Fermion scattering off dilatonic black holes

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The scattering of massless fermions off magnetically charged dilatonic black holes is reconsidered and a violation of unitarity is found. Even for a single species of fermion it is possible for a particle to disappear into the black hole with its information content. The same conclusion is arrived at for chiral fermions as well.

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I. INTRODUCTION

In recent years there has been much interest in the physics of black holes. The issue which has engaged the attention of most workers is that of possible information loss. Matter falling into a black hole carries some information with it. This becomes inaccessible to the rest of the world, but may in principle be supposed to be stored inside the black hole in some sense. A problem arises when the black hole evaporates through the process of thermal Hawking radiation. The information does seem to be lost now [1].

Although there have been attempts at studying this problem in its full complexity [2], most authors have considered simplified models of black holes as in [3, 4]; see [5] for a review. We shall consider the extremal magnetically charged black hole solution of dilatonic gravity. This is a four-dimensional model involving an extra field, the dilaton, but for s-wave scattering of particles in the field of this black hole, the angular coordinates are not relevant and a two-dimensional effective action can be used [6]. If the energies involved are not too high, the metric and the dilaton field can be treated as external classical quantities and an amusing version of electrodynamics emerges, where the kinetic energy of the gauge field has a position-dependent coefficient [7].

The scattering of massless fermions has been considered in this context. The model admits a solution which is very close to the conventional solution of the Schwinger model, i.e., two-dimensional massless electrodynamics. In this solution, there is a massive free particle, but in the present case its mass becomes position dependent [7, 8]. To be precise, the mass vanishes near the *mouth* of the black hole but increases indefinitely as one goes into the *throat*. (The dilatonic field increases linearly with distance in the throat.) This is interpreted to mean that massless fermions proceeding into the black hole cannot go very far and have to turn back with probability one. Thus the danger of information loss is averted very simply.

In this article, we first reexamine the model by taking into account the possibility of alternative solutions. The Schwinger model possesses other solutions in addition to the conventional one, although this is not universally known. These correspond to different quantum theories built from the same classical theory. Different quantum theories can be constructed without violating gauge invariance by changing the definition of the point split fermionic currents [9]. By considering this freedom, we shall demonstrate that the problem of information loss can in fact appear even in the extremal magnetically charged dilatonic black hole.

Although the scattering of a classical *chiral* fermion was considered in [7], the quantum field theoretic discussion was restricted to Dirac fermions to avoid a gauge anomaly. However, the treatment of gauge anomalies due to chiral fermions is straightforward if bosonization techniques are used [10, 11]. Using these now standard ideas, we analyze the interaction of a chiral fermion with the black hole and find once again that it can fall in, so that the problem of information loss is present here too.

II. DIRAC FERMIONS IN A DILATON BACKGROUND

The model is described by the Lagrangian density [7, 8]

$$\mathcal{L} = \overline{\psi}(i\not{\partial} + e\not{A})\psi - \frac{1}{4}e^{-2\varphi(x)}F^{\mu\nu}F_{\mu\nu}, \tag{1}$$

where the Lorentz indices take the values 0,1 corresponding to a (1+1)-dimensional spacetime, e measures the coupling of the vector current corresponding to the massless fermion ψ to the gauge field A, and there is a dilatonic background $\varphi(x)$ whose dynamics we do not go into. It is clear that if $\varphi(x)$ vanishes, we get the well-known Schwinger model [12–14]. The model with nonvanishing $\varphi(x)$ has also been solved [7, 8] with the help of the usual scheme of bosonization. Here we discuss a solution in a different framework, which leads to vastly altered physical consequences.

In two dimensions we can always set

$$A_\mu = -\frac{\sqrt{\pi}}{e}(\widetilde{\partial}_\mu\sigma + \partial_\mu\widetilde{\eta}), \tag{2}$$

where

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$$\tilde{\partial}_\mu = \epsilon_{\mu\nu}\partial^\nu, \tag{3}$$

with $\epsilon_{01} = +1$ and $\sigma, \tilde{\eta}$ are scalar fields.

