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Inflation and primordial black holes as dark matter

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We discuss the hypothesis that a large (or even a major) fraction of dark matter in the Universe consists of primordial black holes (PBH's). PBH's may arise from adiabatic quantum fluctuations appearing during inflation. We demonstrate that the inflation potential $V(\varphi)$ leading to the formation of a great number of PBH's should have a feature of the "plateau"-type in some range $\varphi_1 < \varphi < \varphi_2$ of the inflation field φ. The mass spectrum of PBH's for such a potential is calculated.

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I. INTRODUCTION

The nature of dark matter (DM) in the Universe is one of the greatest puzzles of modern cosmology. The DM may consist of baryons, weakly interacting massive exotic particles predicted by grand unified theory (GUT), primordial black holes, or some combination of these species.

In this paper we shall consider the hypothesis that the DM consists mainly of primordial black holes (PBH's). (The earlier works on PBH's are [1,2] see also [3] and [4].)

Recently the possible discovery of microlensing of stars in the Large Magellanic Cloud by massive compact halo objects (MACHO's) with probable masses ~ 0.1 solar mass was reported [5,6]. It was supposed (among other possibilities) that such objects might be black holes. We would like to emphasize that black holes with masses of the order of $0.1M_\odot$ can only be of primordial origin. Thus, this discovery gives additional arguments for considering the possibility of the PBH nature of DM.

Let us consider the conditions for PBH formation in the early Universe. The simple estimates (see, for example, [4,7]) show that for the formation of PBH's with a total mass density close to the critical one ($\Omega_{\rm PBH} \approx 1$), and with a mass $M_{\rm PBH}$ around $0.1M_\odot$ one needs a rms amplitude $\delta_{\rm rms}(0.1M_\odot)$ of the Gaussian distribution of the scalar metric fluctuations of the order of $\delta_{\rm rms}^{\rm crit}(0.1M_\odot) \approx 0.06$. This estimate depends on $\Omega_{\rm PBH}$ and $M_{\rm PBH}$ only logarithmically. For example, $\delta_{\rm rms}^{\rm crit} = 0.04$ at 10^{15} g and $\delta_{\rm rms}^{\rm crit} = 0.08$ at $10^6 M_\odot$. On the other hand, the Cosmic Background Explorer (COBE) measurements of the anisotropy of the cosmic microwave background radiation and other satellite, balloon, and ground-based radio telescope measurements, and also deep surveys of galaxy distributions, strongly indicate that on scales of galaxies and greater scales (up to the horizon scale) the amplitude of $\delta_{\rm rms}$ was significantly less, probably around 10^{-5}–5×10^{-6}.

It is worth noting that COBE data are compatible with a power spectrum of the adiabatic perturbations $P(k) \propto k^n$ with $n = 1.15^{+0.45}_{-0.65}$ (see [8]). This means that a direct extrapolation of the COBE data to smaller scales even with the maximal possible value $n \approx 1.6$, can give $\delta_{\rm rms}$ great enough for the formation of an essential number of black holes only for $M_{\rm PBH}$ less than 10^{15} g [4]. However, such small PBH's would have evaporated a long time ago and could not contribute to DM [1].[1] Notice that if we believe that the main part of a PBH has some specific mass M_*, then the spectrum of the primordial fluctuations must have a decrease or a cutoff from the side of smaller mass at $M_{\rm PBH} \approx M_*$.

Thus, for the hypothesis of PBH DM one needs the following behaviors of the spectrum of the primordial scalar metric perturbations. The rms amplitude must be the order of 10^{-5} at large scales, must increase by a factor 10^4 at the scales corresponding to the masses of the PBH, and must decrease at smaller scales.

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[1]Note that if one supposes that evaporating PBH's leave stable Planck mass relics, these relics could contribute to DM and constrain the spectrum [32], but we shall not discuss this possibility below.

