COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review.** *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Collective modes in dense neutrino systems"

Alan S. Raskin and Peter G. Blunden
Department of Physics, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2
(Received 1 June 1993)

A previous study of dense neutrino matter indicated that the behavior of such a system becomes nonperturbative at relatively low density under certain kinematic conditions. A detailed examination of the analysis indicates errors in the formulation and in some of the expressions used. We present a reformulation of the problem, and find no such physical effect.

PACS number(s): 05.30.Fk, 05.70.Fh, 11.30.Qc

In a recent paper [1], Chakrabarti and Kögerler investigated the behavior of a dense gas of neutrinos interacting via Z^0 exchange. Their analysis was based on a similar calculation by Chakrabarti [2] of the boson self-energy (vacuum polarization) tensor $\Pi_{\mu\nu}$ for a dense system of massless up and down quarks, whose interaction was mediated instead by the $W^{\pm}$ bosons. They found that under certain kinematic conditions there exists a phase characterized by a spatially uniform, lepton-number-violating particle-hole condensate, and conjectured that this phase would appear at relatively low densities corresponding to a neutrino chemical potential $\mu < M_Z$.

If correct, such an effect would have profound consequences for phenomena of astrophysical and cosmological interest. However, we have concluded that there are a number of errors in Refs. [1,2] in the expressions used for the polarization tensor, and in certain assumptions about its structure. These errors seem to invalidate the major conclusions of both papers.

To lowest order in the $\nu\nu Z^0$ interaction, the Z^0 polarization tensor $\Pi_{\mu\nu}(q)$ is given by

$$-i\Pi_{\mu\nu}(q) = \int \frac{d^4k}{(2\pi)^4}\, \mathrm{Tr}[J_\mu G(k+q) J_\nu G(k)], \tag{1}$$

with the current $J_\mu = \hat{g}\gamma_\mu(1-\gamma_5)/2$, and $\hat{g} = g/2\cos\theta_W$ [3]. The propagator for massless neutrinos ($E_k = |\mathbf{k}|$) has the form

$$\begin{aligned} G(k) &= \gamma\cdot k\left[\frac{1}{k^2+i\epsilon} + \frac{i\pi}{E_k}\delta(k_0-E_k)\theta(\mu-E_k)\right] \\ &\equiv \gamma\cdot k\; \tilde{G}(k). \end{aligned} \tag{2}$$

It includes both the density-independent Feynman term and a density-dependent piece corresponding to the propagation of neutrinos below the Fermi surface, characterized by the chemical potential μ [4]. We assume that there is no significant antineutrino density.

On taking the trace in Eq. (1), the integrand becomes

$$2\hat{g}^2[2k_\mu k_\nu + k_\mu q_\nu + q_\mu k_\nu - g_{\mu\nu}(k\cdot q + k^2) + i\varepsilon_{\mu\nu\alpha\beta}k^\alpha q^\beta]\tilde{G}(k+q)\tilde{G}(k). \tag{3}$$

in the conventional metric of Ref. [5]. The polarization tensor satisfies the constraint $q^\mu\Pi_{\mu\nu} = \Pi_{\mu\nu}q^\nu = 0$, due to the fact that for massless neutrinos, both vector and axial vector currents are conserved. In a frame with $(q^\mu) = (q_0, |\mathbf{q}|, 0, 0)$, the polarization tensor separates into longitudinal and transverse components, and takes the form

$$\mathbf{\Pi}(q) = \begin{pmatrix} \Pi_{00} & \Pi_{01} & 0 & 0 \\ \Pi_{10} & \Pi_{11} & 0 & 0 \\ 0 & 0 & \Pi_{22} & \Pi_{23} \\ 0 & 0 & \Pi_{32} & \Pi_{33} \end{pmatrix}. \tag{4}$$

For undamped modes, we are interested in the kinematical regime $q_0, |\mathbf{q}| < \mu$, and $q_0/|\mathbf{q}| \equiv x > 1$, where the real components of the polarization tensor are

$$\Pi_{00}(q) = \frac{\hat{g}^2\mu^2}{4\pi^2}\left(x\ln\left|\frac{x+1}{x-1}\right| - 2\right) \equiv \frac{\hat{g}^2\mu^2}{4\pi^2}f_1(x), \tag{5a}$$

$$\Pi_{11}(q) = -x\Pi_{01}(q) = -x\Pi_{10}(q) = x^2\Pi_{00}(q), \tag{5b}$$

$$\begin{aligned} \Pi_{22}(q) &= \Pi_{33}(q) \\ &= \frac{\hat{g}^2\mu^2}{4\pi^2}x\left(x - \frac{1}{2}(x^2-1)\ln\left|\frac{x+1}{x-1}\right|\right) \\ &\equiv \frac{\hat{g}^2\mu^2}{4\pi^2}x f_2(x), \end{aligned} \tag{5c}$$

$$\Pi_{23}(q) = -\Pi_{32}(q) = -i\frac{\hat{g}^2\mu^2}{4\pi^2}f_2(x). \tag{5d}$$

Note that $f_2(x)$ remains finite as $x \to 1$ and $x \to \infty$, whereas $f_1(x)$ converges in the second limit but in fact diverges in the first one.

