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Measurement of the kaon content of three-prong τ decays

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We present branching fraction measurements of three-prong decays of the τ^- lepton based on data from the TPC/Two-Gamma detector at SLAC PEP. The decays are classified according to the identities of the charged tracks to give simultaneous measurements of $B[\tau^- \to \nu_\tau K^- \pi^+ \pi^-] = 0.58^{+0.15}_{-0.13}\%$ and $B[\tau^- \to \nu_\tau K^- K^+ \pi^-] = 0.15^{+0.09}_{-0.07}\%$, and upper limits of $B[\tau^- \to \nu_\tau \pi^- K^+ \pi^-] < 0.25\%$, $B[\tau^- \to \nu_\tau K^- \pi^+ K^-] < 0.09\%$, and $B[\tau^- \to \nu_\tau K^- K^+ K^-] < 0.21\%$, where additional neutrals may be present in each case. The branching fractions are normalized to the world average three-prong topological branching fraction and there is an estimated 20% systematic error in addition to the listed statistical errors. We find the mass distributions in the $K^- \pi^+ \pi^-$ decay are consistent with K_1 dominance, and obtain branching fractions of $B[\tau^- \to \nu_\tau K_1^-(1270)] = 0.41^{+0.41}_{-0.35}\%$ and $B[\tau^- \to \nu_\tau K_1^-(1400)] = 0.76^{+0.40}_{-0.33}\%$.

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Decays of τ lepton pairs produced at e^+e^- colliders provide an exceptionally clean environment for studies of the weak charged current and of the strong interaction below the τ mass. The well-measured hadronic decays of the τ^- lepton are observed to proceed through coupling of the W^- boson to $\bar{u}d$ or $\bar{u}s$ resonant states. (Charge conjugate processes are implicit.) The $(\pi\pi)^-$ and $(K\pi)^-$ decays are dominated by the ρ^- and K^{*-} vector channels [1], while the $(\pi\pi\pi)^-$ decays occur predominantly through $(\rho\pi)^-$ in s wave, consistent with a_1^- axial-vector dominance [2]. However, the resonance structure of the $(K\pi\pi)^-$ decays is still an open question due to their small branching fractions and the difficulty of identifying or reconstructing the strange mesons. The obvious candidates for $(K\pi\pi)^-$ resonances are the $K_1^-(1270)$ and the $K_1^-(1400)$ [3], which are mixtures of the $\bar{u}s$ analogs of the a_1^- and b_1^-.

We describe a set of measurements of τ^- lepton branching fractions in which all three-prong decays, those with three charged particles among the decay products, are classified according to the identities of the charged tracks. We also examine the resonance structure of the $(K\pi\pi)^-$ decay through the three-charged mode, $K^-\pi^+\pi^-$, and report branching fractions for $\tau^- \to \nu_\tau K_1^-(1270)$ and $\tau^- \to \nu_\tau K_1^-(1400)$.

The data were recorded with the TPC/Two-Gamma detector facility [4] at the SLAC e^+e^- collider PEP during the 1982–1983 and 1984–1986 runs. The total sample has an integrated luminosity of 140 pb^{-1} at an e^+e^- center of mass energy of 29 GeV. Central tracking was performed by a time projection chamber (TPC) which identifies charged particles through simultaneous measurements of momentum and ionization energy loss, dE/dx. At PEP energies the decay products of the $\tau^+\tau^-$ events are well separated into two hemispheres, giving a distinct signature for the selection of τ_{1+3} events, where the subscript gives the number of charged particles for each of the τ decays. The event selection is based entirely on kinematic and particle identification information from the charged tracks observed in the TPC. In addition to our basic set of τ_{1+3} selection criteria [5], we apply the following requirements in order to minimize $q\bar{q}$ and radiative Bhabha backgrounds, and to ensure efficient $\pi^\pm/K^\pm$ separation. There must be no reconstructed tracks in addition to the one-prong and three-prong candidates. The dE/dx of each of the three-prong tracks must satisfy

