Instabilities of the Cauchy horizon in Kerr black holes

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A previously developed Cauchy horizon stability conjecture is used to investigate the stability of the Cauchy horizon in the Kerr geometry when various fields are introduced. In particular, the effects of an electromagnetic field, infalling null dust, and combined infalling and outgoing null dust are studied. Stability predictions are made and in one case verified. The nature of any resulting singularities is predicted.

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I. INTRODUCTION

The Kerr geometry of a rotating black hole has two horizons: an outer, event horizon and an inner, Cauchy horizon. The Cauchy horizon (CH) is the boundary of points within the Cauchy development of the world outside the event horizon. The stability of this CH is interesting because it is a gateway to regions containing singularities and alternative external worlds in the complete analytic extension of the geometry. Observers falling through the event horizon of a Kerr black hole would subsequently pass through the CH as well, entering a region in which they could see the singularity, in violation of the strong cosmic censorship hypothesis. Instabilities of the CH in realistic circumstances might close the gate, protecting strong cosmic censorship by introducing impenetrable singularities.

In a number of papers [1–8], we have developed stability conjectures for the investigation of mild singularities and CH's in solutions of Einstein's equations. We look at the behavior of test fields in the vicinity of the singularity or CH, and based upon this behavior, we predict what should become of the singularity or CH if the fields are allowed to influence the geometry through back reaction calculations using Einstein's equations. In a few cases, these back reaction calculations have actually been carried out [3,5,8]; in each of these cases, the results agree with the predictions of the conjectures.

In this paper we use the CH conjecture [8] to investigate the stability of the CH in the Kerr geometry when various fields are added. We are able to test our conjecture for one of these fields by comparing with the results of a back reaction calculation.

In Sec. II we define singularity types and review the strong cosmic censorship hypothesis of Penrose. We then state the stability conjectures and tests of the conjectures. In Sec. III we begin by reviewing the properties of Kerr spacetime and review a previous investigation of the CH stability in this geometry. Then in Sec. III A we derive a prediction in the case of the lowest electromagnetic-field mode and compare with the Kerr-Newman spacetime. In Sec. III B we derive a prediction in the case of infalling null dust. In Sec. III C we derive a prediction in the case of both infalling and outgoing null dust. In Sec. IV we summarize our conclusions.

II. SINGULARITY CLASSIFICATION AND STABILITY CONJECTURES

We use a singularity classification scheme based on one devised by Ellis and Schmidt [9]. They classified singularities in maximal spacetimes into three basic types: quasiregular, nonscalar curvature, and scalar curvature. The mildest singularity is quasiregular and the strongest is scalar curvature. At a scalar curvature singularity, physical quantities such as energy density and tidal forces diverge in the frames of all observers who approach the singularity. At a nonscalar curvature singularity, there exist curves through each point arbitrarily close to the singularity such that observers moving on these curves experience perfectly regular tidal forces [9,10]. For a quasiregular singularity, no observers see physical quantities diverge, even though their world lines end at the singularity in a finite proper time.

Our version of the Ellis-Schmidt classification scheme can be expressed mathematically. We define singular points simply as the end points of incomplete geodesics in maximal spacetimes; Ellis and Schmidt use instead a b-boundary construction to define the singular points. In our scheme a singular point q is a quasiregular singularity if all components of the Riemann tensor R_{abcd} evaluated in an orthonormal frame parallel propagated along an incomplete geodesic ending at q are C^0 (or C^{0-}). In other words, the Riemann tensor components tend to finite limits (or are bounded). On the other hand, a singular point q is a curvature singularity if some components are not bounded in this way. If all scalars in g_{ab}, the antisym-

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metric tensor η_{abcd}, and R_{abcd} nevertheless tend to a finite limit (or are bounded), the singularity is nonscalar, but if any scalar is unbounded, the point q is a scalar curvature singularity.

We have previously used stability conjectures [1–7] to test the stability of quasiregular singularities, nonscalar curvature singularities, and Cauchy horizons. For singularities our conjecture states the following.

Conjecture 1. If a test-field stress-energy tensor evaluated in a parallel-propagated orthonormal (PPON) frame mimics the behavior of the Riemann tensor components which indicate a particular type of singularity, then a complete nonlinear back reaction calculation would show that this type of singularity occurs.

For Cauchy horizons, the conjecture is slightly modified [8] to state the following.

