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Relativistic scattering and bound-state properties in a special Hamiltonian model

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We present numerical results for fermion-boson scattering cross sections and bound-state form factors in an elementary model of relativistic renormalized light-front Hamiltonian dynamics. The model Hamiltonian describes a fermion emitting and absorbing one scalar boson. Renormalization of the coupling constant leads to triviality and the cutoff cannot be arbitrarily large. Nevertheless, the resulting total fermion-boson scattering cross section is found to be practically independent of the cutoff within the triviality bounds. We also study cutoff dependence of the fermion-boson bound-state form factors. The cutoff dependence is negligible for boson masses considerably smaller than the fermion mass even for momentum transfers exceeding many times the fermion mass, as long as the momentum transfer is small compared to the cutoff. For heavy bosons the cutoff dependence of the fermion-boson sector contribution is stronger but the bound-state structure is dominated by the cutoff-independent bare fermion component. Thus, the model Hamiltonian leads to almost cutoff-independent results in a whole range of mass and coupling parameters within the triviality bounds.

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I. INTRODUCTION

The goal of light-front QCD is to compute wave functions of hadrons by solving for the eigenvalues and eigenstates of the QCD Hamiltonian [1,4]. Physical observables which characterize hadrons, such as form factors and structure functions, can be calculated once these wave functions are known [3,6,16,19]. However, light-front Hamiltonians for quantum field theories pose very difficult renormalization problems [2,7]. It is useful to do model studies of renormalized light-front Hamiltonian dynamics which help in building the intuition and understanding required to attack light-front QCD.

Recently, Głazek and Perry [8] have constructed a relativistic model Hamiltonian with a Yukawa interaction in 3+1 dimensions which yields a nontrivial covariant fermion-boson scattering amplitude and a fermion-boson bound state. The Hamiltonian acts in a space of two Fock sectors: a sector with one fermion and a sector with one fermion and one boson. This feature makes the model resemble a sector of the Lee model [9,10]. The major difference is that the Lee model or its relativistic extensions are usually constructed using an equal-time form of dynamics while this model is constructed using renormalized light-front dynamics. Fuda has considered a similar model [11] but did not discuss renormalization issues.

Strong limitations on the space in which the model Hamiltonian acts violate rotational invariance, which has to be restored. There are two kinds of terms in the model light-front Hamiltonian which restore covariance: seagull terms, which correspond to the fermion-antifermion pair creation in the equal-time dynamics, and renormalization counterterms, which remove divergences and whose finite parts have to be carefully chosen in order to obtain results which possess the full symmetry required by special relativity.

The model exhibits a special kind of triviality due to the restriction on the number of particles. It should be stressed that the triviality of the truncated model does not imply that the full Yukawa theory is trivial. In order to prove that the full theory is trivial, one would have to exclude the possibility of the existence of an ultraviolet fixed point. This cannot be done without solving the full theory. The triviality of the model presented in Ref. [8] and here is in qualitative agreement with the perturbative leading log calculation. A similar but different type of triviality appears in the case of two-fermion bound-state Hamiltonians [5].

The triviality bound forces us to keep the ultraviolet relative transverse momentum cutoff finite. One cannot make the cutoff go to infinity, as is possible in the case of asymptotically free theories. This raises the question of how strong the dependence of observables on the large but finite cutoff actually is. The answer determines whether it is useful to consider such cutoff models without asymptotic freedom. We are primarily concerned with making physical observables independent of the cutoff when the invariant mass of a state under consideration is well below the cutoff. We do not try to remove the cutoff dependence as the cutoff is approached. This would require arbitrarily many irrelevant operators even in an asymptotically free theory, and it is not possible in a theory plagued by triviality.

In this paper we present results of numerical studies of the model from Ref. [8]. We present formulas for renormalized scattering amplitudes and the bound-state form factors. Then we present the triviality curves, total fermion-boson scattering cross sections, and numerical results for the form factors for various choices of masses, couplings, and cutoffs.

The paper is organized as follows. In Sec. II we review the model, calculate the triviality bounds on various parameters in the Hamiltonian, and evaluate the fermion-boson scattering cross section. We demonstrate that the cross section is insensitive to the cutoff, even for very large center of mass momenta of the incoming fermion-boson states. Section III presents form factors of the fermion-boson bound state, considering only the + component of the current, $j^+(q)$, with the momentum transfer q chosen such that $q^+ = 0$. We calculate form factors $F_1(q^2)$ and $F_2(q^2)$ for various choices of the fermion and boson masses, and study their dependence on the ultraviolet cutoff in order to establish in what range of parameters and momentum transfers the model can provide cutoff-independent predictions within the triviality bounds. Section IV concludes the paper.