In the inflationary scenario of the early Universe the spectrum of the primordial perturbations is determined by the potential $V(\varphi)$ of the scalar field φ ("inflaton"). Note that in more complicated theories it can be several inflatons (see for example [9–14]). In the most simple theory with smooth, featureless $V(\varphi)$ the spectrum very slightly depends on scale [15–20] and cannot produce PBH's in a large amount. The requirement that the spectrum increases with a decrease of scale as a power leads to special "trigonometric" potentials [4] and also cannot explain the large PBH production (see above and [4]). The introduction of two or more inflatons or taking the potential to have one break [21] may produce the bump in the spectrum, but such a type of spectrum possesses additional power at large scales [10,21]. Thus normalized at COBE data, spectra of this type seem not to produce a large amount of PBH's. From this discussion it follows that the most natural way for large PBH production to occur is to introduce the specially engineering local feature to the inflation potential at PBH scales. Although the known particle physics may not support such features, the possible discovery of PBH's may turn the problem around and demand the existence of such features in any realistic particle physics. The purpose of our paper is the following. We shall demonstrate that an inflation potential $V(\varphi)$ leading to the formation of a great number of PBH's must have a feature of the "plateau"-type in some range $\varphi_1 < \varphi < \varphi_2$, and we shall calculate the mass spectrum of PBH's for such a $V(\varphi)$.

Qualitatively the conclusion about the plateau in $V(\varphi)$ follows from a well-known estimate for the spectrum of primordial metric fluctuations in the model of chaotic inflation assuming the friction-dominated and slow-roll conditions, $|\ddot{\varphi}| \ll H|\dot{\varphi}|$ and $\dot{\varphi}^2 \ll V(\varphi)$, respectively. Here the overdot denotes differentiation with respect to time, and H is the expansion rate. The power spectrum $P(k)$ in this case can be written as [9]

$$P(k) \sim k \frac{V^3}{(\partial V/\partial\varphi)^2}\bigg|_{k=aH(\varphi)} , \tag{1}$$

where $H(\varphi)$ is the value of the Hubble parameter at the moment when the Universe has the value φ of the inflaton field and a is the scale factor. If the potential $V(\varphi)$ has a plateau in the range $\varphi_1 < \varphi < \varphi_2$, $V(\varphi) \approx$ const and $\partial V/\partial\varphi \to 0$, then the spectral amplitude $P(k)$ is strongly increased [see the formula (1)]. Outside the range $\varphi_1 < \varphi < \varphi_2$, $V(\varphi)$ has a standard (for example a power law) shape. In the range $k \ll k_2$ and $k \gg k_1$, where $k_i = a(\varphi_i)H(\varphi_i)$, the corresponding $P(k)$ has also a standard shape [for example it can be the Harrison-Zeldovich spectrum $P(k) = A^2 k$, with $A \approx 5\times 10^{-6}$].

The structure of the paper is the following. In Sec. II the modification of the inflaton scenario with the plateau-type peculiarity in $V(\varphi)$ is discussed, and we calculate the distortion of the spectrum of the primordial metric fluctuations due to this peculiarity. For simplicity we shall use the simple approximation with two breaks for potential form (see Fig. 1). The spectrum of adiabatic perturbations in such type theories was calculated by Starobinsky [21] (for a potential with one break), and Demiansky, Ivanov, and Novikov [22] for any number of breaks. Sec. III is devoted to the analysis of the mass spectrum of the PBH's. In Sec. IV we discuss the possible role of the "gas" of PBH's in the origin of the large-scale structure of the Universe, and summarize the main conclusions.

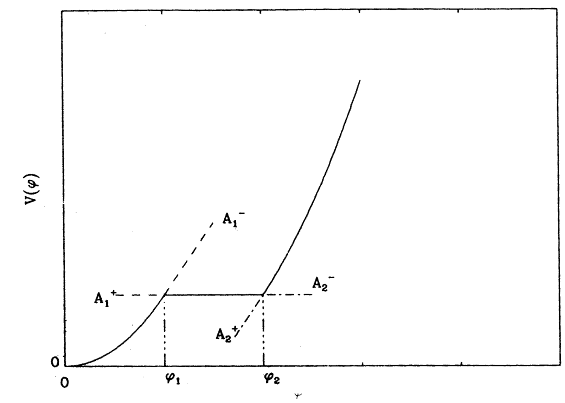

FIG. 1. Schematic representation of the potential $V(\varphi)$ of the scalar field φ (inflaton). The potential has a plateau in the range $\varphi_1 < \varphi < \varphi_2$ and is of the power-law type outside of this range. The breaks of the potential are smoothed out in small ranges $\Delta\varphi_1 \ll \varphi_1$ and $\Delta\varphi_2 \ll \varphi_2$ around φ_1 and φ_2 correspondingly.

