Dirac quantization of gravity-Yang-Mills systems in 1+1 dimensions

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In two dimensions a large class of gravitational systems including, e.g., R^2 gravity can be quantized exactly also when coupled dynamically to a Yang-Mills theory. Some previous considerations on the quantization of pure gravity theories are improved and generalized.

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In recent years the study of two-dimensional (2D) exactly solvable field theories has attracted considerable interest. One of the areas of investigations is 2D Yang-Mills (YM) theory (on a cylinder in a Hamiltonian approach [1,2] or on an arbitrary Riemann surface when evaluating the partition function [3]); other models of interest are gravitational ones such as the one for a 2D black hole [4], the Jackiw-Teitelboim model [5], or the Katanaev-Volovich (KV) model [6]. The first main purpose of this work is to show that the exact quantum integrability extends to the combined treatment of the YM theory and a large class of gravitational systems.[1]

The gravitational part of the action considered in this work will be

$$S_G = \int [\pi_\omega d\omega + \pi_a De^a - V(\pi_\omega, \pi^2)\varepsilon], \qquad V = v(\pi_\omega) + \frac{\tau}{2}\pi^2 \ , \tag{1}$$

in which the basic fields are the zweibein and spin-connection one-forms e^a and ω, respectively, as well as the functions π_a and π_ω. $De^a \equiv de^a + \varepsilon^a{}_b\omega \wedge e^b$ is the torsion two-form, $\pi^2 \equiv \pi^a\pi_a \equiv 2\pi_+\pi_-$, and $\varepsilon \equiv e^+ \wedge e^- \equiv ed^2x$ with $e \equiv \det(e_\mu{}^a)$ denotes the ε tensor or metric-induced volume form. v is some potential and τ a constant. For the case that v is chosen as $-(1/4\gamma)(\pi_\omega)^2 + \lambda$ and $\tau \neq 0$ our action (1) yields, after elimination of π_a and π_ω (use $*\varepsilon = -1$), 2D gravity with torsion [6,8,9]:

$$S_G^{\mathrm{KV}} = \int \left(\gamma d\omega \wedge *d\omega - \frac{1}{2\tau} De^a \wedge *De_a - \lambda\varepsilon\right) \ , \tag{2}$$

the most general Lagrangian in two dimensions yielding second-order differential equations for e^a and ω. The same v but with τ=0 is analogously found to describe torsionless R^2 gravity [10]. For $V \propto \pi_\omega$ the action S_G describes de Sitter gravity (the Jackiw-Teitelboim model [5,11]), whereas $V \propto (\pi_\omega)^{-2}$ effectively describes 4D spherical symmetrical gravity. V= const, furthermore, yields a gravity theory basically equivalent [12] to the string-inspired 2D black hole gravity [4] for a redefined metric; this equivalence, however, loses its attractiveness when one couples the action to nonconformal matter using the redefined metric. Most of the specific models have been quantized in a Dirac approach already (cf. citations above); moreover, this is also true for the general action S_G in the torsionless case τ=0 [13]. It is the second main purpose of this work that these quantizations, which came down to the quantization of a one-dimensional point particle system, in many cases have to be supplemented by appropriate discrete indices, originating from nontrivial topological properties of the constraint surface.

The Yang-Mills part of our action has the standard form $(1/4\kappa^2)\int \mathrm{tr}(F \wedge *F)$, where $F = dA + A \wedge A$ and the trace is taken in the adjoint representation. Rewriting this action in first-order form, it reads

$$S_{\mathrm{YM}} = \int \mathrm{tr}(EF + \kappa^2 E^2 \varepsilon) \ , \tag{3}$$

the "electric fields" E begin (Lie-algebra-valued) functions. The coupling to the gravity sector is seen to be separated to the second term now. For simplicity we will assume the gauge group G to be compact and simply connected, which implies also that G is simple. But it would be straightforward to generalize what follows, e.g., to arbitrary compact groups G (gaining a Θ angle for every U(1) factor, cf., e.g., [2]).

