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Possible resolution of the black hole information puzzle

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The problem of information loss is considered under the assumption that the process of black hole evaporation terminates in the decay of the black hole interior into a baby universe. We show that such theories can be decomposed into superselection sectors labeled by eigenvalues of the third-quantized baby universe field operator, and that scattering is unitary within each superselection sector. This result relies crucially on the quantum-mechanical variability of the decay time. It is further argued that the decay rate in the black hole rest frame is necessarily proportional to $e^{-S_{\rm tot}}$, where $S_{\rm tot}$ is the total entropy produced during the evaporation process, entailing a very long-lived remnant.

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In the 1970's, Hawking [1] made the profound discovery that quantum-mechanical black holes evaporate. Hawking went on to claim [2] that black holes ultimately disappear, and take with them most of the information contained in the initial state which formed the black hole. This claim ignited a controversy which has continued up to the present. Three main schools of thought have emerged on this black hole information puzzle: (I) information is destroyed in quantum processes involving black holes; (II) a very careful analysis will reveal that the information comes back out; (III) the information is stored in an eternal or long-lived remnant. In this paper we shall present a fourth alternative, which might be described as (IV) all of the above.

In our proposal information is lost in the sense that arbitrarily precise knowledge of the local laws of physics is insufficient to predict the outcome of gravitational collapse. Additional coupling constants (relatives of the α parameters of wormhole physics [3]) are required which can only be measured by forming black holes and watching them evaporate. After a very large number of experiments, these parameters can be determined to within a finite accuracy. Less and less information is then lost in each successive experiment, and asymptotically the outcome becomes completely predictable. A key ingredient providing for the self-consistency of our picture is a remnant which remains after the black hole horizon has shrunk to zero (or Planckian) size. Compatability with information bounds on remnant lifetimes [4] imply that these remnants must be very long lived. We indeed give a dynamical argument that in the models under consideration the decay time is proportional to $e^{S_{\rm tot}}$, where $S_{\rm tot}$ is the total entropy in the Hawking radiation produced during the evaporation process.

Our analysis assumes the qualitative features[1] of black hole evaporation depicted in Fig. 1. A sufficiently energetic incoming pulse of matter collapses into a black hole. The apparent horizon subsequently shrinks, as expected from the usual semiclassical reasoning. Eventually the

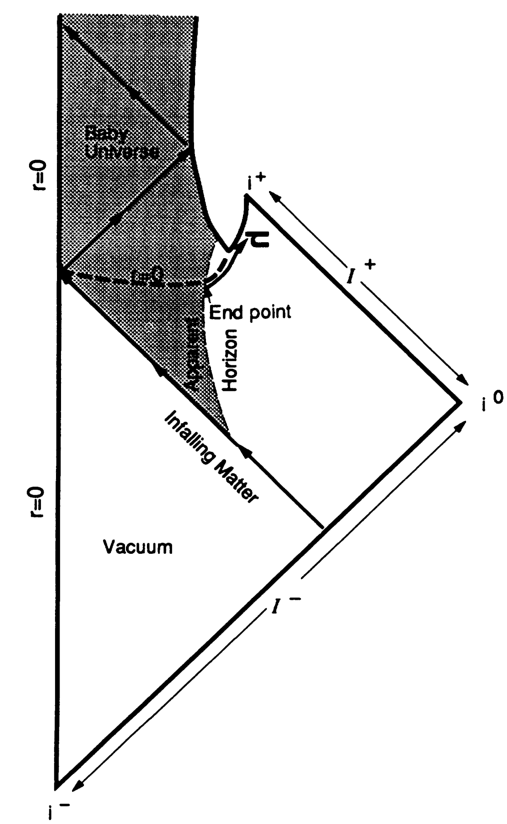

FIG. 1. A large infalling matter pulse forms a black hole (shaded region) which evaporates down to zero size at the end point. Shortly thereafter, the black hole interior splits off from the exterior spacetime. The exterior spacetime settles back to the vacuum, and the Bondi mass accordingly vanishes at i^+. τ measures the proper time after the end point along the world line indicated.

[1] An attempt to construct a two-dimensional model incorporating these features was made in [5].