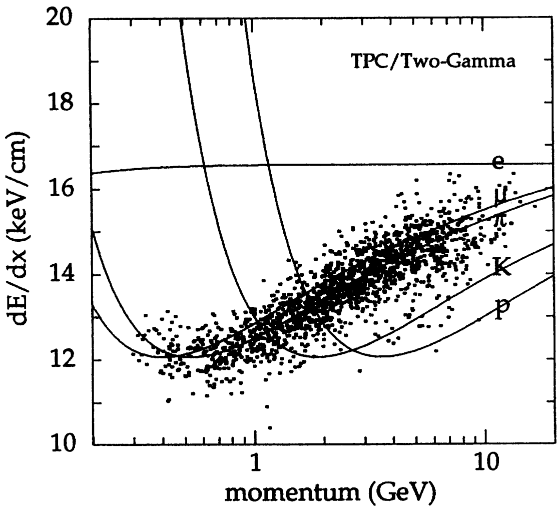

FIG. 1. dE/dx vs momentum for the three-prong tracks in the final τ_{1+3} event sample, demonstrating the $\pi^{\pm}/K^{\pm}$ separation achieved. The expected curves for various particle species are also shown.

$\chi^2_\pi<9.0$ or $\chi^2_K<9.0$, where χ^2_i is the goodness of fit of the measured dE/dx and momentum to the expected dE/dx vs momentum curve for particle species i. Finally, each of the three-prong tracks with momentum greater than 2 GeV must have at least 80 wire dE/dx samples. The resulting τ_{1+3} sample consists of 518 events, with an estimated purity of 99.1%. Background estimates, based on Monte Carlo simulation, are $3.5q\bar{q}$ and $1.3\tau_{1+1}$ events with negligible contributions from Bhabha, $\mu^+\mu^-$, and two-photon events.

The separation of the three-prong tracks into $\pi^{\pm}$ and $K^{\pm}$ candidates is based on their momentum and dE/dx values, shown in Fig. 1. The dE/dx distributions for charged particles are known to be Gaussian out to three standard deviations, with a well-understood resolution that is a function of the track angle and number of wire samples [4]. The average dE/dx resolution is 3.2% for the tracks in our sample with momentum above 2 GeV. Tracks below 2 GeV are considered to be $\pi^{\pm}$, since Monte Carlo simulations predict that fewer than 4% of the $K^{\pm}$ from three-prong τ^- decays will have such low momenta. Protons from background sources pose a particular concern since they would likely be misidentified as $K^{\pm}$ due to their low dE/dx values through the high momentum region. However, Monte Carlo simulations predict fewer than one proton in the sample from $q\bar{q}$ background, while protons from nuclear interactions tend to have much lower momenta and would be easily distinguishable. Moreover, the tracks in our sample with momentum above 2 GeV and dE/dx below the expected $K^{\pm}$ value show no preference for positive charge.

To determine the decay populations in the sample we use both an extended maximum likelihood fit and an identification matrix inversion technique [6]. The statistical nature of these methods requires that we consider eight decay classes, since each of the randomly ordered tracks may be π or K. The population estimates for $K^-\pi^+\pi^-$ and $\pi^-\pi^+K^-$ are then added together, as are those for $K^-K^+\pi^-$ and $\pi^-K^+K^-$, to give event populations for the six physically distinct decay modes.

For the *likelihood method* [7], we derive the expression

$$\mathscr{L}_E=\exp\left(-\sum_j N_j\right)\prod_a\left(\sum_j N_j f_j(I_1^a,I_2^a,I_3^a)\right),$$

where N_j is the expected number of events in decay class j, the index a runs over the events, and the I_i^a are the measured dE/dx values for the three tracks in an event. The distribution f_j is taken to be a product of three Gaussian distributions, each centered on the expected dE/dx value given the momentum and particle species that define class j; momentum uncertainties are negligible. The exponential factor takes account of Poisson fluctuations in the sample. Using the package MINUIT [8], we determine the values of N_j that give a global maximum for the expression.

