

## Stability of the black hole horizon and the Landau ghost

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(Received 14 June 1994)

The stability of the black hole horizon is demanded by both cosmic censorship and the generalized second law of thermodynamics. We test the consistency of these principles by attempting to exceed the black hole extremality condition in various processes in which a  $U(1)$  charge is added to a nearly extreme Reissner-Nordström black hole charged with a *different* type of  $U(1)$  charge. For an infalling spherical charged shell the attempt is foiled by the self-Coulomb repulsion of the shell. For an infalling classical charge it fails because the required classical charge radius exceeds the size of the black hole. For a quantum charge the horizon is saved because, in order to avoid the Landau ghost, the effective coupling constant cannot be large enough to accomplish the removal of the horizon.

PACS number(s): 04.20.Dw, 04.70.Bw, 11.10.Jj, 12.20.Ds

### I. INTRODUCTION

Notwithstanding various possible exceptions, the principle of cosmic censorship is a popular tenet of belief in black hole physics. This principle rules that the black hole event horizon cannot be removed because that would expose naked singularities to distant observers. Likewise, the disappearance of the event horizon would violate the generalized second law of thermodynamics inasmuch as the horizon area is associated with entropy which would thereby disappear without any obvious way to compensate for its loss. For these reasons processes which seem to have a chance of eliminating the event horizon must be unphysical. Devising candidate processes and finding out how they fail turns out to be a source of physical insight into black holes, and even into more mundane physics.

In this paper we inquire into the physics that defends the horizon from attempts to transcend the extremality condition for a Reissner-Nordström black hole. As is well known, for such a black hole the charge must not exceed the mass (in units with  $G = c = \hbar = 1$ ); otherwise the Reissner-Nordström solution contains no horizon. Attempts to violate this condition by adding to a nearly extreme Reissner-Nordström black hole a particle with charge of the same sign as the hole's and with charge-to-mass ratio larger than unity are known to be defeated by Coulomb repulsion. In fact, the energy required to get the particle to surmount the potential barrier surrounding the black hole is found to be enough to make the mass of the hole grow more than its charge, so that the hole becomes further removed from extremality.

But suppose there exist two types of local charge, type-

$\epsilon \in U(1)$  and type- $q \in U'(1)$ , e.g., electric and magnetic charges, which always reside in different particles. The black hole is assumed to contain a total  $U(1)$  charge  $\epsilon$  which is close to its mass  $M$ , but no  $U'(1)$  charge to start with, so that it is not endowed with a  $U'(1)$  gauge field. Thus an infalling  $q$ -type charge encounters no repulsive electrostatic potential barrier and, on first sight, is not hindered from crossing the horizon. Now, for two charge types the condition for the Reissner-Nordström horizon to continue to exist after the assimilation is

$$\epsilon^2 + q^2 \leq M^2. \quad (1)$$

What physics prevents the added charge  $q$  from violating this condition? We shall study two distinct *gedanken* experiments. In the first a  $U'(1)$  charge  $Q$  is allowed to fall on the black hole as a spherical shell concentric with the black hole. The calculation can be carried out exactly, and shows that, in fact, if the shell's charge is large enough to lead to a violation of Eq. (1), the shell's own self-repulsion prevents it from reaching the black hole. This is an extension of the usual mechanism that safeguards the horizon with one kind of charge present.

In our second *gedanken* experiment we consider an infalling pointlike  $q$ -type charge of mass  $\mu$ . Again, it meets no repulsion from the hole's field, but neither does self-repulsion play any visible role in preventing its assimilation by the black hole. In fact, we find that the condition for transcending Eq. (1) is precisely that the classical charge radius  $r_c = q^2/\mu$  of the charge be bigger than the black hole. Thus if the particle is classical, it cannot get into the black hole, and the attempt fails. If the particle is an elementary quantum charge, its size is set by the Compton length  $1/\mu$ . The condition for removal of the horizon then translates into  $q^2 > 2$ , meaning that the  $U(1)$  gauge theory must be strongly coupled.

As is well known, the vacuum polarization required by QED or its analogue makes the charge associated with a particle significantly dependent on the length scale at which it is looked at: at large distances most of the charge

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is screened. The condition that, on the scale of the black hole horizon,  $q^2 > 2$  means that the Landau ghost would show up at measurable scales, an intolerable situation. Thus if we require that the U(1) gauge theory in question be described by a consistent effective theory, the conditions for removal of the black hole horizon by addition of  $q$ -type charge cannot be satisfied. The event horizon is truly stable.