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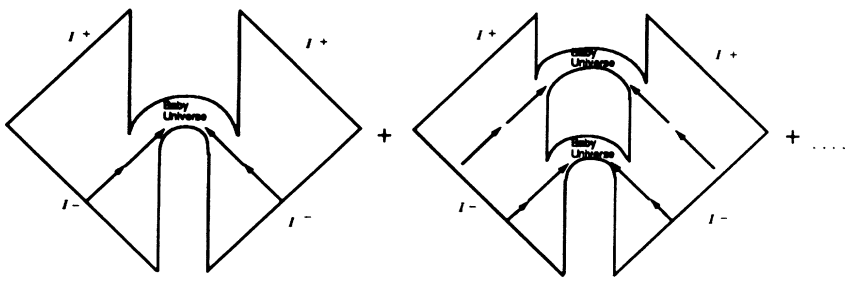

FIG. 2. In Hawking's proposal, a \$ matrix is formed by tracing over everything which falls into the black hole. This trace effectively sews together the left and right portions (representing S and $S^\dagger$) of each diagram. Contributions to \$ arising from one or two black holes are depicted.

apparent horizon reaches zero size. The spatial geometry (for an appropriate slicing) then contains an exterior, asymptotically flat region connected to the black hole interior by an umbilical cord of Planckian dimensions. The umbilical cord breaks with an amplitude proportional to g_s per unit proper time along its world line. The black hole interior then becomes a baby universe (rather than simply terminating at a singularity), and the exterior spacetime eventually settles back to the vacuum. A second assumption is that quantum fluctuations of the geometry are small and the notion of an approximate semiclassical geometry can be employed. This assumption follows formally from a $1/N$ expansion, where N is the number of matter fields.

g_s is a new parameter in the theory, which only affects topology-changing processes. If (unnaturally) set to zero, the umbilical cord can never break, and information is stored within an eternal remnant. We find this behavior implausible: in quantum mechanics what is not forbidden is compulsory, and there is no conservation law which forbids disassociation of the (neutral) black hole interior.

In such ($g_s \neq 0$) models there is an "S matrix," denoted S, which maps the incoming Hilbert space (on $\mathcal{I}^-$) to the tensor product of the outgoing (on $\mathcal{I}^+$) and baby universe Hilbert spaces.[2] Important subtleties arise in utilizing S to describe scattering from $\mathcal{I}^-$ to $\mathcal{I}^+$. As discussed in [5], there are at least two inequivalent proposals.

The first proposal, advocated by Hawking, amounts to throwing away whatever falls in to the black hole. For a single black hole, one forms a \$ matrix by simply tracing (denoted tr_i) over the internal and unobservable baby universe Hilbert space:

$$\$_1 = \mathrm{tr}_i[SS^\dagger]. \tag{1}$$

$\$_1$ acts on density matrices, and in general maps pure states to mixed states. In general for an arbitrary number n of black holes, \$ is defined by

$$\$ = \sum_{n=0}^{\infty} \mathrm{tr}_{i_1}\mathrm{tr}_{i_2}\cdots\mathrm{tr}_{i_n}[SS^\dagger], \tag{2}$$

with a separate trace for each internal Hilbert space.

[2]We assume here for convenience that no baby universes are present initially. A different choice of initial state would affect the measure in Eq. (6) below, but not our final conclusions.

This proposal has been criticized [6] on the grounds that it will inevitably violate energy conservation. This criticism invoked results of [7]. However, [7] considered only unitarity-violating dynamics which are strictly local in time. Black hole formation and evaporation requires a finite time and so is not local in this sense. If we try to derive a local description by looking at time scales long compared to the formation and/or evaporation time, the incoming states which create the black holes in the first place are no longer present in the effective field theory. Thus, while we agree that energy conservation is an important issue here, we know of no regime in which the results of [7] are directly applicable (although perhaps an adaptation of their arguments can be applied). Hawking's proposal therefore remains a logical contender for a consistent description of quantum black hole processes. It is of course of utmost importance to determine whether or not this proposal is fully consistent, but we shall not attempt to do so here.