II. SPECTRUM OF SCALAR METRIC PERTURBATIONS IN THE INFLATIONARY SCENARIO WITH A "PLATEAU" IN THE POTENTIAL $V(\varphi)$

The simple approach to the inflaton based on one scalar field φ is to specify the physics by choosing an appropriate form for $V(\varphi)$ and assuming the friction-dominated and slow-roll conditions [9]:

$$|\ddot{\varphi}| \ll 3H|\dot{\varphi}| , \quad (\dot{\varphi})^2 \ll 2V(\varphi) , \tag{2}$$

where $H = \dot{a}/a$; $a(t)$ is the scale factor. In this regime Fourier components of the scalar metric perturbations are δ-correlated random values with a Gaussian distribution.

Our task is to increase the spectral amplitude in some range $k_2 < k < k_1$, where k is a wave number, without changing the standard spectrum of perturbations outside this range. We propose to introduce the potential $V(\varphi)$ of the inflaton φ, which is depicted in Fig. 1. This potential has a plateau in the range $\varphi_1 < \varphi < \varphi_2$ and is a power-law type outside of this range.

There are two breaks of the potential at $\varphi = \varphi_1$ and $\varphi = \varphi_2$. We suppose that these breaks are smoothed out in small ranges $\Delta\varphi_1 \ll \varphi_1$ and $\Delta\varphi_2 \ll \varphi_2$ around φ_1 and φ_2 correspondingly (see Fig. 1).

The conditions (2) are violated in these ranges. Starobinsky has pointed out [10] that this violation results is a nonmonotonic spectrum of perturbations. In the vicinities of breaks of the potential $V(\varphi)$, but outside of $\Delta\varphi_1$ and $\Delta\varphi_2$, the potential can be described by

$$V(\varphi \sim \varphi_i) = V(\varphi_i) + v(x_i) \ ,$$
$$v(x_i) = \begin{cases} A_i^+ x_i & \text{if } x_i \gg \varphi_i, x_i > 0, \\ A_i^- x_i & \text{if } |x_i| \gg \varphi_i, x_i < 0, \end{cases} \tag{3}$$

where $x_i = \varphi - \varphi_i$, $i = 1,2$, $\varphi_2 > \varphi_1$, or it can be rewritten as

$$V(\varphi \sim \varphi_i) = V(\varphi_i) + A_i^- x_i + (A_i^+ - A_i^-)\Theta(x_i)x_i, \tag{4}$$

where $\Theta(x)$ is the Heaviside function. Notice, that in the general case the shape of the potential at $\varphi_1 < \varphi < \varphi_2$ can be complicated enough. However, for our purpose it is enough to choose $V(\varphi)$ =const at $\varphi_1 < \varphi < \varphi_2$. In this model $A_1^-, A_2^+ \neq 0; A_2^- = A_1^+ = 0$. Evolution of the scalar field φ is governed by the equation [9]

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi},$$
$$3H^2 = 8\pi[\dot{\varphi}^2 + V(\varphi)], \tag{5}$$

where $\dot{\varphi} \equiv d\varphi/dt$. In the case of the evolution of φ when $\varphi > \varphi_2$, from (5) we have

$$\dot{\varphi} \approx -\frac{A_2^+}{3H}. \tag{6}$$

We suppose that $|A_2^+ - A_2^-| > H^2\Delta\varphi_2$ and $A_2^+\Delta\varphi_2 \ll V(\varphi_2)$. In this case for the dynamics of φ one can neglect $v(x_2)$ in (3) compared with $V(\varphi_2)$, and the regime $a(t) \propto \exp(Ht)$ goes on after the field φ passes the break of the potential at φ_2.

After this passage the field evolves along the plateau, and the solution of the equations (5) can be written as

$$\dot{\varphi} = -\frac{A_2^+}{3H} e^{-3H(t-t_2)}, \tag{7}$$

where t_2 is the moment of time, when $\varphi(t_2) = \varphi_2$. At the moment when φ comes to the point $\varphi_1 = \varphi(t_1)$, its "velocity" is

$$\dot{\varphi}|_{t\simeq t_1} \approx -\frac{A_2^+}{3H}\left(\frac{k_2}{k_1}\right)^3, \tag{8}$$

where $k_i = a(t_i)H(t_i)$, $i = 1,2$. Using (6) and (8) we have the following expression for $\partial^2 V/\partial\varphi^2$, which determines the dynamics of the generation of adiabatic perturbations in the vicinity of the break of the potential [21]:

$$\frac{\partial^2 V}{\partial \varphi^2} = \frac{3H}{a}\left[\delta(\zeta - \zeta_2) - \frac{A_1^-}{A_2^+}\left(\frac{k_1}{k_2}\right)^3 \delta(\zeta - \zeta_1)\right], \tag{9}$$

where $\zeta = \int(dt/a)$, and ζ_1 and ζ_2 correspond to t_1 and t_2 (see [15–17,22]).

