Operator ordering problem of the nonrelativistic Chern-Simons theory

Mu-In Park and Young-Jai Park*

Department of Physics and Basic Science Research Institute, Sogang University, C.P.O.Box 1142, Seoul 100-611, Korea

(Received 31 May 1994)

The operator ordering problem due to the quantization or regularization ambiguity in the Chern-Simons theory exists. However, we show that this can be avoided if we require Galilei covariance of the nonrelativistic Abelian Chern-Simons theory even at the quantum level for the extended sources. The covariance can be recovered only by choosing some particular operator orderings for the generators of the Galilei group depending on the quantization ambiguities of the *gauge-matter* commutation relation. We show that the desired ordering for the unusual prescription is not the same as the well-known normal ordering but still satisfies all the necessary conditions. Furthermore, we show that the equations of motion can be expressed in a similar form regardless of the regularization ambiguity. This suggests that the different regularization prescriptions do not change the physics. On the other hand, for the case of point sources the regularization prescription is uniquely determined, and only the orderings, which are equivalent to the usual one, are allowed.

PACS number(s): 11.10.Lm, 11.30.−j

I. INTRODUCTION

Recently, the Chern-Simons gauge theory has captured considerable interest due to the fact that it is still the only known example of Galilei-covariant gauge theory [1], or it can realize Wilczek's charge-flux composite model of the anyon [2,3]. Furthermore, it has been proposed as a toy model of quark confinement in D=2+1 [4]. It has been also extensively studied as the topologically massive gauge theory with the Maxwell term [5]. But quantal analysis of the Chern-Simons theory in [1,3,6,7] shows that there is the operator ordering problem due to the quantization or regularization ambiguity in defining the quantization rule, i.e., commutation relation of gauge-matter. Although they chose a particular regularization prescription such that there is no operator ordering problem, it is still unclear whether one can avoid the ordering problem even for more general regularization prescriptions, or determine the correct prescriptions from some first principles [3].

In this paper we show that the ordering problem can be avoided, and the key resides in an unusual property of the nonrelativistic Chern-Simons gauge theory. In contrast with our common expectation, the Galilei covariance of the nonrelativistic Chern-Simons gauge theory in the Galilei-covariant gauges is not a trivial matter. The covariance can be only recovered by choosing some particular operator orderings for the generators of the Galilei group. The extended and point source systems are separately analyzed. Since the point source systems with more general regularization prescriptions need much more care than the extended ones, we will first treat the extended case. In Sec. II the Galilei covariance of the nonrelativistic Abelian Chern-Simons gauge theory in the Coulomb gauge is examined for the extended sources. We explicitly show that the covariance can be recovered only by choosing some particular operator orderings depending on the quantization ambiguities of the gauge-matter commutation relations. Moreover, we point out that the desired ordering for the unusual prescription is not the same as the well-known normal ordering but still satisfies all the necessary conditions, while the desired ordering for the usual prescription is the same as the usual one. In Sec. III the operator equations of motion for the properly ordered generators for the extended sources are examined. As a result, all the equations of motion corresponding to different orderings can be expressed in a similar form. In Sec. IV the ordering problem for the point source system is examined, and we show that the usual prescription is the only possible one. Hence the orderings, which are equivalent to the usual one, are only allowed. Section V is devoted to summary and remarks about several generalizations of our analysis.

II. GALILEI COVARIANCE IN COULOMB GAUGE

The nonrelativistic Abelian Chern-Simons gauge theory on the plane is described by the Lagrangian

$$\mathcal{L} = \frac{\kappa}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + i\phi^* D_t\phi - \frac{1}{2m}(D_k\phi)^*(D_k\phi), \qquad (1)$$

where $\epsilon^{012}=1$, $g_{\mu\nu}=\text{diag}(1,-1,-1)$, and $D_\mu=\partial_\mu+iA_\mu$. It is invariant up to the total divergence under the gauge transformations

$$\phi \rightarrow \exp[-i\Lambda]\phi, \quad A_\mu \rightarrow A_\mu + \partial_\mu\Lambda, \qquad (2)$$

where Λ is a well-behaved function such that $\epsilon^{\mu\nu\lambda}\partial_\mu\partial_\nu\Lambda=0$. Then the classical equations of motion are

$$B \equiv \epsilon^{ij}\partial_i A^j = -\frac{1}{\kappa}J_0, \qquad (3)$$

*Electronic address: yjpark@ccs.sogang.ac.kr

0556-2821/94/50(12)/7584(8)/$06.00

$$E^i \equiv -\partial_i A^0 - \partial_t A^i = \frac{1}{\kappa}\epsilon^{ij} J^j, \tag{4}$$

where $J_0 = \phi^*\phi$, $J^i = (1/2mi)\{\phi^* D_i\phi - (D_i\phi^*)\phi\}$, and $\epsilon^{0ij} \equiv \epsilon^{ij}$. We will quantize this model by defining usual equal-time commutation relations for the matter field as

$$[\phi(\mathbf{x}), \phi^\dagger(\mathbf{x}')] = \delta^2(\mathbf{x}-\mathbf{x}'),$$
$$[\phi(\mathbf{x}), \phi(\mathbf{x}')] = 0, \quad [\phi^\dagger(\mathbf{x}), \phi^\dagger(\mathbf{x}')] = 0 \tag{5}$$

by considering ϕ to be a quantum field and ϕ^* its dagger $\phi^\dagger$. Note that although several authors have considered the usual Dirac procedure [8] or symplectic quantization method [9], we consider here the safest approach in order not to miss the ordering problem following Refs. [1,3,4,7]. In this approach one adopts quantum rules such as (5), which do not possess the ordering ambiguities as the fundamental ones.

