i \mu_i^2\Phi_i^\dagger\Phi_i+\sum_{i,j}\lambda_{ij}(\Phi_i^\dagger\Phi_i)(\Phi_j^\dagger\Phi_j)\\
&+\sum_{i\neq j}\eta_{ij}(\Phi_i^\dagger\Phi_j)(\Phi_j^\dagger\Phi_i)\\
&+[\tilde{\mu}^2\Phi_3^\dagger\Phi_5+\zeta(\Phi_0^\dagger\Phi_3)(\Phi_0^\dagger\Phi_5)+\text{H.c.}],
\end{aligned}
\tag{2}
$$

where λ_{ij} and η_{ij} are symmetric. Note that the ζ term is Z_8 invariant, while the gauge invariant "mass" $\tilde{\mu}^2$ transforms as ω^2. Since only $\mu_0^2<0$, while μ_3^2 and $\mu_5^2>0$, $\phi_0^0\to(v+H_0+i\chi_0)/\sqrt{2}$, and $\phi_i^0\to(h_i+i\chi_i)/\sqrt{2}$ for $i=3,5$. The quadratic part of V is

$$
\begin{aligned}
V^{(2)}={}&\lambda_{00}v^2H_0^2+\sum_{i\neq 0}[\tfrac{1}{2}(\mu_i^2+\lambda_{0i}v^2+\eta_{0i}v^2)(h_i^2+\chi_i^2)\\
&+(\mu_i^2+\lambda_{0i}v^2)|\phi_i^+|^2]+\tilde{\mu}^2(h_3h_5+\chi_3\chi_5\\
&+\phi_3^-\phi_5^++\phi_5^-\phi_3^+)+\tfrac{1}{2}\zeta v^2(h_3h_5-\chi_3\chi_5).
\end{aligned}
\tag{3}
$$

The standard Higgs boson H_0 couples only diagonally to heavy particles. The nonstandard scalars $(\phi_3^\pm,\phi_5^\pm)$, (h_3,h_5), and (χ_3,χ_5) mix via $\tilde{\mu}^2$ and ζ terms. Rotating by θ_+, θ_H, and θ_A, we obtain the mass basis (H_1^+,H_2^+), (H_1,H_2), and (A_1,A_2), respectively. It is clear that $\sin\theta_+\to 0$, $(\theta_A,m_{A_1},m_{A_2})\to(-\theta_H,m_{H_1},m_{H_2})$ as $\tilde{\mu}^2\to 0$, while in the limit $\zeta\to 0$, $(\theta_A,m_{A_1},m_{A_2})\to(+\theta_H,m_{H_1},m_{H_2})$. These two limits restore the two extra U(1) symmetries of the doublets Φ_3 and Φ_5. As we shall see, fermion mass generation is due to mixing and nondegeneracy of the two charged scalars, and especially the four real scalar fields.

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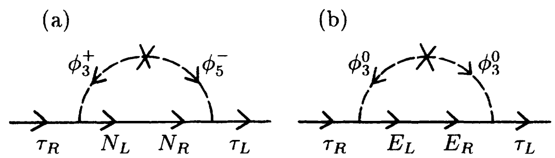

FIG. 1. Mechanism for m_τ.

The τ lepton acquires mass via the one-loop diagram shown in Fig. 1:

$$
\begin{aligned}
m_{33}=&\left(\frac{\tilde{f}_{34}f_{43}}{32\pi^2}\right)\sin 2\theta_+[G(m_{H_1^+}/m_N)-G(m_{H_2^+}/m_N)]m_N\\
&+\left(\frac{f_{34}f_{43}}{32\pi^2}\right)[\cos^2\theta_H G(m_{H_1}/m_E)+\sin^2\theta_H G(m_{H_2}/m_E)\\
&-\cos^2\theta_A G(m_{A_1}/m_E)-\sin^2\theta_A G(m_{A_2}/m_E)]m_E, \qquad (4)
\end{aligned}
$$

where $G(x)=(x^2\ln x^2)/(x^2-1)$, while $m_{34}=m_{43}=0$. As a numerical exercise, let $f_{43}=\tilde{f}_{34}=f_{44}=\tilde{f}_{44}\sim\sqrt{2}$ so $\tilde{f}_{34}f_{43}/4\pi\simeq 1/2\pi$ and m_N, $m_E\cong 246$ GeV. Then $\sin 2\theta_+\,[G(x_1)-G(x_2)]\simeq 0.6$ would make $m_\tau=m_{33}\simeq 1.8$ GeV, if the m_E term contributes as much as the m_N term, and with the same sign. There are separate Glashow-Iliopoulos-Maiani- (GIM-) like cancellation mechanisms rooted in charged and neutral scalar mixing. In general $G(x_1)-G(x_2)$ is regulated by $\sin 2\theta_+$; hence typically $\sin 2\theta_+[G(x_1)-G(x_2)]\lesssim 1$. Similar statements can be made for the neutral scalar contribution. This implies that $f_{43}, \tilde{f}_{34}, f_{44}, \tilde{f}_{44}\sim f\gtrsim 1$ is *natural* in our model.

