ARTICLES

Direct detection of supersymmetric dark matter and the role of the target nucleus spin

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We investigate the role of nuclear spin in elastic scattering of dark matter (DM) neutralinos from nuclei in the framework of the minimal supersymmetric standard model (MSSM). The relative contribution of spin-dependent axial-vector and spin-independent scalar interactions to the event rate in a DM detector has been analyzed for various nuclei. Within general assumptions about the nuclear and nucleon structure we find that for nuclei with atomic weights $A > 50$ the spin-independent part of the event rate $R_{\rm SI}$ is larger than the spin-dependent one $R_{\rm SD}$ in the domain of the MSSM parameter space allowed by the known experimental data and where the total event rate is $R = R_{\rm SD} + R_{\rm SI} > 0.01$ events/(kg day). The latter condition reflects realistic sensitivities of present and near future DM detectors. Therefore we expect equal chances for discovering a DM event either with spin-zero or with spin-nonzero isotopes if their atomic weights are $A_1 \sim A_2 > 50$. We discuss several examples of spin-nonzero nuclei (^{19}F, ^{23}Na, ^{73}Ge, ^{127}I, ^{129}Xe) as a target material for DM detectors and compare their axial-vector couplings to the neutralino.

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I. INTRODUCTION

The analysis of the data on the distribution and motion of astronomical objects within our Galaxy and far beyond indicates the presence of a large amount of nonluminous dark matter (DM). According to estimations, DM may constitute more than 90% of the total mass of the Universe if a mass density ρ of the Universe close to the critical value $\rho_{\rm crit}$ is assumed. The exact equality $\Omega = \rho/\rho_{\rm crit} = 1$, corresponding to a flat universe, is supported by naturalness arguments and by inflation scenarios. Also, in our Galaxy most of the mass should be in the form of a spherical dark halo.

The theory of primordial nucleosynthesis restricts the amount of baryonic matter in the Universe to $\sim 10\%$. Thus a dominant component of DM is nonbaryonic. The recent data from the Cosmic Background Explorer (COBE) satellite [1] on anisotropy in the cosmic background radiation and the theory of the formation of large scale structures of the Universe lead to the conclusion that nonbaryonic DM itself consists of a dominant (70%) "cold" DM (CDM) and smaller (30%) "hot" DM (HDM) component [2,3]. At present the neutralino χ is a favorable candidate for CDM. This is a Majorana ($\chi^c = \chi$) particle with spin $\frac{1}{2}$ predicted by supersymmetric (SUSY) models.

There are four neutralinos in the minimal supersymmetric extension of the standard model (MSSM) (see [4]). They are a mixture of gauginos ($\tilde{W}_3, \tilde{B}$) and higgsinos ($\tilde{H}_{1,2}$) being SUSY partners of gauge (W_3, B) and Higgs ($H_{1,2}$) bosons. The DM neutralino χ is assumed to be the lightest supersymmetric particle (LSP) and therefore is stable in SUSY models with R-parity conservation.

In the galactic halo, neutralinos are assumed to be Maxwellian distributed in velocities with a mean velocity in the Earth frame $v \approx 320$ km/sec [5]. Their mass density in the Solar System is expected to be about $\rho \approx 0.3$ GeV cm^{-3}. Therefore, neutralinos might produce at the Earth surface a substantial flux ($\Phi = \rho v/M$) of $\Phi > 10^7$ cm^{-2} sec^{-1} for a particle mass of $M \sim 1$ GeV. In view of this, one may hope to detect DM particles directly, for instance, through elastic scattering from nuclei inside a detector. The problem of direct detection of the DM neutralino χ via elastic scattering off nuclei has been considered by many authors and remains a field of great experimental and theoretical activity [6–22].

The final goal of theoretical calculations in this problem is the event rate R for elastic χ-nucleus scattering. In general, it contains contributions from the spin-dependent ($R_{\rm SD}$) and spin-independent ($R_{\rm SI}$) neutralino-nucleus interactions: $R = R_{\rm SD} + R_{\rm SI}$. Obviously, $R_{\rm SD}$ vanishes for spinless nuclei, while both terms contribute in the case of spin-nonzero nuclei. This fact is often regarded as a reason to assert spin-nonzero nuclei to be

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the most favorable target material for DM neutralino detectors as giving a larger event rate in comparison with spinless nuclei. However, this is right only if the spin-dependent interaction dominates in elastic neutralino scattering off nuclei with nonzero spin.