Let us turn to the phase-space structure of the theory. Since $S = S_G + S_{\mathrm{YM}}$ is already in first-order form, we can read off the Poisson brackets and constraints directly. The canonically conjugates are $(e_1^a, \omega_1, A_1; \pi_a, \pi_\omega, E)$, respectively, whereas the zero components of the basic one-forms enforce the constraints $(\partial \equiv \partial/\partial x^1)$

$$G_a = \partial\pi_a + \varepsilon_a{}^b\pi_b\omega_1 - \varepsilon_{ab}e_1{}^b[V - \kappa^2 trE^2] \approx 0 \ , \tag{4}$$

$$G_\omega = \partial\pi_\omega + \varepsilon^a{}_b\pi_a e_1{}^b \approx 0 \ , \tag{5}$$

$$G = \nabla_1(A)E \equiv \partial_1 E + [A_1, E] \approx 0 \ , \tag{6}$$

and can be regarded as arbitrary Lagrange multipliers within the Hamiltonian

$$H = -\oint dx^1 e_0{}^a G_a + \omega_0 G_\omega + \mathrm{tr}(A_0 G) \ . \tag{7}$$

We observe that in two dimensions the addition of a dy-

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[1]The classical local integrability of YM coupled to the KV model, defined through S_G^{KV} below, has been observed already in [7].

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namical gravity sector leaves the Yang-Mills Gauss law $G \approx 0$ completely unchanged. This is in contrast with four space-time dimensions where the covariant derivative ∇ contains also a gravitational connection resulting from the fact that the electric fields are not functions there but one-forms on a three-manifold (c.f., e.g., [14]). Since, furthermore, $\partial(\mathrm{tr}E^2) = \mathrm{tr}GE/2 \approx 0$, on shell the YM theory modifies the gravitational theory only via dynamically shifting the cosmological constant of the gravity sector by the YM Hamiltonian[2] $H_{YM}^{(0)} \equiv -\kappa^2 \oint \mathrm{tr}E^2 dx^1$.

The constraints (4–6) are first class: A straightforward computation yields $\{G_a, G_b\} = -\varepsilon_{ab}(dV/d\pi_i)G_i\delta$, where $\pi_i \equiv (\pi_a, \pi_\omega)$ and the arguments on the circle S^1 have been suppressed [i.e., $\delta \sim \delta(x^1 - y^1)$, etc.]. Furthermore, one finds $\{G_a, G_\omega\} = \varepsilon_a{}^b G_b \delta$ as G_ω generates the local frame rotations, whereas the $\{G, G\}$ brackets follow the Lie algebra of the chosen gauge group.

To quantize the system we choose our wave functionals to depend on π_a, π_ω, A_1 and replace $e_1{}^a, \omega_1, E$ by the appropriate derivative operators. Our ordering prescription for the quantum constraints is to put all derivative operators to the right. This reproduces (up to a factor $i\hbar$ for the structure functions) the classical constraint algebra and thus does not lead to an anomaly. Obviously this is always the case with the above ordering prescription when one deals with classical constraints that, as here, are at most linear in those phase-space coordinates which are represented by the derivative operators.[3]

For the solution of the quantum Gauss law $G\Psi = 0$ we can refer the reader to the extensive literature. The basic result is (cf., e.g., [2]) that the functional $\Psi[\pi_b, \pi_\omega, A_1(x^1)]$ can be written as a function $\Psi[\pi_b, \pi_\omega, a]$ of a constant element a of the corresponding Cartan subalgebra (CSA) which is gauge related to $A_1(x^1)$; due to a residual gauge freedom, Ψ is, moreover, invariant under affine Weyl transformations so that the fundamental domain of definition of Ψ as a function of a is the Weyl cell of the CSA. For the simplest case of SU(2), e.g., Ψ is a periodic function of $a \in R$, and similarly for the case of SU(3) a function on a triangle, "periodically" continued to the plane via Weyl reflections and translations. The Hamilton operator of pure Yang-Mills theory $H_{\mathrm{YM}}^{(0)}$ projected onto this physical subspace is proportional to the ordinary Laplacian on the CSA. The same projection yields also a natural measure $\mu(a)$ with respect to which $H_{\mathrm{YM}}^{(0)}$ is self-adjoint.

Because of the finite size of the Weyl cell $H_{\mathrm{YM}}^{(0)}$ has a discrete spectrum ϵ_k, $k \in N$, and we can expand our gravity-Yang-Mills functional Ψ onto an orthogonal set of eigenfunctions $\chi_{k,l}(a)$, l labeling possible degeneracies of ϵ_k:

$$\Psi[\pi_+, \pi_-, \pi_\omega, a] = \sum_{k,l} \psi_{k,l}[\pi_+, \pi_-, \pi_\omega]\chi_{k,l}(a) \ . \tag{8}$$

