

Ehlers-Harrison-type transformations in dilaton-axion gravity

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The ten-parametric internal symmetry group is found in the $D = 4$ Einstein-Maxwell-dilaton-axion theory restricted to space-times admitting a Killing vector field. The group includes dilaton-axion $SL(2, R)$ duality and Harrison-type transformations which are similar to some target-space duality boosts, but act on a different set of variables. A new symmetry is used to derive a seven-parametric family of rotating dilaton-axion Taub-NUT dyons.

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I. INTRODUCTION

Two notable symmetries of the bosonic part of compactified low-energy heterotic string effective theory were widely discussed recently [1–3] and used to generate new classical solutions [4, 5]. One of them is *target space* duality $O(d, d + p)$, which is valid (in particular) for a D -dimensional Einstein-Maxwell-dilaton-axion (EMDA) system with p Abelian vector fields whenever variables are independent of d space-time coordinates [1, 3]. The (primitive) set of variables on which the group $O(d, d + p)$ acts consists of the stringy frame space-time metric, the Kalb-Ramond field $B_{\mu\nu}$, the vector fields A_μ^a , $a = 1, \dots, p$, and the dilaton ϕ , from which the corresponding matrix representation is built up. The group mixes the metric with the vector fields, the dilaton, and the axion. The second symmetry is a *dilaton-axion* (or electric-magnetic) duality $SL(2, R)$, which arises in the case $D = 4$ for which the Kalb-Ramond field can be transformed into the Peccei-Quinn axion κ . It says that a pair ϕ, κ parametrizes the $SL(2, R)/SO(2)$ coset. These two symmetry groups apparently were regarded as unrelated to each other at least in the context of the EMDA theory [3].

Here we show that for $D = 4$, $p = 1$, $d = 1$, symmetries of both kinds can be combined within a larger group. Our approach is similar to that used earlier for the Einstein-Maxwell (EM) system [6–8]. It consists in reduction from 4 to 3 dimensions preserving three-covariance and involving dualization of nondiagonal metric components and the magnetic part of the Maxwell tensor. This leads to the gravity-coupled three-dimensional σ model (not to be confused with the initial *string* σ model) with a six-dimensional real target space. The latter turns out to possess a 10-parameter isometry group including the $SL(2, R)$ duality as a subgroup. The group also contains Harrison-type transformations, similar to some target-space duality boosts, but now acting on a different set of variables related to the primitive

variables in a nonpointlike way.

Remarkably, our group is larger than the product of both target-space duality [in this case $O(1,2)$] and dilaton-axion duality. Its nontrivial part generalizes Ehlers-Harrison transformations known in the EM theory [9, 10]. The group also contains scale and gauge transformations. New symmetries open a very simple way to construct dilaton-axion counterparts to any stationary solution of the vacuum Einstein equations. As an example we derive a seven-parametric family of charged rotating Taub-NUT (Newman-Unti-Tamburino) dyons endowed with dilaton and axion fields. Some future prospects are briefly discussed.

II. σ -MODEL REPRESENTATION

We start with the $D = 4$, $p = 1$ EMDA action in the Einstein frame

$$S = \frac{1}{16\pi} \int \left\{ -R + 2\partial_\mu \phi \partial^\mu \phi + \frac{1}{2} e^{4\phi} \partial_\mu \kappa \partial^\mu \kappa - e^{-2\phi} F_{\mu\nu} F^{\mu\nu} - \kappa F_{\mu\nu} \tilde{F}^{\mu\nu} \right\} \sqrt{-g} d^4 x, \quad (1)$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2} E^{\mu\nu\lambda\tau} F_{\lambda\tau}$, $F = dA$, and consider a space-time possessing (at least) one Killing vector field which we choose here to be timelike. Then it is standard to present an interval in terms of a three-metric h_{ij} , a rotation one-form ω_i ($i, j = 1, 2, 3$), and a scalar f depending only on spacelike coordinates x^i :

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = f(dt - \omega_i dx^i)^2 - \frac{1}{f} h_{ij} dx^i dx^j. \quad (2)$$