In this paper we address the question of the role of nuclear spin in the DM neutralino detection. We investigate this problem in the framework of the MSSM. We avoid using specific nuclear and nucleon structure models but rather base our consideration on the known experimental data about nuclei and nucleon properties. We pay special attention to uncertainties in nucleonic matrix elements, contributing to the spin-dependent and to the spin-independent neutralino-nucleon scattering. We undertake a systematic exploration of a broad domain of the MSSM parameter space restricted by experimental constraints on SUSY-particle and Higgs boson masses as well as by the cosmological bounds on a neutralino relic abundance in the Universe.

We have found that the $R_{\rm SI}$ contribution dominates in the total event rate R for nuclei with atomic weight $A > 50$ in the region of the MSSM parameter space, where $R = R_{\rm SD} + R_{\rm SI} > 0.01$ events/(kg day). The lower bound 0.01 events/(kg day) seems to be far below the sensitivity of realistic present and near future DM detectors. Therefore, one can ignore the region where $R < 0.01$ events/(kg day) as invisible for these detectors.

In view of this result we do *not* expect a crucial dependence of the DM event rate on the nuclear spin for detectors with target nuclei having an atomic weight larger than 50. In other words, we expect essentially *equal chances* for $J = 0$ and $J \neq 0$ detectors to discover DM events.

In particular, this conclusion supports the idea that presently operating $\beta\beta$ detectors with spinless nuclear target material can be successfully used for DM neutralino search. These highly developed setups (for a review see [16]), operating under extremely low background conditions, use a detection technology that is suitable for the DM search.

In Sec. II we analyze in certain approximations general properties of the neutralino-nucleus interactions relevant for the event rate calculations and for a further discussion of the role of nuclear spin in direct DM detection. In Sec. III we explain details of the supersymmetric model we use for calculation of the effective neutralino-quark interactions at low energies. Section IV is devoted to the numerical analysis of the MSSM parameter space and to the calculations of the event rate for various nuclei. Here we discuss our results and compare some nuclei as target material for DM detectors. Section V gives a conclusion.

II. GENERAL PROPERTIES OF THE NEUTRALINO-NUCLEUS INTERACTIONS

A DM event is elastic scattering of a DM neutralino from a target nucleus producing a nuclear recoil, which can be detected by a detector. The event rate per unit mass of the target material depends on the distribution of the DM neutralinos in the solar vicinity and the cross section $\sigma_{\rm el}(\chi A)$ of neutralino-nucleus elastic scattering. One can calculate $\sigma_{\rm el}(\chi A)$ starting from the neutralino-quark effective Lagrangian. In the most general form it can be given by the formula

$$L_{\rm eff} = \sum_q \left(\mathcal{A}_q \bar{\chi}\gamma_\mu\gamma_5\chi\bar{q}\gamma^\mu\gamma_5 q + \frac{m_q}{M_W}\mathcal{C}_q\bar{\chi}\chi\bar{q}q \right) + O\left(\frac{1}{m_{\tilde{q}}^4}\right) , \qquad (1)$$

where terms with the vector and pseudoscalar quark currents are omitted being negligible in the case of the nonrelativistic DM neutralino with typical velocities $v_\chi \approx 10^{-3}c$.

We also neglect in the Lagrangian (1) terms, which appear in supersymmetric models at the order of $1/m_{\tilde{q}}^4$ and higher, where $m_{\tilde{q}}$ is the mass of the scalar superpartner $\tilde{q}$ of the quark q. These terms, as recently pointed out by Drees and Nojiri [13], are potentially important in the spin-independent neutralino-nucleon scattering, especially in domains of the MSSM parameter space, where $m_{\tilde{q}}$ is close to the neutralino mass M_χ. Below we adopt the approximate treatment of these terms proposed in Ref. [13], which allows to absorb them "effectively" into the coefficients $\mathcal{C}_q$ in a wide region of the SUSY model parameter space. The coefficients $\mathcal{A}_q, \mathcal{C}_q$ depend on the specific SUSY model and will be considered in the next section.

Here we survey general properties of neutralino-nucleus (χA) scattering following from the Lagrangian (1). To calculate $\sigma_{\rm el}(\chi A)$ one should average the χq interactions sequentially over the nucleon and the nuclear structure. The first and the second terms in $L_{\rm eff}$ (1) averaged over the nucleon states give the spin-dependent and the spin-independent matrix elements $\mathbf{M}_{\rm SD}$ and $\mathbf{M}_{\rm SI}$, respectively.