Applying next the remaining quantum constraints $G_\pm, G_\omega$ to this expansion, we see that each of the $\psi_{k,l}$ has to be annihilated by the corresponding operators in which $-\kappa^2 \mathrm{tr}E^2(x^1)$ has been replaced by ϵ_k. Let us denote these modified operators by $G_{\pm,(k)}$ and analogously $v_{(k)} := v + \epsilon_k$, $V_{(k)} := V + \epsilon_k \equiv v_{(k)}(\pi_\omega) + \tau\pi^2/2$. Next one finds the combination

$$\pi^a G_{a,(k)} + V_{(k)} G_\omega = \tfrac{1}{2}\partial(\pi^2) + V_{(k)}\partial\pi_\omega \tag{9}$$

to act in a purely multiplicative way on the wave functionals. Multiplying (9) from the left by the integrating factor $\exp(\tau\pi_\omega)$, this yields (no sums)

$$\partial Q_{(k)}\psi_{k,l} = 0 \ , \tag{10}$$

$$Q_{(k)}(\pi^2, \pi_\omega) = \tfrac{1}{2}\pi^2 \exp(\tau\pi_\omega) + \int^{\pi_\omega} v_{(k)}(u)\exp(\tau u)du \ . \tag{11}$$

Equation (10) is a restriction to the support or the domain of definition of $\psi_{k,l}$. Viewing $\psi_{k,l}$ as a functional of (parametrized and connected) loops in the three-dimensional target space $(\pi_+, \pi_-, \pi_\omega)$, this constraint also has a simple geometrical interpretation: The domain of definition of $\psi_{k,l}$ is restricted to those loops which lie entirely within one of the two-surfaces $\mathcal{M}_q$ generated by setting $Q_{(k)}$ to some constant q.

There is some more or less marginal additional restriction to the domain of definition of $\psi_{k,l}$ concerning the (generically isolated) target space points $\pi_a = 0$, $\pi_\omega = \alpha_c$, where α_c labels the zeros of $v_{(k)}$, i.e., $v_{(k)}(\alpha_c) = 0$. Loops in support of $\psi_{k,l}$ may not pass such points, which we will call "critical" in the following; an exception to this are the constant loops coinciding with a critical point. Clearly this restriction, which is an obvious consequence of the quantum version of the constraints (4) and (5), concerns only loops on a target space surface $\mathcal{M}_q$ with a "critical value" $q_c \equiv Q_{(k)}(0, \alpha_c)$ of q.

In this restricted domain of definition the wave functional $\exp(\phi[\pi_+, \pi_-, \pi_\omega])$ with

$$\phi = \begin{cases} 0, & \pi_+ \equiv \pi_- \equiv 0, \\ -\frac{i}{\hbar}\oint \ln|\pi_+| d\pi_\omega = \frac{i}{\hbar}\oint \ln|\pi_-| d\pi_\omega, & \text{otherwise}, \end{cases} \tag{12}$$

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[2] This suggest also that there should be some connection to [15] where the authors allowed for a dynamical cosmological constant within the reformulated 2D black hole gravity so as to reinterpret the resulting theory as a connection flat gauge theory for the centrally extended Poincaré group. Indeed, choosing as our potential $V = 1/4\kappa^2$, as YM gauge group the real line, and shifting E by $-1/2\kappa^2$, the limit $\kappa \to 0$ reproduces that theory.

[3] One of the referees suggested to cite here [16] where it is shown [for the case of model (2)] that also a naively Hermitian operator ordering of the quantum constraints is anomaly-free (in any reasonable regularization). The resulting difference in the solution of the quantum constraints may be, however, absorbed when constructing the inner product [9] and we will not comment here further on this more complicated operator ordering.

can be seen to be a particular (continuous) solution to the quantum constraints (cf. also [9,10]). Thus the general solution to the gravity constraints can be written as

$$\psi_{k,l} = \exp(\phi)\tilde{\psi}_{k,l} \tag{13}$$

with a $\tilde{\psi}_{k,l}$ which is invariant under the Lie derivative part of the $G_{a,(k)}, G_\omega$ constraints, i.e., under infinitesimal classical gauge transformations. Now, on any of the surfaces $\mathcal{M}_q$, introduced above, the flow of the constraints is transitive, except precisely for the critical points, which are fixed points under this flow. This is most easily seen by noting that on any connected part of this surface where $\pi_+ \neq 0$ ($\pi_- \neq 0$) we can use $(G_\omega/\pi_+, G_+/\pi_+)$ $[(G_\omega/\pi_-, G_-/\pi_-)]$ as conjugate variables to the local coordinates (π_+, π_ω) $[(\pi_-, \pi_\omega)]$ of $\mathcal{M}_q$; furthermore $\{\pi_a, G_b\} = -\varepsilon_{ab}V\delta$. However, the wave functions $\tilde{\psi}_{k,l}$ do not depend only on q as one might suppose at first sight. This is so because certainly only loops from the same homotopy class and the same component of $\mathcal{M}_q$ can be deformed into each other by means of the constraints. Thus $\tilde{\psi}_{k,l}$ is a function of q, $\pi_0(\mathcal{M}_q)$, and $\pi_1(\mathcal{M}_q)$ (as well as the fixed points, if $q = q_c$). Labeling the elements of the latter two discrete groups by n_0^q and n_1^q, and suppressing the fixed points for a moment, we find