Let us consider the origin of adiabatic metric perturbations at the epoch of the inflation. The evolution of the gauge-invariant scales quantity v_k, which describes the adiabatic perturbations [18,23] is governed by the equation [18]

$$v_k'' + [k^2 - (2/\zeta^2) + V_{,\phi\phi}a^2]v_k = 0, \tag{10}$$

where a prime denotes $\partial/\partial\zeta$. Using this equation it is possible to get the spectrum in a form

$$P(k) = A^2 k D(k), \tag{11}$$

where A is the amplitude of the spectrum (it can be normalized at COBE data),

$$D(k) = |\alpha - \beta|^2 \tag{12}$$

is the modulation function [when $D(k) = 1$, we have the standard flat Harrison-Zeldovich spectrum], and α, β are the coefficients of the decomposition of v_k on the "standard" vacuum solution $v_{k_{\rm in}}$ of Eq. (10) with a potential without breaks:

$$v_k = \alpha(k)v_{k_{\rm in}} + \beta(k)v_{k_{\rm in}}^*, \tag{13}$$

where

$$v_{k_{\rm in}} = \frac{1}{(2k)^{1/2}} e^{-ik\eta}\left(-1 + \frac{i}{k\eta}\right). \tag{14}$$

The decomposition is made after ϕ passes the feature area. Using the linearity and invariance under complex conjugate of Eq. (10), we can get α and β in the form

$$\alpha = \alpha_2\alpha_1 + \beta_2\beta_1^*,$$
$$\beta = \alpha_2\beta_1 + \beta_2\alpha_1^*, \tag{15}$$

where α_j and β_j are the coefficients of the decomposition of (13) in the case of one break only. They were calculated by Starobinsky in 1992:

$$\alpha_j = 1 - \frac{3i}{2}C_j y_j^{-1}(1 + y_j^{-2}),$$
$$\beta_j = \frac{3i}{2}C_j \exp(2iy_j)y_j^{-1}(1 + iy_j^{-1}), \tag{16}$$

$y_j = R_j k$, R_j is the wavelength of perturbation entering the horizon at the moment when $\phi = \phi_j$; $C_2 = 1$; $C_1 = -(A_1^-/A_2^+)(k_1/k_2)^3$; $j = 1,2$. Taking into account (12), (15), and (16), we get the explicit form for $D(k)$:

$$D(k) = D_1 D_2 + D_{\rm int},$$
$$D_j = 1 + \frac{3C_j}{y_j}\left[\left(1 - \frac{1}{y_j^2}\right)\sin 2y_j + \frac{2}{y_j}\cos 2y_j\right] + \frac{9}{2}C_j\frac{1}{y_j^2}\left(1 + \frac{1}{y_j^2}\right)\left[1 + \frac{1}{y_j^2} + \left(1 - \frac{1}{y_j^2}\right)\cos 2y_j - \frac{2}{y_j}\sin 2y_j\right], \tag{17}$$

$$D_{\rm int} = 4(a_1 - d_1)[(a_1 - d_1)(b_2 + a_2 d_2) - (1 - b_1)(d_2 - a_2 b_2)],$$
$$a_j = {\rm Im}(\alpha_j), \quad b_j = {\rm Re}(\beta_j), \quad d_j = {\rm Im}(\beta_j), \quad j = 1, 2,$$

where D_j is the modulation function in the theory with one break, $D_{\rm int}$ is the interference term.

Asymptotic behaviors of $D(k)$ are

$$D(0) = \left(\frac{A_1^-}{A_2^+}\right)^2, \quad D(\infty) = 1. \tag{18}$$

The function $D(k)$ is depicted in Fig. 2. Notice the oscillations in the spectrum related to each break in the potential $V(\varphi)$. These oscillations were discussed in [22,24].