For the spin-dependent matrix element we have [6,7]

$$\mathbf{M}_{\rm SD}^{p(n)} = 4\vec{S}_\chi \vec{S}_{p(n)} \sum_{q\in p(n)} \mathcal{A}_q \Delta q^{p(n)} , \qquad (2)$$

where $\vec{S}_\chi$ and $\vec{S}_{p(n)}$ are the neutralino and proton (neutron) spin operators; summation over the quark content of the proton (neutron) is assumed; $\Delta q^{p(n)}$ are the fractions of the proton (neutron) spin carried by the quark q. The standard definition is

$$\langle p(n)|\bar{q}\gamma^\mu\gamma_5 q|p(n)\rangle = 2S^\mu_{p(n)}\Delta q^{p(n)} , \qquad (3)$$

where $S^\mu_{p(n)} = (0, \vec{S}_{p(n)})$ is the 4-spin of the nucleon. The parameters $\Delta q^{p(n)}$ can be extracted from data on polarized nucleon structure functions [23,24] and hyperon semileptonic decay data [25].

It has been recently recognized [26] that the new preliminary Spin Muon Collaboration (SMC) measurements [24] of the spin structure function of the proton at $Q^2 = 10.3$ GeV^2 may have dramatic implications for calculations of the spin-dependent neutralino-nucleus scattering cross section. The values of Δq extracted from these new data in comparison with previous European Muon Collaboration (EMC) [23] data are closer to SU(3) naive quark model (NQM) predictions [6,27]. This gives rise to a small enhancement of the spin-dependent cross section for nuclei with an unpaired proton and a strong (by a factor of about 30) suppression for nuclei with an unpaired neutron. In view of this we use in the analysis Δq values extracted both from the EMC [23] and from SMC [24] data. The values of Δq^p for the proton, taken from Ref. [26], we present in the Table I. For comparison, results of the NQM are also displayed. The relevant values of Δq^n for the neutron can be obtained from those in the Table I by the isospin symmetry substitution $\Delta u \to \Delta d,, \Delta d \to \Delta u$.

The spin-independent matrix element can be written in the form [8,10,11]

$$\mathbf{M}_{\rm SI} = \left[\hat{f}\frac{m_u\mathcal{C}_u + m_d\mathcal{C}_d}{m_u + m_d} + f_s\mathcal{C}_s + \frac{2}{27}(1 - f_s - \hat{f})(\mathcal{C}_c + \mathcal{C}_b + \mathcal{C}_t)\right]\frac{M_{p(n)}}{M_W}\bar{\chi}\chi\bar{\Psi}\Psi \ , \tag{4}$$

where the parameters f_s and $\hat{f}$ are defined as

$$\langle p(n)|(m_u + m_d)(\bar{u}u + \bar{d}d)|p(n)\rangle = 2\hat{f}M_{p(n)}\bar{\Psi}\Psi \ ,$$
$$\langle p(n)|m_s\bar{s}s|p(n)\rangle = f_s M_{p(n)}\bar{\Psi}\Psi \ . \tag{5}$$

The values extracted from the data under certain theoretical assumptions are [28,29]

$$\hat{f} = 0.05 \text{ and } f_s = 0.14 \ . \tag{6}$$

The strange quark contribution f_s is known to be uncertain to about a factor of 2. Therefore we take its values in the analysis within the interval $0.07 < f_s < 0.3$ [30,28].

Averaging (2) and (4) over the nuclear state $|A\rangle$ we deal with the following matrix elements at vanishing momentum transfer:

$$\langle A|M_{p(n)}\bar{\Psi}\Psi|A\rangle = M_A\bar{A}A \ ,$$
$$\langle A|\vec{S}_{p(n)}|A\rangle = \lambda\langle A|\vec{J}|A\rangle \ . \tag{7}$$

TABLE I. Quark spin content of the proton, determined from the SU(3) naive quark model (NQM) [6,27], from the EMC [23], and from the preliminary SMC [24] measurements of the spin-dependent structure functions. In the latter case the central values, and values using the 1σ error on $\Delta\Sigma$ are also presented.