$$\tilde{\psi}_{k,l} = \tilde{\psi}_{k,l}(q, n_0^q, n_1^q) \ , \tag{14}$$

which together with (8), (12), and (13) describe the general solution of the quantum constraints (4)–(6).

To illustrate the above considerations, let us regard some examples: $V_{(k)} = \pi_\omega$ (for some fixed k, dropping this index further on within the paragraph) implies $Q = \frac{1}{2}[\pi^2 + (\pi_\omega)^2]$. Putting this to a constant q, we obtain the typical Lorentz orbits in a three-dimensional "Minkowski space,"[4] i.e., two-sheet hyperboloids for $q > 0$, which implies $n_0^+ \in \{1,2\}$ and $n_1^+ = 0$, one-sheet hyperboloids for $q < 0$, which implies $n_0^- = 0$ and $n_1^- \in Z$, as well as the future and past light cone "separated" by the origin for the critical value $q = q_c = 0$. Because of the latter parts the resulting orbit space is non-Hausdorff (such as the Lorentz orbit space), and there arises some arbitrariness in determining $n_{0,1}^0$ for this value of q: The origin and the light cones have no disjoint neighborhoods so that, e.g., continuous functions on this space would identify them ($\Rightarrow n_0^0 = 0$, $n_1^0 = 0$); on the other hand we know that there are no loops passing through the origin since it is critical ($\Rightarrow n_0^0 \in \{1,2\}$, $n_1^0 \in Z$, plus the origin as an own orbit).

As a second example let us consider R^2 gravity with potential $V = 3\pi_\omega^2/2 - \frac{3}{4}$ coupled to SU(2)-YM, yielding $\epsilon_k = k^2/4$ (for $\kappa^2 = 2$, if we choose $x^1 = 0 \sim x^1 = 1$). We obtain

$$2Q_{(k)} = \pi^2 + \pi_\omega^3 + (2\epsilon_k - 3/2)\pi_\omega .$$

For $k = 1$ there are two critical values of q: $\alpha_c = \pm 1/\sqrt{3} \to q_c = \mp 1/\sqrt{27}$, whereas for $k = 2, 3, \ldots$ there are no critical values of q. In the latter case the resulting two-surfaces are all connected and simply connected. This is also true for $\epsilon = \epsilon_1 = \frac{1}{4}$ and $q \notin [-1/\sqrt{27}, 1/\sqrt{27}]$, whereas in the case $k = 1$ and $q \in]-1/\sqrt{27}, 1/\sqrt{27}[$ the $Q_0 = q$ surface is connected but has the fundamental group of a pointed torus. At $q = q_c$ there arises a similar situation as in the first example. Thus in this example the wave functions have the form

$$\Psi = \exp(\phi)\left(\tilde{\psi}_1(q, n_1)\chi_1(a) + \sum_{k\geq 2}\tilde{\psi}_k(q)\chi_k(a)\right) \ , \tag{15}$$

where ϕ is defined in (12), χ_k is a periodic function of one argument, and ψ_k is a function of one unbounded variable except for $k = 1$, $n_1 \neq 0$, in which case it has support $[-1/\sqrt{27}, 1/\sqrt{27}]$.

More generally the situation can be depicted as follows: For any fixed value q and ϵ_k Eq. (11) induces a curve in a π^2 over π_ω diagram. If this curve has no intersections with the π_ω axis, $\mathcal{M}_q$ has two simply connected components. Otherwise (and for $q \neq q_c$) $\mathcal{M}_q$ is always connected and the number of basic noncontractible loops is by one larger than the number of intersections of the $\pi^2(\pi_\omega)$ curve with the π_ω axis. A change of this number can occur only at critical values of q, which correspond to curves having at least one sliding intersection with the π_ω axis. All these surfaces $\mathcal{M}_q$ are noncompact and the spectrum of q ranges over all of $\mathbb{R}$.[5]