Now let us consider Eqs. (3) and (4) as the operator equations. If we choose the Coulomb gauge, the solutions for $\mathbf{A}$ and A^0 are given by [1,7]

$$\mathbf{A}(\mathbf{x},t) = \overline{\nabla}\frac{1}{\kappa}\int d^2x'\mathcal{D}(\mathbf{x}-\mathbf{x}')J_0(\mathbf{x}',t), \tag{6}$$

$$A^0(\mathbf{x},t) = -\frac{1}{\kappa}\int d^2x'\mathbf{J}(\mathbf{x}',t)\cdot\overline{\nabla}'\mathcal{D}(\mathbf{x}-\mathbf{x}'), \tag{7}$$

where $\overline{\nabla}^i = \epsilon^{ij}\partial_j$, having the property $\overline{\nabla}^2 = \nabla^2$, and $\mathcal{D}(\mathbf{x})$ is defined by

$$\nabla^2\mathcal{D}(\mathbf{x}) = \delta^2(\mathbf{x}) \tag{8}$$

and has the well-known solution

$$\mathcal{D}(\mathbf{x}) = \frac{1}{4\pi}\ln\mathbf{x}^2\ , \tag{9}$$

by neglecting the trivial constant term. Then, the gauge potentials $\mathbf{A}$ and A^0 become the quantum operators due to the operator nature of the densities J_0, $\mathbf{J}$. Although solutions (6) and (7) are generally valid for both point and extended sources, we first consider the extended one in this section. Moreover, since all the gauge fields $\mathbf{A}$ and A^0 are completely expressed by the matter fields, we may expect that the commutation relations of the gauge-gauge and gauge-matter should be completely determined by the matter-matter commutation relation (5). But as was pointed out in Refs. [6] and [7], there is quantization ambiguity in the gauge-matter commutation relation at the same point,

$$[\mathbf{A}(\mathbf{x}), \phi(\mathbf{x})] = \left[-\frac{1}{\kappa}\overline{\nabla}\mathcal{D}(\mathbf{x}-\mathbf{x}')\mid_{\mathbf{x}=\mathbf{x}'}\right]\phi(\mathbf{x}), \tag{10}$$

since $\overline{\nabla}^i\mathcal{D}(\mathbf{x}) = (1/2\pi)\epsilon^{ij}x^j/\mathbf{x}^2$ is ill defined at the origin. Although several authors [1,6,7], by choosing a particular regularization preserving the antisymmetry of $\overline{\nabla}\mathcal{D}$ under space reflection, have assumed $\overline{\nabla}\mathcal{D}(\mathbf{x}-\mathbf{x}')|_{\mathbf{x}=\mathbf{x}'} = 0$ such that there is no operator ordering problem, it is still unclear whether the ordering problem can be also avoided for even more general regularization prescriptions, or whether the correct prescriptions can be determined from some first principles [3].

In the following, we present a novel feature of the nonrelativistic Abelian Chern-Simons theory, which is that the Galilei covariance in the Galilei-covariant gauges, like the Coulomb gauge, is spoiled for incorrect orderings depending on the regularization prescriptions. To this end, we first write Eq. (10) as

$$[\mathbf{A}(\mathbf{x}), \phi(\mathbf{x})] = \mathbf{F}\phi(\mathbf{x}), \tag{11}$$

where $\mathbf{F}$ denotes $-(1/\kappa)\overline{\nabla}\mathcal{D}(\mathbf{x}-\mathbf{x}')\mid_{\mathbf{x}=\mathbf{x}'}$, which is real and should be constant in both space and time such that the quantization rule (11) is covariant under space and time translation [10]. Then, there exists the nontrivial ordering ambiguity in defining the covariant derivatives $\mathbf{D}\phi$ and $\mathbf{D}\phi^\dagger$; i.e., we can define covariant derivatives as

$$\mathbf{D}^{(1)}\phi(\mathbf{x}) \equiv \nabla\phi(\mathbf{x}) - i\mathbf{A}(\mathbf{x})\phi(\mathbf{x}),$$
$$\mathbf{D}^{(1)}\phi^\dagger(\mathbf{x}) \equiv \nabla\phi^\dagger(\mathbf{x}) - i\mathbf{A}(\mathbf{x})\phi^\dagger(\mathbf{x}),$$
$$\mathbf{D}^{(2)}\phi(\mathbf{x}) \equiv \nabla\phi(\mathbf{x}) - i\phi(\mathbf{x})\mathbf{A}(\mathbf{x}),$$
$$\mathbf{D}^{(2)}\phi^\dagger(\mathbf{x}) \equiv \nabla\phi^\dagger(\mathbf{x}) - i\phi^\dagger(\mathbf{x})\mathbf{A}(\mathbf{x}). \tag{12}$$

Then, the differences between these two different definitions $\mathbf{D}^{(1)}\phi$(or $\phi^\dagger$) and $\mathbf{D}^{(2)}\phi$(or $\phi^\dagger$) are given by the field-dependent terms as

$$[\mathbf{D}^{(1)} - \mathbf{D}^{(2)}]\phi(\mathbf{x}) = -i\mathbf{F}\phi(\mathbf{x}),$$
$$[\mathbf{D}^{(1)} - \mathbf{D}^{(2)}]\phi^\dagger(\mathbf{x}) = -i\mathbf{F}\phi^\dagger(\mathbf{x}). \tag{13}$$

As a result, one can easily expect that these produce the nontrivial operator ordering problem to the generators of the current operator $\mathbf{J}$ and Galilei group H (time translation), $\mathbf{P}$ (space translation), $\mathbf{G}$ (Galilei boost), and J (rotation).

A. Hamiltonian operator

The classical Hamiltonian in our analysis is given by

$$\begin{aligned} H &= \int d^2x T^{00} \\ &= \int d^2x \frac{1}{2m}[\mathbf{D}\phi(\mathbf{x})]^*\cdot[\mathbf{D}\phi(\mathbf{x})] \\ &= \int d^2x \frac{1}{2m}[\nabla\phi(\mathbf{x}) \\ &\quad -i\mathbf{A}(\mathbf{x})\phi(\mathbf{x})]^*\cdot[\nabla\phi(\mathbf{x}) - i\mathbf{A}(\mathbf{x})\phi(\mathbf{x})]\ , \end{aligned} \tag{14}$$

using the nonrelativistic energy momentum tensor $T^{\mu\nu}$ [7]:

$$T^{00} = \frac{1}{2m}(D_i\phi)^*(D_i\phi),$$

$$T^{0i} = \frac{1}{2i}[\phi^* D_i\phi - (D_i\phi)^*\phi],$$

$$T^{i0} = -\frac{1}{2m}[(D_t\phi)^*(D_i\phi) + (D_t\phi)(D_i\phi)^*],$$

$$\begin{aligned} T^{ij} = \frac{1}{2m}&[(D_i\phi)^*(D_j\phi) + (D_i\phi)(D_j\phi)^* \\ &-\delta_{ij}(D_k\phi)^*(D_k\phi)] \\ &+\frac{1}{4m}(\delta_{ij}\nabla^2 - 2\partial_i\partial_j)J_0 + \delta_{ij}T^{00}. \end{aligned}$$