	NQM	EMC	$\mathrm{SMC}_{\rm mean}$ (prelim)	$\mathrm{SMC}_{\Delta\Sigma+1\sigma}$ (prelim)
Δu	0.93	0.78	0.79	0.725
Δd	−0.33	−0.50	−0.36	−0.095
Δs	0	−0.16	−0.07	−0.07

Here $M_A, \vec{J}$ are nuclear mass and spin. On the basis of the odd-group shell model [31] (essentially a somewhat relaxed single particle shell model) the parameter λ can be related to the nuclear magnetic moment μ as

$$\lambda J = \frac{\mu - g^l J}{g^s - g^l} \ , \tag{8}$$

where $g^l = 1(0)$ and $g^s = 5.586\ (-3.826)$ are orbital and spin proton (neutron) g factors. Then one can extract values of λ for various nuclei from the experimental data on nuclear magnetic moments. We use in this paper the values of λ as presented in [12,3]. For nuclei of interest in the DM search they are given in the Table II.

For large M_χ and M_A the momentum transfer may be comparable to the inverse radius of a nucleus and then one should take into account the finite size effect. It can be done by introducing the coherence loss factor, which is the squared nuclear form factor integrated over the Maxwellian velocity distribution of the incident neutralino. Taking an empirical Gaussian parametrization for the form factor one can write the coherence loss factor in the form [5,12]

$$\zeta(r) = \frac{0.573}{b}\left(1 - \frac{\exp(-b/1+b)}{\sqrt{1+b}}\frac{\mathrm{erf}(\sqrt{1/1+b})}{\mathrm{erf}(1)}\right) \ , \tag{9}$$

where

$$b = \frac{8}{9}\sigma^2 r^2\frac{M_\chi^2 M_A^2}{(M_\chi + M_A)^2} \ .$$

Here σ^2 is the dispersion of the Maxwellian neutralino velocity distribution. To obtain the coherence loss factor for spin-independent scattering we take $r = r_{\rm charge}$ in (9), where $r_{\rm charge}$ is the rms charge radius of the nucleus A [32]:

$$r_{\rm charge} = (0.3 + 0.89A^{1/3})\ \mathrm{fm} \ . \tag{10}$$

The coherence loss factor for spin-dependent scattering is given by (9) with $r = r_{\rm spin}$. For the rms spin radius of the nucleus A we use the values from harmonic well potential calculations [12]. The ratio $r_{\rm spin}/r_{\rm charge}$ for various nuclei obtained in this approach is given in the Table II.

Recently a noticeable progress in detailed calculations of nuclear matrix elements relevant for DM detection has been achieved. An approach based on the theory of finite Fermi systems is advocated in Ref. [33]. Calculations for some nuclei of interest in the DM search have been made [34] also within the conventional shell model. However, for our purposes the above-described semiempirical scheme is sufficient.

On this basis one can arrive at the formulas for the event rate of elastic neutralino-nucleus scattering in the detector per day and unit mass of the target material:

$$R = R_{\rm SI} + R_{\rm SD} \ , \tag{11}$$

where the spin-dependent ($R_{\rm SD}$) and spin-independent

TABLE II. Parameters for event rate calculations.