Conjecture 2. For all maximally extended spacetimes with CH's, the back reaction due to a field (whose test-field stress-energy tensor is $T_{\mu\nu}$) will affect the horizon in the following manner: (1) If both T^{μ}_{μ} and $T_{\mu\nu}T^{\mu\nu}$ are finite and if the stress-energy tensor $T_{(\alpha\beta)}$ in all PPON frames is finite, then the CH will remain nonsingular; (2) if both T^{μ}_{μ} and $T_{\mu\nu}T^{\mu\nu}$ are finite but $T_{(\alpha\beta)}$ diverges in some PPON frame, then a nonscalar curvature singularity will be formed at the CH; (3) if either T^{μ}_{μ} or $T_{\mu\nu}T^{\mu\nu}$ diverges, then a scalar curvature singularity will be formed at the CH.

Conjecture 1 has been tested in several cases, as reviewed in a previous paper [8]. Conjecture 2 has been tested so far only in the Reissner-Nordström spacetime [8]. The conjecture predicts that the addition of infalling null dust with a power-law tail produces a nonscalar curvature singularity at the CH in the Reissner-Nordström spacetime. The prediction was verified using a Reissner-Nordström-Vaidya spacetime studied by Hiscock [11]. The conjecture also predicts that a combination of infalling and outgoing null dust produces a scalar curvature singularity at the CH. This prediction was verified using the mass inflation results of Poisson and Israel [12]. Finally, the conjecture predicts that the addition of infalling scalar or electromagnetic waves produces a scalar curvature singularity at the CH; there are no exact solutions with which to verify the conjecture in these cases.

Conjecture 2 bears on the question of cosmic censorship. There are two versions of the cosmic censorship conjecture [13]. The strong version states that there are no nontrivial CH's, that is, that the entire spacetime is globally hyperbolic. One can reduce the question of strong cosmic censorship down to a question: Does there exist at least one stable CH? Or does there exist at least one observer who sees a singularity but does not run into it? If so, strong cosmic censorship is violated. The weak version states that all "physically realistic" singularities are hidden by black hole event horizons. In other words, all breakdowns of global hyperbolicity occur inside black holes.

We are concerned here with strong cosmic censorship. Three exact solutions are counterexamples: Taub-Newman-Unti-Tamburino (Taub-NUT), Reissner-Nordström, and Kerr. The maximal extensions of all these spacetimes are extendible through their maximal Cauchy developments. However, many authors have shown that perturbations of these spacetimes destroy the extendibility of the CH's by turning them into singularities [13].

Here we are interested in perturbing the CH in Kerr and in predicting not only whether a singularity forms but predicting what kind of singularity forms. In other words, we are not only interested in answering the question of strong cosmic censorship in this case, but also in answering the question of the nature of the singularity if one forms.

III. STABILITY TESTS OF THE KERR CAUCHY HORIZON

The extended Kerr geometry of a rotating uncharged black hole is described in Boyer-Lindquist coordinates [14] by the metric

$$\begin{aligned} ds^2 = &-\left[1-\frac{2mr}{\Delta}\right]dt^2-\frac{2a\sin^2\theta}{\rho^2}(2mr)dt\,d\phi \\ &+\frac{\rho^2}{\Delta}dr^2+\rho^2 d\theta^2 \\ &+\left[r^2+a^2+\frac{a^2\sin^2\theta}{\rho^2}(2mr)\right]\sin^2\theta\,d\phi^2\,, \end{aligned} \tag{1}$$

where $\Delta=r^2+a^2-2mr$ and $\rho^2=r^2+a^2\cos^2\theta$. There is an outer event horizon at $r=r_+$ and an inner (Cauchy) horizon at $r=r_-$, where $r_\pm$ are the zeros of Δ. A portion of the extended Kerr geometry for $a^2<m^2$ is shown in Fig. 1. Observers falling into the black hole through r_+ see light from the entire future history of the region outside the black hole, as they approach r_-.

Evidence has accumulated [13] showing that this CH, like the Reissner-Nordström CH, is unstable. Fields falling in the vicinity of r_- from outside the black hole or

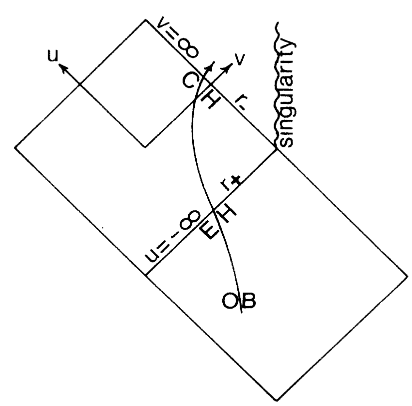

FIG. 1. Conformal diagram of a portion of the Kerr spacetime. An observer (OB) is shown falling through the event horizon (EH) at $r=r_+$ into the interior region and then through the Cauchy horizon (CH) at $r=r_-$. Just before reaching the CH, the observer can receive light from the entire future of the exterior region; after passing the CH, the observer can view the singularity.