For the *identification matrix inversion method*, each of the three-prong tracks is initially classified as a $\pi^{\pm}$ or $K^{\pm}$. Each track with momentum greater than 2 GeV and with dE/dx that satisfies $\chi^2_K<(\chi^2_\pi-4.0)$ is classified as $K^{\pm}$, while all other tracks are classified as $\pi^{\pm}$. From the assumption of Gaussian dE/dx distributions for charged particle species, an identification matrix

$$\mathbf{Q}^a=\begin{pmatrix} Q^a_{\pi\leftarrow\pi} & Q^a_{\pi\leftarrow K}\\ Q^a_{K\leftarrow\pi} & Q^a_{K\leftarrow K}\end{pmatrix}$$

is generated for each of the tracks, where the element $Q_{K\leftarrow\pi}$, for example, is the probability of $\pi^{\pm}$ being misidentified as $K^{\pm}$. These elements are functions of the momentum and dE/dx resolution of the track, with an average value for $Q_{\pi\leftarrow\pi}$ of 0.99 and for $Q_{K\leftarrow K}$ of 0.86. The outer product of the three *track* identification matrices gives an *event* identification matrix

$$\mathbf{P}^a(p_1,p_2,p_3)\equiv\mathbf{Q}^a(p_1)\otimes\mathbf{Q}^a(p_2)\otimes\mathbf{Q}^a(p_3),$$

where a is an event label and p_i are the measured parameters for track i of the event. For event a, $\mathbf{P}^a$ relates the true decay class, given by vector $\mathbf{N}^a$, to the assigned decay class, given by vector $\mathbf{M}^a$, according to

$$\mathbf{M}^a=\mathbf{P}^a\mathbf{N}^a.$$

The element of $\mathbf{M}^a$ that corresponds to the assigned decay class is set to one, while all other elements are set to zero. We then solve for $\mathbf{N}^a$ to obtain a vector of weights for the possible decay modes. The summation of these weights over all of the events,

$$N_j=\sum_a N_j^a,$$

provides another estimate of the decay populations N_j within the sample.

The decay class populations for the two methods, given in Table I, agree very well. Note that these decay classes are defined by the identities of their charged particles and may include decays with different numbers of neutrals. To deter-

TABLE I. Event populations with statistical errors for τ^- decay classes. The branching fractions are normalized to the world average three-prong topological branching fraction. There is an additional systematic error of 20% for TPC/Two-Gamma and 23% for DELCO. The upper limits are at the 95% confidence level, including systematic uncertainties.

Decay	Population		Branching fraction (%)	
(≥0 neutrals)	Matrix	Likelihood	TPC/2γ	DELCO
$\nu_\tau\,\pi^-\pi^+\pi^-$	489.6±22.6	$492.4^{+22.7}_{-22.1}$	$13.29^{+0.27}_{-0.27}$	
$\nu_\tau\,K^-\pi^+\pi^-$	19.7±5.9	$19.9^{+5.4}_{-4.6}$	$0.58^{+0.15}_{-0.13}$	$0.22^{+0.16}_{-0.13}$
$\nu_\tau\,\pi^-K^+\pi^-$	3.5±3.4	1.6±2.9	<0.25	
$\nu_\tau\,K^-K^+\pi^-$	5.4±2.9	$4.1^{+2.5}_{-1.8}$	$0.15^{+0.09}_{-0.07}$	$0.22^{+0.17}_{-0.11}$
$\nu_\tau\,K^-\pi^+K^-$	−0.1±0.1	0.0±0.5	<0.09	
$\nu_\tau\,K^-K^+K^-$	0.0±0.0	0.0±0.5	<0.21	

