Evolution of topological defects during inflation

Rama Basu and Alexander Vilenkin

Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155

(Received 28 February 1994)

Topological defects can be formed during inflation by phase transitions as well as by quantum nucleation. We study the effect of the expansion of the Universe on the internal structure of the defects. We look for stationary solutions to the field equations, i.e., solutions that depend only on the proper distance from the defect core. In the case of very thin defects, whose core dimensions are much smaller than the de Sitter horizon, we find that the solutions are well approximated by the flat-space solutions. However, as the flat-space thickness parameter δ_0 increases we notice a deviation from this, an effect that becomes dramatic as δ_0 approaches $H^{-1}/\sqrt{2}$. Beyond this critical value we find no stationary solutions to the field equations. We conclude that only defects that have flat-space thicknesses less than the critical value survive, while thicker defects are smeared out by the expansion.

PACS number(s): 98.80.Cq, 11.27.+d

I. INTRODUCTION

The inflationary epoch in cosmology increases the size of the Universe by a huge factor [1], implying that the present observable Universe originated from a tiny initial region. Any topological defects formed at the onset of inflation, or before the start of inflation would be inflated away, implying that the only defects that could possibly be observable at the present time would be those formed at or near the end of the inflationary epoch. Topological defects can be continuously formed during the course of inflation by quantum mechanical tunneling processes [2], and defects formed during inflation by this mechanism could be present after inflation with appreciable densities. Phase transitions could also occur during inflation if the symmetry-breaking field φ is coupled to the inflaton field [3]. The characteristic length scales of the defects formed in such phase transitions increases exponentially due to inflation. However, if the phase transition occurs close enough to the end of inflation, so that this length scale does not exceed the size of the presently observable Universe, then these defects are not diluted away. Defects could also be formed during inflation by quantum fluctuations [4] in the case where the symmetry is broken before or at the beginning of inflation.

All of these defects would have been exposed to the exponential expansion of the Universe long enough to make the question of what happens to the internal structure of these defects during inflation significant. In this paper we will address this question. The inflationary Universe is approximated by de Sitter spacetime, which has a constant expansion rate H. We can therefore look for stationary solutions to the scalar field equations for domain walls, strings, and monopoles.

Intuitively, one expects that the defect structure in de Sitter space will be essentially the same as in flat space if the flat-space thickness of the defects δ_0 is much smaller than the de Sitter horizon H^{-1}. On the other hand, it is hard to see how a coherent defect structure can be

sustained on scales greater than H^{-1}, and we expect that stationary solutions do not exist when δ_0 exceeds some critical value $\delta_c \sim H^{-1}$, and that for $\delta_0 > \delta_c$, the defects are smeared by the expansion of the Universe. We shall see that these expectations are indeed correct.

This paper is organized as follows. We study the structure of defects using a simple scalar field model; Sec. II deals with the structure of domain wals, for which we obtain numerical solutions. We also obtain analytic solutions in two asymptotic regimes corresponding, respectively, to the case of very thick walls with flat-space wall thickness comparable to the critical value and to the case of very thin walls. We do a similar analysis for strings and monopoles in Sec. III. Our conclusions are summarized in Sec. IV.

II. DOMAIN WALLS

The spacetime of the inflationary Universe is approximately de Sitter spacetime, and the metric with flat-spatial sections is given by

$$ds^2 = dt^2 - e^{2Ht}[dx^2 + dy^2 + dz^2] . \tag{1}$$

We consider a one component scalar field theory with a simple double-well potential

$$V(\varphi) = \frac{\lambda}{2}(\varphi^2 - \eta^2)^2 . \tag{2}$$

The scalar field equation is given by

$$\frac{1}{\sqrt{-g}}\partial_\mu(g^{\mu\nu}\partial_\nu\varphi) = -2\lambda\varphi(\varphi^2 - \eta^2) . \tag{3}$$