II. THE MODEL

The model Hamiltonian has the form [8]

$$H = H_0^f + H_0^{fb} + H_1 + H_2 + H_3,$$

$$H_0^f = \sum_\lambda \int [dp] |p\lambda\rangle \left[\frac{p^{\perp 2} + m_1^2 + g^2\omega^2}{p^+}\right] \langle p\lambda|,$$

$$H_0^{fb} = \sum_\lambda \int [dp][dk] |p\lambda, k\rangle \left[\frac{p^{\perp 2} + m^2}{p^+} + \frac{k^{\perp 2} + \mu^2}{k^+}\right] \langle p\lambda, k|,$$

$$H_1 + H_2 = g \sum_{\lambda\sigma} \int [dp][dp'][dk] \Theta(\Lambda^2 - \kappa^2) 2(2\pi)^3 \delta^3(p + k - p')$$
$$\times \left\{ |p\lambda, k\rangle \bar{u}(p,\lambda) \left[1 + \frac{\delta m}{2p'^+}\gamma^+\right] u(p',\sigma) \langle p'\sigma| + |p'\sigma\rangle \bar{u}(p',\sigma) \left[1 + \frac{\delta m}{2p'^+}\gamma^+\right] u(p,\lambda) \langle p\lambda, k| \right\},$$

$$H_3 = g^2 \sum_{\lambda\sigma} \int [dp_1][dp_2][dk_1][dk_2] \Theta(\Lambda^2 - \kappa_1^2) \Theta(\Lambda^2 - \kappa_2^2) 2(2\pi)^3 \delta^3(p_1 + k_1 - p_2 - k_2)$$
$$\times |p_2\lambda, k_2\rangle \bar{u}(p_2,\lambda) \frac{\gamma^+}{2(p_1^+ + k_1^+)} u(p_1,\sigma) \langle p_1\sigma, k_1|, \qquad (1)$$

where m is the mass of the fermion in the fermion-boson sector, which equals the mass of the physical fermion, m_1 is the bare mass of the fermion in the one-fermion Fock sector, μ is the mass of the boson, p, p' denote momenta of fermions, k denotes the boson momentum, $[dp] \equiv \frac{dp^+ d^2p^\perp}{2(2\pi)^3 p^+}$,

$$|p\lambda\rangle = b^\dagger(p,\lambda)|0\rangle, \quad |p\lambda, k\rangle = b^\dagger(p,\lambda) a^\dagger(k)|0\rangle,$$

κ is the transverse relative momentum, $\kappa^\perp = \frac{p^+k^\perp - k^+p^\perp}{p^+ + k^+}$ and $\delta m = m_1 - m$.

We follow here the notation from Ref. [8], except for a change to the convention that the fermion carries x and boson $(1-x)$ fraction of the total P^+.

Notice that the cutoff Λ limits the relative transverse momentum but the total momentum is not limited. There are additional cutoffs imposed. Namely, all boson's longitudinal momenta must be greater than b^+, and all fermion's longitudinal momenta must be greater than f^+.

A mass counterterm provided by $g^2\omega^2$ and the ratio of infrared cutoffs b^+ and f^+ are chosen in such a way that the fermion self-energy has a covariant form. Then both b^+ and f^+ are allowed to go to zero with their ratio fixed. Recently, Burkhart and Langnau studied similar fermion self-interactions in perturbation theory [12].

The self-energy given in Ref. [8] has the covariant form

$$\Sigma(P) = \alpha(M)\not{P} + \beta(M)m, \qquad (2)$$

where $\alpha(M)$ and $\beta(M)$ are

$$\alpha(M) = -\frac{1}{4(2\pi)^2} \int d\kappa^2 \Theta(\Lambda^2 - \kappa^2) \int_0^1 dx \frac{x}{-M^2x(1-x) + \mu^2 x + m^2(1-x) + \kappa^2 - i\epsilon}, \qquad (3)$$

and

$$\beta(M) = -\frac{1}{4(2\pi)^2}\int d\kappa^2\Theta(\Lambda^2-\kappa^2)\int_0^1 dx\frac{1}{-M^2x(1-x)+\mu^2x+m^2(1-x)+\kappa^2-i\epsilon}. \tag{4}$$

The T matrix for the fermion boson scattering is [13]

$$\begin{aligned}\langle\phi_i|T(E)|\phi_j\rangle &= \langle\phi_i|H_I + H_I\frac{1}{E-H_0+i\epsilon}T(E)|\phi_j\rangle,\\ &= \langle\phi_i|H_I + H_I\frac{1}{E-H_0+i\epsilon}H_I+\cdots|\phi_j\rangle,\\ &= 2(2\pi)^3\delta^3(P_i-P_j)g^2\bar{u}_m(p_i,\sigma_i)\frac{1}{\not{P}-m_1-g^2\Sigma(P)+i\epsilon}u_m(p_j,\sigma_j).\end{aligned} \tag{5}$$

Notice that $\alpha(M)$ and $\beta(M)$ are divergent functions of the cutoff Λ.

The physical fermion is an eigenstate of the Hamiltonian. The bare mass satisfies the relation

$$m_1 = \left\{1-g^2\left[\alpha(m)+\beta(m)\right]\right\}m \tag{6}$$

in order to cancel the divergence in $\beta(M)$ in the self-energy in the T matrix. The remaining divergence due to $\alpha(M)$ is canceled by introducing the renormalized coupling constant

$$\begin{aligned}\tilde{g}^2(M) &= \frac{g^2}{1-g^2\alpha(M)},\\ &= \frac{\tilde{g}^2(m)}{1-\tilde{g}^2(m)\left[\alpha(M)-\alpha(m)\right]}.\end{aligned} \tag{7}$$

Substituting this expression into Eq. (6) one obtains the running mass

$$\begin{aligned}\tilde{m}(M) = \{1+\tilde{g}^2(M)[\alpha(M)-\alpha(m)+\beta(M)\\ -\beta(m)]\}m.\end{aligned} \tag{8}$$

The renormalized scattering amplitude is given in Eq. (11) below.