Then, four different Hermitian operator forms of the Hamiltonian are possible:

$$H_a \equiv \int d^2x \frac{1}{2m}[\mathbf{D}^{(1)}\phi(\mathbf{x})]^\dagger \cdot [\mathbf{D}^{(1)}\phi(\mathbf{x})]$$
$$= \int d^2x \frac{1}{2m}[\nabla\phi(\mathbf{x}) - i\mathbf{A}(\mathbf{x})\phi(\mathbf{x})]^\dagger \cdot [\nabla\phi(\mathbf{x}) - i\mathbf{A}(\mathbf{x})\phi(\mathbf{x})],$$
$$H_b \equiv \int d^2x \frac{1}{2m}[\mathbf{D}^{(2)}\phi(\mathbf{x})]^\dagger \cdot [\mathbf{D}^{(2)}\phi(\mathbf{x})]$$
$$= H_a + \mathbf{F}\cdot\int d^2x \mathbf{J}^{(1)} - \frac{1}{2m}\mathbf{F}^2 Q,$$
$$H_c \equiv \int d^2x \frac{1}{2m}[\mathbf{D}^{(1)}\phi(\mathbf{x})]\cdot[\mathbf{D}^{(1)}\phi(\mathbf{x})]^\dagger$$
$$= H_b + \frac{\delta^2(0)}{2m}\int d^2x \mathbf{A}^2(\mathbf{x}),$$
$$H_d \equiv \int d^2x \frac{1}{2m}[\mathbf{D}^{(2)}\phi(\mathbf{x})]\cdot[\mathbf{D}^{(2)}\phi(\mathbf{x})]^\dagger$$
$$= H_c + \mathbf{F}\cdot\int d^2x \mathbf{J}^{(1)} - \frac{1}{2m}\mathbf{F}^2 Q - \frac{\delta^2(0)}{m}\mathbf{F}\cdot\int d^2x \mathbf{A} + \frac{\mathbf{F}^2}{2m}V, \tag{15}$$

where $\mathbf{J}^{(1)} = (1/2mi)[\phi^\dagger(\mathbf{D}^{(1)}\phi) - (\mathbf{D}^{(1)}\phi)^\dagger\phi]$, $\delta^2(0) = \delta^2(\mathbf{x}-\mathbf{x})$, $Q = \int d^2x J^0$, and $V = \int d^2x$. The corresponding quantum field equations of motion for the matter field are given by

$$(i\partial_t\phi(\mathbf{x}))_a \equiv [\phi(\mathbf{x}), H_a] = -\frac{1}{2m}\mathbf{D}^{(1)^2}\phi(\mathbf{x}) + A^{0^{(1)}}(\mathbf{x})\phi(\mathbf{x}) + \frac{1}{2m\kappa^2}\int d^2x'[\overline{\nabla}'\mathcal{D}(\mathbf{x}'-\mathbf{x})]^2 J_0(\mathbf{x}')\phi(\mathbf{x}),$$
$$(i\partial_t\phi(\mathbf{x}))_b \equiv [\phi(\mathbf{x}), H_b]$$
$$= [\phi(\mathbf{x}), H_a] - \frac{i}{m}\mathbf{F}\cdot\mathbf{D}^{(1)}\phi(\mathbf{x}) + \frac{1}{2m}\mathbf{F}^2\phi(\mathbf{x}) - \frac{1}{m\kappa}\mathbf{F}\cdot\int d^2x'[\overline{\nabla}'\mathcal{D}(\mathbf{x}'-\mathbf{x})]J_0(\mathbf{x}')\phi(\mathbf{x}),$$
$$(i\partial_t\phi(\mathbf{x}))_c \equiv [\phi(\mathbf{x}), H_c]$$
$$= [\phi(\mathbf{x}), H_b] + \frac{\delta^2(0)}{m\kappa}\int d^2x'[\overline{\nabla}'\mathcal{D}(\mathbf{x}'-\mathbf{x})]\cdot\mathbf{A}(\mathbf{x}')\phi(\mathbf{x}) + \frac{\delta^2(0)}{2m\kappa^2}\int d^2x'\overline{\nabla}\mathcal{D}(\mathbf{x}'-\mathbf{x})]^2\phi(\mathbf{x}),$$
$$(i\partial_t\phi(\mathbf{x}))_d \equiv [\phi(\mathbf{x}), H_d]$$
$$= [\phi(\mathbf{x}), H_c] - \frac{i}{m}\mathbf{F}\cdot\mathbf{D}^{(1)}\phi(\mathbf{x}) + \frac{1}{2m}\mathbf{F}^2\phi(\mathbf{x}) - \frac{1}{m\kappa}\mathbf{F}\cdot\int d^2x'[\overline{\nabla}'\mathcal{D}(\mathbf{x}'-\mathbf{x})]J_0(\mathbf{x}')\phi(\mathbf{x})$$
$$-\frac{\delta^2(0)}{m\kappa}\mathbf{F}\cdot\int d^2x'[\overline{\nabla}'\mathcal{D}(\mathbf{x}'-\mathbf{x})]\phi(\mathbf{x}), \tag{16}$$

where the scalar potential $A^{0^{(1)}}$ is

$$A^{0^{(1)}}(\mathbf{x},t) = -\frac{1}{\kappa}\int d^2x'\mathbf{J}^{(1)}(\mathbf{x}',t)\cdot\overline{\nabla}'\mathcal{D}(\mathbf{x}-\mathbf{x}'). \tag{17}$$

Note that it is clear from the expression of $\mathbf{J}^{(1)}$ that there is also quantization ambiguity in an A^0-matter commutation relation. This problem is related to the nonuniqueness of the current operator and will be treated later. At present, the **A**-matter commutation relation is only needed in calculating the generators of the Galilei group. Moreover, we note that the last term in the first equation of (16) is the usual Jackiw-Pi quantum correction term from reordering [7]. Furthermore, Eqs. (15) and (16) show additional quantum effects due to the regularization ambiguity by the explicit appearance of **F**-dependent terms and highly divergent terms proportional to $\delta^2(0)$ due to additional reordering.

B. Angular momentum operator

The classical angular momentum in our analysis is given by

$$J = \int d^2x\, (\mathbf{x}\times\mathbf{T})$$
$$= \int d^2x\, \frac{1}{2i}\mathbf{x}\times[\phi^*\mathbf{D}\phi - (\mathbf{D}\phi)^*\phi]$$
$$= L + S, \tag{18}$$

where $\mathbf{T}^i \equiv T^{0i}$, the orbital angular momentum L is given by

$$L = \int d^2x\, \frac{1}{2i}\mathbf{x}\times[\phi^*\nabla\phi - (\nabla\phi^*)\phi], \tag{19}$$

and S is the well-known Hagen anomalous spin angular momentum [1,2,4]:

$$S = -\int d^2x\, (\mathbf{x}\times\mathbf{A})J^0 = \frac{1}{4\pi\kappa}Q^2. \tag{20}$$