The vector field may be fully described by two real functions: an electric potential v ,

$$F_{i0} = \partial_i v / \sqrt{2}, \quad (3)$$

and a magnetic one u ,

$$e^{-2\phi} F^{ij} + \kappa \tilde{F}^{ij} = f \epsilon^{ijk} \partial_k u / \sqrt{2h} \quad (4)$$

(note, that v is but u is not a component of the four-potential). Instead of ω_k a twist potential χ is then intro-

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duced in accordance with the Einstein constraint equations [7]

$$\tau_i = \partial_i \chi + v \partial_i u - u \partial_i v, \quad \tau^i = -f^2 \epsilon^{ijk} \partial_j \omega_k / \sqrt{h}. \quad (5)$$

Here and below three-indices are raised and lowered using the metric h_{ij} and its inverse h^{ij} .

A new set of variables consists of the three-metric h_{ij} and six "material" fields $\varphi^A = (\kappa, \phi, f, v, u, \chi)$, $A = 1, \dots, 6$. It is straightforward to check that the field equations following from the action (1) are identical to the equations of motion of a curved space three-dimensional σ model possessing a six-dimensional target space $\{\varphi^A\}$, together with the three-dimensional Einstein equations for h_{ij} with the energy-momentum tensor built from φ^A . The corresponding action is

$$S_\sigma = \int (\mathcal{R} - \mathcal{G}_{AB} \partial_i \varphi^A \partial_j \varphi^B h^{ij}) \sqrt{h} d^3 x, \quad (6)$$

where \mathcal{G}_{AB} is the target-space metric to be read off from the line element

$$dl^2 = 2d\phi^2 + \frac{1}{2} e^{4\phi} d\kappa^2 + \frac{1}{2f^2} \{df^2 + (d\chi + vdu - u dv)^2\} - \frac{1}{f} \{e^{-2\phi} dv^2 + e^{2\phi} (du - \kappa dv)^2\}. \quad (7)$$

Similar representation has been derived for the stationary EM system by Neugebauer and Kramer [6]. Note, that the EMDA theory does not include an EM one as a particular case. Indeed, setting $\phi = \kappa = 0$ gives two constraints $F^2 = 0 = F\bar{F}$.

III. ISOMETRIES OF THE TARGET SPACE

The target space possesses a 10-parametric isometry group. Seven of its elements can be easily found from the direct inspection of the metric (7). They include the following.

(i) *The scale transformation:*

$$f = f_0 e^{2\lambda_1}, \quad \chi = \chi_0 e^{2\lambda_1}, \quad v = v_0 e^{\lambda_1}, \quad u = u_0 e^{\lambda_1}, \quad (8)$$

leaving κ and ϕ unchanged (here and in what follows λ_s , $s = 1, \dots, 10$, are real group parameters).

(ii) *The electromagnetic and gravitational gauge transformations:*

$$u = u_0 + \lambda_2, \quad \chi = \chi_0 + v_0 \lambda_2, \quad (9)$$

$$v = v_0 + \lambda_3, \quad \chi = \chi_0 - u_0 \lambda_3, \quad (10)$$

$$\chi = \chi_0 + \lambda_4 \quad (11)$$

(all other quantities being unchanged) leaving the metric and the Maxwell tensor invariant.

(iii) *The $SL(2, R)$ dilaton-axion duality subgroup:*

$$z = e^{-2\lambda_5} z_0, \quad u = u_0 e^{-\lambda_5}, \quad v = v_0 e^{\lambda_5}, \quad (12)$$

$$v = v_0, \quad u = u_0 + v_0 \lambda_6, \quad z = z_0 + \lambda_6, \quad (13)$$

$$u = u_0, \quad v = v_0 + u_0 \lambda_7, \quad z^{-1} = z_0^{-1} + \lambda_7, \quad (14)$$

where $z = \kappa + ie^{-2\phi}$ is the complex axion-dilation field.

It can be verified that seven generators of the above transformations form a closed algebra thus giving no indications on the existence of further symmetries. However three more generators can be found by solving Killing equations for the target space. They correspond to a nontrivial Ehlers-Harrison-type part of the full symmetry group.