For scattering states, both the renormalized mass and the coupling constant become complex. Expressions for their real and imaginary parts are presented in Appendix A.

A. Triviality limits

The requirement that the bare coupling constant is finite and real in Eq. (7) imposes triviality limits on the maximal value of the renormalized coupling constant at a given cutoff:

$$\alpha_{\max} \equiv \frac{\tilde{g}^2_{\max}(m)}{4\pi} = -\frac{1}{4\pi\alpha(m)}. \tag{9}$$

Figure 1 shows the triviality curves for different values of the boson mass. The curves behave like $\left(\ln\frac{\Lambda}{\mu}\right)^{-1}$ for large Λ. This implies that for larger boson masses stronger renormalized couplings are allowed.

B. Total cross section

The total cross section in the center of mass frame is

$$\sigma_{fi} = \frac{1}{64\pi^2M^2}\int d\Omega\sum_{\text{pol}}|M_{fi}|^2, \tag{10}$$

where M_{fi} is the invariant scattering amplitude and M is the invariant mass of the scattering state. In this case, M_{fi} is just T_{fi} from Eq. (5) without factors $2(2\pi)^3\delta^3(P_f-P_i)$. Substituting renormalized quantities from Eqs. (7) and (8) into Eq. (5) one finds

$$M_{fi} = \bar{u}_m(p',\lambda')\frac{\tilde{g}^2(M)}{\not{P}-\tilde{m}(M)}u_m(p,\lambda)\ , \tag{11}$$

where $P = P_i = P_f$ and $P^2 = M^2$. This leads to the total cross section,

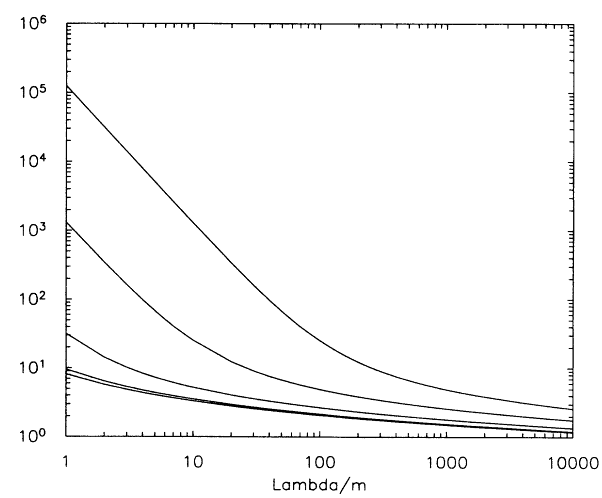

FIG. 1. Triviality curves [i.e., maximum allowed coupling $\alpha(m)$ versus the cutoff Λ] for mass of the boson $\mu = 0.01m$, $0.1m$, m, $10m$, $100m$, respectively. The uppermost curve corresponds to the largest ratio μ/m and lower curves to lower values, successively. The maximum allowed coupling increases with increased boson mass and behaves like $(\ln\frac{\Lambda}{\mu})^{-1}$ for large Λ.

$$\sigma = \frac{1}{8\pi M^2}\Theta(\Lambda^2 - \kappa^2)\Theta(\Lambda^2 - \kappa'^2)\frac{|\tilde{g}(M)|^4}{|M^2 - \tilde{m}^2(M)|^2} \times \left\{ (2m^2 + p^2)[M^2 + |\tilde{m}(M)|^2] + 4mM\sqrt{m^2+p^2}\,\mathrm{Re}\tilde{m}(M) \right\}, \tag{12}$$

where p^2 is the center of mass momentum, and

$$M = \sqrt{m^2 + p^2} + \sqrt{\mu^2 + p^2}.$$

Figures 2(a), 2(b), and 2(c) show the total fermion-boson scattering cross section as a function of the center of mass momentum. We use $\Lambda = 10m$, $50m$, $100m$, and vary the coupling α up to the triviality limits. The cross sections are practically cutoff independent up to the center of mass momenta comparable to the cutoff. The cutoff dependence, although negligible, increases with increasing boson mass and for the boson mass fixed, with increasing coupling, as one can see especially well on Fig. 2(c). The extreme case is when $\mu = 10m$, and the coupling constant can reach values up to 25 for $\Lambda = 10m$.

III. FORM FACTORS

We consider fermions to carry a charge $e = 1$ and bosons to be uncharged. The physical fermion state is a superposition of a bare fermion state and bare fermion-boson states:

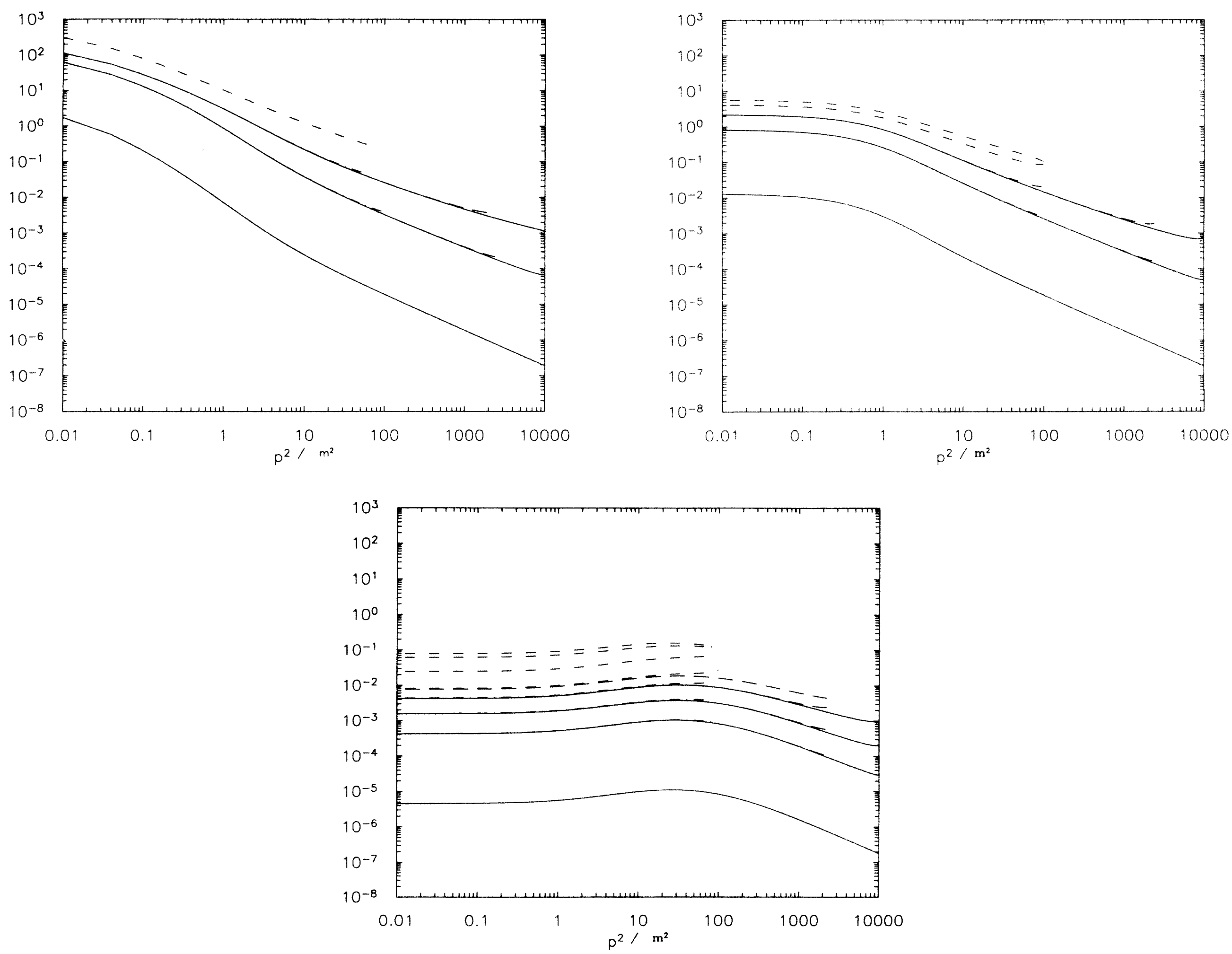

FIG. 2. Total cross sections for unpolarized fermion boson scattering as a function of the center of mass momentum for different values of coupling constant and cutoff. The uppermost curves correspond to the largest value of coupling constant and lower curves to lower values, successively. The full line is used for $\Lambda = 100m$, dashed line for $\Lambda = 50m$, and dot-dashed line for $\Lambda = 10m$. (a) Boson mass $\mu = 0.1m$, $\alpha = 0.1$, 1.0, 2.0 for each of the following values of cutoff: $\Lambda = 10m$, $50m$, $100m$; and $\alpha = 3.5$ for $\Lambda = 10m$. (b) Boson mass $\mu = m$, $\alpha = 0.1$, 1.0, 2.0 for each of the following values of cutoff: $\Lambda = 10m$, $50m$, $100m$; and $\alpha = 3.5$, 5.0 for $\Lambda = 10m$. (c) Boson mass $\mu = 10m$, $\alpha = 0.1$, 1.0, 2.0, 3.5 for each of the following values of cutoff: $\Lambda = 10m$, $50m$, $100m$; $\alpha = 5.0$ for $\Lambda = 10m$, $50m$; and $\alpha = 10.0$, 20.0, 25.0 for $\Lambda = 10m$.

$$|P,\lambda\rangle = Nb^\dagger(P,\lambda)|0\rangle + \sum_\sigma \int [d^3p][d^3k]2P^+(2\pi)^3$$
$$\times\delta^3(P-p-k)f_\sigma^\lambda b^\dagger(p,\sigma)a^\dagger(k)|0\rangle, \quad (13)$$

where

$$f_\sigma^\lambda = Ng\frac{\Theta(\Lambda^2-\kappa^2)}{m^2-\frac{\kappa^2+m^2}{x}-\frac{\kappa^2+\mu^2}{1-x}}\bar{u}_m(p,\sigma)u_m(P,\lambda),$$

and N is the state normalization constant, which also gives the result $F_1(0)=1$.