Let us conclude with some remarks. First, already the example of Lorentz transformations in a Minkowski space shows that in general orbits cannot be (uniquely) characterized by means of continuous invariants only: In addition to the invariant length of a vector of this space, one needs also some (discontinuous) sign functions and, to distinguish the origin from the light cones, even a distribution. This is the reason for the quantum numbers n_0 and n_1 within the wave functions $\psi_{k,l}$: $Q_{(k)}$ is the continuous invariant on the underlying function space, n_0 and n_1 correspond to discontinuous (invariant) functions on the latter, and the critical points are the counterpart to the origin of the Minkowski space example above. From this perspective it comes at no surprise that in addition to (14) also

$$\sum_c C_c\delta[\pi_+]\delta[\pi_-]\delta[\pi_\omega(x^1) - \alpha_c], \quad C_c \in \mathbb{C} \tag{16}$$

[4]This has to do with the fact that the gravitational action for the above potential can be reinterpreted as the one of a πF theory for gauge group $SO(2,1)_e \sim PSL(2,R)$ [11], or rather its universal covering, as pointed out in [17,18].

[5]This picture is changed when regarding the gravitational theories corresponding to a Euclidean signature. Some values of q generate compact target space surfaces $\mathcal{M}_q$ then; on the latter the Wick rotated phase factor (12) is globally defined only for some values of q, which leads to a discretization of the corresponding part of the spectrum [18–20].

solves the quantum constraints, where, as defined already above, the set $\{\alpha_c\}$ labels the zeros of $v_{(k)}$.

Second, we still have to define an inner product for the $\tilde{\psi}_{k,l}$ [for fixed k and l only, since the $\chi_{k,l}$ in (8) are orthogonal by construction]. On parts of the phase space which do not contain critical points the Dirac observable conjugate to $q = \oint Q_{(k)} dx^1$ can be put into the form

$$p = \oint \exp(-\tau\pi_\omega)\frac{e_1^-}{\pi_+}dx^1 \sim \oint \exp(-\tau\pi_\omega)\frac{{e_1}^+}{\pi_-}dx^1. \qquad (17)$$

Replacing ${e_1}^\pm$ by the corresponding functional derivative operator, it acts as $(\hbar/i)(d/dq)$ on $\tilde{\psi}_{k,l}$. Requiring that this fundamental Dirac observable shall be represented by a Hermitian operator, restricts the measure to be proportional to dq within any interval of q not containing a critical value q_c. The implementation of the quantum numbers n_0 and n_1, however, seems not determined by this procedure and a further investigation of this point would be interesting.

Let me remark that it is probably incorrect to just neglect the solutions (16). This becomes most apparent in the extreme case $V_{(k)} \equiv 0$, where any constant loop on the π_ω axis becomes critical; on the classical level these solutions are pared with the compactifications of Minkowski space along the boost orbits, yielding Misners' two-dimensional analogue of a Taub-NUT (Newman-Unti-Tamburino) space [21]. So, neglecting the solutions (16) in this case comes down to throwing away about a third of the reduced phase space. [The remaining "two-thirds," represented by (14) on the quantum level, correspond to a Minkowski space factored along the translational isometries of the flat metric, thus, in part, also incorporate classical solutions with closed timelike curves.] In the more generic case $V_{(k)} \not\equiv 0$ the neglecting of (16) would still change the degeneracy of the spectrum of the Dirac observable q. [In the Euclidean formulation of the theory the omission of (16) in some case even leads to a change of the values appearing in the spectrum of q [18].]

In this paper we carefully constructed the general solutions to the quantum constraints of many two-dimensional gravity theories coupled to YM theories. Open technical questions concern the construction of an inner product and an inclusion of the solutions (16). Furthermore, the treatment of conceptual questions of quantum gravity seems rewarding at this point: First, since the classical solutions include black-hole-type solutions for some choices of V, and, second, since S reduces to a reparametrization invariant formulation of a pure 2D YM theory for $V \equiv 0$ so that the models comprised in the action (1) and (3) may well be used for testing and developing concepts to solve the "problem of time" [22] (cf. [9,18]). From the mathematical point of view the evaluation of the partition function for S_G would be an interesting open task (cf. also [23]).

Most noteworthy, however, is the recent observation [18,19] that the gravity action (1) as well as any Yang-Mills action (3) may be seen to be a special case of a σ model characterized by a Poisson structure on the target space. This not only allows for a simplification of some of the considerations performed in the present paper, but also for a generalization of them to a new class of models defined on some arbitrary two-dimensional world-sheet manifold.

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