In this paper we will develop an alternate proposal based on third quantization [8] of the baby universe Hilbert space, and partially inspired by an analogy to string theory [5]. In this formalism, baby universes are created and annihilated by operators which act on the third-quantized Hilbert space. For the case of a single black hole, this alternate proposal is indistinguishable from Hawking's proposal. However, for multiple black holes, (2) is replaced by

$$\begin{aligned}\$ &= \sum_{\{n_k\}} \langle\{n_k\}|S|\{0\}\rangle\langle\{0\}|S^\dagger|\{n_k\}\rangle \\ &= \sum_{n=0}^{\infty} \mathrm{tr}_{i_1}\mathrm{tr}_{i_2}\cdots\mathrm{tr}_{i_n}\left(\sum_{j=1}^{n!} P_j SS^\dagger\right),\end{aligned} \tag{3}$$

where $|\{n_k\}\rangle$ is the third-quantized state with n_k baby universes in the kth single-baby-universe state, the operator P_j generates the jth permutation of the n baby universes and the initial baby universe state $|\{0\}\rangle$ is made explicit in the middle expression. These permutations arise because the third-quantized baby universes are treated like indistinguishable particles, unlike in (2) where they are effectively treated as distinguishable. The difference between the two proposals is schematically illustrated in Figs. 2 and 3.

In Ref. [6], expression (3) was criticized on the grounds that the probabilities do not properly cluster: The last diagram in Fig. 3 represents interactions between widely separated experiments. This lack of clustering accu-

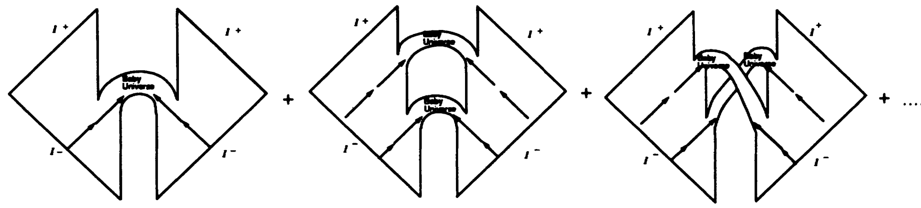

FIG. 3. Third quantization implies that the baby universes are distinguishable only by their internal state, and not by the spacetime location of the black hole from which they were created. One accordingly must sew the left and right halves of the diagrams together in all possible ways. This has no consequence for the one-black-hole sector of \$, but for two black holes, there is one extra diagram, as illustrated.

rately illustrates the difficulties in trying to use (3) to describe a theory with information loss. However, the violations of clustering are physically unobservable because the Hilbert space divides into \$-matrix superselection sectors, in each of which clustering is valid. To see this, following [3], let ϕ_i denote the third-quantized operator which creates and annihilates a single baby universe in the ith state. Consider an "α-basis" $|\{\alpha\}\rangle \equiv |\alpha_1, \alpha_2, \ldots\rangle$ for the baby universe sector of the third-quantized Hilbert space whose elements obey

$$\phi_i|\{\alpha\}\rangle = \alpha_i|\{\alpha\}\rangle. \tag{4}$$

In this basis, the \$ matrix (3) becomes

$$\$ = \int \prod_j d\alpha_j \langle\{\alpha\}|S|\{0\}\rangle\langle\{0\}|S^\dagger|\{\alpha\}\rangle. \tag{5}$$

The interaction which describes the creation of a baby universe in the ith state by a black hole is linear in the operator ϕ_i. The operator S therefore has vanishing matrix elements between different α states. This allows us to write

$$\$ = \int \prod_j \left(\frac{d\alpha_j}{\sqrt{2\pi}} e^{-\alpha_j^2/2}\right) S_{\{\alpha\}} S^\dagger_{\{\alpha\}}, \tag{6}$$

where

$$\langle\{\alpha\}|S|\{\alpha'\}\rangle \equiv \delta(\alpha - \alpha') S_{\{\alpha\}}. \tag{7}$$

The physical content of (6) is that the theory decomposes into superselection sectors parametrized by $\{\alpha\}$, i.e., the values of these parameters are not changed in any scattering experiment. Furthermore, within each superselection sector, \$ factorizes into the product of matrices $S_{\{\alpha\}}$.

This observation has been made previously by many people following [3] (perhaps in a slightly different form) and is not new to the present work. On its own this does not provide a resolution of the information puzzle because it is far from obvious that the $S_{\{\alpha\}}$ are unitary matrices. Indeed, previous estimates of $S_{\{\alpha\}}$ (unpublished, or as obtained by the rules of [6]) would seem to indicate that it does not even conserve probability. In this case one would have to conclude that third quantization simply makes no sense in the context of black hole physics (no one promised us it would). However, in the following we shall see that a careful evaluation of $S_{\{\alpha\}}$ in the type of models under consideration does in fact yield a unitary matrix. The key feature (not considered previously) essential for unitarity is that baby universe formation occurs with a finite quantum-mechanical amplitude per unit proper time, rather than instantaneously at the black hole endpoint.