We extract the form factors from matrix elements of the current $j^\mu = \bar{\psi}\gamma^\mu\psi$ for $\mu = +$, between states with the same P^+ so that the momentum transfer $q^+=0$. This is a standard procedure, see Ref. [16] and references therein. In this way one can avoid contributions from the pair creation by the current (see Refs. [14] and [15] for numerical significance of this choice). Explicitly,

$$\langle P',\lambda'|j^+(0)|P,\lambda\rangle = \bar{u}_m(P',\lambda') \times\left[F_1(q^2)\gamma^+ + \frac{F_2(q^2)}{2m}q_\nu i\sigma^{\nu+}\right] \times u_m(P,\lambda), \quad (14)$$

where $q = P'-P$ and $\sigma^{\nu\mu} \equiv \frac{i}{2}[\gamma^\nu,\gamma^\mu]$. The bare matrix elements are

$$\langle 0|b(P',\lambda')j^+(0)b^\dagger(P,\lambda)|0\rangle = \bar{u}_m(P',\lambda')\gamma^+u_m(P,\lambda) \quad (15)$$

and

$$\langle 0|b(p',\sigma')a(k')j^+(0)b^\dagger(p,\sigma)a^\dagger(k)|0\rangle$$
$$= 2(2\pi)^3k^+\delta^3(\mathbf{k}-\mathbf{k}')\bar{u}_m(p',\sigma')\gamma^+u_m(p,\sigma). \quad (16)$$

Then, $j^+(0)$ matrix elements between the physical fermion states are

$$\langle P',\lambda'|j^+(0)|P,\lambda\rangle = \bar{u}_m(P',\lambda')eN^2\Bigg[\gamma^+ + \frac{g^2}{2(2\pi)^3}\int\frac{dx}{x^2(1-x)}d^2\kappa^\perp\frac{\theta'\theta}{D'D} \times\left(\not{p}'+m\right)\gamma^+\left(\not{p}+m\right)\Bigg]u_m(P,\lambda), \quad (17)$$

where $\theta \equiv \theta(\Lambda^2-\kappa^2)$, $\theta' \equiv \theta(\Lambda^2-\kappa'^2)$, and D, D' are the light-front energy denominators, $D \equiv m^2 - \frac{\kappa^2+m^2}{x} - \frac{\kappa^2+\mu^2}{1-x}$, $D' \equiv m^2 - \frac{\kappa'^2+m^2}{x} - \frac{\kappa'^2+\mu^2}{1-x}$, and $\kappa'^\perp = \kappa^\perp - (1-x)q^\perp$. Calculations can be simplified using the facts that the spinors $u_m(P',\lambda')$ and $u_m(P,\lambda)$ are solutions of the Dirac equation and that $(\gamma^+)^2=0$ which implies that neither p^- nor p'^- contribute to Eq. (17). Namely,

$$\bar{u}_m(P',\lambda')(\not{p}'+m)\gamma^+(\not{p}+m)u_m(P,\lambda)$$
$$= \bar{u}_m(P',\lambda')\left[(1+x)m+\kappa'^\perp\gamma^\perp\right]\gamma^+ \times\left[(1+x)m+\kappa^\perp\gamma^\perp\right]u_m(P,\lambda). \quad (18)$$

Substituting $\kappa^\perp = u^\perp + \frac{1}{2}(1-x)q^\perp$ and using commutation relations for γ matrices, the above expression can be written as

$$\bar{u}_m(P',\lambda')\left\{\left[(1+x)^2m^2 - \frac{1}{4}(1-x)^2q^2 + {u^\perp}^2\right]\gamma^+ + i\frac{q^\perp}{m}\sigma^{\perp+}(1-x^2)m^2\right\}u_m(P,\lambda). \quad (19)$$

We use this result in Eq. (17) and perform the angular integration. By comparison with Eq. (14) one can identify the form factors