We shall restrict ourselves to the Lorentz gauge. From (2) we see that the field $\tilde{\eta}$ can be taken as a massless field with $\Box\tilde{\eta} = 0$. We introduce its dual through

$$\tilde{\partial}_\mu \eta(x) = \partial_\mu \tilde{\eta}(x). \tag{4}$$

These massless fields have to be regularized [13] but we shall not need the explicit form of the regularization.

The Dirac equation in the presence of the gauge field is

$$[i\partial\!\!\!/ + eA\!\!\!/]\,\psi(x) = 0. \tag{5}$$

This equation is satisfied by

$$\psi(x) = :\, e^{i\sqrt{\pi}\gamma_5[\sigma(x)+\eta(x)]} :\psi^{(0)}(x), \tag{6}$$

where $\psi^{(0)}(x)$ is a free fermion field satisfying $i\partial\!\!\!/\psi^{(0)}(x) = 0$.

We can calculate the gauge-invariant current using the point-splitting regularization. While constructing a gauge-invariant bilinear of fermions which in the limit of zero separation would give the usual fermion current, we can generalize the conventional construction [12]. We take

$$J^{\rm reg}_\mu(x) = \lim_{\epsilon\to 0}[\overline{\psi}(x+\epsilon)\gamma_\mu \;:e^{ie\int_x^{x+\epsilon} dy^\rho \left\{A_\rho(y) - 2\partial^\nu[\Phi(y)F_{\rho\nu}(y)]\right\}} : \psi(x) - \mathrm{VEV}]\,, \tag{7}$$

where Φ is a field which we shall relate to φ later on. The addition of the term containing Φ in the exponent represents a generalization of Schwinger's regularizing phase factor [9]. It preserves gauge invariance, Lorentz invariance, and even the linearity of the theory. The explicit coordinate dependence of Φ may come as a surprise, but it must be remembered that the model under discussion does not possess translation invariance because of the factor containing $\varphi(x)$ in the Lagrangian density (1). When φ is made dynamical, the related field Φ also becomes dynamical, and there is no conflict with translation invariance. In fact this freedom can be used to simplify the solution of the model enormously, as we shall see. Now using (2) and (6) together with

$$F_{\mu\nu} = \frac{\sqrt{\pi}}{e}\epsilon_{\mu\nu}\Box\sigma \tag{8}$$

we obtain the current which, up to an overall wave function renormalization, is equal to

$$J^{\rm reg}_\mu(x) \approx \;:\overline{\psi^{(0)}}(x)\gamma_\mu\psi^{(0)}(x): - i\sqrt{\pi}\lim_{\epsilon\to 0}\langle 0 \mid \overline{\psi^{(0)}}(x+\epsilon)\gamma_\mu\left[(\gamma_5\epsilon\cdot\partial + \epsilon\cdot\tilde{\partial})(\sigma+\eta) + 2\epsilon\cdot\tilde{\partial}(\Phi\Box\sigma)\right]\psi^{(0)}(x) \mid 0\rangle \tag{9}$$

$$= \;:\overline{\psi^{(0)}}(x)\gamma_\mu\psi^{(0)}(x): - \frac{1}{\sqrt{\pi}}\left[\frac{\epsilon_\mu\epsilon_\nu - \tilde{\epsilon}_\mu\tilde{\epsilon}_\nu}{\epsilon^2}\tilde{\partial}^\nu(\sigma+\eta) + 2\frac{\epsilon_\mu\epsilon_\nu}{\epsilon^2}\tilde{\partial}^\nu(\Phi\Box\sigma)\right], \tag{10}$$

where we have used the identity

$$\langle 0 \mid \overline{\psi^{(0)}}_\alpha(x+\epsilon)\psi_\beta(x) \mid 0\rangle = -i\frac{\epsilon\!\!/_{\beta\alpha}}{2\pi\epsilon^2}. \tag{11}$$