As a warm-up to computation of $S_{\{\alpha\}}$, we first mention some features of an initial massive particle $|I\rangle$ at rest which decays to a number of possible final states $|F_i(t)\rangle$ of outgoing particles with decay constants g_i. (Of course one can always set all but one of the g_i's to zero by a basis rotation.) At the moment t_0 at which the decay occurs, the outgoing particles are created in some state $|F_i(0)\rangle$. At some later time t they will be in a different state (by virtue of their motion) $|F_i(t - t_0)\rangle$. If the outgoing particles promptly disperse after they are created, and the decay times are long compared to other scales in the problem, we may make the approximation that states at different times are orthogonal:

$$\langle F_i(t')|F_j(t)\rangle = \delta_{ij}\delta(t' - t). \tag{8}$$

In this same approximation the outgoing and initial state do not interact after the decay has occurred, and the interaction Hamiltonian is characterized by the matrix elements

$$\langle I|H_{\text{int}}|F_j(t)\rangle = ig_j\delta(t). \tag{9}$$

Solving Schrödinger's equation we then find that the full quantum state is[3]

$$|\psi(t)\rangle = a(t)|I\rangle - \int_0^t dt_0 a(t_0) \sum_j g_j |F_j(t - t_0)\rangle,$$
$$a(t) = e^{-\sum_i g_i^2 t/2}. \tag{10}$$

The decay of a black hole spacetime to an exterior spacetime plus a baby universe is similar, except that one of the decay products is emitted into an α state (rather than the vacuum) and there is an additional suppression arising from overlap of the initial and final state wave functions. A natural time parameter is the proper time τ from the black hole endpoint along the world line of the umbilical cord (see Fig. 1). After the umbilical cord

[3]The result (10) is equivalent to Fermi's golden rule, but adapted to the situation that the system has a semiclassical motion parametrized by t. Defining energy eigenstates $|E, i\rangle = \int_{-\infty}^{\infty} dt\, e^{iEt} F_i(t)$, the golden rule decay rate $\Gamma = 2\pi\rho(E_I)\sum_i |\langle E, i|H_{\text{int}}|I\rangle|^2$, where $\rho(E_I) = 1/2\pi$, is the same as in (10).

breaks, τ is chosen to be the proper time at the (newly formed) origin. (It will not be necessary to choose a specific time parameter prior to the end point.) We consider a Hamiltonian which evolves the system along a sequence of asymptotically flat spacelike slices labeled by the value of τ at which the slice intersects the umbilical cord or the origin. For $g_s = 0$ (so that decay is suppressed) the quantum state $|I(\tau)\rangle$ on a slice at time τ can be written as a state in the tensor product of the Hilbert space inside and outside the umbilical cord:

$$|I(\tau)\rangle = \sum_{m,J} \rho_{mJ,I}(\tau)|m\rangle|J\rangle, \tag{11}$$

where m (J) is an index in the internal (external) Hilbert space.

We wish to evolve the full state $|\psi(\tau)\rangle$ for $g_s \neq 0$ (so that decay can occur) in a baby universe α state. The (third-quantized) interaction Hamiltonian $H_{\rm int}$ contains a piece which destroys the incoming state from $\mathcal{I}^-$ and creates a baby universe and a state outgoing to $\mathcal{I}^+$. It is accordingly linear in the the baby universe field operator ϕ_i. In an α state, ϕ_i can be replaced by its eigenvalue α_i. With this replacement, the interaction Hamiltonian describes the decay of the incoming state $|I(\tau)\rangle$ to an outgoing state in the exterior spacetime:

$$\langle J(\tau')|H_{\rm int}|I(\tau)\rangle = ig_s \sum_i \alpha_i \rho_{iJ,I}(\tau)\delta(\tau'). \tag{12}$$

$|J(\tau')\rangle$ here is the unitary evolution of the detached exterior state $|J\rangle$ (using the Hamiltonian which incorporates the appropriate reflecting boundary conditions [9, 10] at the newly formed origin) for a time τ' after the decay has occurred, which approximately (at large N) obeys $\langle J(\tau')|J(\tau)\rangle = \delta(\tau' - \tau)$ as in (8). The i index here runs over a smaller set of values than the corresponding m index in (11) because it only includes states obeying the appropriate boundary conditions at the umbilical cord. This distinction will be further discussed below.