For the hypothesis of PBH DM, the case $\left(\frac{A_1^-}{A_2^+}\right) \le 1$ is especially interesting. Under this condition, at $k/k_2 > 4$, $D(k)$ can be described with accuracy better than 5% by the fitting formula

$$D(k) \approx \left[1 + \frac{A_1^-}{A_2^+}\gamma^3\right]^2 \left[1 + 3\frac{\sin 2kR_2}{kR_2}\right],$$

$$\gamma = \frac{k_1}{k_2}, \quad \gamma \gg 1. \tag{19}$$

One can see in Fig. 2 that in the case $\gamma \gg 1$ at $k_2 < k < k_1$ there is a great increase of the spectral amplitude by a factor $D^{1/2}(k) \sim (A_1/A_2)\gamma^3$. The two lowest curves in Fig 2 show the character of approach of $D(k)$ to its asymptotic value at $k \to 0$. One can see that between the long-wavelength asymptotic of $D(k)$ and the range of the strong increase of the spectral amplitude there is a range where the amplitude is suppressed.

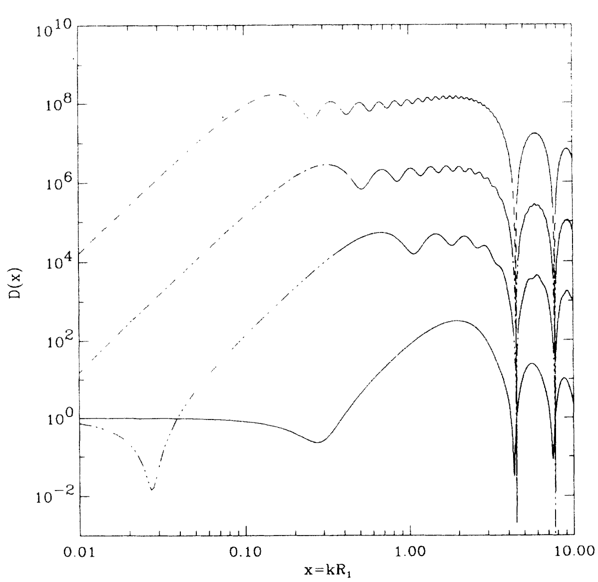

FIG. 2. Result of the computations of $D(x)$, $x = kR_1$. The dashed, dashed-dotted, dashed-triple-dotted, and solid lines correspond to $\gamma = 20$, $\gamma = 10$, $\gamma = 5$, and $\gamma = 2$, respectively.

III. MASS SPECTRUM OF PBH's

In our approach to the calculation of the mass spectrum of PBH's we focus on the peaks of the Gaussian random field of the primordial adiabatic metric perturbations [2].[2]

A PBH can arise when the space scale of the peak scalar metric perturbations of the order of 1 becomes smaller than a particle horizon $\lambda_H \approx t$ but still is greater than the Jeans' radius $\lambda_J = \lambda_H/\sqrt{3}$. The masses of PBH's, $M_{\rm BH}$, are proportional to the moment $t_{\rm BH}$ of their formation $M_{\rm BH} \propto t_{\rm BH}$. The shapes of the peaks of the fluctuations play an important role in the formation of PBH's [26,27]. Some shapes can result in the dissipation of the peaks due to pressure gradients.

Zabotin and Naselsky [27] have pointed out that in the case of the Harrison-Zeldovich spectrum of the primordial fluctuations (and spectra, which are close to them) the most probable distribution of the matter inside the peak is favorable for the formation of PBH's. Moreover, one can calculate the mass spectrum of PBH's in the framework of the model of the homogeneous collapse proposed in the work [28].

Note that the process of PBH formation is certainly nonlocal because it includes a volume of the radius R. In order to take into account this nonlocality, one needs to use the characteristics averaged over the sphere with a Gaussian filtering function [29].

In addition, it is necessary to take into account that for the perturbations with a wavelength of more than the particle horizon evolution of the density contrast $\delta(\mathbf{r}, t)$ can be written in the form $\delta(\mathbf{r}, t) = \delta(\mathbf{r})\Phi(t)$, where $\delta(\mathbf{r})$ is determined by the spectrum of the initial metric perturbations, and $\Phi(t)$ corresponds to the growing mode of gravitational instability. Because of this factorization it is enough to analyze the statistical behavior of the peaks of the function $\delta(\mathbf{r})$.