Now we take the symmetric limit, i.e., average over the point-splitting directions ϵ and finally obtain

$$J^{\rm reg}_\mu(x) = -\frac{1}{\sqrt{\pi}}\tilde{\partial}_\mu(\phi + \sigma + \Phi\Box\sigma + \eta), \tag{12}$$

where ϕ is a free massless bosonic field satisfying

$$-\frac{1}{\sqrt{\pi}}\tilde{\partial}_\mu\phi = \;:\overline{\psi}^{(0)}(x)\gamma_\mu\psi^{(0)}(x): \tag{13}$$

and thus representing the conventional bosonic equivalent of the free fermionic field $\psi^{(0)}$ [15]. We find

$$J^{\rm reg}_{\mu 5}(x) = \epsilon_{\mu\nu}J^\nu_{\rm reg}(x) \tag{14}$$

$$= -\frac{1}{\sqrt{\pi}}\partial_\mu(\phi + \eta + \sigma + \Phi\Box\sigma). \tag{15}$$

This implies that the anomaly in this regularization is

$$\partial^\mu J^{\rm reg}_{\mu 5} = -\frac{1}{\sqrt{\pi}}\Box(\phi + \eta + \sigma + \Phi\Box\sigma). \tag{16}$$

Note now that Maxwell's equation with sources, viz.,

$$\partial_\nu\left(\frac{F^{\nu\mu}}{g^2}\right) + eJ^\mu_{\rm reg} = 0, \tag{17}$$

where

$$g^2(x) = e^{2\varphi(x)}, \tag{18}$$

can be converted to the pair of equations

$$\left[\left(\frac{1}{g^2} + \frac{e^2}{\pi}\Phi\right)\Box + \frac{e^2}{\pi}\right]\sigma = 0 \tag{19}$$

and

$$\phi + \eta = 0. \tag{20}$$

The first equation (19), which depends on the choice of Φ, determines the spectrum of particles in the theory. The other equation (20), relating two massless free fields, has to be satisfied in a weak sense by imposing a subsidiary condition

$$(\phi+\eta)^{(+)} \mid \mathrm{phys}\rangle = 0 \tag{21}$$

to select out a physical subspace of states. One can ensure that $\phi+\eta$ creates only states with zero norm by taking η to be a negative metric field, i.e., by taking its commutators to have the "wrong" sign. The subsidiary condition then separates out a subspace with non-negative metric as usual.

Φ is as yet undetermined. We shall consider a few possible choices. The conventional choice [7, 8] is zero. (19) then becomes

$$\left[\Box + \frac{e^2 g^2}{\pi}\right]\sigma = 0. \tag{22}$$

This describes a particle of mass $\frac{eg(x)}{\sqrt{\pi}}$. Now g is related to φ, which is taken to vary linearly with distance in the throat of the black hole. The situation envisaged is that g vanishes at the mouth of the black hole, but rises indefinitely as one proceeds into the interior. The effect is that the mass of the particle vanishes at the mouth but rises indefinitely inside the throat. Since massless scalars are equivalent to massless fermions in two dimensions, it follows that one can think of an initial condition where a massless fermion starts at the mouth of the black hole and proceeds inwards. The fact that the mass involved in the equation of motion rises indefinitely means that the fermion cannot go arbitrarily far and is reflected back with unit probability. Thus the scattering of the fermion off the black hole is unitary and information is not lost.

On the other hand, if Φ is chosen to satisfy the condition

$$\frac{e^2}{\pi}\Phi = g^2, \tag{23}$$

(19) simplifies to

$$\left[\Box + \frac{e^2}{\pi(g^2 + g^{-2})}\right]\sigma = 0. \tag{24}$$

The mass of the particle now vanishes not only at the mouth, but also asymptotically in the interior of the black hole. In fact, the mass has a maximum somewhere in between. Therefore it is possible for a massless fermion to exist both at the mouth and in the interior, and the height of the barrier being finite, there is a finite amplitude for the fermion to go in and get lost. Thus the danger of information loss is *not* averted in this case.