The full quantum state is determined from the Schrödinger-Wheeler-DeWitt equation to be

$$|\psi(\tau)\rangle = |\psi_1(\tau)\rangle + |\psi_2(\tau)\rangle, \tag{13}$$

where

$$|\psi_1(\tau)\rangle = \sum_{I'} |I'(\tau)\rangle a_{I'I}(\tau),$$
$$|\psi_2(\tau)\rangle = -g_s \int_0^\tau d\tau_0 \sum_{i,J,I'} \alpha_i \rho_{iJ,I'} |J(\tau - \tau_0)\rangle a_{I'I}(\tau_0), \tag{14}$$

with $a_{I'I}(\tau)$ the time-ordered exponential

$$a(\tau) = Te^{-g_s^2 \int_0^\tau d\tau' \, \Gamma(\tau')/2},$$
$$\Gamma_{I'I}(\tau) = \sum_{J,i,i'} \alpha_{i'} \alpha_i \rho^*_{i'J,I'}(\tau)\rho_{iJ,I}(\tau). \tag{15}$$

By construction $\langle\psi(\tau)|\psi(\tau)\rangle = 1$, so that expression (13) provides a unitary description of black hole formation and/or evaporation.

Note that the α parameters are not directly equal to in-out S matrix elements, but rather enter them in a complicated and indirect fashion in (13) through the decay constants. It should now be evident that a quantum-mechanically variable decay time is crucial for unitarity of the $S_{\{\alpha\}}$. For example in the Russo-Susskind-Thorlacius (RST) model [11], where decay occurs instantaneously at the evaporation end point, the $S_{\{\alpha\}}$ defined in (7) would not be unitary.

The probability that the interior eventually splits off in to a baby universe is $1 - a_{II}^2(\infty)$. It is important that this is unity, in order to avoid an eternal remnant with probability one. It appears from (13) that this will indeed be the case for generic values of the α parameters. However, it is worth noting that counterexamples are easily constructed if the α parameters respect global symmetries. For example, suppose that the α parameters were flavor blind and took the same values for a black hole formed by a collapsing $|\text{chocolate}\rangle$ state and a collapsing $|\text{vanilla}\rangle$ state. Then they will all vanish for the collapsing state $(|\text{chocolate}\rangle - |\text{vanilla}\rangle)/\sqrt{2}$, which accordingly never decays. Consistency of our picture thus requires a genericity condition on the α parameters.

In practice, the α parameters are not known initially. If only a small number (relative to the number of relevant α parameters) of experiments are performed, the results predicted by our formulas are indistinguishable from those of Hawking's. Differences will emerge only when the number of experiments is of order the number of relevant α parameters. In fact, since there are an infinite number they can never all be measured. However, "most" of the α parameters have a very small effect on the outgoing quantum state because the incoming state has a very small amplitude for producing the corresponding baby universe. These parameters will be very hard to measure but, by the same token, they will have little effect on the out state. We expect that if one repeatedly prepares identical collapsing states, the outcome will be increasingly predictable. This is the case in any real experiment: the outcome is affected by an infinite number of higher-dimension operators whose coefficients we do not know, but which have little effect on the outcome. However, a precise understanding of how the predictability increases is lacking at present.

It would be of great interest to obtain a measure of the number of α parameters relevant for the prediction of the out state associated with a given in state. An upper bound on the number can be estimated as follows. The information arrives at $\mathcal{I}^+$ in the decay time τ_D, using radiation with total energy and angular momentum of order one (in powers of M). Standard thermodynamic estimates imply that there are of order $e^{\sqrt{N\tau_D}}$ such states. So the number of relevant parameters is at most $e^{\sqrt{N\tau_D}}$. The number of baby universe states may be greater than this, but only this finite number of linear combinations of the α_i is relevant. The number of baby universe states depends on the detailed dynamics. In the model of [5], the baby universe continues to expand after the end point forms, so the number of states is comparable to the maximum above. With different dynamics, such that the baby universe did not continue to expand, the number would be much smaller and fewer experiments would be needed.

The notion that information is not really lost in black hole formation and/or evaporation has been previously advocated by a number of authors [12–15]. In these works it was argued that precise knowledge of the local laws of physics (e.g., string theory) would eventually enable one to unitarily predict the out state from the in state. In our proposal, this is not possible. Additional input, namely, the values of the α parameters, is required.[4] A further distinction is that in these previous works the information comes back out before the end point, whereas in our picture (as we shall see) it comes out after the end point. Thus there is no obvious connection of our results with these previous works. Nevertheless it is possible that future work will reveal a unified treatment of these different pictures.