A pair of Harrison-type transformations mix metric functions with electromagnetic potentials, a dilaton, and an axion. Generically, they produce charged solutions from uncharged ones. The first (electric) leaves invariant the quantities

$$f e^{-2\phi} \equiv f_0 e^{-2\phi_0}, \quad \tilde{\chi} = \chi - \tilde{u}v \equiv \chi_0 - \tilde{u}_0 v_0, \quad (15)$$

where $\tilde{u} = u - \kappa v$, whereas other variables transform as

$$\tilde{u} = \tilde{u}_0 + \tilde{\chi}_0 \lambda_8, \quad \kappa = \kappa_0 + 2\tilde{u}_0 \lambda_8 + \tilde{\chi}_0 \lambda_8^2,$$

$$(\sqrt{f} e^\phi \pm v)^{-1} = (\sqrt{f_0} e^{\phi_0} \pm v_0)^{-1} m p \lambda_8. \quad (16)$$

The second (magnetic) also leaves two combinations invariant:

$$q = f^{-1/2} |z| e^\phi \equiv f_0^{-1/2} |z_0| e^{\phi_0}, \quad (17)$$

$$p = f^{-1} u^+ u^- \equiv f_0^{-1} u_0^+ u_0^-,$$

where

$$u^\pm = u \pm qf = \frac{u_0^\pm}{1 - \lambda_9 u_0^\pm}, \quad (18)$$

while other transformations read

$$\chi = k_+ u^+ + k_- u^- + k q f, \quad v = k_+ \frac{u^+}{u^-} + k_- \frac{u^-}{u^+},$$

$$z = \frac{f q^2}{dq - i}, \quad d = k + k_+ \frac{u^+}{u^-} - k_- \frac{u^-}{u^+}, \quad (19)$$

$$k_\pm = \frac{u_0^\mp}{2u_0^\pm} \left(v_0 \pm \frac{\kappa_0 f_0 e^{2\phi_0} - \chi_1}{2q f_0} \right),$$

$$k = \frac{\kappa_0 f_0 e^{2\phi_0} + \chi_1}{2q f_0},$$

where $\chi_1 = \chi_0 - u_0 v_0$.

A commutator of two Harrison-type generators gives a generator of the Ehlers-type transformation. This last transformation, which closes the full isometry group, has three real,

$$w = e^{2\phi} - v^2 f^{-1} \equiv e^{2\phi_0} - v_0^2 f_0^{-1},$$

$$1 - \beta = f^{-1} |\Phi|^2 e^{2\phi} \equiv f_0^{-1} |\Phi_0|^2 e^{2\phi_0},$$

$$\gamma = f^{-1} (\chi^2 + \beta f^2) \equiv f_0^{-1} (\chi_0^2 + \beta f_0^2), \quad (20)$$

and one complex,

$$\nu = v + (if - \chi)\Phi^{-1} \equiv v_0 + (if_0 - \chi_0)\Phi_0^{-1}, \quad (21)$$

invariants, where $\Phi = u - zv$, and

$$f = \chi\xi^{-1} = \gamma(\beta + \xi^2)^{-1}, \quad \xi = \chi_0 f_0^{-1} - \lambda_{10}\gamma,$$

$$\Phi^{-1} = \Phi_0^{-1} + \nu\lambda_{10}, \quad z = z_0 - \nu^{-1}(\Phi - \Phi_0). \quad (22)$$

There is a certain similarity between Harrison-type and some of the string target-space duality transformations. Both generate charged solutions to the EMDA theory starting from vacuum solutions of the Einstein equations. However, our group acts on a different set of variables related to the string σ -model variables by nonpointlike transformations. In the present formulation dilaton-axion $SL(2, R)$ duality enters into the same symmetry group. This group is the symmetry of the σ -model action (6) and hence that of the *equations of motion* of the initial theory. Note that in the *static* case there seems to exist an analogue of the Harrison transformation for the Einstein-Maxwell-dilaton system (without an axion) too [11].