$$F_1(q^2) = \frac{1+g^2I(q)}{1+g^2I(0)} \quad (20)$$

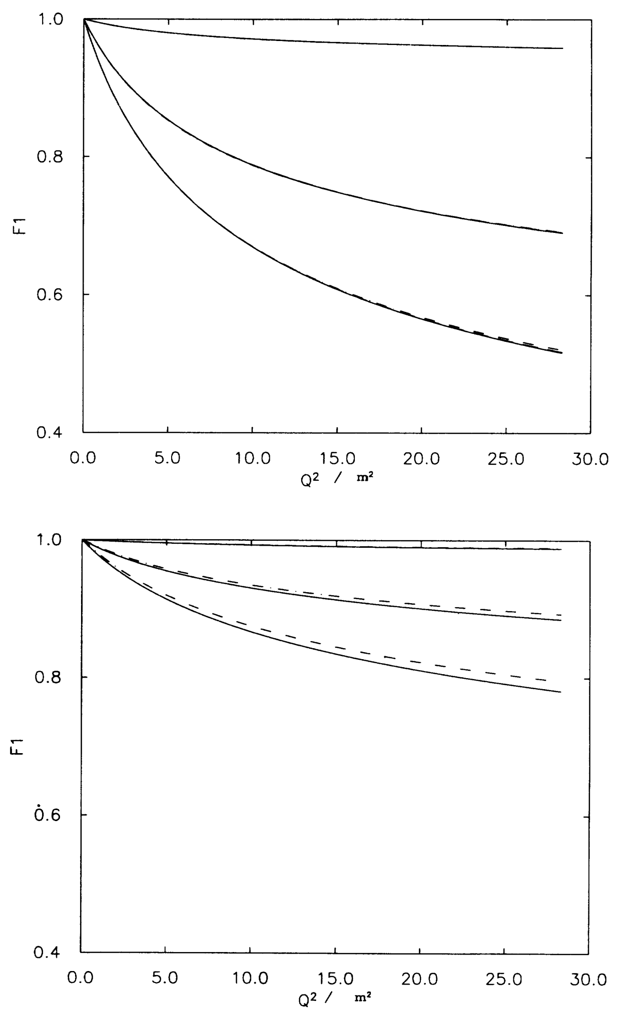

FIG. 3. Form factors $F_1(q^2)$ versus Q^2/m^2 for $\alpha = 0.1$, 1.0, 2.0. The uppermost curve corresponds to the smallest value of coupling constant and lower curves to larger values, successively. Full line is used for $\Lambda = 100m$, dashed line for $\Lambda = 50m$, and dot-dashed line for $\Lambda = 10m$. The dashed line concises with the full line. (a) Boson mass $\mu = 0.1m$, cutoff $\Lambda = 10m$, $50m$, $100m$. (b) Boson mass $\mu = m$, cutoff $\Lambda = 10m$, $100m$.

and

$$F_2(q^2) = \frac{g^2 J(q)}{1 + g^2 I(0)}. \tag{21}$$

Expressions for the functions I and J are given in Appendix B.

The denominator in Eqs. (20) and (21) is provided by the normalization factor N^2. Note that the form factors are expressed through the bare coupling constant. We have to express them in terms of the renormalized coupling constant, and after that they become

$$F_1(q^2) = \frac{1 + \tilde{g}^2 I(q) + \tilde{g}^2 \alpha(m)}{1 + \tilde{g}^2 I(0) + \tilde{g}^2 \alpha(m)} \tag{22}$$

and

$$F_2(q^2) = \frac{\tilde{g}^2 J(q)}{1 + \tilde{g}^2 I(0) + \tilde{g}^2 \alpha(m)}, \tag{23}$$

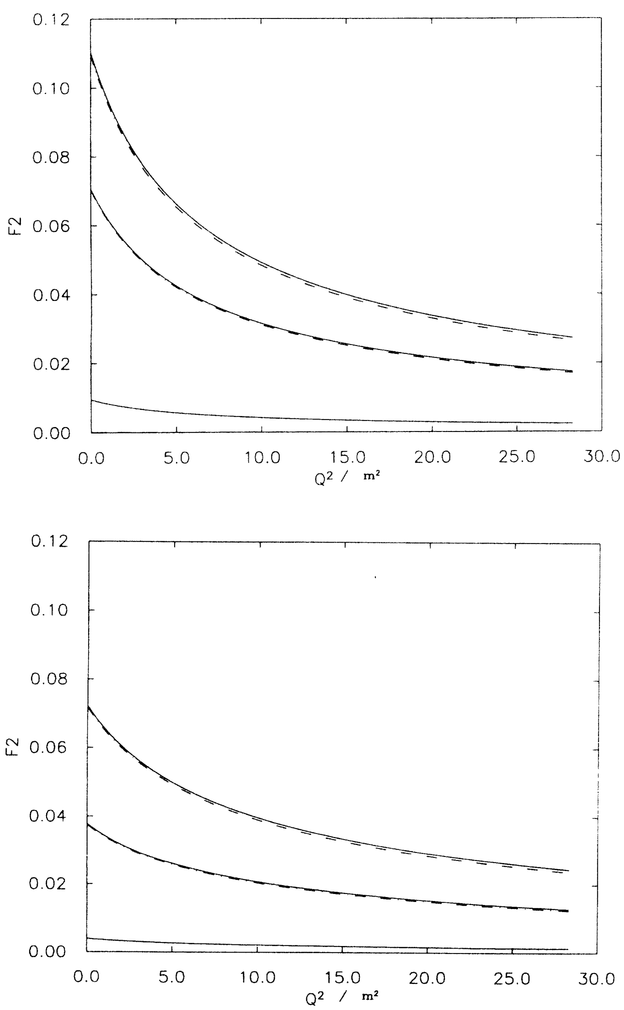

FIG. 4. Form factors $F_2(q^2)$ versus Q^2/m^2 for $\alpha = 0.1$, 1.0, 2.0. The lines correspond to cutoffs as in other figures, although the dashed line is not visible. (a) Boson mass $\mu = 0.1m$, cutoff $\Lambda = 10m$, $50m$, $100m$. (b) Boson mass $\mu = m$, cutoff $\Lambda = 10m$, $100m$.

where $\tilde{g} \equiv \tilde{g}(m)$ and $\alpha(m)$ is given in the previous section.