We consider a plane domain wall situated at $z = 0$. This is not as special a case as it appears because a recoordinization of de Sitter space in the calculations of [2], reveals [5] that the spherical domain walls nucleating during inflation are equivalent to plane domain walls appear-

EVOLUTION OF TOPOLOGICAL DEFECTS DURING INFLATION

ing at $z = 0$ in the new coordinates. We can write out the field equation explicitly as

$$\frac{\partial^2 \varphi}{\partial t^2} + 3H \frac{\partial \varphi}{\partial t} - \exp\left(-2Ht\right)\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right\}\varphi$$
$$= -2\lambda\varphi(\varphi^2 - \eta^2) . \quad (4)$$

If the wall is not smeared by the expansion, it should be described by a stationary field configuration, in terms of the proper distance from the wall. Accordingly, we choose the following ansatz for φ:

$$\varphi = \varphi(u), \quad \text{where } u = Hz \exp(Ht) . \quad (5)$$

In terms of the variable u, the field equation may be rewritten as

$$(1 - u^2)\frac{\partial^2 y}{\partial u^2} - 4u\frac{\partial y}{\partial u} = 2Cy(y^2 - 1) , \quad (6)$$

where $y = \varphi/\eta$, and $C = H^{-2}/\lambda^{-1}\eta^{-2}$, where $\delta_0 = (\sqrt{\lambda})^{-1}$ is the flat-space wall thickness. The solution must obey the boundary conditions

$$y(0) = 0, \quad y(\pm\infty) = \pm 1 . \quad (7)$$

Equation (6) cannot be solved in closed form. We therefore used a shooting routine to obtain numerical solutions. The numerical results are graphically depicted in Fig. 1. As expected, at large u the field φ approaches its vacuum expectation value (VEV) η. However, the solutions exhibit an aperiodical damped oscillatory behavior as φ approaches the VEV, as opposed to the monotonic approach to the VEV in flat space. At large values of C, when $\delta_0 \ll H^{-1}$, the solution is essentially identical to the flat-space solution $y = \tanh(\sqrt{C}u)$. [In this case most of the variation of y between 0 and 1 occurs at small values of u, so that the terms proportional to u^2 and u in

FIG. 1. The scalar field φ/η as a function of $zH \exp(Ht)$, shown for different values of the flat-space thickness parameter, $C = 10, 4, 2.5, 2.05,$ and 2.001.

Eq. (6) are negligible, and it reduces to the corresponding flat-space equation.] As $C \to 2$, or $\delta_0 \to H^{-1}/\sqrt{2}$, the solution approaches $y = 0$ over the whole range of integration. We found no nontrivial solutions to the field equations when $\delta_0 \geq H^{-1}/\sqrt{2}$. We now wish to examine the asymptotic behavior of the solutions more closely. Accordingly, we consider two asymptotic regimes, one where $y \ll 1$ corresponding to very thick walls with flat-space wall thickness $\delta_0 \approx H^{-1}/\sqrt{2}$ and the other where $|y - 1| \ll 1$, which corresponds to large distances from the core.

A. Large-distance asymptotic

In this asymptotic region, where u is large, the field equation can be written as

$$u^2 \frac{\partial^2 y}{\partial u^2} + 4u \frac{\partial y}{\partial u} + 2Cy(y^2 - 1) = 0. \quad (8)$$

Furthermore, we can assume that $y = 1 - f$ where $f \ll 1$, in the region of interest. Substituting this in Eq. (8) and discarding higher-order terms in f we have

$$u^2 \frac{\partial^2 f}{\partial u^2} + 4u \frac{\partial f}{\partial u} + 4Cf = 0 . \quad (9)$$

Now, making a change of variables $v = \ln u$, we have

$$\frac{\partial^2 f}{\partial v^2} + 3\frac{\partial f}{\partial v} + 4Cf = 0 . \quad (10)$$

The solution is of the form $\exp(\alpha v)$, with

$$\alpha^2 + 3\alpha + 4C = 0 . \quad (11)$$

It follows that

$$\alpha = -\frac{3}{2} \pm \frac{\sqrt{9 - 16C}}{2} . \quad (12)$$

The solution for f is therefore given by

$$f = Au^{-3/2}\cos(\tfrac{1}{2}\sqrt{16C - 9}\ln u) . \quad (13)$$

This indicates that the field φ approaches the vacuum expectation value η for large u as expected. In agreement with our numerical results, the field exhibits a damped oscillatory behavior about the vacuum expectation value for large u.

