PHYSICAL REVIEW D VOLUME 50, NUMBER 3 1 AUGUST 1994

Test of factorization in $D \to \pi\pi$ and $D \to K\bar{K}$ decays

A. N. Kamal
Department of Physics and Theoretical Physics Institute, University of Alberta, Edmonton, Alberta, Canada T6G 2J1

T. N. Pham
Centre de Physique Théorique, Centre National de la Recherche Scientifique, UPR A0014, Ecole Polytechnique, 91128 Palaiseau Cedex, France

(Received 21 January 1994)

We propose a method that involves quantities independent of strong interaction phases to test the factorization model in $D \to \pi\pi$ and $K\bar{K}$ decays. The method allows us to conclude that the factorization model correctly predicts $|A_2^{\pi\pi}|$, overestimates $|A_0^{\pi\pi}|$ and $|A_1^{K\bar{K}}|$, and underestimates $|A_0^{K\bar{K}}|$. We rule out the Penguin mechanism as the solution to the disagreement between theory and experiment. We believe inelastic final state interactions are responsible for bridging the gap between theory and experiment.

PACS number(s): 13.25.Ft, 14.40.Lb

With the knowledge of $D \to K\bar{K}$ branching ratios in all the charged modes [1] and the availability of new data on all the charged modes in $D \to \pi\pi$ decays [2,3], we claim it is now possible to test the factorization model in these decays purely from hadronic measurements. In contrast, the tests of factorization used hitherto involve a comparison between hadronic and semileptonic branching ratios [2,4,5]; such tests are generally rendered untrustworthy due to final-state interaction (FSI) interference effects. Our proposal is to test quantities that are independent of the strong interaction phases against data. As a consequence our method shifts the emphasis from the decay amplitudes for particular decay channels to the decay amplitudes in particular isospin states.

$D \to \pi\pi$ decays. In terms of isospin amplitudes, the decay amplitudes are

$$A(D^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{6}} A_0^{\pi\pi} \exp(i\delta_0^{\pi\pi}) + \frac{1}{\sqrt{12}} A_2^{\pi\pi} \exp(i\delta_2^{\pi\pi}),$$

$$A(D^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{6}} A_0^{\pi\pi} \exp(i\delta_0^{\pi\pi}) - \frac{1}{\sqrt{3}} A_2^{\pi\pi} \exp(i\delta_2^{\pi\pi}), \qquad (1)$$

$$A(D^+ \to \pi^0\pi^+) = -\frac{\sqrt{3}}{2\sqrt{2}} A_2^{\pi\pi} \exp(i\delta_2^{\pi\pi}).$$

Our definition in Eq. (1) appears to be different from that used in Refs. [2,3]; the reason is that in our definition the rate for $D^0 \to \pi^0\pi^0$ is obtained by squaring the amplitude and *dividing* the phase space by 2 due to the identity of the pions while in the definitions of Refs. [2,3] the phase space is not to be so divided. Our amplitudes $A_0^{\pi\pi}$ and $A_2^{\pi\pi}$ are *twice* those in Refs. [2,3] leaving the ratio A_2/A_0 the same.

From Eq. (1) it is evident that there are two phase-independent quantities:

$$\sum B(D^0 \to \pi\pi) \equiv B(D^0 \to \pi^+\pi^-) + B(D^0 \to \pi^0\pi^0) \qquad (2)$$

and $B(D^+ \to \pi^0\pi^+)$.

In the factorization model the decay amplitudes are given by [6] [we are suppressing an overall factor $(G_F/\sqrt{2}) V_{ud} V_{cd}^*$]

$$A(D^0 \to \pi^+\pi^-) = a_1 f_\pi F_0^{D\pi}(m_\pi^2)(m_D^2 - m_\pi^2),$$

$$A(D^0 \to \pi^0\pi^0) = -a_2 f_\pi F_0^{D\pi}(m_\pi^2)(m_D^2 - m_\pi^2),$$

$$A(D^+ \to \pi^0\pi^+) = -\frac{a_1 + a_2}{\sqrt{2}} f_\pi F_0^{D\pi}(m_\pi^2)(m_D^2 - m_\pi^2), \qquad (3)$$

where the negative sign in $A(D^0 \to \pi^0\pi^0)$ comes from the matrix element of the $\bar{d}d$ left-handed current between π^0 and the vacuum. We use $f_\pi = 131$ MeV. a_1 and a_2 are parameters related to the Wilson coefficients c_+ and c_- [7,8] and $F_0^{D\pi}$ the relevant $D \to \pi$ transition form factor [7,8]. As we argue below, through a theoretical analysis of CLEO data [3], one can get a good handle on the parameters entering Eq. (3).