## II. INFALL OF CHARGED SHELL

The Reissner-Nordström metric [1] must be the exterior metric of a spherical distribution with two different U(1) charges  $Q$  and  $\epsilon$  [2]:

$$ds^2 = - \left( 1 - \frac{2M}{r} + \frac{\epsilon^2 + Q^2}{r^2} \right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r} + \frac{\epsilon^2 + Q^2}{r^2}} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (2)$$

It displays an event horizon only if condition (1) holds. The electric potential of the  $q$ -type charge is [1]

$$\Phi = Q/r. \quad (3)$$

Let us start with a black hole of mass  $M$  and  $\epsilon$ -type charge  $\epsilon$ , but with vanishing  $q$ -type charge, and consider the radial infall into it of a thick spherical shell concentric with it. The shell is made up of identical particles, each bearing a  $q$ -type charge with a specific charge  $q/\mu$ . The total  $q$ -type charge of the shell is  $Q$  and its total rest mass  $m$ . The initial conditions are that the shell starts off at very large distance  $r$  from the hole, and with each of its constituents having the same given inward velocity. Consequently, the specific energy at infinity is a fixed quantity  $E > 1$  for all particles. We neglect pressure in the shell; i.e., we assume random velocities remain negligible. This means that the shell has conserved energy  $mE$  in the field of the black hole.

The equation of motion of a particle at the outer edge of the shell is that of a point charge with a specific charge  $q/\mu$  moving in the metric (2) with mass  $M' = M + mE$ , the mass of the hole plus shell, and in the potential  $\Phi(r)$  of the shell itself. We shall assume that the same particles stay at the edge of the shell throughout its motion. This assumption can be violated if there is shell crossing, i.e., if a shell of charged particles gets inside an initially smaller shell in the course of the motion. Such shell crossing is rampant in the collapse of thick shells of uncharged dust. Pressure militates against it since in general shell crossing would result in an unbounded rise in the local pressure. By analogy we expect that for "charged dust," the subject of our inquiry, the rising Coulomb potential when two charged shells approach each other should mitigate shell crossing. It will actually be sufficient for all that follows if for the outer edge particles shell crossing does not take place. Then the edge of our shell can be treated as a charge radially moving in the Reissner-Nordström geometry and field, as explained above.

If  $\tau$  denotes the charge's proper time, the conservation of the specific energy  $E$  of a particle at the edge is written as [1]

$$\left( 1 - \frac{2M'}{r} + \frac{\epsilon^2 + Q^2}{r^2} \right) \frac{dt}{d\tau} + \frac{(q/\mu)Q}{r} = \text{const} = E. \quad (4)$$

This equation may be used to eliminate  $dt/d\tau$  in the normalization of the velocity

$$- \left( 1 - \frac{2M'}{r} + \frac{\epsilon^2 + Q^2}{r^2} \right) \left( \frac{dt}{d\tau} \right)^2 + \frac{(dr/d\tau)^2}{1 - \frac{2M'}{r} + \frac{\epsilon^2 + Q^2}{r^2}} = -1. \quad (5)$$

The result is

$$\left( \frac{dr}{d\tau} \right)^2 - \frac{2}{r} \left[ M + mE \left( 1 - \frac{Q^2}{m^2} \right) \right] + \frac{1}{r^2} \left[ \epsilon^2 + Q^2 \left( 1 - \frac{Q^2}{m^2} \right) \right] = E^2 - 1 > 0, \quad (6)$$

where we have replaced  $q/\mu \rightarrow Q/m$ , as well as  $M' \rightarrow M + mE$ . This first quadrature for the problem has the form of an energy conservation equation. We refer to the terms following  $(dr/d\tau)^2$  as the potential.

With the first quadrature for the shell edge's motion on hand, we now examine the most interesting possibility: the shell falls whole into the black hole and the resulting configuration transcends Eq. (1), thus signaling the destruction of the horizon. Of course other cases also leading to the same outcome are possible. For example, part of the shell may be repulsed, while the other part falls in and removes the horizon. Rather than considering all such possibilities, we concentrate on the first one mentioned. Not only is it the simplest, but if any of the others should be able to lead to the removal of the horizon, it is difficult to see how the whole shell's infall should fail to do so too.