B. Near-critical behavior ($\delta_0 \approx H^{-1}/\sqrt{2}$)

The numerical results presented earlier in this section indicate that $y = \varphi/\eta$ becomes very small over the whole range of integration of Eq. (6) when $C \approx 2$, i.e., when $\delta_0 \approx H^{-1}/\sqrt{2}$. In this regime, we are therefore justified to assume that $y \ll 1$ up to large values of u. The field equation can then be linearized to become

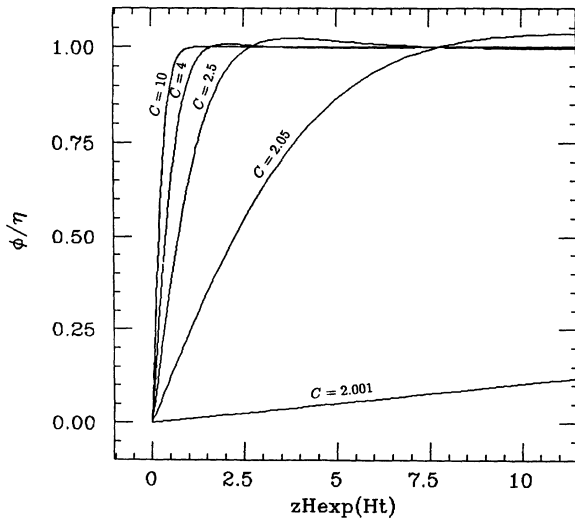$$(1 - u^2)\frac{\partial^2 y}{\partial u^2} - 4u\frac{\partial y}{\partial u} + 2Cy = 0 . \quad (14)$$

Now making a change of dependent variable $y = w/\sqrt{(1-u^2)}$, and replacing it in Eq. (14), we have

$$(1-u^2)\frac{\partial^2 w}{\partial u^2} - 2u\frac{\partial w}{\partial u} + \{2(C+1) - (1-u^2)^{-1}\}w = 0. \tag{15}$$

This is precisely the associated Legendre equation [6]. The general solution to this equation is

$$w(u) = AP_\nu^\mu + BQ_\nu^\mu \tag{16}$$

with $\mu = 1$ and $\nu(\nu+1) = 2(C+1)$.

In order that the solutions to this equation be bounded at $u = \pm 1$, ν is constrained to assume only integral values. The value of interest to us is $\nu = 2$, which corresponds to $C = 2$. With this value, the nonsingular solution of Eq. (15) satisfying the boundary condition (7) at $u = 0$ is

$$y = A(1-u^2)^{-1/2}P_2^1(u) = Au , \tag{17}$$

where A is a constant. Although (17) solves Eq. (15) only for $C = 2$, we expect it to be approximately valid for $C \approx 2$.

The constant A was approximately evaluated in our earlier paper [7] as

$$A^2 = \frac{7}{3C}(C-2) . \tag{18}$$

In that paper we studied instanton solutions of the scalar field equations in Euclideanized de Sitter space. These instantons describe nucleation of spherical domain walls during inflation. The subsequent evolution of the walls can be found by analytically continuing the instanton solutions. Moreover, by a suitable choice of coordinates, an expanding nucleated wall can be transformed into a planar wall (5). [This transformation is similar to the transformation from the de Sitter metric with closed spatial sections to the spatially flat form.] This leads to the conclusion that expression (18) for A is still applicable for Lorentzian wall solutions.