scattering from within the black hole are blueshifted; the magnitude and/or energy density of these fields diverges as measured by observers falling toward the CH. McNamara [15] analytically evolved gravitational and electromagnetic perturbations in Kerr space, determining the effect of the perturbations on the CH to first order. He argued that the entire CH would become singular. While this result does not prove that the CH is unstable, since it does not include a complete back reaction calculation of the field effects on the geometry, it does point to a likelihood of instability. Many other researchers use the instability of the Reissner-Nordström CH to indicate the instability of the Kerr CH because of the similar geometries (see the review in [13]).

We now explore the Kerr CH using our conjecture. Radial geodesics with angular momentum $L=aE$ in the equatorial plane $\theta=\pi/2$ obey

$$\begin{aligned} \dot{t}&=\frac{r^2+a^2}{\Delta}E\ ,\\ \dot{r}&=-\left[E^2-\frac{\Delta}{r^2}\right]^{1/2}\ ,\\ \dot{\theta}&=0\ ,\\ \dot{\phi}&=\frac{aE}{\Delta}\ , \end{aligned} \tag{2}$$

where E is the energy. We will use only PPON frame vectors $E^{\mu}_{(a)}$ for infalling timelike $L=aE$ geodesics in the equatorial plane; general vectors are more complicated. These vectors, satisfying $E^{\mu}_{(0);\nu}E^{\nu}_{(a)}=0$ and $E_{(a)\mu}E^{\mu}_{(b)}=\delta_{(ab)}$, are

$$\begin{aligned} E^{\mu}_{(0)}&=\begin{bmatrix}(r^2+a^2)E/\Delta\\ -\sqrt{E^2-\Delta/r^2}\\ 0\\ aE/\Delta\end{bmatrix}\ ,\\ E^{\mu}_{(1)}&=\begin{bmatrix}\frac{a}{r}-\frac{r^2+a^2}{\Delta}\sqrt{E^2-\Delta/r^2}\\ E\\ 0\\ \frac{1}{r}-\frac{a}{\Delta}\sqrt{E^2-\Delta/r^2}\end{bmatrix}\ ,\\ E^{\mu}_{(2)}&=\begin{bmatrix}0\\ 0\\ 1/r\\ 0\end{bmatrix}\ ,\\ E^{\mu}_{(3)}&=\begin{bmatrix}-\frac{a}{r}-\frac{r^2+a^2}{\Delta}\sqrt{E^2-\Delta/r^2}\\ E\\ 0\\ -\frac{1}{r}-\frac{2}{\Delta}\sqrt{E^2-\Delta/r^2}\end{bmatrix}\ , \end{aligned} \tag{3}$$

in the order t,r,θ,ϕ.

A. Electromagnetic field

The only nonzero Maxwell scalar in the Newman-Penrose formalism for a monopole electric and dipole magnetic field in the Kerr metric is [16]

$$\phi=-\frac{Q}{4}[r-ia\cos\theta]^{-2}\ . \tag{4}$$

The corresponding electromagnetic-field tensor in a coordinate frame is

$$F_{\mu\nu}=2(\phi+\bar{\phi})[n_\mu l_\nu-l_\mu n_\nu]+2(\phi-\bar{\phi})[m_\mu\bar{m}_\nu-\bar{m}_\mu m_\nu]\ , \tag{5}$$

in terms of the null tetrad $l_\mu,n_\mu,m_\mu,\bar{m}_\mu$ for the Kerr metric [16,17]. One may then calculate the stress-energy tensor $T_{\mu\nu}$ in a coordinate frame. Its nonzero components are

$$\begin{aligned} T_{00}&=Q(\Delta+a^2\sin^2\theta)/2\rho^6\ ,\\ T_{03}&=-Qa\sin^2\theta(\Delta+mr)/\rho^6\ ,\\ T_{11}&=-Q/2\rho^2\Delta\ ,\\ T_{22}&=Q/2\rho^2\ ,\\ T_{33}&=Q\sin^2\theta[(r^2+a^2)^2+\Delta a^2\sin^2\theta]/2\rho^6\ , \end{aligned} \tag{6}$$

for which the scalar $T^{\mu}{}_{\mu}=0$ and the scalar $T^{\mu\nu}T_{\mu\nu}=16Q/\rho^8$. Since both are finite, a scalar curvature singularity should not form at the CH, according to the conjecture. What about a nonscalar curvature singularity?