Isotope	J	λ^2	Unpaired p/n	$\frac{r_{\rm spin}}{r_{\rm charge}}$	Isotope	J	λ^2	Unpaired p/n	$\frac{r_{\rm spin}}{r_{\rm charge}}$
^{1}H	1/2	1.0	p	1.00	^{99}Ru	5/2	0.0045	n	1.19
^{3}He	1/2	1.2373	n	1.00	^{101}Ru	5/2	0.0056	n	1.19
^{7}Li	3/2	0.1096	p	1.17	^{107}Ag	1/2	0.0720	p	1.06
^{9}Be	3/2	0.0768	n	1.12	^{109}Ag	1/2	0.0760	p	1.06
^{11}B	3/2	0.0299	p	1.09	^{111}Cd	1/2	0.0960	n	1.17
^{15}N	1/2	0.1160	p	1.06	^{113}Cd	1/2	0.1053	n	1.17
^{17}O	5/2	0.0391	n	1.25	^{115}Sn	1/2	0.2307	n	1.16
^{19}F	1/2	0.8627	p	1.21	^{117}Sn	1/2	0.2733	n	1.16
^{23}Na	3/2	0.0109	p	1.16	^{121}Sb	5/2	0.0057	p	1.15
^{27}Al	5/2	0.0099	p	1.13	^{123}Sb	7/2	0.0035	p	1.15
^{29}Si	1/2	0.0840	n	1.12	^{127}I	5/2	0.0026	p	1.15
^{31}P	1/2	0.0760	p	1.11	^{129}Xe	1/2	0.1653	n	1.14
^{35}Cl	3/2	0.0096	p	1.10	^{131}Xe	3/2	0.0147	n	1.14
^{47}Ti	5/2	0.0067	n	1.20	^{133}Cs	7/2	0.0033	p	1.14
^{49}Ti	7/2	0.0068	n	1.20	^{139}La	7/2	0.0020	p	1.13
^{51}V	7/2	0.0106	p	1.19	^{155}Gd	3/2	0.0021	n	1.22
^{55}Mn	5/2	0.0069	p	1.17	^{157}Gd	3/2	0.0035	n	1.22
^{59}Co	7/2	0.0049	p	1.15	^{183}W	1/2	0.0040	n	1.19
^{67}Zn	5/2	0.0083	n	1.13	^{191}Ir	3/2	0.0387	p	1.08
^{69}Ga	3/2	0.0056	p	1.13	^{193}Ir	3/2	0.0379	p	1.08
^{71}Ga	3/2	0.0237	p	1.13	^{199}Hg	1/2	0.0693	n	1.17
^{73}Ge	9/2	0.0026	n	1.25	^{201}Hg	3/2	0.0096	n	1.17
^{79}Br	3/2	0.0077	p	1.11	^{203}Tl	1/2	0.2400	p	1.07
^{81}Br	3/2	0.0125	p	1.11	^{205}Tl	1/2	0.2467	p	1.07
^{91}Zr	5/2	0.0186	n	1.21	^{207}Pb	1/2	0.0960	n	1.17
^{93}Nb	9/2	0.0065	p	1.20	^{209}Bi	9/2	0.0002	p	1.16

($R_{\rm SI}$) terms are (see also [8,13])

$$R_{\rm SD}^{p(n)} = 4\zeta(r_{\rm spin})\lambda^2 J(J+1)|\mathcal{M}_{\rm SD}^{p(n)}|^2\mathcal{D}\frac{\rm events}{\rm kg\,day}\ , \qquad (12)$$

$$R_{\rm SI} = \zeta(r_{\rm charge})\left(\frac{M_A}{M_W}\right)^2|\mathcal{M}_{\rm SI}|^2\mathcal{D}\frac{\rm events}{\rm kg\,day}\ . \qquad (13)$$

The common kinematic factor $\mathcal{D}$ and the properly normalized nucleon matrix elements $\mathcal{M}_{\rm SI}, \mathcal{M}_{\mathcal{D}}$ are defined as

$$\mathcal{D} = 1.8\times10^{11}\ {\rm GeV}^4\left[\frac{4M_\chi M_A}{\pi(M_\chi+M_A)^2}\right]\left[\frac{\rho}{0.3\ {\rm GeV\,cm^{-3}}}\right]\times\left[\frac{\langle|\vec{v}_E|\rangle}{320\ {\rm km\,sec^{-1}}}\right]\ , \qquad (14)$$

$$\mathbf{M}_{\rm SD}^{p(n)} = 4\mathcal{M}_{\rm SD}^{p(n)}\vec{S}_\chi\vec{S}_{p(n)}\ , \qquad (15)$$

$$\mathbf{M}_{\rm SI} = \mathcal{M}_{\rm SI}\frac{M_{p(n)}}{M_W}\bar{\chi}\chi\bar{\Psi}\Psi\ , \qquad (16)$$

For the definition of $\mathbf{M}_{\rm SD}, \mathbf{M}_{\rm SI}$ see formulas (2),(4). Note that the dimension of the matrix elements $\mathcal{M}_{\rm SD,SI}$ is ${\rm GeV}^{-2}$; thus, the event rate R in Eq. (11) is measured in events/(kg day). In Eq. (14) $\rho\approx0.3$ GeV cm^{-3} is the DM neutralino density in the solar vicinity and $\langle|\vec{v}_E|\rangle\approx320$ km/sec is the averaged velocity of the DM neutralino at Earth's surface.

To study the role of nuclear spin in elastic χ-nucleus scattering we consider the ratio

$$\eta = R_{\rm SD}/R_{\rm SI} \qquad (17)$$

characterizing the relative contribution of spin-dependent and spin-independent interactions. The quantity $\eta+1$ determines the expected relative sensitivity of DM detectors with spin-nonzero ($J\neq0$) to those with spin-zero ($J=0$) nuclei as target material, if their atomic masses are close in value. If $\eta<1$, then detectors with spin-nonzero and spin-zero target materials have approximately equal sensitivities to the DM signal; otherwise, if $\eta>1$, the spin-nonzero detectors are more sensitive than the spin-zero ones.