The possibility of baby universe formation has no effect on the quantum state $|\psi\rangle$ in (13) prior to the future light cone of the black hole endpoint, since formation cannot occur prior to this point. The entropy on $\mathcal{I}^+$ prior to the future of the end point accordingly is independent of g_s and the α parameters. It follows (as expected from causality) that the information contained in the collapsing state cannot have been returned to $\mathcal{I}^+$ prior to the future of the end point. The rate at which it can come out after the end point encoded in the finite amount of available energy is highly constrained by entropy and/or energy bounds [4]. Consistency with these bounds then implies that all models of the type we discuss have long-lived remnants. This is not obvious from Eq. (16). However, in two dimensions it can be seen explicitly, as follows.

Before the decay, the quantum state of the matter fields is in the Hilbert space $\mathcal{H}$ of states on the half-line extending from the origin. Momentarily after the decay, it is in the product space $\mathcal{H}'$ of baby universe and exterior Hilbert spaces, which obey, e.g., Neumann boundary conditions along the umbilical cord. $\mathcal{H}'$ may be regarded as a subspace of $\mathcal{H}$, and the quantum state prior to the decay is a general state in $\mathcal{H}$ which has a component in $\mathcal{H}'$. Decay can occur only through this component. Denoting the projection from $\mathcal{H}$ to $\mathcal{H}'$ by $\mathcal{P}$, the decay rate will accordingly contain a factor $\langle I|\mathcal{P}|I\rangle$. More explicitly, the α ensemble average of the decay rate is

$$\int \prod_j \left(\frac{d\alpha_j}{\sqrt{2\pi}} e^{-\alpha_j^2/2} \right) g_s^2 \sum_J \left| \sum_i \alpha_i \rho_{iJ,I} \right|^2 = g_s^2 \langle I|\mathcal{P}|I\rangle. \tag{16}$$

The projection $\langle I|\mathcal{P}|I\rangle$ can be represented as a Euclidean path integral with initial and final boundary conditions corresponding to the state $|I\rangle$. The intermediate projection is represented by the insertion of a circular puncture at the boundary of which Neumann boundary conditions are imposed on the matter fields, together with appropriate boundary conditions on the gravitational fields. To compute this path integral we must choose a coordinate system. The simplest choice is "σ coordinates," in which the matter vacuum is simply the state annihilated by positive $\sigma^{\pm}$ Fourier components of the matter fields. The metric is $ds^2 = -e^{2\rho_\sigma} d\sigma^+ d\sigma^-$, and ρ_σ goes to zero on $\mathcal{I}^-$. The path integral is then proportional to the determinant of the matter Laplacian regulated with respect to ρ_σ:

$$\langle I|\mathcal{P}|I\rangle \sim \det[\Box]_{\rho=\rho_\sigma}. \tag{17}$$

The formula for the trace anomaly on flat manifolds with curved boundaries then implies

$$\det[\Box]_{\rho=\rho_\sigma} \sim e^{-N\oint K\rho_\sigma/12\pi} \det[\Box]_{\rho=0}, \tag{18}$$

where the exponent contains the integral of the extrinsic curvature K around the boundary of the puncture, and N is the central charge of the matter fields. The determinant evaluated at $\rho = 0$ is independent of the black hole mass to leading order and does not concern us. For a small puncture, ρ is nearly constant along the boundary, and we may approximate

$$\langle I|\mathcal{P}|I\rangle \sim e^{-N\rho(\sigma_0)/6}, \tag{19}$$

where σ_0 is the location of the puncture.[5] Explicit computation [11, 16] reveals that near the evaporation end point

$$N\rho_\sigma(\sigma_0)/6 \sim NM/3 \sim S_{\rm tot}, \tag{20}$$

where $S_{\rm tot}$ here is the fine-grained "entropy of entanglement" $(-{\rm tr}\rho_{\rm out} \ln \rho_{\rm out})$ of the quantum state $\rho_{\rm out}$ outside the black hole. $\rho_{\rm out}$ is obtained by tracing over the portion of the quantum state inside the black hole, and $S_{\rm tot}$ is the total entropy in outgoing Hawking radiation. The exponentially large (in M) value of e^ρ near the end point is equivalent to the well-known fact that (in any dimension) frequencies of field modes undergo exponentially large redshifts in propagation from $\mathcal{I}^-$ to the vicinity of the end point. So we see that

$$\langle I|\mathcal{P}|I\rangle \sim e^{-S_{\rm tot}} \ll 1, \tag{21}$$

and the decay time $\sim e^{S_{\rm tot}}/g_s^2$ is extremely long.[6,7]

[4]Although in the proposal of [15] these might be viewed as nonperturbative parameters of string theory.