A somewhat mundane case is when Φ is such that

$$\left(\frac{1}{g^2} + \frac{e^2}{\pi}\Phi\right) = 1, \tag{25}$$

and (19) simplifies to

$$\left[\Box + \frac{e^2}{\pi}\right]\sigma = 0. \tag{26}$$

This means that the usual massive free scalar field of the Schwinger model is recovered. The modified Schwinger model thus accommodates the unmodified solution with this altered definition of currents. In this case, the mass of the scalar field does not vanish anywhere, so there is no fermion in the spectrum and one speaks of the fermion as being confined. Hence, if the scattering of fermions is to be considered, this choice of Φ is not relevant.

A more dramatic case is when Φ is allowed to go to infinity. In this case the free scalar field becomes exactly massless. So the fermion is massless at all positions. This fermion travels freely into the black hole and *all* information is lost.

III. THE CASE OF CHIRAL FERMIONS

If we replace the Dirac fermion considered above by a right moving chiral fermion, the Lagrangian density can be written as

$$\mathcal{L} = i\bar{\psi}_R \not{D} \psi_R + \frac{1}{2}e^{-2\varphi(x)}F_{01}^2. \tag{27}$$

We shall use the method of bosonization, to take the anomaly into account. The fermion is integrated out, so that an effective action is obtained. The definition of the effective action involves a regularization and the result depends on which one is chosen. In any case, the effective action is nonlocal, but can be made local by introducing an auxiliary bosonic field. If a Dirac fermion were integrated out, a full boson field would emerge, but as a chiral fermion is involved, a chiral boson emerges. The action that is obtained is given by the Lagrangian density [11]

$$\mathcal{L} = -\dot{\phi}\phi' - (\phi')^2 - 2\tilde{e}\phi'(A_0 + A_1) - \frac{1}{2}\tilde{e}^2(A_0 + A_1)^2 + \frac{1}{2}a\tilde{e}^2 A_\mu A^\mu \frac{1}{2}e^{-2\varphi(x)}F_{01}^2, \tag{28}$$

where ϕ is the auxiliary boson field. The free part of the Lagrangian for this field has the standard form for right moving chiral bosons [16]. The quantity $\tilde{e}$ stands for $\frac{e}{2\sqrt{\pi}}$ and a is a parameter [10] that distinguishes the members of a family of regularizations. Time and space derivatives are indicated by overdots and primes, respectively. The theory described by this Lagrangian density is equivalent at the quantum level to some regularized version of the fermionic theory (27) we started with. The advantage of using this formulation is that the anomaly is incorporated into it, so that even tree level results of the bosonic theory would show effects of the anomaly.

Accordingly, we proceed to carry out a Hamiltonian analysis of the theory (28). The canonical momenta are

$$\pi_\phi = -\phi' \tag{29}$$

for the chiral boson and

$$\pi_0 = 0, \tag{30}$$

$$\pi_1 = e^{-2\varphi}F_{01} \tag{31}$$

for the gauge fields. The first two of these are constraint equations. The canonical Hamiltonian density is

$$\mathcal{H} = \frac{1}{2}e^{2\varphi}\pi_1^2 + \pi_1 A_0' + (\phi')^2 + 2\tilde{e}\phi'(A_0 + A_1) + \frac{1}{2}\tilde{e}^2(A_0 + A_1)^2 - \frac{1}{2}a\tilde{e}^2 A_\mu A^\mu. \tag{32}$$