Semileptonic measurements on $D \to K l \nu$ [9–11] allow one to extract [2]

$$F_0^{DK}(0) = 0.77 \pm 0.04. \qquad (4)$$

Experiments on $D \to \pi l \nu$ are beginning to measure $F_0^{D\pi}(0)$ [2]:

$$F_0^{D\pi}(0)/F_0^{DK}(0) = \begin{cases} 1.0 \pm^{0.6}_{0.3} \pm 0.1 & \text{(Mark III [12])}, \\ 1.29 \pm 0.21 \pm 0.11 & \text{(CLEO [13])}. \end{cases} \qquad (5)$$

CLEO data [13], in particular, suggest that $F_0^{D\pi}(0)$ could be larger than $F_0^{DK}(0)$.

It has recently been inferred by Chau and Cheng [14] that $F_0^{D\pi}(0) \approx 0.83$ is needed to understand the ratio of the rates $\Gamma(D^+ \to \pi^0\pi^+)/\Gamma(D^+ \to \bar{K}^0\pi^+)$. The data, thus, provide strong evidence that $F_0^{D\pi}(0) > F_0^{DK}(0)$ which is also consistent with a recent theoretical prediction $F_0^{D\pi}(0)/$

$F_0^{DK}(0)=1.18$ based on heavy quark symmetry and chiral perturbation theory [14,15], though one may question the validity of the extrapolation from $q^2=q^2_{\rm max}$ to $q^2=0$ for these form factors.

We show in the following that this narrowing down of $F_0^{DK}(0)$ and $F_0^{D\pi}(0)$ allows us to test the factorization model in $D\to\pi\pi$ and $K\bar{K}$ decays.

In our analysis we use the parameters [1,14]

$$a_1=1.26\pm0.05,\quad a_2=-0.51\pm0.05,$$

$$F_0^{D\pi}(0)=0.83\pm0.08,\quad F_0^{DK}(0)=0.76\pm0.04,$$

$$V_{ud}=0.975,\quad V_{cd}\approx -V_{us}=0.220,$$

$$\tau_{D^0}=4.2\times10^{-13}\ \text{s},\quad \tau_{D^+}=10.66\times10^{13}\ \text{s}. \tag{6}$$

In the past one of us [5] had assigned independent errors of 10% to both a_1 and a_2, but we shall argue later that data require a much tighter error. Hence we assign a 10% error to a_2 and $F_0^{D\pi}(0)$ but a smaller error to a_1. Furthermore, it has been shown in Ref. [14] that the ratio $B(D^+\to\pi^0\pi^+)/B(D^+\to\bar{K}^0\pi^+)$ is very sensitive to the ratio $F_0^{D\pi}(0)/F_0^{DK}(0)$, and that one needs this ratio to be greater than unity. In fact, the use of the form factors of Ref. [8] where this ratio is equal to or less than unity fares poorly in explaining the measured value for this ratio. Though the error in the measurement of $F_0^{D\pi}(0)/F_0^{DK}(0)$ is large, we show later that data demand a much tighter error.

Now, since $\Sigma B(D^0\to\pi\pi)$ and $B(D^+\to\pi^0\pi^+)$ depend only on $|A_0^{\pi\pi}|$ and $|A_2^{\pi\pi}|$, and not on $\delta^{\pi\pi}=\delta_0^{\pi\pi}-\delta_2^{\pi\pi}$ we can calculate these combinations of branching ratios from Eq. (3) using the parameter set in Eq. (6) to test if the factorization model generates $|A_0^{\pi\pi}|$ and $|A_2^{\pi\pi}|$ correctly. A simple calculation using Eq. (3) and Eq. (6) yields

$$\sum B(D^0\to\pi\pi)=(0.51\pm0.01)\%, \tag{7}$$

while CLEO data yield [3]

$$\sum B(D^0\to\pi\pi)=(0.207\pm0.025)\%. \tag{8}$$

The factorization model obviously fails for the isospin amplitudes. Indeed, one ought to have suspected this would be the case from the ratio A_2/A_0. The factorization model gives, independent of the form factor $F_0^{D\pi}(q^2)$,

$$A_2^{\pi\pi}/A_0^{\pi\pi}=\sqrt{2}\,\frac{a_1+a_2}{2a_1-a_2}=0.35\pm0.035, \tag{9}$$

while CLEO data yield [3]