[5]This calculation hides a divergent, cutoff-dependent factor which is absorbed by multiplicative renormalization of g_s.

[6]To those familiar with string theory, the preceding discussion is just the usual statement that the string coupling is field dependent.

[7]It is possible that the suppression of the decay rate can alternately be understood as a consequence of the need to conserve energy, and in this way is related to results of [17]. In order to carry away the infalling information, a large number of low energy outgoing particles are needed. A phase space suppression might be then be expected, and this would make its appearance in the matter determinant. We further note that information flow in and out of the black hole is accompanied by energy flow, in harmony with remnant constraints discussed in [17].

We stress that what we have computed is the decay time in the rest frame of the umbilical cord. In general there will be a dilation factor relating this to the decay time as seen at $\mathcal{I}^+$. The precise form of this factor appears rather model dependent, but we do not expect it to affect the exponentially growing behavior. Compatability with the information and/or energy bounds [4] only requires that the decay time as seen at $\mathcal{I}^+$ must grow as a (dimension dependent) power of $S_{\rm tot}$.

Relation (21) states that the entanglement of the interior and exterior of the black hole slows down the decay rate. A heuristic understanding of this can be obtained without resorting to explicit calculation, as follows. Divide the matter field modes into those which are fully inside or outside the apparent horizon of the black hole and those which overlap the horizon. As discussed in detail in [16], the entropy obtains contributions only from the overlapping modes (which contribute to the entanglement) and can be written as a sum with equal contributions from each mode. The sum diverges and an ultraviolet regulator is needed to define it. The regulator dependence can be absorbed by a shift in the zero of the entropy. The physically relevant finite part of the entanglement entropy increases as the black hole evaporates because the increasing redshift of field modes at the horizon relative to $\mathcal{I}^-$ causes increasing numbers of them to contribute to the entropy.

Similarly, $\mathcal{P}$ is a product of projection operators which are unity except for the overlapping modes. $\ln\langle I|\mathcal{P}|I\rangle$ is then a sum over modes which obtains contributions only from the overlapping modes. At very high frequencies there should be equal contributions from each mode. One thus expects that $\ln\langle I|\mathcal{P}|I\rangle$ is proportional to $S_{\rm tot}$, as we have verified by explicit calculation.

The heuristic arguments of the preceding two paragraphs are still applicable in $3+1$ dimensions with slight modifications (and conceivably might be crystallized in to a precise calculation). Thus we expect that the decay time is still of order $e^{S_{\rm tot}}$, where the fine grained entropy produced in $3+1$ dimensions is $S_{\rm tot} = 16\pi M^2/3$. This greatly exceeds the information bound of M^4 found in [4].

Two main objections have been raised in the past to proposed resolutions of the information puzzle which involve remnants. The first is the problem of huge pair production rates or virtual loop effects associated with the large numbers of long-lived states. Although this issue deserves further scrutiny in the present context, it appears plausible to us that these effects will be suppressed by a mechanism of the type described in [18,17]: Roughly speaking, the large number of states are in a distant region deep inside the black hole and most of them cannot be accessed (by causality) in any finite time process.

The second objection has been the lack of a good dynamical reason why a long-lived remnant would stay around long enough to reemit the information. Naively, the cost in action for a (neutral) Planckian remnant to disappear is of order one and it should therefore disappear in a time of order the Planck time. In this paper we have not only found a general dynamical origin of the long decay time, we have also described the actual mechanism by which the information is reemitted.

In conclusion, third quantization appears to offer a viable resolution to the black hole information puzzle. We find it fascinating that the consistency of quantum mechanics and gravity in our own Universe may require the existence of other universes.