If $\tilde{\mu}^2\to 0$, both terms would vanish and $m_\tau=0$. It is interesting to note that even if $\tilde{\mu}^2\neq 0$, the neutral scalar contribution would vanish if $\zeta=0$. This is of crucial importance for muon and electron mass generation, for they receive radiative masses at two- and three-loop order, respectively, via neutral scalar loops that are similar to Fig. 1(b). These "nested" diagrams are illustrated in Fig. 2. The upshot then is that $\zeta v^2/2$ should be of similar order of magnitude as $\tilde{\mu}^2$, which in turn implies that μ_3^2 and μ_5^2 should also be of order v^2. Hence, nonstandard Higgs boson masses cannot be too far above the electroweak scale.

Since off-diagonal mass terms m_{24} and m_{13} are also at two- and three-loop order, respectively, we have the mass hierarchy $m_E:m_\tau:m_\mu:m_e\sim 1:\lambda:\lambda^2:\lambda^3$, where λ is the loop expansion parameter. That is, $m_i/m_{i+1}\sim f_{i,i+1}f_{i+1,i}/32\pi^2$ or more. If $(f_{i,i+1}f_{i+1,i})^{1/2}\sim 1$ for all $i=1$–3, the mass hierarchy of order 10^{-1}–10^{-2} can be realized. Together with f_{44}, $\tilde{f}_{44}\sim 1$, we see that Yukuwa couplings could be generation blind. It is amusing that in our model, *all* dimensionless couplings seem to be of order one, and *all* scale parameters are of order v.

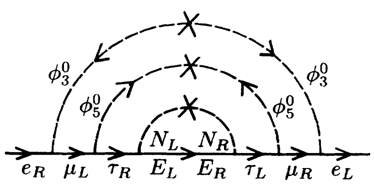

FIG. 2. Mechanisms for m_μ and m_e via "nested" neutral scalar diagrams.

It is intriguing that the model could account for m_e/m_μ $(\sim m_\tau/m_E)\sim 1/200\ll m_\mu/m_\tau\sim 1/20$. Concentrating on neutral scalar loops, from Figs. 1 and 2 we see that m_e, m_τ come from m_μ, m_E seeds via "ϕ_3^0" loop, while m_μ arises from m_τ seed via "ϕ_5^0" loop. With obvious notation, we estimate $m_e/m_\mu\simeq\lambda[s_H^2\ln(m_{H_2}^2/m_{H_1}^2)-s_A^2\ln(m_{A_2}^2/m_{A_1}^2)+\ln(m_{H_1}^2/m_{A_1}^2)]$, while for m_μ/m_τ one has $s_{H,A}^2\leftrightarrow c_{H,A}^2$. As an example, we could have $m_{H_1}\sim m_{A_1}\sim m_{A_2}$, then $(m_\mu/m_\tau)/(m_e/m_\mu)\cong\cot^2\theta_H\simeq 12$, and $\sin\theta_H\simeq 0.28$ which is rather reasonable.

Note that N_R is introduced solely for the purpose of satisfying CERN e^+e^- collider LEP bounds [4]. Having just a massive E would have been sufficient for charged lepton mass generation. However, N_R could in principle have Majorana mass m_R, which could serve as seed for radiatively generating Majorana mass for the three left-handed neutrinos. We emphasize that $m_R\gg m_N$ is not allowed [4], for then the seesaw mechanism [5] would drive N_L mass effectively to zero, violating LEP bounds. Rough estimates of loop-induced Majorana neutrino masses indicate that m_R should be rather small, and we set $m_R=0$ in the present work. However, our model provides interesting, new mechanisms, details of which will be reported elsewhere.

We turn to phenomenological prospects. These depend on the lowest allowed mass(es) for the charged or flavor-changing neutral current (FCNC) neutral Higgs bosons. Mixing effects in the charged current [4] can be ignored since they are radiatively generated. As discussed earlier, radiative mass generation for μ and e suggest that nonstandard Higgs boson masses should not be too far above the electroweak scale v. Hence, one might worry about low energy FCNC effects. Very stringent limits exist for $\mu\to e\gamma$ [3]. Interestingly, our model has a dichotomy [6] of leptons: E, N, μ, and ν_μ are even under Z_2, whereas τ, e, ν_τ, and ν_e are odd. For scalars, Φ_0 is even, while Φ_3 and Φ_5 are odd. Hence, $\mu\to e\gamma$ is *forbidden* in our model, since the photon is Z_2 invariant. Similarly, $\tau\to\mu\gamma$ is forbidden, but $\tau\to e\gamma$ is allowed, as we shall discuss later.