In Refs. [1,2], it was assumed that the off-diagonal parts of $\Pi_{\mu\nu}$ vanish, and that $\Pi_{11} = \Pi_{22} = \Pi_{33}$. This renders Chakrabarti's procedure [2] for calculating the Z^0

propagator in the "ring" or "random phase" approximation [6] invalid. Furthermore, our function $f_1(x)$ defined in Eq. (5a) has a power series expansion in $1/x$ which goes as $2/3x^2 + \mathcal{O}(1/x^4)$, so that Π_{11} approaches a constant for large values of x. By contrast, the expressions equivalent to Eq. (5a) given in Refs. [1,2] have a term $-\frac{4}{3}$ rather than -2, leading to an unphysical x^2 divergence.

Thus, the additional collective state which Chakrabarti and Kögerler [1] deduce from their calculated divergence of Π_{11} at large x (and the resulting pole in the two-point function $G_{\mu\nu}$ of Ref. [1]), as well as the corresponding new scale which is dynamically generated at a lower energy (i.e., for $\mu < M_Z$), no longer appear in the theory. However, the logarithmic divergences in Π_{00} and Π_{11} as $x \to 1$ persist when D_{00} and D_{11} are calculated in the ring approximation.

Summing the contributions of all of the ring diagrams results in the "dressed" Z^0 propagator $D^R_{\mu\nu}(q)$ (equivalent to the function $G_{\mu\nu}$), satisfying the usual Schwinger-Dyson equation

$$D^R_{\mu\nu}(q) = D^0_{\mu\nu}(q) + D^0_{\mu\alpha}(q)\ \Pi^{\alpha\beta}(q)\ D^R_{\beta\nu}(q), \tag{6}$$

where

$$D^0_{\mu\nu}(q) = -\frac{g_{\mu\nu}}{q^2 - M_Z^2 + i\epsilon} \approx \frac{g_{\mu\nu}}{M_Z^2}$$

is the bare Z^0 propagator. Any gauge terms in the bare propagator will not give any additional contribution to $D^R_{\mu\nu}$ because of current conservation $(q_\mu \Pi^{\mu\nu} = 0)$.

Equation (6) reduces to a simple matrix equation and can be solved quite readily. Introducing the dimensionless constant $\beta = (\hat{g}\mu/2\pi M_Z)^2$, we find

$$D^R_{00}(q) = \frac{1 + \beta x^2 f_1(x)}{M_Z^2 \epsilon_L(x)}, \tag{7a}$$

$$D^R_{11}(q) = -\frac{1 - \beta f_1(x)}{M_Z^2 \epsilon_L(x)}, \tag{7b}$$

$$D^R_{01}(q) = D^R_{10}(q) = -\frac{\beta x f_1(x)}{M_Z^2 \epsilon_L(x)}, \tag{7c}$$

$$D^R_{22}(q) = D^R_{33}(q) = -\frac{1 + \beta x f_2(x)}{M_Z^2 \epsilon_T(x)}, \tag{7d}$$

$$D^R_{23}(q) = -D^R_{32}(q) = -i\frac{\beta f_2(x)}{M_Z^2 \epsilon_T(x)}. \tag{7e}$$

All of the other components of the propagator are zero in the kinematic limit under consideration. We have defined the generalized dielectric functions for the longitudinal and transverse modes as

$$\epsilon_L(x) \equiv 1 + \beta(x^2 - 1) f_1(x), \tag{8a}$$

$$\epsilon_T(x) \equiv \left[1 + \beta(x+1) f_2(x)\right]\left[1 + \beta(x-1) f_2(x)\right], \tag{8b}$$

respectively. Any poles in the propagator will appear as zeros in the dielectric functions. It is straightforward to verify that both $\epsilon_L(x)$ and $\epsilon_T(x)$ are finite and nonzero over the entire range $x \geq 1$. For stability, the dielectric functions must be nonzero for imaginary x (i.e., $q_0^2 < 0$ for real $\mathbf{q}^2$). This criterion is indeed satisfied, indicating that the uniform ground state is stable against collective modes of this type.

Hence the transverse components of $D^R_{\mu\nu}$ (and $\Pi_{\mu\nu}$ itself) are finite in the kinematic regime q^0, $|\mathbf{q}| \to 0$, while the longitudinal ones have the usual collective mode similar to zero sound [7], as indicated by the breakdown in the perturbation theory as $x \to 1$. No new phase transition or any other such interesting physical effect seems to manifest itself in dense neutrino matter at the lower densities being considered. Unfortunately, it seems that we must look elsewhere for new and interesting phenomena in the neutrino sector of astrophysics and cosmology.

[1] J. Chakrabarti and R. Kögerler, Phys. Rev. D **45**, 2086 (1992).

[2] J. Chakrabarti, Phys. Rev. D **35**, 2622 (1987).

[3] T.-P. Cheng and L.-F. Li, *Gauge Theory of Elementary Particle Physics* (Oxford University Press, New York, 1984).

[4] R. L. Bowers and R. L. Zimmerman, Phys. Rev. D **7**, 296 (1973).

[5] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[6] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[7] S. A. Chin, Ann. Phys. (N.Y.) **108**, 301 (1977).