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[1] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
[2] S. W. Hawking, Phys. Rev. D **14**, 2460 (1976).
[3] S. Coleman, Nucl. Phys. **B307**, 864 (1988); S. B. Giddings and A. Strominger, *ibid.* **B307**, 854 (1988).
[4] Y. Aharonov, A. Casher, and S. Nussinov, Phys. Lett. B **191**, 51 (1987); R. D. Carlitz and R. S. Willey, Phys. Rev. D **36**, 2336 (1987).
[5] A. Strominger (unpublished).
[6] L. Susskind, "Comment on a Proposal By Strominger," Report No. SU-ITP-94-14, hep-th/9405103 (unpublished).
[7] T. Banks, M. Peskin, and L. Susskind, Nucl. Phys. **B244**, 125 (1984).
[8] K. Kuchar, J. Math. Phys. **22**, 2640 (1981); N. Cadderni and M. Martellini, Int. J. Theor. Phys. **23**, 23 (1984); A. Jevicki, in *Frontiers in Particle Physics '83*, edited by D. Sijacki *et al.* (World Scientific, Singapore, 1984); I. Moss, in *Field Theory, Quantum Gravity and Strings II*, Proceedings of the Seminar Series, Meudon and Paris, France, 1985–1986, edited by H. de Vega and N. Sanchez, Lecture Notes in Physics Vol. 280 (Springer, Berlin, 1987); T. Banks, Nucl. Phys. **B309**, 643 (1988); V. Rubakov, Phys. Lett. B **214**, 503 (1988); S. B. Giddings and A. Strominger, Nucl. Phys. **B321**, 481 (1989); W. Fischler, I. Klebanov, J. Polchinski, and L. Susskind, *ibid.* **B327**, 157 (1989); A. Strominger, Philos. Trans. R. Soc. London A **329**, 395 (1989).
[9] T. D. Chung and H. Verlinde, Nucl. Phys. **B418**, 305 (1994).
[10] A. Strominger and L. Thorlacius, Phys. Rev. D **50**, 5177 (1994).
[11] J. G. Russo, L. Susskind, and L. Thorlacius, Phys. Rev. D **46**, 3444 (1993); **47**, 533 (1993).
[12] D. Page, Phys. Rev. Lett. **44**, 301 (1980).

[13] G. 't Hooft, Nucl. Phys. **B335**, 138 (1990).
[14] E. Verlinde and H. Verlinde, Nucl. Phys. **B406**, 43 (1993).
[15] L. Susskind, L. Thorlacius, and J. Uglum, Phys. Rev. D **48**, 3743 (1993).
[16] T. Fiola, J. Preskill, A. Strominger, and S. Trivedi, Phys. Rev. D **50**, 3987 (1994).
[17] S. B. Giddings, Phys. Rev. D **49**, 4078 (1994); and (private communication).
[18] T. Banks, M. O'Loughlin, and A. Strominger, Phys. Rev. D **47**, 4476 (1993).

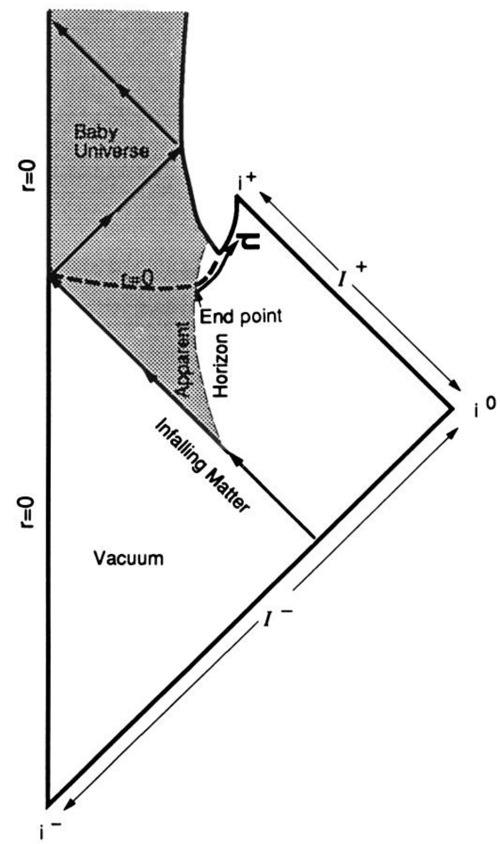

FIG. 1. A large infalling matter pulse forms a black hole (shaded region) which evaporates down to zero size at the end point. Shortly thereafter, the black hole interior splits off from the exterior spacetime. The exterior spacetime settles back to the vacuum, and the Bondi mass accordingly vanishes at i^+. τ measures the proper time after the end point along the world line indicated.