Although it contains an explicit space dependence through φ, it is not an explicit function of time. Hence the Hamiltonian is in fact preserved in time. The requirement that the constraints (29) and (30) are also preserved in time is found to lead to only one additional constraint provided $a \neq 1$. This is the Gauss law

$$\pi_1' - 2\tilde{e}\phi' + \tilde{e}^2[(a-1)A_0 - A_1] = 0. \tag{33}$$

If $a \neq 1$, these three constraints form a second class set consistent with the dynamics and can be used to eliminate the variables π_0, A_0, π_ϕ. The remaining fields ϕ, A_1, π_1 are governed by the reduced Hamiltonian obtained from (32) by applying the constraints (29), (30), (33):

$$\mathcal{H}_{\rm red} = \frac{1}{2}e^{2\varphi}\pi_1^2 + \frac{1}{2(a-1)\tilde{e}^2}(\pi_1')^2 + \frac{a+1}{a-1}(\phi')^2 - \frac{2}{(a-1)\tilde{e}}\phi'\pi_1' + \frac{2\tilde{e}a}{a-1}\phi' A_1 - (a-1)^{-1}A_1\pi_1' + \frac{a^2\tilde{e}^2}{2(a-1)}A_1^2. \quad (34)$$

However, the elimination of π_ϕ through the constraint (29) leaves behind the noncanonical Dirac brackets

$$\{\phi(x), \phi(y)\}_D = \frac{1}{4}\epsilon(x^1 - y^1) \quad (35)$$

characteristic of chiral bosons. Here, ϵ stands for the sign function and the other Dirac brackets are canonical.

Using these brackets and the reduced Hamiltonian, one obtains the equations of motion

$$\dot{\phi} = -\frac{a+1}{a-1}\phi' + \frac{1}{(a-1)\tilde{e}}\pi_1' - \frac{a\tilde{e}}{a-1}A_1, \quad (36)$$

$$\dot{A}_1 = e^{2\varphi}\pi_1 - \frac{1}{(a-1)\tilde{e}^2}\pi_1'' + \frac{2}{(a-1)\tilde{e}}\phi'' + (a-1)^{-1}A_1', \quad (37)$$

$$\dot{\pi}_1 = -\frac{a^2\tilde{e}^2}{a-1}A_1 - \frac{2a\tilde{e}}{a-1}\phi'(a-1)^{-1}\pi_1'. \quad (38)$$

A little algebra now shows that

$$\dot{\phi} - \frac{1}{a\tilde{e}}\dot{\pi}_1 = -\phi' + \frac{1}{a\tilde{e}}\pi_1', \quad (39)$$

$$\left(\Box + \frac{a^2\tilde{e}^2}{a-1}e^{2\varphi}\right)\pi_1 = 0, \quad (40)$$

$$\left(\Box + \frac{a^2\tilde{e}^2}{a-1}e^{2\varphi}\right)\left(A_1 + \frac{2}{a\tilde{e}}\phi' - \frac{1}{a^2\tilde{e}^2}\pi_1'\right) = 0. \quad (41)$$

This means that the theory contains a right moving chiral boson $\phi - \frac{1}{a\tilde{e}}\pi_1$ and a massive boson π_1 (together with its conjugate momentum) with position-dependent mass $\frac{a\tilde{e}}{\sqrt{a-1}}e^{\varphi}$. Here it is assumed that $a > 1$. Indeed, for the foregoing constraint analysis to be valid, $a \neq 1$. Furthermore, the Hamiltonian density can be written as

$$\mathcal{H}_{\rm red} = \frac{a^2\tilde{e}^2}{2(a-1)}\left[A_1 + \frac{2}{a\tilde{e}}\phi' - \frac{1}{a^2\tilde{e}^2}\pi_1'\right]^2 + \frac{a-1}{2a^2\tilde{e}^2}(\pi_1')^2 + \frac{1}{2}e^{2\varphi}\pi_1^2 + \left(\phi' - \frac{1}{a\tilde{e}}\pi_1'\right)^2, \quad (42)$$

which shows that the Hamiltonian is positive definite if and only if $a > 1$. In this case, the theory is unitary in spite of the anomaly [10]. The case $a = 1$ can be analyzed separately, but does not lead to anything essentially different. The massive boson disappears (it may be considered to become infinitely massive), but the chiral boson remains.