If the whole shell falls into the black hole, the potential barrier should not be able to turn  $r(\tau)$  back at any time. Since the black hole existed to start with,  $\epsilon \leq M$ . The horizon is assumed to cease to be after the assimilation of the shell; thus  $(M + mE)^2 < \epsilon^2 + Q^2$ . Combining these inequalities tells us that

$$(Q/m)^2 > E^2 + 2EM/m. \quad (7)$$

Thus the specific charge  $Q/m = q/\mu$  must be large. It also follows from Eq. (7) that

$$M + mE(1 - Q^2/m^2) < M(1 - 2E^2) < 0 \quad (8)$$

and

$$\epsilon^2 + Q^2(1 - Q^2/m^2) < M^2(1 - 4E^2) < 0. \quad (9)$$

Thus the quantities in the square brackets in Eq.(6) are both negative if the shell is capable of removing the horizon. This means that the potential has a hump which could well block the shell from continuing on its way into the black hole.

A simple calculation shows that the peak of the potential term is

$$V_{\text{peak}} = \frac{[mE(Q^2/m^2 - 1) - M]^2}{Q^2(Q^2/m^2 - 1) - \epsilon^2} \quad (10)$$

and lies at

$$\begin{aligned} r_{\text{peak}} &= \frac{Q^2(Q^2/m^2 - 1) - \epsilon^2}{mE(Q^2/m^2 - 1) - M} \\ &= \frac{Q^2}{mE} + \frac{\frac{MQ^2}{mE} - \epsilon^2}{mE(Q^2/m^2 - 1) - M}. \end{aligned} \quad (11)$$

It now follows from inequalities (7) and (8) that  $r_{\text{peak}} > 2M$  so that the shell will certainly reach the potential barrier before reaching the original horizon. Thus in order for the *whole* shell (with parameters capable of leading to a removal of the horizon) to actually fall into the hole, it is necessary for  $E^2 - 1 \geq V_{\text{peak}}$ .

Let us introduce the variables  $\alpha$  and  $\beta$  by

$$Q^2 = m^2 E^2 + 2EmM + m^2 \alpha, \quad \epsilon^2 = (1 - \beta)M^2. \quad (12)$$

Trivially  $0 \leq \beta \leq 1$ , while inequality (7) guarantees that  $\alpha > 0$  in our case where the shell's parameters are appropriate for removing the horizon. Using Eq. (10) we write

$$V_{\text{peak}} + 1 - E^2 = \frac{(E^2 \beta - E^2 - \beta)M^2 - (E^2 m^2 - m^2 + \alpha m^2 + 2EmM)\alpha}{\epsilon^2 m^2 - Q^4 + m^2 Q^2} m^2. \quad (13)$$

A look at inequality (9) shows that the denominator here is negative. In view of the ranges of  $\alpha$  and  $\beta$  and the fact that  $E \geq 1$ , the numerator is also negative, making the whole expression positive. Thus by Eq. (6) the outer edge of the shell must reach a turning point before it reaches the maximum of the potential. This means that, contrary to assumption, part of the shell must be turned back.

Thus if the change in black hole parameters that would have resulted from assimilation of the shell sufficed to remove the horizon, that whole shell cannot reach the black hole. The contrapositive of this is thus true: if the shell's parameters are contrived so that all of it can reach the black hole, it cannot remove the horizon. Thus the classical process envisaged respects the horizon's existence, cosmic censorship, and the generalized second law.

### III. INFALL OF POINT CHARGE

Now let a pointlike  $q$ -type charge of mass  $\mu$  and charge  $q$  fall radially into a Reissner-Nordström hole of mass  $M$  and  $\epsilon$ -type charge  $\epsilon$  satisfying the second of Eqs. (12). If we treat the charge as a classical test particle ( $\mu \ll M$  and  $q \ll \epsilon$ ), its motion  $\{t(\tau), r(\tau)\}$  will again be described by Eqs. (4) and (5), but with  $Q = 0$  and  $M' = M$  since the black hole bears no  $q$ -type charge and the particle's influence on the background is being neglected. Combining the equations as in Sec. I, we find the first quadrature,

$$(dr/d\tau)^2 - 2M/r + \epsilon^2/r^2 = E^2 - 1 > 0. \quad (14)$$

The particle will move inward until it bumps into the rising potential ( $\epsilon^2/r^2$  term). The turning point is

$$r_{\text{turn}} = M \frac{[1 + (E^2 - 1)(1 - \beta)]^{1/2} - 1}{E^2 - 1}. \quad (15)$$

It is easy to see that  $r_{\text{turn}} < M/2$  for any choice of  $E$  since  $0 \leq \beta \leq 1$ . Hence the distortion of spacetime due

to the black hole charge  $\epsilon$  cannot prevent the particle with charge  $q$  from falling into it.