Similar to the case of the Hamiltonian operator, four Hermitian different operator forms of angular momentum are also possible as follows:

$$
\begin{aligned}
J_a &\equiv \int d^2x\ \frac{1}{2i}\mathbf{x}\times[\phi^\dagger\mathbf{D}^{(1)}\phi-(\mathbf{D}^{(1)}\phi)^\dagger\phi]\\
&=J+\mathbf{F}\times\int d^2x\ \mathbf{x}J^0,\\
J_b &\equiv \int d^2x\ \frac{1}{2i}\mathbf{x}\times[\phi^\dagger\mathbf{D}^{(2)}\phi-(\mathbf{D}^{(2)}\phi)^\dagger\phi]\\
&=J_a-\mathbf{F}\times\int d^2x\ \mathbf{x}J^0,\\
J_c &\equiv \int d^2x\ \frac{1}{2i}\mathbf{x}\times[(\mathbf{D}^{(1)}\phi)\phi^\dagger-\phi(\mathbf{D}^{(1)}\phi)^\dagger]\\
&=J_b-\delta^2(0)\int d^2x\ (\mathbf{x}\times\mathbf{A}),\\
J_d &\equiv \int d^2x\ \frac{1}{2i}\mathbf{x}\times[(\mathbf{D}^{(2)}\phi)\phi^\dagger-\phi(\mathbf{D}^{(2)}\phi)^\dagger]\\
&=J_c-\mathbf{F}\times\int d^2x\ \mathbf{x}J^0,
\end{aligned}
\tag{21}
$$

where $J = \int d^2x\ (1/2i)\mathbf{x}\times[\phi^\dagger\nabla\phi-(\nabla\phi^\dagger)\phi] + (1/4\pi\kappa)Q^2$. Then the corresponding infinitesimal rotations of the matter field are

$$
\begin{aligned}
\delta_a\phi &\equiv [J_a,\phi(\mathbf{x})]\\
&= i\mathbf{x}\times\nabla\phi-\frac{Q}{2\pi\kappa}\phi+(\mathbf{x}\times\mathbf{F})\phi,\\
\delta_b\phi &\equiv [J_b,\phi(\mathbf{x})]\\
&=\delta_a\phi-(\mathbf{x}\times\mathbf{F})\phi,\\
\delta_c\phi &\equiv [J_c,\phi(\mathbf{x})]\\
&=\delta_b\phi-\frac{\delta^2(0)}{\kappa}\int d^2x'\mathbf{x}'\cdot\overline{\nabla}'\mathcal{D}(\mathbf{x}'-\mathbf{x})\phi(\mathbf{x}),\\
\delta_d\phi &\equiv [J_d,\phi(\mathbf{x})]\\
&=\delta_c\phi-(\mathbf{x}\times\mathbf{F})\phi.
\end{aligned}
\tag{22}
$$

Here we also see the iterative changes of the angular momentum operators and their rotational anomalies due to the reordering and regularization ambiguity.

C. Linear momentum and Galilei boost operators

The classical linear momentum is given by

$$
\begin{aligned}
\mathbf{P} &= \int d^2x\ \mathbf{T}\\
&=\int d^2x\ \frac{1}{2i}[\phi^*\nabla\phi-(\nabla\phi^*)\phi]-\int d^2x\mathbf{A}J^0\\
&=\int d^2x\ \frac{1}{2i}[\phi^*\nabla\phi-(\nabla\phi^*)\phi]\,.
\end{aligned}
\tag{23}
$$

In the last step we used the fact $\int d^2x\mathbf{A}J^0$ vanishes due to the symmetry property. However, this step would be illegitimate for the case of point sources, i.e., sum of δ functions. This matter will be treated in Sec. IV. The corresponding possible four Hermitian operator forms are

$$
\begin{aligned}
\mathbf{P}_a &\equiv \int d^2x\ \frac{1}{2i}[\phi^\dagger\mathbf{D}^{(1)}\phi-(\mathbf{D}^{(1)}\phi)^\dagger\phi]\\
&=\mathbf{P}-\mathbf{F}Q,\\
\mathbf{P}_b &\equiv \int d^2x\ \frac{1}{2i}[\phi^\dagger\mathbf{D}^{(2)}\phi-(\mathbf{D}^{(2)}\phi)^\dagger\phi]\\
&=\mathbf{P}_a+\mathbf{F}Q,\\
\mathbf{P}_c &\equiv \int d^2x\ \frac{1}{2i}[(\mathbf{D}^{(1)}\phi)\phi^\dagger-\phi(\mathbf{D}^{(1)}\phi)^\dagger]\\
&=\mathbf{P}_b-\delta^2(0)\int d^2x\mathbf{A},\\
\mathbf{P}_d &\equiv \int d^2x\ \frac{1}{2i}[(\mathbf{D}^{(2)}\phi)\phi^\dagger-\phi(\mathbf{D}^{(2)}\phi)^\dagger]\\
&=\mathbf{P}_c+\mathbf{F}Q+\delta^2(0)\mathbf{F}V,
\end{aligned}
\tag{24}
$$

where $\mathbf{P} = \int dx^2(1/2i)[\phi^\dagger\nabla\phi-(\nabla\phi^\dagger)\phi]$, which is the usual momentum operator. Although the anomalous terms in the Hamiltonian and angular momentum may not be harmful in these cases because the anomaly can be attributed to some exotic property of field itself, this is not the case for the linear momentum. In this case, it can produce space-translationally noninvariant theory, which cannot be allowed even for any exotic fields.

The corresponding four space-translational operations of the matter field are

$$
\begin{aligned}
&[\mathbf{P}_a,\phi(\mathbf{x})]=i\nabla\phi(\mathbf{x})-\mathbf{F}\phi(\mathbf{x}),\\
&[\mathbf{P}_b,\phi(\mathbf{x})]=i\nabla\phi(\mathbf{x}),\\
&[\mathbf{P}_c,\phi(\mathbf{x})]=i\nabla\phi(\mathbf{x})+\frac{\delta^2(0)}{\kappa}\int d^2x'\overline{\nabla}'\mathcal{D}(\mathbf{x}-\mathbf{x}')\phi(\mathbf{x}),\\
&[\mathbf{P}_d,\phi(\mathbf{x})]=i\nabla\phi(\mathbf{x})+\mathbf{F}\phi\\
&\qquad+\frac{\delta^2(0)}{\kappa}\int d^2x'\overline{\nabla}'\mathcal{D}(\mathbf{x}-\mathbf{x}')\phi(\mathbf{x})\ ,
\end{aligned}
\tag{25}
$$

showing the space-translational anomalies, which should vanish for the true space-translational generators. We can classify true momentum operators according to two possible values of $\mathbf{F}$: i.e., $\mathbf{F}=0$ and $\mathbf{F}\neq 0$. For $\mathbf{F}=0$, the true momentum operators are