$$A_2^{\pi\pi}/A_0^{\pi\pi}=0.72\pm0.13\pm0.11. \tag{10}$$

To trace the source of the problem let us calculate $B(D^+\to\pi^0\pi^+)$ which depends only on $|A_2^{\pi\pi}|$. Using Eq. (3) and Eq. (6) we get

$$B(D^+\to\pi^0\pi^+)=(0.21\pm0.07)\% \tag{11}$$

in excellent agreement with data [3]:

$$B(D^+\to\pi^0\pi^+)=(0.22\pm0.05\pm0.05)\%. \tag{12}$$

The conclusion is that the factorization model predicts $|A_2^{\pi\pi}|$ quite reliably. The reason for this success is that there are no Penguin or annihilation terms in this channel and π-π scattering in the $L=0$, $I=2$ state is elastic and relatively weak, the scattering phase shift being about $-30°$ [16]. Thus the magnitude of the amplitude is little affected by the rescattering in $I=2$ state. Our test has essentially served as a diagnostic tool to lay the blame for the disagreement between Eq. (7) and Eq. (8) on the failure of the factorization model to generate $|A_0^{\pi\pi}|$ correctly—theory predicting too large a value for $|A_0^{\pi\pi}|$.

As inelastic final-state interactions mix $\pi\pi$ and $K\bar{K}$ systems in $I=0$ state, we will return to a full discussion of the failure of the factorization model for $|A_0^{\pi\pi}|$ after we have discussed $D\to K\bar{K}$ decays. For the moment we simply mention two possible effects that could be responsible for lowering $|A_0^{\pi\pi}|$—Penguin diagrams and inelastic final-state interactions.

We end the discussion of $D\to\pi\pi$ decay with a remark on permissible errors in the parameters introduced in Eq. (6). The branching ratio for $D^+\to\pi^0\pi^+$ given in Eq. (12) requires (we have combined the errors in quadrature)

$$(a_1+a_2)F_0^{D\pi}(0)=(0.634\pm0.100); \tag{13}$$

i.e., the combined error in (a_1+a_2) and $F_0^{D\pi}(0)$ is 15%. This is how we have assigned errors to a_1, a_2, and $F_0^{D\pi}(0)$ in Eq. (12). Note that Eq. (13) also implies that for any reasonable set of values for a_1 and a_2, $F_0^{D\pi}(0)$ would have a value larger than $F_0^{DK}(0)$ given in Eq. (12).

$D\to K\bar{K}$ decays. Let us define the decay amplitudes in terms of the isospin amplitudes as,

$$A(D^0\to K^+K^-)=\frac{1}{\sqrt{2}}[A_0^{K\bar{K}}\exp(i\delta_0^{K\bar{K}})+A_1^{K\bar{K}}\exp(i\delta_1^{K\bar{K}})],$$

$$A(D^0\to K^0\bar{K}^0)=\frac{1}{\sqrt{2}}[A_0^{K\bar{K}}\exp(i\delta_0^{K\bar{K}})-A_1^{K\bar{K}}\exp(i\delta_1^{K\bar{K}})], \tag{14}$$

$$A(D^+\to\bar{K}^0K^+)=\sqrt{2}A_1^{K\bar{K}}\exp(i\delta_1^{K\bar{K}}).$$

We find from Eq. (14) that the following two quantities are independent of the phases:

$$\sum B(D^0\to K\bar{K})\equiv B(D^0\to K^+K^-)+B(D^0\to K^0\bar{K}^0) \tag{15}$$

and $B(D^+\to\bar{K}^0K^+)$.

In the factorization model one obtains [an overall factor $(G_F/\sqrt{2})V_{us}V_{cs}^*$ is being suppressed]

$$A(D^0 \to K^+K^-) = -a_1 f_K F_0^{DK}(m_K^2)(m_D^2 - m_K^2),$$

$$A(D^0 \to K^0\bar{K}^0) = 0,$$

$$A(D^+ \to \bar{K}^0K^+) = A(D^0 \to K^+K^-). \tag{16}$$

We use $f_K = 161$ MeV and obtain $F_0^{DK}(m_K^2)$ by extrapolating $F_0^{DK}(0)$ of Eq. (6) with a monopole formula with mass 2.6 GeV [8]. We believe we have very good control over all the parameters in Eq. (16). Using Eq. (16) and the set of parameters in Eq. (6) we obtain

$$\sum B(D^0 \to K\bar{K}) = (0.48 \pm 0.05)\%. \tag{17}$$

Experiments yield [1]