After the infall the black hole mass is  $M + \mu E$ . The condition for removal of the horizon is  $\epsilon^2 + q^2 = M^2(1 - \beta) + q^2 > (M + \mu E)^2$ . Since  $E \geq 1$ ,  $q^2 > 2M\mu + \mu^2 + M^2\beta$ . Since we can make  $\beta$  arbitrarily small, and  $\mu \ll M$ , to remove the horizon we need at least that

$$q^2 > 2M\mu. \quad (16)$$

What physics prevents a particle with  $q^2 > 2M\mu$  from accreting onto the black hole? Let us consider some options.

As a charge is lowered towards a black hole, it polarizes the hole in such a way that from far away the source of its field looks more spread out around the hole than the particle [3]. Could this effective spreading stop the "dangerous" particle from falling in? No. One can view the spreading as resulting from image charges induced on the black hole's surface by the approaching charge. Just under the charge the image charges are of opposite sign. Around the hole they are of the same sign. Obviously, the effect of the image charges should be to pull in the charge even more strongly than gravity alone. Thus this phenomenon cannot help to prevent assimilation of the charge  $q$  by the black hole.

The black hole might discharge its  $\epsilon$ -type charge in the manner of Schwinger sufficiently rapidly to offset the push beyond extremality by the added charge. Schwinger-type charge emission would depend on the  $\epsilon$ -type electric field of the black hole, which is of order  $\epsilon/M^2 \approx 1/M$  near the black hole. This field can tear the virtual pairs in the  $\epsilon$  vacuum if the work done by it on an elementary  $\epsilon$ -type charge  $e$  over its Compton length  $1/m$  amounts to at least the mass of a pair  $2m$ . Thus Schwinger discharge will be exponentially suppressed unless  $e/m > 2mM$ . Now suppose that there exist in nature  $\epsilon$ -type elementary charges with  $e/m \approx 1$ . We can then make a black hole by collapsing a large number of these unmixed with other stuff. In spherical collapse there is no

energy loss to waves, so that  $\epsilon/M = e/m \approx 1$ , and we can indeed form a nearly extreme black hole. If the charges, whose Compton length is  $1/m$ , are to fit into the black hole of size  $M$ , we must demand  $mM > 1$ . But then it is impossible to satisfy the condition for Schwinger emission. Thus one can imagine black holes that cannot be saved from destruction by Schwinger-type discharge.

Hawking thermal emission preferentially carries charges of the same sign as  $\epsilon$ ; it might thus drive the black hole below extremality before the added charge drove the hole over it. But since the black hole is assumed near extreme, its Hawking temperature is very small so that the emission is unimportant. For precisely the same reason, radiation pressure in the “photons” of the  $U'(1)$  gauge field can be regarded as weak compared to gravity, and is powerless to prevent infall of the charge  $q$ .

Our persistent failure to find a mechanism that prevents ingestion by the black hole of a “dangerous” charge leads us to the conclusion that there must be some basic physical reason why condition (16) cannot be satisfied for a charge that is able to fall into the black hole. For a classical charge  $q$  the reason is not far to seek. We note that its classical charge radius (analogous to the classical electron radius) is  $r_c = q^2/\mu$ , and condition (16) simply says that  $r_c > 2M$ . A classical particle which does not contain a negative energy density region somewhere in it must be larger than  $r_c$  since the electrostatic energy residing outside  $r_c$  would already account for all of the rest mass  $\mu$ . Thus if the charge is capable of fitting in the black hole ( $r_c < M$ ), it cannot satisfy (16), and cannot be used to remove the horizon.

However, for an elementary charge, e.g., an electron,  $r_c$  is not the measure of particle size. In fact, for  $U(1)$  charges found free in nature (weak coupling constant  $q^2 \ll 1$ ),  $r_c$  is far smaller than the Compton length  $1/\mu$ , the true quantum measure of particle size. Thus the charge can fall into the black hole only if  $r_c < 1/\mu < M$ . But then condition (16) cannot be satisfied, and we recover our previous conclusion that the horizon cannot be removed.

To satisfy condition (16) together with the requirement that the particle fit in the black hole,  $M > 1/\mu$ , means that we must consider strongly coupled QED-type theory ( $q^2 > 1$ ). An elementary charge in such theory has  $r_c > 1/\mu$ , and we cannot rule out condition (16) from the requirement that the particle can fit into the black hole,  $1/\mu < M$ . We thus look more carefully at what strongly coupled  $U(1)$  gauge theories are like.