IV. APPLICATION TO VACUUM SOLUTIONS

Any solution to the vacuum Einstein equations is a solution of the present theory with $v = u = \kappa = \phi = 0$. Therefore using the above transformations an axion-dilaton counterpart can be found to any stationary vacuum solution. In this case the above formulas simplify considerably. The first Harrison transformation will read

$$\frac{f}{f_0} = \frac{\chi}{\chi_0} = e^{2\phi} = \frac{1}{1 - \lambda_8^2 f_0}, \quad (23)$$

$$v = \lambda_8 f, \quad u = \lambda_8 \chi, \quad \kappa = \lambda^2 \chi_0.$$

If the seed solution is asymptotically flat, and one wishes to preserve this property, it has to be accompanied by the scale transformation (8) with the parameter $e^{2\lambda_1} = 1 - \lambda^2$. The result can be concisely expressed in terms of the Ernst potential $\mathcal{E} = f + i\chi$:

$$\mathcal{E} = \frac{\sqrt{1 - \lambda_8^2}}{\lambda_8} (v + iu) = \frac{(1 - \lambda_8^2)\mathcal{E}_0}{1 - \lambda_8^2 \text{Re}\mathcal{E}_0}, \quad (24)$$

$$z = i(1 - \lambda_8^2 \mathcal{E}_0).$$

A similar combined transformation via (17)–(19) reads

$$\mathcal{E} = \frac{\sqrt{1 - \lambda_9^2}}{\lambda_9} (u - iv) = \frac{(1 - \lambda_9^2)\mathcal{E}_0}{1 - \lambda_9^2 \text{Re}\mathcal{E}_0}, \quad z = \frac{i}{(1 - \lambda_9^2 \mathcal{E}_0)}. \quad (25)$$

In both cases the metric rotation function is simply rescaled $\omega_i = (1 - \lambda^2)^{-1}\omega_{0i}$, where λ is either λ_8 or λ_9 .

Remarkably, the axion-dilation Ehlers-type transformation reduces exactly to the original Ehlers transformation [9] when applied to purely vacuum solutions. In-

deed, in this case $\beta = 1$ and from (22) we get $\mathcal{E} = \mathcal{E}_0(1 + i\lambda_{10}\mathcal{E}_0)^{-1}$, while v, u, ϕ, κ remain zero.

The scale transformation (8) being applied to vacuum solutions reduces to that of the vacuum Einstein gravity, while the transformations (9)–(14) trivialize. Therefore, the only nontrivial effect on vacuum solutions is produced by the Harrison-type transformations. Generically they give rise to charged configurations endowed with dilaton and axion fields.

V. DILATON-AXION KERR-NUT DYON

Starting with the vacuum Kerr-NUT solution

$$ds_0^2 = \frac{\Delta_0 - a^2 \sin^2 \theta}{\Sigma_0} (dt - \omega_0 d\varphi)^2 - \Sigma_0 \left(\frac{dr_0^2}{\Delta_0} + d\theta^2 + \frac{\Delta_0 \sin^2 \theta}{\Delta_0 - a^2 \sin^2 \theta} d\varphi^2 \right), \quad (26)$$

where

$$\begin{aligned} \Delta_0 &= r_0(r_0 - 2M) + a^2 - N_0^2, \\ \Sigma_0 &= r_0^2 + \delta^2, \quad \delta = a \cos \theta + N_0, \\ \omega_0 &= \frac{2}{a^2 \sin^2 \theta - \Delta_0} \\ &\quad \times [N_0 \Delta_0 \cos \theta + a \sin^2 \theta (M_0 r_0 + N_0^2)], \end{aligned} \quad (27)$$

with the corresponding Ernst potential $\mathcal{E}_0 = 1 - 2(M_0 + iN_0)(r_0 + i\delta)^{-1}$ (a is Kerr rotation parameter, N_0 is NUT parameter), it is a simple matter to construct its axion-dilaton counterpart. We will do it in two steps. First, we perform a constant shift (11) of the twist potential $\mathcal{E}_0 \rightarrow \mathcal{E}_0 + i\lambda_4$ in order to have one free parameter more (this will ensure electric and magnetic charges in the resulting axion-dilaton solution to be independent), and then make either electric or magnetic Harrison-type transformation accompanied by a suitable scale transformation (24) and (25) (both lead to the same final form of the axion-dilaton solution). Furthermore, the electromagnetic gauge freedom (9) and (10) is used to remove constant asymptotic values of electric and magnetic potentials, and axion-dilaton rescaling (12) is performed to make the dilaton asymptotically zero too. As a result, the following electric and magnetic potentials will be obtained at this step:

$$v^{(0)} = -\frac{2\lambda}{\Sigma} (Mr_0 + N\delta), \quad (28)$$

$$a^{(0)} = \frac{2\lambda}{\Sigma} \left(M\delta - Nr_0 - \frac{\lambda_4 r_-}{2M_0} (Mr_0 + N\delta) \right),$$

where $M = M_0(1 - \lambda^2)^{-1}$, $N = N_0(1 - \lambda^2)^{-1}$ are rescaled mass and NUT parameters and $\Sigma = \Sigma_0 + r_0 r_- - 2N_- \delta$, $r_- = 2\lambda^2 M$, $N_- = \lambda^2 N$.

At the second step we consider (26)–(28) as new seed solution and perform axion-dilaton duality transformations (12) and (14) with the parameters

$$e^{-2\lambda_6} = \frac{|\mathcal{M}\mathcal{Q}|^2 e^{-2\phi_0}}{[\text{Im}(\mathcal{M}z_\infty \mathcal{Q})]^2}, \quad \lambda_7 = \frac{\text{Im}(\mathcal{M}\mathcal{Q})}{\text{Im}(\mathcal{M}z_\infty \mathcal{Q})}, \quad (29)$$

which are now expressed through physical quantities: a complex mass $\mathcal{M} = M + iN$, an electromagnetic charge $\mathcal{Q} = Q - iP$, an axion-dilaton charge $\mathcal{D} = D + iA$, and an asymptotic value of the axion-dilaton z_∞ . Similarly, for λ_4 one has

$$\frac{\lambda_4 r_-}{2M_0} = \frac{\text{Re}(\mathcal{M}z_\infty \mathcal{Q})}{\text{Im}(\mathcal{M}z_\infty \mathcal{Q})}. \quad (30)$$

The transformed metric can be written in the same form as (26):

$$ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} (dt - \omega d\varphi)^2 - \Sigma \left(\frac{dr^2}{\Delta} + d\theta^2 + \frac{\Delta \sin^2 \theta}{\Delta - a^2 \sin^2 \theta} d\varphi^2 \right), \quad (31)$$

where now

$$\begin{aligned} \Delta &= (r - r_-)(r - 2M) + a^2 - (N - N_-)^2, \\ \Sigma &= r(r - r_-) + (a \cos \theta + N)^2 - N_-^2, \end{aligned} \quad (32)$$

$$\omega = \frac{2}{a^2 \sin^2 \theta - \Delta} \{ N \Delta \cos \theta + a \sin^2 \theta [M(r - r_-) + N(N - N_-)] \}.$$

The corresponding electric and magnetic potentials and the axion-dilaton field are

$$v = \frac{\sqrt{2}e^{\phi_\infty}}{\Sigma} \text{Re}[\mathcal{Q}(r - r_- + i\delta)], \quad (33)$$

$$u = \frac{\sqrt{2}e^{\phi_\infty}}{\Sigma} \text{Re}[\mathcal{Q}z_\infty(r - r_- + i\delta)],$$

$$z = \frac{z_\infty \rho + \mathcal{D}z_\infty^*}{\rho + \mathcal{D}}, \quad \rho = r - \frac{\mathcal{M}^* r_-}{2M} + i\delta. \quad (34)$$

Here a new radial coordinate is introduced $r = r_0 + r_-$, parameters r_- and N_- in terms of the physical charges read

$$r_- = \frac{M|\mathcal{Q}|^2}{|\mathcal{M}|^2}, \quad N_- = \frac{N|\mathcal{Q}|^2}{2|\mathcal{M}|^2}, \quad (35)$$

and δ is the same as in (27) (note that $N_0 = N - N_-$). The following expression for the real dilaton function is also useful:

$$e^{2(\phi - \phi_\infty)} = \frac{1}{\Sigma} \left| r + i\delta - \frac{Q\mathcal{Q}^*}{\mathcal{M}} \right|^2. \quad (36)$$