Following the work [29], let us introduce a new field $F(\mathbf{r}, R)$, which is the result of Gaussian smoothing of the random field $\delta(\mathbf{r})$ on the scale R,

$$f(\mathbf{r}, R) = \frac{1}{(2\pi R^2)^{3/2}} \int d^3\mathbf{r}' \delta(\mathbf{r}') \exp\left(-\frac{|\mathbf{r} - \mathbf{r}'|^2}{2R^2}\right), \tag{20}$$

and consider the correlation function

$$C(R, x) = \frac{1}{2\pi^2} \int_0^\infty dk\, k^2 P(k) \exp(-k^2 R^2) \frac{\sin kx}{kx},$$

where $x^2 = \sum_{\alpha=1}^3 \left(x_1^{(\alpha)} - x_2^{(\alpha)}\right)^2$. In the notation of $x_l^{(\alpha)}$, the upper indices $\alpha = 1, 2, 3$ label coordinates, and the lower ones $l = 1, 2$ label points in space.

In order to make further calculations the dispersion $C_0(R) \equiv C(R, 0)$ is required. For the analytic estimates we assume $\gamma \gg 1$, $R_1 \ll R \le R_2$ and we shall use ex-

[2]For an alternative approach to the mechanism for PBH formation, see the work by Dolgov and Silk [25].

pression (19) for the spectrum. In such an approximation we get

$$C(0,R) = \frac{1}{2\pi^2}\int_0^\infty dk\, k^2 P(k) e^{-k^2R^2}$$
$$\approx \frac{A^2\gamma^6}{2\pi^2 R^4}\int_0^\infty dy\, y^3 e^{-y^2}\left[1+\frac{\sin(2R_2/R)y}{(R_2/R)y}\right]$$
$$= \frac{A^2\gamma^6}{2\pi^2 R^4}\left[1+6\,{}_1F_1\left(3,\frac{3}{2},-\frac{R_2^2}{R^2}\right)\right], \quad (21)$$

where ${}_1F_1(a,b,x)$ is the degenerated hypergeometric function and A is the amplitude of the Harrison-Zeldovich spectrum on large scales (it can be normalized to COBE data).

Formula (21) is correct for the range $R_1 \le R \le R_2$. For the range $R \ll R_1$ and $R \gg R_2$, the dispersion $C_0(R)$ is negligible compared with the dispersion in the range $R_1 \le R \le R_2$, and we can put $C_0(R) \simeq 0$ at $R \ll R_1$ and $R \gg R_2$.

Thus the spectrum of Gaussian random matter density perturbations smoothed by the Gaussian filter on the scale R can be given by the simple formula

$$C_0(R) = \varepsilon^2\left(\frac{R_1}{R}\right)^4 F(R_2/R), \quad (22)$$

where $\varepsilon^2 = (A^2/4\pi^2)(\gamma^6/R_1^4)$ is a measure of the spectral amplitude at $R = R_1$ and

$$F(R_2/R) = 1 + 6\,{}_1F_1\left(3,\frac{3}{2},-\frac{R_2^2}{R^2}\right).$$

During its formation each PBH absorbs mass from the region with the comoving scale $R \propto M_{\rm PBH}^{1/2}$. Since $F(R_2/R)$ varies in the limits of order 1, one can use the following approximate estimate of the fraction of the total matter $\beta(R)$ collapsing into PBH's with mass $M_{\rm PBH}$:

$$\beta(R) \simeq \varepsilon\left[F\left(\frac{R_2}{R}\right)\right]^{1/2}\exp\left[-\frac{1}{18\varepsilon^2 F(R_2/R)}\right]. \quad (23)$$

We performed numerical computations of $\beta(R)$ using Eqs. (10) and (11) and estimated for calculation $\beta(R)$ given by [4]. The results of these computations are presented on Fig. 3 for $\gamma = 16.5$, 18, and 20. As seen from Fig. 3, the asymptotic rough analytic formula (21) is valid at $R_1/R \le R_2$ only. Numerical computations show two maxima that correspond to two scales, R_1 and R_2, of the initial spectrum $P(k)$ of perturbations. The mass spectrum of PBH's is determined by the function $\beta(M_{\rm BH})$:

$$F(M_{\rm BH}) \propto \frac{1}{M_{\rm BH}}\frac{d}{dM_{\rm BH}}[\beta(M_{\rm BH})M_{\rm BH}^{-1/2}], \quad M_{\rm BH} \propto R^2.$$

Thus, varying the main parameters of the model one can vary the possible values for PBH masses in very broad limits.