$$\sum B(D^0 \to K\bar{K}) = (0.52 \pm 0.06)\%. \tag{18}$$

Thus there appears to be a reasonable agreement between the factorization model prediction for $\Sigma B(D^0 \to K\bar{K})$ and experiment. However, if we calculate $B(D^+ \to \bar{K}^0K^+)$ from Eq. (16), we get

$$B(D^+ \to \bar{K}^0K^+) = (1.22 \pm 0.12)\% \tag{19}$$

while experiments yield [1]

$$B(D^+ \to \bar{K}^0K^+) = (0.73 \pm 0.18)\%. \tag{20}$$

Equations (19) and (20) imply that the factorization model *overestimates* $|A_1^{K\bar{K}}|$ and since Eqs. (17) and (18) are in rough agreement, one also concludes that the factorization model *underestimates* $|A_0^{K\bar{K}}|$.

Let us now consider the mechanism that could lower $|A_0^{\pi\pi}|$ and $|A_1^{K\bar{K}}|$, and raise $|A_0^{K\bar{K}}|$. There are two candidate mechanisms, Penguin diagrams and inelastic final-state interactions, that need be considered.

First, the Penguin contribution. Penguin terms have the following characteristics: (i) They have the same sign for $D^0 \to K^+K^-, \pi^+\pi^-, \pi^0\pi^0$, and $D^+ \to \bar{K}^0K^+$ decays and (ii) they do not generate $K^0\bar{K}^0$. If the Penguin mechanism is used to lower $|A_1^{K\bar{K}}|$ it will have the following undesirable effects: (a) It will also lower $|A_0^{K\bar{K}}|$ since in the factorization model $A_0^{K\bar{K}} = A_1^{K\bar{K}}$, whereas we need to *raise* $|A_0^{K\bar{K}}|$; (b) it will raise $\Sigma B(D^0 \to \pi\pi)$ by *raising* $A_0^{\pi\pi}$ due to the fact that in the factorization model $A_0^{\pi\pi}$ and $A_0^{K\bar{K}}$ have opposite signs due to $V_{cd} \approx -V_{us}$. This will worsen the disagreement between Eqs. (7) and (8). It will make the ratio $A_2^{\pi\pi}/A_0^{\pi\pi}$ in Eq. (9) even lower, thereby aggravating the disagreement between theory and experiment.

This conclusion remains valid even if the Penguin contribution has an absorptive part as discussed in Ref. [17] since the imaginary part does not interfere with the real part of $|A_0^{\pi\pi}|$ and $|A_0^{K\bar{K}}|$.

We conclude that the Penguin mechanism, which could solve some problems in isolation, is not the solution to the global problems in $D \to \pi\pi$ and $K\bar{K}$ decays.

We believe that inelastic final-state interactions are the most likely mechanism to resolve the disagreements between theory and experiment. For example, a coupling of $K\bar{K}$ channel in $I=1$ state to, say, $K^*\bar{K}^*$ channel could lower $|A_1^{K\bar{K}}|$. As for raising $|A_0^{K\bar{K}}|$ and simultaneously lowering $A_0^{\pi\pi}$, Ref. [6] discusses a coupled channel FSI scenario that does just that; the model parameters in this calculation were fixed by fitting π-π scattering data in the $L=0$, $I=0$ state close to the D mass.

In summary, we have proposed a test of the factorization model in $D \to \pi\pi$ and $K\bar{K}$ decays which is independent of the FSI phases. The method tests if the factorization model correctly predicts the isospin amplitudes. When applied to $D \to \pi\pi$ decays, we find that factorization model correctly predicts $|A_2^{\pi\pi}|$ but overestimates $|A_0^{\pi\pi}|$. In $D \to K\bar{K}$ decays, the method allows us to conclude that the factorization model overestimates $|A_1^{K\bar{K}}|$ and underestimates $|A_0^{K\bar{K}}|$. We rule out Penguin mechanism to bridge the gap between theory and experiment. We believe that inelastic FSI are a likely mechanism to bring theory in agreement with experiment.

We have applied this method to other two-body decays of D meson. The results will be submitted elsewhere for publication.

It should be mentioned here that the role of inelastic final-state interactions and Penguins in $D \to \pi\pi$ and $K\bar{K}$ decays has also been discussed previously in Ref. [18].

A.N.K. thanks the Theory Group at Ecole Polytechnique for their hospitality where most of this work was done and the Natural Sciences and Engineering Research Council of Canada for a research grant which partly supported this work.

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