$$
\mathbf{P}_a=\mathbf{P}_b=\int d^2x\ \frac{1}{2i}[\phi^\dagger\nabla\phi-(\nabla\phi^\dagger)\phi]=\mathbf{P}, \tag{26}
$$

while, for $\mathbf{F}\neq 0$, they are

$$
\mathbf{P}_b=\int d^2x\ \frac{1}{2i}[\phi^\dagger\nabla\phi-(\nabla\phi^\dagger)\phi]=\mathbf{P}, \tag{27}
$$

and all others cannot be accepted as the true momentum operators for the given values of $\mathbf{F}$ due to wrong space-translational forms of the matter field. This result means that the linear momentum operators can be considered as the correct space-translational generators only with appropriate operator orderings for a given regularization or quantization ambiguity $\mathbf{F}$. However surprisingly, the final forms of linear momentum operators are all the same, and uniquely defined to be the canonical one which does not have the operator ordering problem. Hence, the quantization ambiguity of the gauge-matter commutation relation (11) does not induce the operator ordering ambiguity for the linear momentum.

Now, by defining the Galilei boost operators corresponding to four momentum operators of (23),

$$\mathbf{G}_k = t\mathbf{P}_k - \int d^2x\, \mathbf{x} J_0 \quad (k = a, b, c, d), \tag{28}$$

the true boost operators become

$$\mathbf{G}_a = \mathbf{G}_b = t\mathbf{P} - \int d^2x\ \mathbf{x} J_0 \tag{29}$$

for $\mathbf{F} = 0$, and

$$\mathbf{G}_b = t\mathbf{P} - \int d^2x\ \mathbf{x} J_0 \tag{30}$$

for $\mathbf{F} \neq 0$. Furthermore, the final forms of these boost operators, which are all the same, also have no operator ordering ambiguity.

D. Galilei covariance

The Galilei covariance of the quantum field theory can be expressed by the Galilei group

$$\begin{aligned}
&[P^k, P^l] = 0, \ \ [G^k, G^l] = 0, \ \ [P^k, G^l] = i\delta_{kl} M, \\
&[J, M] = 0, \ \ [H, H] = 0, \ \ [G^k, M] = 0, \ \ [P^k, M] = 0, \\
&[J, J] = 0, \ \ [P^k, J] = -i\epsilon_{km} P^m, \ \ [G^k, J] = -i\epsilon_{km} G^m, \\
&[P^k, H] = 0, \ \ [G^k, H] = -iP^k, \ \ [J, H] = 0,
\end{aligned} \tag{31}$$

where M is the mass operator. In this section we will show that this algebra can be satisfied only for some particular orderings depending the regularization ambiguity $\mathbf{F}$. As a result, our model with inappropriate orderings destroys the Galilei covariance, which has existed at the classical level [7,11,12]. Although there is no *a priori* reason to reject the Galilei anomaly, we only consider the theory without this anomaly in order to determine the orderings of the generators uniquely.

First, by using the properly ordered momentum and Galilei boost operators of Eqs. (24) and (28), we can easily confirm that the first three commutation relations, i.e., **P**-**P**, **G**-**G**, **P**-**G** relations, are satisfied with $M = m\int d^2x J_0$.

Second, by considering the **P**-J relation, the commutation relations for all possible orderings of angular momentum $J_a \sim J_d$ of Eq.(21) are obtained to be

$$\begin{aligned}
&[P^k, J_a] = -i\epsilon_{km} P^m + \mathbf{F} \times \int d^2x\ \mathbf{x}\partial_k J_0, \\
&[P^k, J_b] = -i\epsilon_{km} P^m, \\
&[P^k, J_c] = -i\epsilon_{km} P^m - \delta^2(0) \int d^2x\ (\mathbf{x} \times \partial_k \mathbf{A}), \\
&[P^k, J_d] = -i\epsilon_{km} P^m - \mathbf{F} \times \int d^2x\ \mathbf{x}\partial_k J_0 \\
&\qquad\qquad -\delta^2(0) \int d^2x\ (\mathbf{x} \times \partial_k \mathbf{A}).
\end{aligned} \tag{32}$$

By noting that the anomalous term cannot vanish even if we assume highly localized fields such that the appearing surface terms can be neglected, we see that the appropriate orderings become

$$J_a = J_b = \int d^2x\ \frac{1}{2i}\mathbf{x} \times [\phi^\dagger \nabla\phi - (\nabla\phi^\dagger)\phi] + \frac{1}{4\pi\kappa} Q^2 = J \tag{33}$$

for $\mathbf{F} = 0$, and

$$J_b = \int d^2x\ \frac{1}{2i}\mathbf{x} \times [\phi^\dagger \nabla\phi - (\nabla\phi^\dagger)\phi] + \frac{1}{4\pi\kappa} Q^2 = J \tag{34}$$

for $\mathbf{F} \neq 0$. This result shows that, although the appropriate orderings of angular momentum are determined by the closure property of the Galilei algebra, the final forms of the angular momentum operators are all the same as for the linear momentum operator. Although (33) and (34) contain the usual anomalous spin term, we will use the representation of no rotational anomaly such that we can treat the matter fields as the usual ones most safely in the following. This is possible because the redefined angular momentum without the anomalous spin term also satisfies the Galilei algebra [1,4].

Next, we consider the most nontrivial J-H commutation relations. A lengthy calculation shows that

$$\begin{aligned}
&[J, H_a] = -\frac{i}{m}\mathbf{F} \times \mathbf{P}, \\
&[J, H_b] = 0, \\
&[J, H_c] = \frac{\delta^2(0)}{2m} \int d^2x \int d^2x' \int d^2x'' [\overline{\nabla}\mathcal{D}(\mathbf{x} - \mathbf{x}')] \cdot [\overline{\nabla}\mathcal{D}(\mathbf{x} - \mathbf{x}'')][\mathbf{x}'' \times \nabla'' J_0(\mathbf{x}'')] J_0(\mathbf{x}''), \\
&[J, H_d] = \frac{i}{m}\mathbf{F} \times \mathbf{P} - \frac{\delta^2(0)}{m}\mathbf{F} \cdot \int d^2x \int d^2x' [\overline{\nabla}\mathcal{D}(\mathbf{x} - \mathbf{x}')][\mathbf{x}' \times \nabla' J_0(\mathbf{x}')] \\
&\qquad\qquad + \frac{\delta^2(0)}{2m} \int d^2x \int d^2x' \int d^2x'' [\overline{\nabla}\mathcal{D}(\mathbf{x} - \mathbf{x}')] \cdot [\overline{\nabla}\mathcal{D}(\mathbf{x} - \mathbf{x}'')][\mathbf{x}'' \times \nabla'' J_0(\mathbf{x}'')] J_0(\mathbf{x}'').
\end{aligned} \tag{35}$$