To predict the formation of a nonscalar curvature singularity, we need to calculate $T_{(ab)}$ in the parallel-propagated orthonormal frame of Eqs. (3). The nonzero components are

$$T_{(00)}=T_{(22)}=T_{(13)}/2=T_{(31)}/2=-8/r^4\ , \tag{7}$$

which are nonsingular at the CH. Our conjecture predicts that the CH will remain nonsingular if the $T_{(ab)}$ are nonsingular for all PPON frames approaching the CH. Although we have calculated the $T_{(ab)}$ only for $L=aE$ equatorial-plane geodesics, we expect that the CH is nonsingular in general.

There is a well-known back reaction solution with which to compare the Kerr-Newman spacetime of a rotating black hole with electric charge Q. Its Coulomb electric field and dipole magnetic field lead to the same $F_{\mu\nu}$ and $T_{\mu\nu}$ as those of the test field in the Kerr geometry. The Kerr-Newman spacetime has a CH at $r_-=m-\sqrt{m^2-a^2-Q^2}$, which corresponds to the Kerr CH when the charge $Q\to0$. The only singularity is the scalar curvature ring singularity at $r=0$, $\theta=\pi/2$ as in the Kerr case; in particular, the Kerr-Newman CH is nonsingular. As one would expect, our conjecture is verified in this simple case.

B. Infalling null dust

Now add to a background Kerr spacetime a test field of infalling null dust. The stress-energy tensor has the form

$$T^{\mu\nu}=\rho u^{\mu}u^{\nu} , \tag{8}$$

where $\rho(t,r,\theta)$ is a scalar density and

$$u^{\mu}=\left[\frac{r^2+a^2}{\Delta},-1,0,a/\Delta\right] \tag{9}$$

is the infalling principal null congruence [18]. From the continuity equation $T^{\mu\nu}{}_{;\nu}=0$, we find ρ has the form

$$\rho(t,r,\theta)=\frac{A(\theta)F(v)}{r^2+a^2\cos^2\theta} , \tag{10}$$

where $A(\theta)$ is an arbitrary function of θ and $F(v)$ is an arbitrary function of $v=t+r_*$, with tortoise coordinate

$$r_*=r+\left[\frac{r_+^2+a^2}{r_+-r_-}\right]\ln|r-r_+|-\left[\frac{r_-^2+a^2}{r_+-r_-}\right]\ln|r-r_-| . \tag{11}$$

We can set initial data for ρ on a spacelike hypersurface and will set $A(\theta)=1$.

The scalars $T^{\mu}{}_{\mu}$ and $T^{\mu\nu}T_{\mu\nu}$ vanish everywhere, and so according to our conjecture such null dust should not produce a scalar curvature singularity (SCS) at the CH. To see whether null dust should produce a nonscalar curvature singularity (NSCS) instead, we must compute $T_{(ab)}$ in a PPON frame and see how it behaves as the frame approaches the CH. The PPON frame vector $E^{\mu}_{(0)}$ for equatorial-plane timelike geodesics, with angular momentum and energy related by $L=aE$, is

$$E^{\mu}_{(0)}=\left[\frac{E(r^2+a^2)}{\Delta},-\sqrt{E^2-\Delta/r^2},0,aE/\Delta\right] . \tag{12}$$

Therefore the energy density of infalling radiation measured in the PPON frame is

$$T_{(00)}=\frac{(|E|+\sqrt{E^2-\Delta/r^2})^2F(v)}{\Delta^2} . \tag{13}$$

As $v\to\infty$, the behavior of $T_{(00)}$ is governed by that of $F(v)/\Delta^2$. If this ratio diverges, a NSCS will be formed at the CH.

Define $\varepsilon=r-r_-$; then, $\Delta=-(r_+-r_-)\varepsilon$ to first order in ε. Along an infalling timelike geodesic,

$$\frac{dv}{d\varepsilon}=\frac{dv}{dr}=-(\alpha\varepsilon)^{-1} \tag{14}$$

to leading order, where $\alpha=(r_+-r_-)/2(r_-^2+a^2)$. Therefore $\varepsilon=\varepsilon_0e^{-\alpha v}$ near the CH, where ε_0 is a constant, and so

$$F(v)/\Delta^2\sim F(v)e^{2\alpha v} \tag{15}$$

for large v, which diverges unless $F(v)$ falls off fast enough. That is, according to the conjecture, a NSCS should be formed at the CH by infalling null dust, unless $F(v)$ falls off at least as fast as $F(v)\sim e^{-2\alpha v}$, in which case the CH remains nonsingular.

Sources of infalling radiation outside a Kerr black hole typically produce a power-law tail $F(v)\sim v^{-n}$ inside r_+, due to scattering [19]. If so, $F(v)$ does not fall off fast enough to prevent the formation of a NSCS at the CH.