The wall thickness in de Sitter space can be approximately calculated using the relation

$$y'(0)\delta \sim 1 . \tag{19}$$

Combining Eqs. (17), (18), and (19), we can obtain an approximate expression for the wall thickness in de Sitter space:

$$\delta \sim H^{-1}\{1 - 2(H\delta_0)^2\}^{-1/2} . \tag{20}$$

The effect of de Sitter expansion assumes significance as $\delta_0 \sim H^{-1}$ and becomes dramatic when $\delta_0 \to H^{-1}/\sqrt{2}$. In this limit Eq. (20) indicates that the wall thickness grows without bound. Thicker walls cannot survive in de Sitter space as coherent objects. They are smeared by the expansion of the Universe.

III. STRINGS AND MONOPOLES

A. Strings

We again consider a scalar field theory with a simple double-well potential

$$V(\varphi_a) = \frac{\lambda}{2}(\varphi_a\varphi_a - \eta^2)^2 , \tag{21}$$

where $a = 1,\ldots,N$. The values $N = 2$ and $N = 3$ correspond to strings and monopoles, respectively. The scalar field equation is given by

$$\frac{1}{\sqrt{-g}}\partial_\mu(g^{\mu\nu}\partial_\nu\varphi_a) = -2\lambda\varphi_a(\varphi_b\varphi_b - \eta^2) . \tag{22}$$

We consider an infinite straight string situated along the z axis. This is again a configuration that is equivalent to the nucleating circular loops discussed in [2]. The cylindrical symmetry of this string configuration suggests a recoordination of the de Sitter metric as follows:

$$ds^2 = dt^2 - \exp(2Ht)(d\rho^2 + dz^2 + \rho^2 d\phi^2) . \tag{23}$$

The string is described by a two-component scalar field theory. Since we are looking for stationary solutions to the scalar field equation, we choose the following ansatz for the scalar field:

$$\varphi_1 = f(\rho e^{Ht})\cos(n\phi) ,$$
$$\varphi_2 = f(\rho e^{Ht})\sin(n\phi) . \tag{24}$$

Replacing φ_a from Eq. (24) in Eq. (22), and introducing the dimensionless variables $u = H\rho e^{Ht}A$ and $y = f/\eta$, the two field equations reduce to a single equation for y:

$$(1-u^2)\frac{\partial^2 y}{\partial u^2} + \frac{1-4u^2}{u}\frac{\partial y}{\partial u} - yu^{-2} = 2Cy(y^2-1) , \tag{25}$$

where $C = H^{-2}/\delta_0^2$, and δ_0 is the flat-space thickness of the string core. As we did for walls, we look for solutions to Eq. (25) in two asymptotic regimes, i.e., the region far away from the string core where $y \approx 1$, and thick string asymptotics where $y \approx 0$.

In the large distance distance regime, $u \gg 1$, and $y = 1 + f$, with $f \ll 1$. Then Eq. (25) reduces to the same equation (9) that we obtained in the case of a domain wall. The solution is given by Eq. (13). Once again, instead of a monotonic approach to η as observed in flat space, the string solution exhibits a damped oscillatory behavior.

For a thick string we can assume that $y \ll 1$ and linearize Eq. (25) by discarding the cubic term in y. We expect that in this case, the solution will be well approximated by a linear term,

$$y = Au \tag{26}$$

up to very large values of u. Substituting this in the linearized equation, Eq. (25), we obtain a condition for C, $C = 2$. This indicates that the critical value of the flat-space core thickness is $\delta_0 = H^{-1}/\sqrt{2}$, the same as for the domain wall.