Then the desired Hamiltonian becomes

$$H_a = H_b = \int d^2x \left(\frac{1}{2m}(\nabla\phi^\dagger) \cdot (\nabla\phi) - \mathbf{A} \cdot \mathcal{J} + \frac{1}{2m}\mathbf{A}^2 J_0 \right) \tag{36}$$

for $\mathbf{F} = 0$, and

$$H_b = \int d^2x \left(\frac{1}{2m}(\nabla\phi^\dagger) \cdot (\nabla\phi) - \mathbf{A} \cdot \mathcal{J} + \frac{1}{2m}\mathbf{A}^2 J_0 \right) \tag{37}$$

for $\mathbf{F} \neq 0$. Here the matter current $\mathcal{J}$ is given by

$$\mathcal{J} = \frac{1}{2mi}[\phi^\dagger \nabla \phi - (\nabla \phi^\dagger)\phi].$$

Although the final results of (36) and (37) are formally the same, the ordering contents are very different. (36) can be considered as the normal-ordered Hamiltonian as

$$H_a = \,:H_a:\, = \,:H_b:\, = H_b \tag{38}$$

due to the fact that $\mathbf{A}$ and ϕ(or $\phi^\dagger$) commute in the integration of (36) for the $\mathbf{F} = 0$ case. However, this is not the case for (37). Actually

$$\begin{aligned} H_b = &\,:H_b: + \frac{1}{m}\mathbf{F}\cdot\mathbf{P} - \frac{1}{2m}\mathbf{F}^2 Q \\ &+ \frac{1}{2m\kappa}\int d^2x \int d^2x' \phi^\dagger(\mathbf{x})[\nabla \mathcal{D}(\mathbf{x}-\mathbf{x}')]^2 \\ &\times J_0(\mathbf{x}')\phi(\mathbf{x}) \end{aligned} \tag{39}$$

due to the fact that $\mathbf{A}$ and ϕ(or $\phi^\dagger$), now, do not commute in the integration of (36) for the $\mathbf{F} \neq 0$ case. Note that all the last three terms of (39) cannot be simply subtracted for the Galilei covariance because this cannot be recovered without some of them. Hence the correct ordering of our theory is not the conventional normal ordering. If one insists on conventional normal ordering, there is the Galilei anomaly for all Hamiltonians $H_a \sim H_d$ for $\mathbf{F} \neq 0$. But, since this is not compulsory, a rather more general ordering can be adopted, i.e., $\phi^\dagger(\phi^\dagger\phi + \phi\phi^\dagger)\phi$, $i\phi^\dagger(\phi^\dagger\phi - \phi\phi^\dagger)\phi, \ldots$, which can be considered as a *modified normal ordering.* The modified normal ordering satisfies all the necessary conditions of ordering such that properly ordered generators should be Hermitian and annihilate the vacuum state.

Using the asserted form of H, $\mathbf{P}$, $\mathbf{G}$, and J, it is straightforward to verify that all other commutation relations in Eq. (31) are satisfied. Hence we showed that the operator ordering problem arisen from the quantization ambiguity at the same point of $[\mathbf{A}(\mathbf{x}), \phi(\mathbf{x})]$ for the extended sources can be avoided by considering only the system with no Galilei anomaly.

III. EQUATIONS OF MOTION FOR EXTENDED SOURCES

We now study how the quantum equations of motion behave for our properly ordered generators for the extended sources. To this end we define

$$\begin{aligned} E^{i(1)} &\equiv -\partial_i A^{0(1)} - (\partial_t A^i)^{(1)}, \\ E^{i(2)} &\equiv -\partial_i A^{0(2)} - (\partial_t A^i)^{(2)}, \end{aligned} \tag{40}$$

where

$$\begin{aligned} A^{0(2)}(\mathbf{x},t) &\equiv -\frac{1}{\kappa}\int d^2x' \mathbf{J}^{(2)}(\mathbf{x}',t)\cdot \nabla' \mathcal{D}(\mathbf{x}-\mathbf{x}'), \\ \mathbf{J}^{(2)} &\equiv \frac{1}{2mi}[\phi^\dagger \mathbf{D}^{(2)}\phi - (\mathbf{D}^{(2)}\phi)^\dagger\phi] \\ &= \mathbf{J}^{(1)} + \frac{1}{m}\mathbf{F}J^0, \\ (\partial_t A^i)^{(1)} &\equiv i[H_a, A^i], \quad (\partial_t A^i)^{(2)} \equiv i[H_b, A^i]. \end{aligned} \tag{41}$$

Then it is straightforward to show that

$$\begin{aligned} B &= -\frac{1}{\kappa}J^0, \\ E^{i(1)} &= \frac{1}{\kappa}\epsilon^{ij}J^{j(1)}, \\ E^{i(2)} &= \frac{1}{\kappa}\epsilon^{ij}J^{j(2)}, \end{aligned} \tag{42}$$

using

$$\begin{aligned} (\partial_t J_0)^{(1)} &\equiv i[H_a, J_0] = -\nabla\cdot\mathbf{J}^{(1)}, \\ (\partial_t J_0)^{(2)} &\equiv i[H_b, J_0] = -\nabla\cdot\mathbf{J}^{(2)}. \end{aligned} \tag{43}$$

Now, Faraday's induction laws for each type become

$$\begin{aligned} \nabla\times\mathbf{E}^{(1)} + (\partial_t B)^{(1)} &= 0, \\ \nabla\times\mathbf{E}^{(2)} + (\partial_t B)^{(2)} &= 0. \end{aligned} \tag{44}$$

Moreover, the equations of motion for the matter field for the second type in Eq. (16) reduce to

$$\begin{aligned} (i\partial_t\phi(\mathbf{x}))^{(2)} &\equiv (i\partial_t\phi(\mathbf{x}))_b \\ &= -\frac{1}{2m}\mathbf{D}^{(2)^2}\phi(\mathbf{x}) + A^{0(2)}(\mathbf{x})\phi(\mathbf{x}) + \frac{1}{2m\kappa^2}\int d^2x'[\nabla'\mathcal{D}(\mathbf{x}'-\mathbf{x})]^2 J_0(\mathbf{x}')\phi(\mathbf{x}). \end{aligned} \tag{45}$$

All these results show that all the equations of the motion for both gauge and matter fields can be expressed as the similar form regardless of the types of the orderings or the regularization prescriptions. This result strongly suggests that the different orderings or regularization prescriptions do not change the physics.