The integral $I(q)$ is divergent, but the divergent part of the integral $I(q)$ is canceled by the divergent part of the integral in $\alpha(m)$. This kind of cancellation is familiar from perturbation theory. The divergent logarithmic cutoff dependence vanishes for very large Λ. However, there is still some finite cutoff dependence left since the cutoff cannot be sent to infinity. The cutoff dependence comes from the fermion-boson sector of the theory and increases with the coupling strength for other parameters fixed. To the first order in Λ^{-2} and neglecting spurious sharp cutoff effects, the cutoff dependent terms in $F_1(q^2)$ are

$$\frac{\alpha}{6\pi}\left[\frac{\mu^2}{\Lambda^2} + \frac{1}{2}\frac{q^2}{\Lambda^2} + \frac{1}{4}\frac{m^2}{\Lambda^2}\right].$$

Figures 3(a) and 3(b) illustrate these formulas. Figure 3(a) shows F_1 for $\mu = 0.1m$ and different values of α and Λ. For $\mu = m$ the cutoff dependence increases in comparison with the former case and reaches a few percent. The two particle sector which builds the composite structure of the physical fermion, is strongly suppressed for heavy bosons.

Figure 4 illustrates a couple of typical results for F_2. $F_2(q^2)$ is practically cutoff independent since the integral $J(q)$ is finite in the limit of large Λ.

IV. CONCLUSION

The model Hamiltonian is regulated by an ultraviolet cutoff which limits relative transverse momenta, while the total momentum is unlimited. Therefore, the Lorentz covariance can be restored in the fermion-boson scattering amplitude by a special choice of the Hamiltonian counterterms, even for the fermion-boson center of mass momenta up to the order of the cutoff. The model is not asymptotically free, and triviality prevents us from taking the cutoff to infinity. Therefore, there might be important finite cutoff-dependent corrections to physical observables. This raises the question of whether it is at all useful to consider few-body renormalized models.

We have examined the finite cutoff dependence for the fermion-boson scattering cross sections and for the bound state form factors. We find a very small cutoff dependence of these quantities, on the order of a few percent or less, provided that the mass of the boson is small. The cutoff dependence is smaller for a smaller boson mass and is very weak for light bosons, which is a physically interesting regime. On the other hand, for heavy bosons the cutoff dependence is strong. The cutoff dependence originates from the momentum-dependent fermion spin effects. For light bosons fermions do not move much and for heavy bosons fermions move a lot with their momenta ranging up to the order of the boson mass. However, when the boson is heavy, the contribution of the two-particle Fock sector is strongly suppressed. Even though the triviality allows much stronger coupling constants in this case, the allowed increase in the coupling does not compensate for the decrease of the probability for emission of a heavy boson. In other words, the bound state approaches a pointlike fermion as the boson mass goes to infinity, and the form factor F_1 is consistently close to unity no matter what the cutoff is.

In conclusion, our study shows that it is reasonable to consider trivial renormalized light-front Hamiltonian models with finite cutoffs and without asymptotic freedom. This is encouraging from the point of view of relativistic nuclear physics, where asymptotic freedom does not appear.

We would also like to stress the need for extending the current study to the matrix elements of other components of the current operator j^μ for $\mu \neq +$ and to the case when $q^+ \neq 0$. The two major reasons are following. First, our understanding of the composite nature of elementary particles hinges on the quality of our models for relativistic bound states, and we need to learn how to construct models which unambiguously lead to fully covariant and conserved currents in quantum field theory in order to be able to address the basic issues of compositeness of elementary particles [16]. Light-front Hamiltonian approach to this problem is an interesting alternative to other approaches. Second, it is known that naive calculations of matrix elements for $q^+ \neq 0$ or $\mu \neq +$ lead to results which depend on the arbitrary choice of the z axis and the current is not conserved. Renormalization theory suggests that the bare current operator may need a modification and further counterterms may be necessary in a Hamiltonian which includes the coupling to external fields in the bound state dynamics. This kind of problems is known in the equal time dynamics and recently has become more widely recognized in light-front models [11,14,15,17–21,23].

Finally, we wish to mention that the model provides an opportunity to study the bound state deep inelastic structure functions using fully interacting scattering final states. For example, such studies have been carried out in equal-time dynamics using some model bound state equations for scalar particles but avoiding renormalization problems through the use of arbitrarily chosen form factors in the strong interaction vertices [22]. Analytic expressions for deep inelastic structure functions in the present renormalized Hamiltonian model including final state interactions are given in Ref. [24].

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APPENDIX A

Starting with $\alpha(M)$ given in Eq. (3) one finds the imaginary part

$$\mathrm{Im}\alpha(M) = -\frac{1}{16\pi}\left[x - \frac{1}{2}x^2\right]_{x_1}^{x_2} = -\frac{1}{16\pi}\frac{\sqrt{(M^2-m^2+\mu^2)^2-4M^2\mu^2}}{M^2}\left[1-\frac{(M^2-m^2+\mu^2)}{2M^2}\right] \tag{A1}$$

and the real part

$$\mathrm{Re}\alpha(M) = -\frac{1}{4(2\pi)^2}\left\{\int_0^1 dx(1-x)\ln\frac{\Lambda^2+m^2x+\mu^2(1-x)-M^2x(1-x)}{M^2} + x_1\left(1-\frac{1}{2}x_1\right)\ln\frac{(1-x_1)}{x_1} + x_2\left(1-\frac{1}{2}x_2\right)\ln\frac{(1-x_2)}{x_2} + \frac{3}{2} - \frac{1}{2}(x_1+x_2)\right\}, \tag{A2}$$

where x_1, x_2 are solutions to $M^2x(1-x) - m^2x - \mu^2(1-x) = 0$.