The very notion of charge of a point particle in such a theory is dependent on the length scale on which it is measured. In a QED like theory, the relation between the charge of a point particle at two different scales  $\ell$  and  $L$ , with  $L \gg \ell$ , is given by the result from renormalization improved perturbation theory [4]:

$$\frac{1}{q(\ell)^2} = \frac{1}{q(L)^2} - \frac{2}{3\pi} \ln(L/\ell). \quad (17)$$

The physics behind this relation is that at long scales (say macroscopic) the charge is weaker because of vacuum polarization shielding of the charge at small (microscopic)

scales. Evidently for  $q(L) \neq 0$  there exists a sufficiently short scale  $\ell_L$  at which  $q(\ell_L) \rightarrow \infty$ ; this is the Landau ghost. Of course, this behavior is unacceptable. One possible resolution [5] is that QED and similar  $U(1)$  theories are trivial, i.e.,  $q \equiv 0$ . The Landau ghost does not then appear. This is what happens for  $\lambda\phi^4$  theories [6]. Another possibility [7] is that as  $q(\ell)^2$  grows, the theory makes a transition to a new phase so that the Landau ghost never shows up. The new phase is characterized by massive four-fermion interactions and seems to lack a long range force. Since  $q(L)^2 \neq 0$  in the real world, and electrons interact via photon exchange, we can consider QED as an effective theory valid above some short scale cutoff. Both alternatives are consistent with all experimental facts because the Landau ghost occurs at extremely short scales in QED (shorter than the Planck scale).

If a  $U(1)$  gauge theory undergoes a phase transition at strong coupling comparable to that implied by condition (16), we cannot obviously talk about simple charged particles with their attendant Coulomb interaction. Though we are unable to work out the details of the protection mechanism, the horizon will probably be safe. If the theory is trivial, it is certainly safe. We are left with the possibility that the  $U(1)$  theory is an effective theory defined over some finite range of scales. Can a charge in such theory remove the horizon?

In order for us to consider the charge as a quantum particle subject to the effective field theory, that theory must be applicable at scales below the particle’s Compton length, i.e.,  $\ell = 1/(\xi\mu)$  with  $\xi > 1$ . On the other hand, the charge relevant for the motion of the charge in the black hole’s background must be defined on scales larger than  $M$ ; hence we need  $L = \zeta M$  with  $\zeta > 1$ . Finally, for the effective theory to be self-consistent, the Landau ghost must not appear; i.e., the right hand side of Eq. (17) must be positive. We must thus put an upper bound on  $q(\zeta M)^2$ :

$$q(\zeta M)^2 < \frac{3\pi/2}{\ln(\zeta\xi M\mu)}. \quad (18)$$

However, this constraint is in the opposite sense as condition (16) for the removal of the horizon. In fact, they can be compatible only if  $2M\mu \ln(\zeta\xi M\mu) = 2M\mu \ln(M\mu) + 2M\mu \ln(\zeta\xi) < 3\pi/2$ . However, since  $\zeta\xi$  must be a few times unity, this last inequality can be satisfied only if the Compton length  $1/\mu$  is almost as large as  $M$ , the black hole’s radius. Needless to say, the infall of a particle of this size cannot be treated classically; its evolution in the black hole background is in all cases quantum mechanical. Thus we cannot draw the conclusion that the horizon can be removed by a particle obeying condition (16).

Our discussion has been qualitative because the analysis of strongly coupled  $U(1)$  field theory is not yet feasible. It is clear, however, that the physics of strongly coupled  $U(1)$  is just what is needed to protect cosmic censorship and the second law. We find it significant that a classical black hole requires the help of a quantum effect

(vacuum polarization) to preserve its integrity while absorbing charges. Perhaps this was to be expected from the quantum nature of black hole entropy which enters into the second law of thermodynamics for black holes.

It would obviously be interesting to explore further the question with lattice simulations of strongly coupled QED to see if the effective long range charge is indeed kept small enough to comply with considerations raised by our discussion.

#### ACKNOWLEDGMENTS

J.D.B.'s work was supported by a grant from the Israel Science Foundation which is administered by the Israel Academy of Sciences and Humanities. C.R. was supported by the DOE under Contract No. DE-FGO2-85-ER 40231, and also thanks the Racah Institute, Hebrew University for hospitality. We both thank Renaud Parentani for a suggestion.

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