The new feature which is present in this analysis of the scattering of a chiral fermion is the right moving chiral boson. This is exactly massless irrespective of its position and can be used to construct a chiral fermion over all space. This chiral fermion goes into the black hole without any hindrance. Thus, in the case of chiral fermions we find that the problem of information loss becomes very real when standard regularization procedures followed in the study of the chiral Schwinger model are adopted.

The massive boson with position-dependent mass that also occurs in this case is analogous to what is found in the analysis of the scattering of a Dirac fermion by the black hole. The mass of the boson vanishes at the mouth of the black hole, but rises indefinitely inside it, so that any particle which goes in must get totally reflected. Since the massless boson at the mouth can be used to construct a massless fermion, this massless fermion, if traveling to the right, i.e., into the black hole, also gets totally reflected. Thus there are some chiral fermions that do not fall into the black hole. However, as shown above, this argument depends crucially on the definition of composite operators used in the theory and a fermion traveling into the black hole can be easily obtained with a suitably altered regularization. We do not go into details because the main interest of this case arises from the chiral fermion that may be constructed from the chiral boson mentioned above.

IV. CONCLUSION

To summarize, we have looked at the extremal magnetically charged black hole in a dilatonic background. First we considered Dirac fermions using a generalized construction of fermion bilinears. We point split the current which is formally defined as the product of two fermionic operators. Schwinger has prescribed the insertion of an exponential of a line integral of the gauge field to make the product gauge invariant. However, his choice was only one of many possible choices; see, e.g., [9]. We have inserted an extra factor which involves the field strength of the gauge field and a nondynamical function of space-time coordinates and therefore does not interfere with the gauge-invariance of the product. This is not the most general gauge invariant regularization possible in this approach, but is enough to illustrate the range of possibilities. By varying the regularization, the equations of motion of the Schwinger model can be converted to free field equations with the mass exactly as in the usual case, or going to zero at *both* ends of the spatial axis or even vanishing everywhere. In the first case, there is no fermion in the spectrum at all and the question of scattering does not arise. In the other two cases, the massless fermion is *not* totally reflected, so that the problem of information loss appears unless further gravitational effects can change the scenario.

The alteration introduced by us is in the definition of fermion bilinears as composite operators and this is well known to have a lot of flexibility. Formally, in the limit $\epsilon \to 0$, the phase factor does reduce to unity, so that the definition of the bilinears adopted in this paper cannot be thought of as changing the underlying *classical* theory. Only the quantum theory, which is not fully defined until the definition of composite operators is specified, is altered. This alteration takes the form of a renormalization of the effective coupling constant in the theory. The dilatonic field, which entered the model through this coupling constant, can thus be said to get effectively transformed in the quantum theory. However, this change is not a real one as far as the dilatonic field is concerned. This can be seen by considering the kinetic energy term of the dilaton field, which does not get altered but, in the approximation made by us following [7, 8], is simply neglected.

In the second part of the paper, we considered chiral fermions instead of Dirac fermions. This gives rise to an anomalous gauge theory which can be bosonized for a well-known family of regularizations. The only difference from the published literature [10] is that now because of the dilaton field there is a position-dependent factor in the Lagrangian. The calculations go through essentially unchanged and the physical spectrum is seen to contain a chiral boson or equivalently a chiral fermion and a massive boson which now has a position-dependent mass. The unconfined chiral fermion can travel freely into the black hole. Thus, even with a standard regularization, a chiral fermion has a nonvanishing chance of falling into the black hole, thereby causing a loss of information and a violation of unitarity.

It should also be mentioned that if *several* species of fermions are included, the problem of information loss appears automatically [7]. This is in keeping with our finding that magnetically charged black holes do not necessarily behave like elementary particles in scattering incident fermions. Thus there are several models where information loss occurs and unitary S matrices cannot be constructed.

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