The present experimental errors [3] on $g-2$ for e and μ imply [7] lower bounds of a few hundred GeV on the *effective* mass of the exchanged scalar bosons. In principle, $\mu\to e\nu\nu$ and $\tau\to\mu\nu\nu$ decay rates provide sensitive probes of charged Higgs boson contributions [6]. However, with the present level of experimental uncertainties on M_W, these modes do not provide the best constraint on the model. The most stringent constraint on our model turns out to be from leptonic FCNC τ decays: $\tau\to\mu\mu^\mp e^\pm$ (the decay modes $\tau\to ee^\pm\mu^\mp$ are forbidden). Each has four contributions, two of which are shown in Fig. 3 for $\tau^-\to\mu^-\mu^-e^+$. The inverse effective mass squared $1/m^2(\phi_{3,5}^0)$ corresponds to a sum over neutral (pseudo)scalars $\Sigma_{i=1}^4 a_i/m_i^2$ where we now order according to mass m_i, while a_i are mixing factors that should satisfy $0<|a_i|\leq 1$, $\Sigma_i a_i=0$. They are nothing but $\pm s_{H,A}\,c_{H,A}$. Thus, in general $m^2(\phi_{3,5}^0)>m_1^2$, the lightest (pseudo)scalar mass, and could be much larger than v^2. Assuming single (lightest) channel dominance, we find

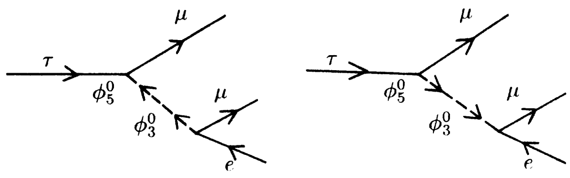

FIG. 3. Some diagrams contributing to $\tau\to\mu\mu^- e^+$.

$$B(\tau^-\to\mu^-\mu^- e^+)\simeq\frac{1}{2}\left(\frac{f_{e\mu}f_{\mu\tau}\,a_1/m_1^2}{g^2/M_W^2}\right)^2 B(\tau\to\mu\nu\bar{\nu}), \tag{5}$$

where $2f_{e\mu}^2 f_{\mu\tau}^2 \equiv \{(f_{23}f_{12})^2+(f_{32}f_{21})^2\}$. We show in Fig. 4 $B(\tau^-\to\mu^-\mu^- e^+)$ vs $(f_{e\mu}f_{\tau\mu}\,|a_1|)^{1/2}$ for $m_1=(0.5,1,2,4)\,v(\cong 125,250,500,1000$ GeV). The present experimental bound of $B(\tau^-\to\mu^-\mu^- e^+)<1.6\times10^{-5}$ [3] is also shown. We see that, because of $|a_1|<1$ and cancellation effects, it is possible to have all Yukawa couplings of order unity while physical nonstandard scalar masses are of order v or greater. Our model could naturally account for $B(\tau^-\to\mu^-\mu^- e^+)$ just below 10^{-5}. Similar results are obtained from $B(\tau^-\to\mu^-\mu^+ e^-)<2.7\times10^{-5}$ [3]. These decays are exceptionally clean, and should be searched for vigorously.

The $\tau^-\to\mu^-\mu^+ e^-$ decay leads to $\tau\to e\gamma$ at one-loop order. Charged Higgs boson loop contributions vanish with neutrino mass. We find

$$\frac{B(\tau\to e\gamma)}{B(\tau\to\mu\bar{\mu}e)}\lesssim 24\frac{\alpha}{\pi}\left(\frac{m_\mu}{m_\tau}\right)^2\left(\ln\frac{m_\mu^2}{m_1^2}\right)^2, \tag{6}$$

where we assume same channel dominance, and drop a constant term accompanying the large logarithm. As an estimate, the ratio is less than 1/21–1/15 for $m_1\simeq 250$ GeV–1 TeV; hence, $\tau\to e\gamma$ is typically just one order of magnitude below $\tau\to\mu\bar{\mu}e$, i.e., at 10^{-6} or lower. The present experimental bound is $\sim10^{-4}$ [3].