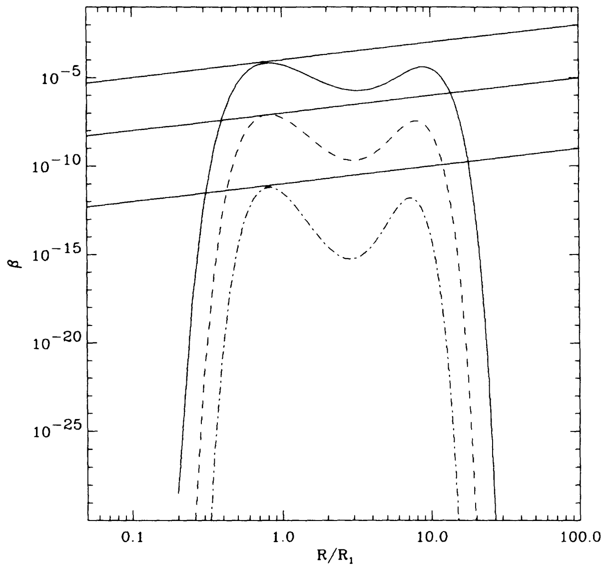

FIG. 3. Function $\beta(y)$, $y = R/R_1$. The solid, dashed, and dashed-dotted lines correspond to $\gamma = 20$, $\gamma = 18$, and $\gamma = 16.5$ respectively. Straight lines, which are tangential to $\beta(y)$ of different models, correspond to the conditions $\Omega_{\rm PBH} = 1$ for the modern Universe. The tangent points determine the corresponding $M_{\rm PBH}$. $M_{\rm PBH} \simeq 10^8 M_\odot$ at $\gamma = 20.0$, $M_{\rm PBH} \simeq 10^2 M_\odot$ at $\gamma = 18.0$, and $M_{\rm PBH} \simeq 10^{-6} M_\odot$ at $\gamma = 16.5$.

IV. ASTRONOMICAL CONSEQUENCES OF THE HYPOTHESIS ABOUT PBH DM AND CONCLUDING REMARKS

We have demonstrated that under some conditions on the inflation potential $V(\varphi)$ (see Sec. III) the matter density of PBH's could be great enough to make up a considerable or even major part of DM in the modern Universe. This imposes a lower limit on the possible parameter R_1 of the model. Indeed, PBH's could not have masses $M_{\rm BH} \le 10^{15}$ g. Such PBH's must evaporate due to Hawking's process, and this gives a strong observational constraint on their density $\Omega_{\rm BH} < 10^{-8}$; see [3]. For $M_{\rm BH} \ge 10^{20}$ g and up to scales of the clusters of galaxies, constraints come only from the inequality $\Omega_{\rm BH} \le 1$ in the modern Universe [3].

One can consider models with $\Omega_{\rm tot} = 1$ and with DM consisting mainly of PBH's, which implies $\Omega_{\rm PBH} \approx 1$, or more complicated models with $\Omega_{\rm PBH} < 1$ and with a Λ term or some hot dark matter (HDM); see [30].

In Fig. 3 we show straight lines corresponding to the conditions $\Omega_{\rm PBH} = 1$ for the modern Universe and which are tangential to the spectrum $\beta(R)$ of different models. The tangent points determine the corresponding effective masses $M_{\rm PBH}$ of the models. As may be seen from Fig. 3, for all interesting ranges of $M_{\rm PBH}$, the parameter γ is $\gamma \simeq 15$–20. The parameter γ determines the distribution of masses of PBH's, and thus it could be a possible test of the nature of DM.

In this paper we do not analyze special behaviors of the formation of the large scale structure (LSS) of the Universe in the framework of the PBH DM model. We note only the following. The main properties of the LSS in this model probably are the same as in different versions of the standard cold dark matter (CDM) model due to the fact that the masses of PBH's are much smaller than the masses of the LSS. On the other hand, the absence of PBH's in the Universe may constrain the plateau-type features in potentials on a wide range of scales.

We want to point out that the condition $\Omega_{\rm PBH} \approx 1$ for PBH's with small masses can be satisfied only by a very "delicate" adjustment of the parameters of the theory. Indeed, in order that the total mass contained in PBH's be close to the critical value, it is necessary that the fraction of the total mass contained in them be sufficiently small (but have some well-defined value) at the period of PBH formation [1,31]. Perhaps the explanation of this "fine-tuning" of the parameters could be related to the anthropic principle.

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