B. Monopoles

We consider a monopole located at the origin. Using spherical polar coordinates to describe the spatial part of the de Sitter metric,

$$ds^2 = dt^2 - \exp(2Ht)(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta \, d\phi^2) \quad (27)$$

we choose the following ansatz for the three components of the scalar field:

$$\varphi_1 = f(re^{Ht}) \cos \theta \ ,$$
$$\varphi_2 = f(re^{Ht}) \sin \theta \, \cos \phi \ ,$$
$$\varphi_3 = f(re^{Ht}) \sin \theta \, \sin \phi \ . \quad (28)$$

The field equation then reduces to

$$(1 - u^2)\frac{\partial^2 y}{\partial u^2} + \frac{2 - 4u^2}{u}\frac{\partial y}{\partial u} - 2yu^{-2} = 2Cy(y^2 - 1) \ , \quad (29)$$

where $u = Hre^{Ht}$, $y = f/\eta$, and C has the same meaning as before. In the large distance limit, we find once again that the asymptotic behavior is described by Eq. (9), with the solution (13). In the thick monopole limit, linearizing Eq. (29), and substituting the linear u dependence (26), we obtain $C = 2$ as in the previous two cases.

Instead of using the linear ansatz (26) for thick monopole and string solutions, we could look for a general solution of the linearized field equation and require regularity at the horizon ($u = 1$), as we did for domain walls in Sec. II. The analysis is essentially identical to that given in Ref. [7] and leads to the same critical value of $C = 2$.

IV. SUMMARY

We studied the effect of the exponential expansion of the Universe on the internal structure of topological defects, concentrating mainly on the case of domain walls. We found that flat-space domain-wall solutions whose thickness δ_0 is much smaller than the de Sitter horizon H^{-1} are not substantially modified when the wall is transplanted to de Sitter space. The main modification is that at distances greater than H^{-1} from the wall, the field exhibits a damped oscillatory approach to the VEV η (in contrast with the monotonic approach to η in flat space). As the flat-space wall thickness δ_0 approaches the critical value

$$\delta_c = H^{-1}/\sqrt{2} \quad (30)$$

the effect of de Sitter expansion becomes predominant. In this limit, the wall thickness grows unboundedly according to the relation (20). No regular solutions to Eq. (3) exist for $\delta_0 > \delta_c$. Very similar results have been obtained for strings and monopoles. In all three cases, when the flat-space thickness of the defect core δ_0 exceeds the critical value (30), no stationary solutions exist indicating that thicker defects cannot survive in de Sitter space as coherent objects. They are smeared out by the expansion of the universe, and their thickness grows as $\delta \sim e^{Ht}$.

The formation of defects by quantum fluctuations during inflation, which was discused in Ref. [4], assumes that the scalar field mass $m_\varphi \ll H$, so that $\delta_0 \sim m_\varphi^{-1} \gg H^{-1}$. Hence, in this case, defects cannot be considered as "formed" until after the end of inflation, and instead of defects it is more appropriate to describe them as "zeros of the field φ." However, since defect formation eventually occurs where there are zeros of φ, the conclusions of Ref. [4] remain essentially unchanged.

[1] For a review of inflation, see e.g., E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, New York, 1990); A. Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic, Chur, Switzerland, 1990); or S. K. Blau and A. H. Guth, in *300 Years of Gravitation*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1987).

[2] R. Basu, A. H. Guth, and A. Vilenkin, Phys. Rev. D **44**, 340 (1991).

[3] Q. Shafi and A. Vilenkin, Phys. Rev. D **29**, 1870 (1984);

L. Kofman and A. D. Linde, Nucl. Phys **B282**, 555 (1987).

[4] E. T. Vishniac, K. A. Olive, and D. Seckel, Nucl. Phys. **B289**, 717 (1987); A. D. Linde and D. H. Lyth, Phys. Lett. B **246**, 353 (1990); H. M. Hodges and J. R. Primack, Phys. Rev. D **43**, 3155 (1991); M. Nagasawa and J. Yokoyama, Nucl. Phys. **B370**, 472 (1992).

[5] J. Garriga and A. Vilenkin, Phys. Rev. D **45**, 3469 (1992).

[6] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1965).

[7] R. Basu and A. Vilenkin, Phys. Rev. D **46**, 2345 (1992).