mine the event selection efficiency for the $\pi^-\pi^+\pi^-$ decay class we use Monte Carlo simulations of $\tau^-\to\nu_\tau\pi^-\pi^+\pi^-$ and $\tau^-\to\nu_\tau\pi^-\pi^+\pi^-\pi^0$, weighted according to current values for their branching fractions. Similarly, the selection efficiencies for the decay classes with $K^\pm$ are based on simulations with and without an additional π^0. The systematic uncertainties in these efficiencies are estimated by allowing the fraction of decays with π^0 to vary between zero and one-half of the total. To calculate the branching fractions, we correct the likelihood populations for variations in acceptance among the different decay classes and normalize to the world average three-prong topological branching fraction of 14.06±0.25% [9]. We estimate a systematic error of 20% for the decay classes with $K^\pm$, primarily due to uncertainties in dE/dx parametrization and in event selection efficiencies. Compared with previous results from DELCO [10], our values for $B_{K^-\pi^+\pi^-}$ and $B_{K^-K^+\pi^-}$ are 1.6σ higher and 0.6σ lower, respectively, while the relative error for the $B_{K^-\pi^+\pi^-}$ decay is reduced from 65% to 25%. Table I also shows our upper limits for $B_{\pi^-K^+\pi^-}$, $B_{K^-\pi^+K^-}$, and $B_{K^-K^+K^-}$.

To examine the resonance structure of the decay $\tau^-\to\nu_\tau K^-\pi^+\pi^-$ [11], we select 23 candidate events where exactly one of the charged tracks satisfies the K^- criteria. The $K^-\pi^+\pi^-$, $K^-\pi^+$, and $\pi^+\pi^-$ invariant mass plots for these events are shown in Fig. 2. Using Monte Carlo methods, we obtain analytical expressions for the expected distributions of $K_1^-(1270)$ and $K_1^-(1400)$ events in the three-dimensional invariant mass space defined by $m^2_{K^-\pi^+\pi^-}$, $m^2_{K^-\pi^+}$, and $m^2_{\pi^+\pi^-}$. Events are generated with the τ^- generator KORALB [12], to which we have added decays to the K_1^- resonances. These events are then passed through a detector simulation in order to model acceptance and resolution effects. For each of the decay modes of the $K_1^-(1270)$ and $K_1^-(1400)$ [9], we fit the analytical expression

$$N\ g(K\pi\pi)\ g(K\pi)\ g(\pi\pi)\left(\frac{m_\tau^2}{m^2_{K\pi\pi}}-1\right),$$

to the invariant mass distribution of the Monte Carlo events that fall within the allowed phase space region of $m_{K\pi\pi}<m_\tau$. Here N is the normalization, and g is a Breit-Wigner function for resonant charged particle combinations and a constant otherwise. The fit determines the best values for the effective mass and width parameters. Finally, these

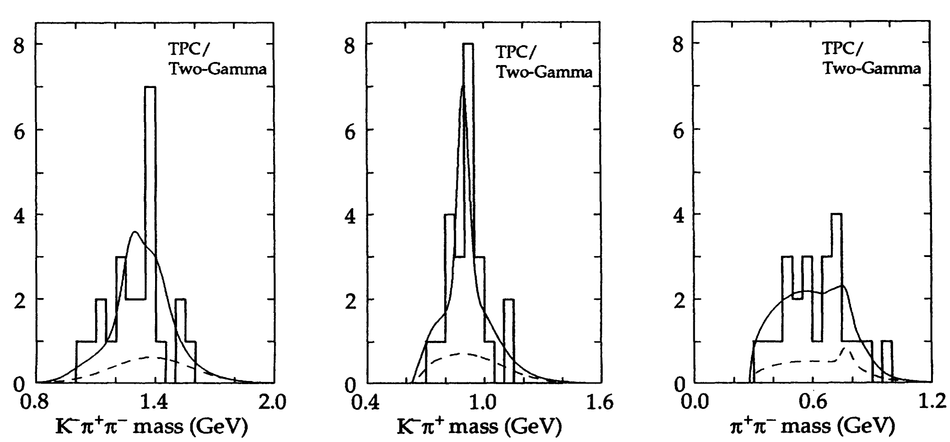

FIG. 2. Invariant mass distributions for the 23 $K^-\pi^+\pi^-$ candidates in the τ_{1+3} sample. The dashed curve shows the estimated background from misidentified $\pi^-\pi^+\pi^-$ events, while the solid curve shows the background plus the best fit combination of $K_1^-(1270)$ and $K_1^-(1400)$.