It seems appropriate to remark that there is also quantization ambiguity for an A^0-ϕ commutation relation at the same point although this does not affect our analysis:

$$\begin{aligned} [A^{0(1)}(\mathbf{x}), \phi(\mathbf{x})] &= -\frac{1}{im}\mathbf{F}\cdot\mathbf{D}^{(1)}\phi(\mathbf{x}) - \frac{1}{m\kappa^2}\int d^2x'' \nabla\mathcal{D}(\mathbf{x}-\mathbf{x}'')\cdot\nabla''\mathcal{D}(\mathbf{x}-\mathbf{x}'')J_0(\mathbf{x}'')\phi(\mathbf{x}), \\ [A^{0(2)}(\mathbf{x}), \phi(\mathbf{x})] &= [A^{0(1)}(\mathbf{x}), \phi(\mathbf{x})] - \frac{1}{m\kappa}\mathbf{F}^2\phi(\mathbf{x}). \end{aligned} \tag{46}$$

IV. ORDERING PROBLEM FOR POINT SOURCES

For the point source system much more care is needed for nonzero $\mathbf{F}$. This is essentially due to the fact that integrals such as $\int d^2x\mathbf{A}J^0$ do not vanish for the point sources, i.e., the sum of δ functions. In this case it becomes

$$\int d^2x\ \mathbf{A}J_0 = -(\mathbf{F}/2\pi)\int d^2xJ_0^2, \tag{47}$$

which does not vanish for a nonzero $\mathbf{F}$. Then the linear momentum operators of (24) are modified as

$$\tilde{\mathbf{P}}_k = \mathbf{P}_k - (\mathbf{F}/2\pi)\int d^2xJ_0^2 \quad (k=a,b,c,d). \tag{48}$$

However, one can easily find that none of these operators can be considered as the true momentum operators for nonzero $\mathbf{F}$ by considering the space-translational operations of the matter field:

$$\begin{aligned}
\left[\tilde{\mathbf{P}}_a,\phi(\mathbf{x})\right] &= i\nabla\phi(\mathbf{x}) - \mathbf{F}\phi(\mathbf{x}) + \frac{\mathbf{F}}{2\pi}J^0(\mathbf{x})\phi(\mathbf{x}),\\
\left[\tilde{\mathbf{P}}_b,\phi(\mathbf{x})\right] &= i\nabla\phi(\mathbf{x}) + \frac{\mathbf{F}}{2\pi}J^0(\mathbf{x})\phi(\mathbf{x}),\\
\left[\tilde{\mathbf{P}}_c,\phi(\mathbf{x})\right] &= i\nabla\phi(\mathbf{x}) + \frac{\delta^2(0)}{\kappa}\int d^2x'\overline{\nabla}'\mathcal{D}(\mathbf{x}-\mathbf{x}')\phi(\mathbf{x})\\
&\quad+\frac{\mathbf{F}}{2\pi}J^0(\mathbf{x})\phi(\mathbf{x}),\\
\left[\tilde{\mathbf{P}}_d,\phi(\mathbf{x})\right] &= i\nabla\phi(\mathbf{x}) + \mathbf{F}\phi\\
&\quad+\frac{\delta^2(0)}{\kappa}\int d^2x'\overline{\nabla}'\mathcal{D}(\mathbf{x}-\mathbf{x}')\phi(\mathbf{x})\\
&\quad+\frac{\mathbf{F}}{2\pi}J^0(\mathbf{x})\phi(\mathbf{x}).
\end{aligned} \tag{49}$$

Any operators in $\tilde{\mathbf{P}}_a \sim \tilde{\mathbf{P}}_d$ do not produce the correct space translation for a nonzero $\mathbf{F}$. Since the regularization prescription of a nonzero $\mathbf{F}$ is not physically allowed for the above reason, the usual prescription of $\mathbf{F}=0$ of Refs. [1,4,7] is the only possible one. This is in sharp contrast to the case of extended source system where any regularization prescriptions are allowed although the proper orderings are determined depending on the prescriptions. Although a similar analysis may be performed for other generators, these are redundant ones because the analysis for the linear momentum operator gives the most restrictive condition for the regularization ambiguity $\mathbf{F}$ already.

Now we discuss the physical implications of this result. It is well known that the nonrelativistic Abelian Chern-Simons gauge theory does not exhibit self-energy at the classical level [7,13]. This is crucially due to the exact cancellation of electric and magnetic field contributions to the Lorentz force by the classical equations motion (3) and (4). By noting that $[\mathbf{A}(\mathbf{x}),\phi(\mathbf{x})]=0$ implies that there is no self-interactions even at the quantum level in our model, one can expect that the usual prescription of $\mathbf{F}=0$ is the only consistent one with the classical results. Actually this can be easily confirmed by the fact that the Schrödinger equation for nonzero $\mathbf{F}$ reveals the self-interaction induced by quantum corrections. Explicit manipulations for the one-body Schrödinger equation give the relations

$$\begin{aligned}
Eu_E^a(\mathbf{x}) &= -\frac{1}{2m}\nabla^2u_E^a(\mathbf{x}),\\
Eu_E^b(\mathbf{x}) &= -\frac{1}{2m}(\nabla+i\mathbf{F})^2u_E^b(\mathbf{x}),\\
Eu_E^c(\mathbf{x}) &= \left(-\frac{1}{2m}(\nabla+i\mathbf{F})^2\right.\\
&\quad\left.+\frac{\delta^2(0)}{2m\kappa^2}\int d^2x'[\overline{\nabla}'\mathcal{D}(\mathbf{x}'-\mathbf{x})]^2\right)u_E^c(\mathbf{x}),\\
Eu_E^d(\mathbf{x}) &= \left(-\frac{1}{2m}(\nabla+2i\mathbf{F})^2-\frac{1}{m}\mathbf{F}^2\right.\\
&\quad+\frac{\delta^2(0)}{2m\kappa^2}\int d^2x'[\overline{\nabla}'\mathcal{D}(\mathbf{x}'-\mathbf{x})]^2\\
&\quad\left.-\frac{\delta^2(0)}{m\kappa}\mathbf{F}\cdot\int d^2x'\overline{\nabla}'\mathcal{D}(\mathbf{x}'-\mathbf{x})\right)u_E^d(\mathbf{x}).
\end{aligned} \tag{50}$$