Similar expressions can be found for $\beta(M)$. The imaginary part is

$$\mathrm{Im}\beta(M) = -\frac{1}{16\pi}[x_2 - x_1] = -\frac{1}{16\pi}\frac{\sqrt{(M^2-m^2+\mu^2)^2-4M^2\mu^2}}{M^2} \tag{A3}$$

and the real part

$$\mathrm{Re}\beta(M) = -\frac{1}{4(2\pi)^2}\left\{\int_0^1 dx\ln\frac{\Lambda^2+m^2x+\mu^2(1-x)-M^2x(1-x)}{m^2} + x_1\ln\frac{(1-x_1)}{x_1} + x_2\ln\frac{(1-x_2)}{x_2} + 2\right\}. \tag{A4}$$

Now we can calculate $\tilde{m}$ and $\tilde{g}^2$ from Eqs. (6) and (7):

$$\mathrm{Re}\tilde{g}^2(M) = \frac{\tilde{g}^2(m)(1-\tilde{g}^2(m)[\mathrm{Re}\alpha(M)-\alpha(m)])}{(1-\tilde{g}^2(m)[\mathrm{Re}\alpha(M)-\alpha(m)])^2 + [\tilde{g}^2(m)\mathrm{Im}\alpha(M)]^2}, \tag{A5}$$

$$\mathrm{Im}\tilde{g}^2(M) = \frac{\tilde{g}^4(m)\mathrm{Im}\alpha(M)}{(1-\tilde{g}^2(m)[\mathrm{Re}\alpha(M)-\alpha(m)])^2+[\tilde{g}^2(m)\mathrm{Im}\alpha(M)]^2}, \quad (A6)$$

$$\mathrm{Re}\tilde{m}(M) = m\left\{1+\mathrm{Re}\tilde{g}^2(M)[\mathrm{Re}\alpha(M)-\alpha(m)+\mathrm{Re}\beta(M)-\beta(m)] - \mathrm{Im}\tilde{g}^2(M)[\mathrm{Im}\alpha(M)+\mathrm{Im}\beta(M)]\right\}, \quad (A7)$$

$$\mathrm{Im}\tilde{m}(M) = m\left\{\mathrm{Re}\tilde{g}^2(M)[\mathrm{Im}\alpha(M)+\mathrm{Im}\beta(M)] + \mathrm{Im}\tilde{g}^2(M)[\mathrm{Re}\alpha(M)-\alpha(m)+\mathrm{Re}\beta(M)-\beta(m)]\right\}. \quad (A8)$$

APPENDIX B

Functions $I(q)$ and $J(q)$ from Eqs. (20) and (21) are

$$I(q) = \frac{1}{(2\pi)^2}\int_0^1 dx\frac{1-x}{2}\int_0^{\sqrt{\Lambda^2-[\frac{1}{2}(1-x)q]^2}} du\left\{\left[(1+x)^2m^2-\frac{1}{4}(1-x)^2q^2+u^2\right]P_1(u,x,m,\mu,q)\right\}$$
$$-\frac{1}{(2\pi)^2}\int_0^1 dx\frac{1-x}{2}\int_{\Lambda-\frac{1}{2}(1-x)q}^{\sqrt{\Lambda^2-[\frac{1}{2}(1-x)q]^2}} du\left\{\left[(1+x)^2m^2-\frac{1}{4}(1-x)^2q^2+u^2\right]P_1(u,x,m,\mu,q)\right.$$
$$\left.\times\frac{2}{\pi}\arctan[P_2(u,x,m,\mu,q)]\right\}$$

and

$$J(q) = \frac{1}{(2\pi)^2}\left\{\int_0^1 dx\frac{(1-x)^2}{2}(1+x)m^2\int_0^{\sqrt{\Lambda^2-[\frac{1}{2}(1-x)q]^2}} duP_1(u,x,m,\mu,q)\right.$$
$$-\int_0^1 dx\frac{(1-x)^2}{2}(1+x)m^2$$
$$\left.\times\int_{\Lambda-\frac{1}{2}(1-x)Q}^{\sqrt{\Lambda^2-[\frac{1}{2}(1-x)q]^2}} duP_1(u,x,m,\mu,q)\frac{2}{\pi}\arctan\left[P_2(u,x,m,\mu,q)\right]\right\}.$$

P_1 and P_2 are

$$P_1(u,x,m,\mu,q) = \frac{u}{\left[(1-x)^2m^2+x\mu^2+\frac{1}{4}(1-x)^2q^2+u^2\right]}$$
$$\times\frac{1}{\left\{(1-x)^2m^2+x\mu^2+[u-\frac{1}{2}(1-x)q]^2\right\}^{\frac{1}{2}}\left[(1-x)^2m^2+x\mu^2+(u+\frac{1}{2}(1-x)q)^2\right]^{\frac{1}{2}}}$$

and

$$P_2(u,x,m,\mu,q) = \frac{\sqrt{\left[(1-x)^2m^2+x\mu^2+(u-\frac{1}{2}(1-x)q)^2\right]\left[(1-x)^2m^2+x\mu^2+(u+\frac{1}{2}(1-x)q)^2\right]}}{(\Lambda^2-(\frac{1}{2}(1-x)q)^2-u^2)\left[(1-x)^2m^2+x\mu^2+\frac{1}{4}(1-x)^2q^2+u^2\right]}.$$

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