The solution obtained may be interpreted as the charged rotating Taub-NUT dyon in dilaton-axion gravity. It contains seven independent real parameters: a mass M , a rotation parameter a , a NUT-parameter N , electric Q and magnetic P charges (defined as in [12] to have the standard asymptotic normalization of the Coulomb energy), and asymptotic values of the axion κ_∞ and the dilaton ϕ_∞ (combined in z_∞). The complex axion-dilaton charge introduced through an asymptotic expansion

$$z = z_\infty - 2ie^{-2\phi_\infty} \frac{\mathcal{D}}{r} + O\left(\frac{1}{r^2}\right) \quad (37)$$

is determined by the electromagnetic charge and the complex mass:

$$\mathcal{D} = -\frac{Q^* a^2}{2\mathcal{M}}. \quad (38)$$

Note that this relation is *independent* of the rotation parameter a .

The new family contains as particular cases many previously known solutions to dilaton-axion gravity. For $N = P = 0$ the metric (31) and (32) corresponds to Sen's solution [4] up to some coordinate transformation (in this case the axion charge $A = 0$). For $a = 0$ (31)–(34) coincides (up to a transformation of the radial coordinate) with the six-parametric solution reported recently by Kallosh *et al.* [12], its three-parametric subfamily was also found by Johnson and Myers [13]. For $N = 0$, $a = 0$ we recover the five-parametric solution presented by Kallosh and Ortin [14], and, if in addition one of the charges Q, P is zero, the solution reduces to the Gibbons-Maeda-Garfinkle-Horowitz-Strominger black hole [15]. Finally, when $P = Q = 0$ we come back to the Kerr-NUT metric (26).

As in vacuum and electrovacuum cases, for $N \neq 0$ our solution cannot be properly interpreted as a black hole because of time periodicity which is to be imposed in the presence of the wire singularity [16]. We will still conserve the notation r_H^\pm for the values of radial coordinate marking positions of the surfaces where $\Delta = 0$:

$$r_H^\pm = M + r_-/2 \pm \sqrt{|\mathcal{M}|^2(1 - r_-/2M)^2 - a^2}. \quad (39)$$

For $N = 0$ the upper value r_H^+ corresponds to the event horizon of a black hole. The timelike Killing vector ∂_t becomes null at the surface $r = r_e(\theta)$,

$$r_e^\pm = M + r_-/2 \pm \sqrt{|\mathcal{M}|^2(1 - r_-/2M)^2 - a^2 \cos^2 \theta}, \quad (40)$$

which marks the boundary of a black hole ergosphere in the case $N = 0$. Inside the two-surface $r = r_e(\theta)$ the Killing vector $\partial_t - \Omega \partial_\varphi$ with some $\Omega = \text{const}$ may still be timelike, the boundary value of Ω at $r = r_H^+$ where it becomes null being

$$\begin{aligned} \Omega_H &= \frac{a}{2} \{ |\mathcal{M}|^2(1 - r_-/2M) \\ &\quad + M \sqrt{|\mathcal{M}|^2(1 - r_-/2M)^2 - a^2} \}^{-1}. \end{aligned} \quad (41)$$

For $N = 0$ this quantity has a meaning of the angular velocity of the horizon. The area of the two-surface $r = r_H^+$ is

$$A = 4\pi a / \Omega_H. \quad (42)$$

The square root in (40) becomes zero for the family of extremal solutions. This corresponds to the following relation between parameters:

$$|\mathcal{D}| = |\mathcal{M}| - a, \quad (43)$$

which defines a four-dimensional hypersurface in the five-dimensional space of Q, P, M, N, a . For extremal solu-

tions we have

$$r_H^{\text{ext}} = 2M - \frac{aM}{|\mathcal{M}|}, \quad \Delta^{\text{ext}} = (r - r_H^{\text{ext}})^2,$$

$$\omega^{\text{ext}} = \frac{2 \{ N \Delta^{\text{ext}} \cos \theta + a \sin^2 \theta [M(r - r_H^{\text{ext}}) + a|\mathcal{M}|] \}}{a^2 \sin^2 \theta - \Delta^{\text{ext}}}, \quad (44)$$

$$\Sigma^{\text{ext}} = 2M(r - r_H^{\text{ext}}) + \Delta^{\text{ext}} - a^2 \sin^2 \theta + 2a(|\mathcal{M}| + N \cos \theta).$$