TABLE II. Branching fractions for decays of the τ^- to $\nu_\tau K_1^-(1270)$, $\nu_\tau K_1^-(1400)$, and total $\nu_\tau K_1^-$, assuming that the process $\tau^- \to \nu_\tau K^-\pi^+\pi^-$ proceeds entirely through the K_1^- channels. The efficiency definition is described in the text.

Decay	Population Likelihood	Efficiency $\epsilon K_1/\epsilon_3$	Branching fraction (%) TPC/2γ
$\nu_\tau K_1^-(1270)$	$5.4^{+5.4}_{-4.6}$	0.36 ± 0.04	$0.41^{+0.41}_{-0.35}$
$\nu_\tau K_1^-(1400)$	$11.0^{+5.8}_{-4.8}$	0.39 ± 0.04	$0.76^{+0.40}_{-0.33}$
$\nu_\tau K_1^-$	$16.4^{+5.7}_{-5.1}$	0.38 ± 0.04	$1.17^{+0.41}_{-0.37}$

expressions for the individual decay modes are combined, according to their relative selection efficiencies and branching fractions, to give net predicted invariant mass distributions for the $K_1^-(1270)$ and the $K_1^-(1400)$.

Using the $\pi^-\pi^+\pi^-$ candidates of the τ_{1+3} data sample, we derive a similar analytical expression for the expected invariant mass distribution for the background. For each π^- with momentum greater than 2 GeV, we replace the π^- mass with that of the K^- and calculate $m^2_{K^-\pi^+\pi^-}$, $m^2_{K^-\pi^+}$, and $m^2_{\pi^+\pi^-}$ for the event. The expression above is then fit to these events, weighted by their misidentification probabilities $Q_{K\leftarrow\pi}$, which range from 0.4% to 2.3%. The values of $Q_{K\leftarrow\pi}$ are also used to obtain a background estimate of 6.3 ± 2.7 events, including systematic uncertainties.

Assuming that the decay $\tau^- \to \nu_\tau K^-\pi^+\pi^-$ is dominated by the two K_1^- channels, we construct an extended maximum likelihood function:

$$\mathscr{L}_E = \exp\left(-\sum_j N_j\right)\prod_a\left(\sum_j N_j h_j(m^a_{K\pi\pi}, m^a_{K\pi}, m^a_{\pi\pi})\right),$$

where a runs over the $K^-\pi^+\pi^-$ candidate events. The N_j and h_j are the expected populations and predicted invariant mass distributions for $K_1^-(1270)$, $K_1^-(1400)$, and background events. Using the background contribution given above, we determine the most probable values for the $K_1^-(1270)$, $K_1^-(1400)$, and total K_1^- populations, listed in Table II. The branching fractions are calculated according to

$$B_{K_1} = B_3(N_{K_1}/N_3)/(\epsilon_{K_1}/\epsilon_3),$$

where B_3 is the topological branching fraction to three prongs, N_3 is the number of events in our τ_{1+3} sample, and $\epsilon_{K_1}/\epsilon_3$ is the selection efficiency for K_1^- events in the $K^-\pi^+\pi^-$ sample relative to that for three-prong events in the τ_{1+3} sample. We estimate a systematic error of 25% from uncertainties in dE/dx parametrization, event selection efficiencies, and branching fractions for the decays of the $K_1^-(1270)$ and $K_1^-(1400)$.