In deriving these equations we use the relations

$$\begin{aligned}
u_E(\mathbf{x}_1,\ldots,\mathbf{x}_N) &= \langle 0 \mid \phi(\mathbf{x})\cdots\phi(\mathbf{x}_N) \mid E,N\rangle\ ,\\
Eu_E(\mathbf{x}_1,\ldots,\mathbf{x}_N) &= \langle 0 \mid [\phi(\mathbf{x})\cdots\phi(\mathbf{x}_N),H] \mid E,N\rangle\ ,
\end{aligned} \tag{51}$$

with the energy and particle number eigenstate $\mid E,N\rangle$ and the vacuum state $\mid 0\rangle$ satisfying

$$\begin{aligned}
&H \mid E,N\rangle = E \mid E,N\rangle,\\
&N \mid E,N\rangle = N \mid E,N\rangle,\\
&\langle 0 \mid \phi^\dagger(\mathbf{x}) = \phi(\mathbf{x}) \mid 0\rangle = 0,\\
&H \mid 0\rangle = N \mid 0\rangle = 0
\end{aligned} \tag{52}$$

for each Hamiltonian of (15) and we use the equations of motion (16), which are valid for the case of the point sources also. The result (50) explicitly shows that only the usual prescription of $\mathbf{F}=0$ exhibits no self-interaction with u_E^a and u_E^b, which are the same in this prescription. Note that, although N-body Schrödinger equation [1,7,14] may be analyzed generally, this does not change the essence of our argument. As a result, we recognize that the usual ordering corresponding to a and b types, which are the same for $\mathbf{F}=0$, is the only possible one due to the unique determination of $\mathbf{F}=0$ for the consistency, i.e., the space-translational invariance of the model at the quantum level.

V. CONCLUSION

In this paper we have shown that the nontrivial operator ordering problem of the nonrelativistic Abelian Chern-Simons theory in the Coulomb gauge can be avoided if we require Galilei covariance even at the quantum level or the consistency. The requirement of Galilei covariance is nontrivial because we do not have any principle to disregard the Galilei anomaly of the model in $D=2+1$ dimension.

Actually, the recovery of the covariance for the extended sources is only possible when we choose some specific orderings, which cannot be the same as the well-known normal ordering but still satisfy all the necessary conditions at the same point of the proper orderings for the unusual prescription $[\mathbf{A}(\mathbf{x}), \phi(\mathbf{x})] \neq 0$. These specific orderings are the same as the usual ordering for the usual prescription $[\mathbf{A}(\mathbf{x}), \phi(\mathbf{x})] = 0$. However, we have shown that both the usual and unusual orderings or regularization prescriptions describe the same physics by noting that all the equations of motion can express the same form regardless of the types of the orderings or regularization prescriptions.

On the other hand, for the point source system, the requirement of consistency, i.e., space-translational invariance, which can be guaranteed by the proper momentum operator, can be satisfied only for the usual prescription and not for the unusual ones. Hence only the orderings, which are equivalent to the usual one, are allowed in this case. Moreover, only for this usual ordering or prescription is the quantum theory consistent with the classical results.

As final remarks, we first note that our analysis for the extended sources may be useful for the relativistic Chern-Simons theory since the sources of the relativistic system are inherently extended although the anyonicity of the model is still debatable. Second, we note that our results are not changed even when the usual quadratic self-interaction term for the matter field are introduced although it may change the structure of the conformal group [7,11,12,15]. Finally, although we only considered the representation without rotational anomaly, it is questionable whether a similar result can be also obtained for the representation with rotational anomaly.

ACKNOWLEDGMENTS

We thank H. Min and W.T. Kim for helpful discussions. The present work was supported in part by the Basic Science Research Institute program, Ministry of Education, Project No. 94-2414, and the Korea Science and Engineering Foundation through the Center for Theoretical Physics.

[1] C. R. Hagen, Phys. Rev. D **31**, 848 (1985).

[2] C. R. Hagen, Phys. Rev. D **31**, 2135 (1985); D. Arovas, J. R. Schrieffer, F. Wilczek, and A. Zee, Nucl. Phys. **B251**, 117 (1985); J. Fröhlich and P.-A. Marchetti, Lett. Math. Phys. **16**, 347 (1988); A. M. Polyakov, Mod. Phys. Lett. A 325 (1988).

[3] G. W. Semenoff, Phys. Rev. Lett. **61**, 517 (1988).

[4] C. R. Hagen, Ann. Phys. (N.Y.) **157**, 342 (1984); Phys. Rev. D **31**, 331 (1985).

[5] J. F. Schonfeld, Nucl. Phys. **B185**, 157 (1981); S. Deser, R. Jackiw, and S. Templeton, Phys. Rev. Lett. **48**, 975 (1982); Ann. Phys. (N.Y.) **140**, 372 (1982); S. Deser and R. Jackiw, Phys. Lett. **139B**, 371 (1984); D. Boyanovsky, R. Blankenbecler, and R. Yabalin, Nucl. Phys. **B270**, 483 (1986).

[6] G. V. Dunne, R. Jackiw, and C. A. Trugenberger, Ann. Phys. (N.Y.) **194**, 197 (1989); C. A. Trugenberger, R. Menikoff, and D. H. Sharp, Phys. Rev. Lett. **67**, 1922 (1991); C. A. Trugenberger, Phys. Rev. D **45**, 3807 (1992).

[7] R. Jackiw and S.-Y. Pi, Phys. Rev. D **42**, 3500 (1900); **48**, 3929(E) (1993).

[8] P. A. M. Dirac, *Lecture on Quantum Mechanics* (Belfer Graduate School of Science, Yeshiva University Press, New York, 1964); P. Gerbert, Phys. Rev. D **42**, 543 (1990); R. Banerjee, Phys. Rev. Lett. **69**, 17 (1992).

[9] L. Faddeev and R. Jackiw, Phys. Rev. Lett. **60**, 1692 (1988); T. Matsuyama, Phys. Lett. B **228**, 99 (1989).

[10] This can be explicitly confirmed by considering the commutation of Eq. (11) with the presumed correct momentum and Hamiltonian operators.

[11] M. Leblanc, G. Lozano, and H. Min, Ann. Phys. (N.Y.) **219**, 328 (1992).

[12] R. Jackiw, Ann. Phys. (N.Y.) **201**, 83 (1990).

[13] Gerbert [8].

[14] C. Kim, C. Lee, P. Ko, B.-H. Lee, and H. Min, Phys. Rev. D **48**, 1821 (1993).

[15] C. R. Hagen, Phys. Rev. D **5**, 377 (1972); R. Jackiw and S.-Y. Pi, Phys. Rev. Lett. **64**, 2969 (1990); D. Z. Freedman, G. Lozano, and N. Rius, Phys. Rev. D **49**, 1054 (1994); O. Bergman and G. Lozano, Ann. Phys. (N.Y.) **229**, 416 (1994); D. Bak and O. Bergman, MIT Report No. CTP No. 2283 (hep-th/9403104) 1994 (upublished).