The metric for the *nonrotating* extremal dilaton-axion Taub-NUT family reads

$$ds^2 = (1 - 2M/r)(dt + 2N \cos \theta d\varphi)^2 - (1 - 2M/r)^{-1} dr^2 - r(r - 2M)(d\theta^2 + \sin^2 \theta d\varphi^2). \quad (45)$$

In this case $r_H^{\text{ext}} = 2M$, this coincides with the curvature singularity. [Note that it is not so if $a \neq 0$, since then $\Sigma^{\text{ext}}(r_H^{\text{ext}}) \neq 0$.] For the dilaton we get, from (36),

$$e^{2(\phi - \phi_\infty)} = \frac{1}{r(r - 2M)} \left| r - 2M + \frac{i(QN - PM)Q^*}{|\mathcal{M}|^2} \right|^2. \quad (46)$$

Comparing (45) and (46) one can see that generically the string metric $ds_{\text{string}}^2 = e^{2\phi} ds^2$ has a nonsingular throat structure. However, if

$$QN = PM, \quad (47)$$

the dilaton factor (46) has zero as $r \rightarrow 2M$ and the string metric will have the same structure as in the case of the static dilaton electrically charged black hole [15] (to which our solution reduces if $N = P = 0$). Hence, regular Taub-NUT string throats form a three-parametric family corresponding to the hypersurface $|\mathcal{D}| = |\mathcal{M}|$ in the parameter space of M, N, P, Q , from which a two-dimensional subspace (47) has to be excluded. As it was shown recently by Johnson [17], some of the family of extremal Taub-NUT solutions have exact gauged Wess-Zumino-Witten (WZW) model counterparts.

In the rotating dyon black hole case $N = 0$, $a \neq 0$ our solution generalizes (and present in more concise form) Sen's solution [4] to include both electric and magnetic charges and, consequently, nonzero axion charge. The entropy can be shown to remain equal to a quarter of the event horizon area (42):

$$S = \pi \left(\mu + \sqrt{\mu^2 - 4J^2} \right), \quad \mu = 2M^2 - Q^2 - P^2, \quad (48)$$

where $J = aM$ is the angular momentum of a hole. The corresponding temperature is

$$T = \frac{\sqrt{\mu^2 - 4J^2}}{4\pi M \left(\mu + \sqrt{\mu^2 - 4J^2} \right)}. \quad (49)$$

In the extremality limit $\mu = 2|J|$, the entropy remains finite $S_{\text{ext}} = 2\pi|J|$ (and vanishing as $a \rightarrow 0$), while the temperature is zero in agreement with previous results [18].

VI. CONCLUSION

Using a three-dimensional σ -model formulation of the stationary $D = 4$, $p = 1$ EMDA theory we have found a ten-parametric noncompact internal symmetry group including dilaton-axion duality and Ehlers-Harrison-type symmetries. In a sense, this group provides a unification of target-space duality and dilaton-axion duality in $D = 4$. It also opens a new simple way to construct stationary solutions to the $D = 4$ EMDA system by transforming stationary vacuum solutions as well as already known solutions to the EMDA theory itself. The above formalism may be generalized to the case of the spacelike Killing vector field too. Furthermore, it can be shown that the target space (7) is a *symmetric Riemannian space*, on which the isometry group acts transitively. This reveals a close similarity between the present group and the Kinnersly $SU(2,1)$ group in the EM theory. It can be anticipated that in the case of *two* commuting Killing vector fields the system will possess an infinite-dimensional internal symmetry group analogous to the Geroch-Kinnersley-Chitre group for electrovacuum. In other words, further restricted to two dimensions, the EMDA system is likely to become fully integrable. This will be discussed in a forthcoming paper.

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