The Monte Carlo invariant mass distributions for the best fit values of $K_1^-(1270)$ and $K_1^-(1400)$ are compared with those of the $K^-\pi^+\pi^-$ candidates in Fig. 2. From the relative magnitude of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|V_{ud}|$ and $|V_{us}|$, the branching fraction for $\tau^- \to \nu_\tau K_1^-$ is expected to be roughly 5% of that for $\tau^- \to \nu_\tau a_1^-$, which is in the range of 16–20%, assuming that the $\nu_\tau\pi^-\pi^+\pi^-$ and $\nu_\tau\pi^-\pi^0\pi^0$ decays are dominated by the a_1^-. This estimate is in good agreement with our measurement for the total branching fraction of $\tau^- \to \nu_\tau K_1^-$, listed in Table II. Moreover, we note that for those events with $K^-\pi^+$ mass in the $\bar{K}^{*0}$ peak, the angular distribution of the $K^-\pi^+$ decay relative to the π^- direction in the K^{*0} rest frame is consistent with isotropy, as expected if the source is predominantly axial-vector K_1^- decaying to $\bar{K}^{*0}\pi^-$ in the s wave. The ratio of the measured $K_1^-(1270)$ and $K_1^-(1400)$ branching fractions provides information for the determination of θ_{K_1}, the mixing angle of the two K_1 states [13].

We check our $K^-\pi^+\pi^-$ sample for the presence of neutrals by looking for calorimeter energy clusters that are in the three-prong half of the event and are not associated with any of the charged tracks. We find that 6 of the 23 events do have an energetic cluster of at least 0.4 GeV. However, Monte Carlo simulations predict that, due to hadronic interactions and clusters that are not correctly associated with charged tracks, about 20% of $K^-\pi^+\pi^-$ decays will include a fake neutral cluster. In addition, we expect the sample to include a few $\pi^-\pi^+\pi^-\pi^0$ background events. Thus, the observed number of events with an energetic cluster is consistent with no significant number of $K^-\pi^+\pi^-\pi^0$ events being present [14]. Finally, we note that a likelihood fit with the 17 events that show no energetic cluster gives $K_1^-(1270)$ and $K_1^-(1400)$ branching fractions that are well within the systematic errors of our reported values based on the full sample.

In order to check for non-K_1^- contributions to the decay $\tau^- \to \nu_\tau K^-\pi^+\pi^-$, we derive expressions for the expected invariant mass distributions of nonresonant $K^-\pi^+\pi^-$, $K^-\rho^0$, and $\bar{K}^{*0}\pi^-$. We perform likelihood fits in which these nonresonant components are included along with $K_1^-(1270)$, $K_1^-(1400)$, and background. The $K^-\pi^+\pi^-$ and $K^-\rho^0$ are strongly excluded by the data because of the presence of a $\bar{K}^{*0}$ peak and absence of a ρ^0 peak in the mass plots of Fig. 2. Since the $\bar{K}^{*0}\pi^-$ and $K^-\rho^0$ decays are related by an SU(3) flavor rotation, we set the coupling of the τ^- current to these channels to be equal. After taking into consideration phase space and isospin factors, we estimate a ratio of 3:1 for the decay rates of the $\bar{K}^{*0}\pi^-$ and $K^-\rho^0$ channels. Using this ratio to constrain the $\bar{K}^{*0}\pi^-$ contribution, we find that the most likely branching fraction value for combined $\bar{K}^{*0}\pi^-$ and $K^-\rho^0$ is $0.03^{+0.25}_{-0.03}\%$, and for $K^-\pi^+\pi^-$ is $0.00^{+0.09}_{-0.00}\%$. Although we are unable to rule out contributions from these nonresonant channels, our overall results are consistent with K_1^- dominance of the $(K\pi\pi)^-$ decays of the τ^- lepton.

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[13] M. Suzuki, Phys. Rev. D **47**, 1252 (1993). Suzuki has shown that the partial decay rates and masses of the $K1(1270)$ and $K1(1400)$ predict a mixing angle of $\theta_{K_1}\approx 33°$ or $\theta_{K_1}\approx 57°$. Our ratio of branching fractions favors $\theta_{K_1}\approx 33°$ for reasonable estimates of SU(3) breaking.
[14] Fitting to the observed number of events with an energetic cluster and using estimates of $\pi^-\pi^+\pi^-$ and $\pi^-\pi^+\pi^-\pi^0$ backgrounds, we obtain an upper limit for the $\tau^-\to\nu_\tau K^-\pi^+\pi^-\pi^0$ branching fraction of 0.40% at the 95% confidence level.