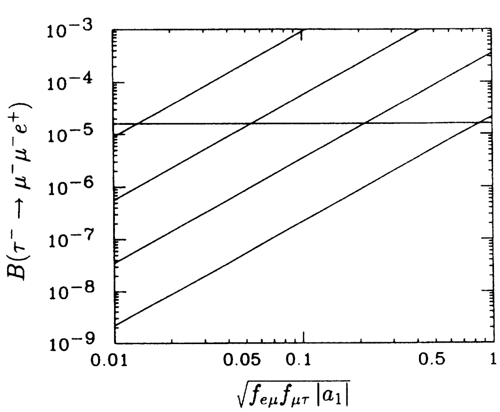

FIG. 4. $B(\tau^-\to\mu^-\mu^- e^+)$ vs $\sqrt{f_{e\mu}f_{\tau\mu}\,|a_1|}$ for (top to bottom) $m_1=$ 125, 250, 500, and 1000 GeV. Horizontal line indicates present limit.

In our model neutral scalars couple to $\mu\bar{e}$ and $\bar{\mu}e$ simultaneously; therefore, they mediate muonium-antimuonium conversion [8] (without doubly charged Higgs bosons). Unlike $\tau\to\mu\mu^{\mp}e^{\pm}$ which is mediated by ϕ_3^0-$\phi_5^{0(*)}$ mixing, here we have a ϕ_3^0 mediated process. The scalar mixing factors a_i are of same sign, i.e., of the form $+c_{H,A}^2$, $+s_{H,A}^2$ and $\Sigma_i a_i=2$. Assuming that the dominating scalar has mass $\sim v$ and the mixing factor ranges from 0.01–1, we estimate that the effective four-Fermi coupling is of order (0.001–0.1) G_F, compared with the present bound of $0.16G_F$ [3]. The limit may be improved to $10^{-3}G_F$ soon [9]. Note that we have an unusual effective interaction $\bar{\mu}(1-\gamma_5)e\;\bar{\mu}(1+\gamma_5)e$. In the same vein, the process $\mu\to e\nu_e\bar{\nu}_\mu$ is mediated by ϕ_3^+, and has a four Fermi coupling of similar order. The present bound is $0.14G_F$, but may be pushed down to $10^{-2}G_F$ [10] in the near future.

Consider now the decays of the heavy lepton E. If $m_E<m_N$, since charged current mixing is loop suppressed, $\phi_{3,5}^{\pm}$-induced $E\to\nu_\tau(e\bar{\nu}_\mu,\mu\bar{\nu}_e,\mu\bar{\nu}_\tau,\tau\bar{\nu}_\mu)$ and $\phi_{3,5}^0$-induced FCNC $E\to\tau(e^{\pm}\mu^{\mp},\mu^{\pm}\tau^{\mp})$ decays could be dominant. They could still be prominent for $m_E\gtrsim m_N$ since W-induced transitions such as $E\to Ne\bar{\nu}_e$ are kinematically suppressed until m_E-m_N approaches M_W. However, for $m_E\gtrsim m_N$, new scalar-induced decays such as $N\to\nu_\tau\mu^{\pm}e^{\mp}$ would dominate N decay. Since the lightest scalar might be lighter than m_E or m_N, it may even be produced directly in E,N decay. These decays would provide dramatic signatures at future colliders.

In summary, we have presented a realistic model for radiative lepton mass generation. The model has Z_8 symmetry and three Higgs doublets with nearest-neighbor Yukawa couplings. Only E and N receive tree level masses upon spontaneous symmetry breaking. When Z_8 is softly broken down to Z_2, they provide the seed for generating, *order by order*, loop masses for τ, μ, and e. Yukawa couplings are naturally of order unity, and the mass pattern $m_E:m_\tau:m_\mu:m_e \simeq 1:\lambda:\lambda^2:\lambda^3$ emerges, which suggests the *possibility of universal Yukawa couplings*. The model could account for $m_\mu/m_\tau \gg m_e/m_\mu$, m_τ/m_E as a consequence of mixing effects among nonstandard Higgs bosons. The residual Z_2 symmetry forbids $\mu\to e\gamma$ and $\tau\to\mu\gamma$ type of transitions. The charged and FCNC neutral Higgs bosons have weak scale masses. However, because of GIM-like cancellations among themselves, they could mimic TeV scale physics. The most promising channels seem to be FCNC τ decays $\tau\to\mu\mu^{\pm}e^{\mp}$ (but not $\tau\to ee^{\pm}\mu^{\mp}$) and $\tau\to e\gamma$ at just below 10^{-5} and 10^{-6}, respectively. Muonium-antimuonium conversion and $\mu\to e\nu_e\bar{\nu}_\mu$ could have strength (0.001–0.1) G_F. FCNC leptonic decays of the fourth generation E and N are likely to be quite prominent. We have consistently assumed that the Majorana mass $m_R=0$ for N_R. A small m_R could induce Majorana masses for left-handed neutrinos via loop processes. The quark sector is clearly richer but more difficult. Work is in progress, and will be reported elsewhere.

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