C. Both infalling and outgoing null dust

Now add "outgoing" null dust to the infalling null dust described in the previous subsection, in the region $r_-<r<r_+$, as shown in Fig. 2. This outgoing radiation moves from lower left to upper right in the figure, approaching the CH at r_-. One would expect such outgoing radiation in realistic circumstances, originating at the surface of the collapsing star. We assume the infalling and outgoing beams do not interact, and so each is separately conserved. We will show that the addition of outgoing radiation produces a SCS at the CH.

The infalling radiation has stress-energy $T_I^{\mu\nu}=\rho_Iu_I^{\mu}u_I^{\nu}$, where $\rho_I(v,r,\theta)$ and u_I^{μ} are given by Eqs. (10) and (9), respectively. The outgoing radiation has the stress-energy

$$T_0^{\mu\nu}=\rho_0(u,r,\theta)u_0^{\mu}u_0^{\nu} , \tag{16}$$

where

$$\rho_0(u,r,\theta)=\frac{G(u)}{r^2+a^2\cos^2\theta} , \tag{17}$$

with $G(u)$ an arbitrary function of $u=r_*-t$. The outgoing null vectors are

$$u_0^{\mu}=\left[-\frac{r^2+a^2}{\Delta},-1,0,-\frac{a}{\Delta}\right] , \tag{18}$$

the appropriate solution of $u_0^{\mu}u_{0\mu}=0$ and $u^{\mu}_{0;\nu}u_0^{\nu}=0$. The outgoing radiation also satisfies the continuity equation $T^{\mu\nu}_{0;\nu}=0$.

The total null dust stress-energy for both infalling and outgoing null dust is

$$T^{\mu\nu}=\rho_I(v,r,\theta)u_I^{\mu}u_I^{\nu}+\rho_0(u,r,\theta)u_0^{\mu}u_0^{\nu} , \tag{19}$$

for which $T^{\mu}{}_{\mu}=0$. Because of cross-product terms, however,

$$T^{\mu\nu}T_{\mu\nu}=\frac{8F(v)G(u)}{\Delta^2} , \tag{20}$$

which diverges as $r\to r_-$ if (i) $G(u)$ is nonzero as $v\to\infty$

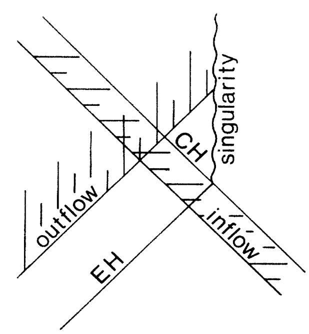

FIG. 2. Inflowing and outflowing radiation in the Ker geometry. Inflowing radiation moves from lower right to upper left, falling through the event horizon into the black hole. Outflowing radiation, originating at the surface of a collapsing star or from backscattering of inflowing radiation, moves from lower left to upper right, passing through the Cauchy horizon.

and (ii) $F(v)/\Delta^2$ diverges. Therefore, if there is at least some outgoing null dust, no matter how small, and the infalling dust has a power-law tail or otherwise fails to fall off too quickly as $v \to \infty$, a SCS is formed at the CH according to our conjecture. The addition of even a tiny amount of outgoing radiation is sufficient to convert the NSCS of the preceding subsection into a SCS. This is related, according to our method, to the nonlinearity of $T^{\mu\nu}T_{\mu\nu}$, which combines properties of the two noninteracting beams.

In the case of the Reissner-Nordström electrically charged, nonrotating black holes, Poisson and Israel have shown that an exact solution of Einstein's equations with both infalling and outgoing null dust has a SCS at the CH [12]. This result is consistent with our conjecture for the stability of CH's [8]. In this paper we have shown that according to the conjecture the same result should pertain to the CH of rotating, uncharged black holes. We expect that solutions containing both infalling and outgoing null dust should display a scalar curvature singularity at the inner horizon.

IV. CONCLUSION AND DISCUSSION

We have used a stability conjecture for Cauchy horizons introduced in a previous paper to predict the fate of the Cauchy horizon in a Kerr black hole under the influence of an added electric charge, infalling null dust, or combination of infalling and outgoing null dust. With the lowest-mode electromagnetic field caused by the electric charge, the Cauchy horizon should remain nonsingular. This result is in agreement with the back reaction calculation embodied in the Kerr-Newman solution of a charged, rotating black hole. With infalling null dust, a nonscalar curvature singularity should be formed, and with the combination of infalling and outgoing null dust, a scalar curvature singularity should be formed.

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