Let us consider separately the dependence of η on the nuclear structure parameters and on the paramete[r] of neutralino-quark interactions determined in a spec[ific] SUSY model. Within our approximations we may w[rite]

$$\eta = \eta_A\eta_{\rm SUSY}^{p(n)}\ ,$$

where

$$\eta_A = 4\lambda^2 J(J+1)\frac{\zeta(r_{\rm spin})M_W^2}{\zeta(r_{\rm charge})M_A^2}\ ,$$

$$\eta_{\rm SUSY}^{p(n)} = \left(\frac{\mathcal{M}_{\rm SD}^{p(n)}}{\mathcal{M}_{\rm SI}}\right)^2\ .$$

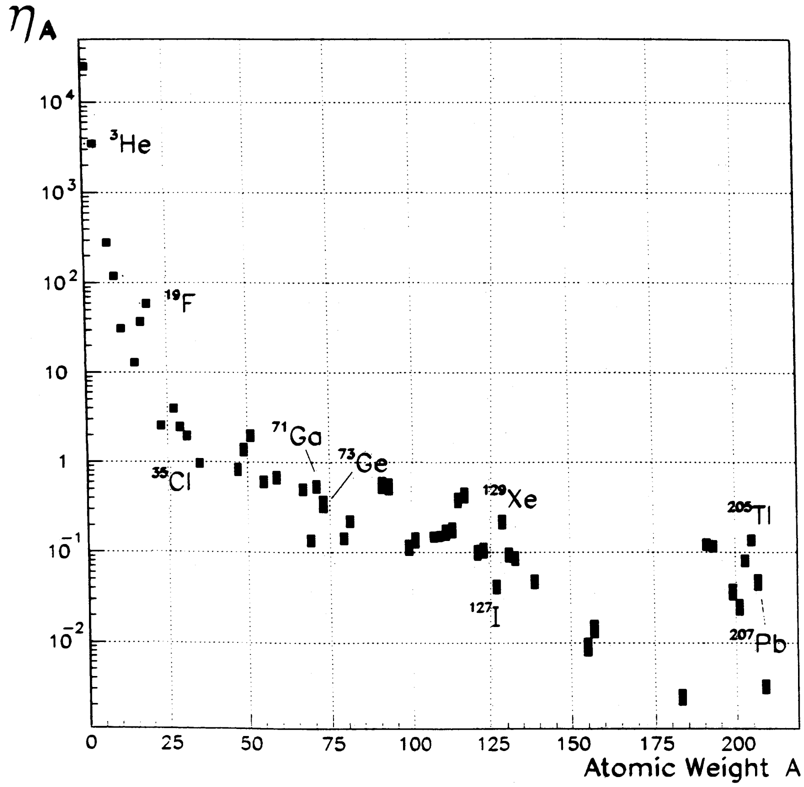

FIG. 1. The nuclear factor η_A vs the atomic weight A for nuclei with nonzero spin. The height of the symbol represents the variation within the interval of the neutralino masses 20 GeV$< M_\chi <$ 500 GeV.

The factorization (18) of the nuclear structure η_A from the supersymmetric part of the neutralino-nucleus interaction $\eta_{\rm SUSY}$ is essentially based on the assumption of the odd-group model [31] about a negligible contribution of the even nucleon group to the total nuclear spin. η_A is a factor depending on the properties of the nucleus A, while $\eta^{p(n)}_{\rm SUSY}$ is defined by the SUSY model, which specifies the neutralino composition and the interactions with matter. The SUSY factor also depends on the nucleon matrix element parameters (3),(5) and on the shell-model class to which nucleus A belongs, being $\eta^n_{\rm SUSY}$ for the "neutron" shell-model (^{3}He,^{29}Si,^{73}Ge, ^{129,131}Xe,...) and $\eta^p_{\rm SUSY}$ for the "proton" shell-model (^{19}F,^{23}Na,^{35}Cl,^{127}I, ^{205}Tl,...).

Figure 1 shows the calculated nuclear factor η_A versus the atomic weight A. The height of the symbols in the picture represents the variation of the ratio η_A within the explored interval of the neutralino mass of 20 GeV$\leq M_\chi \leq$ 500 GeV.

It follows from Fig. 1 that $\eta_A < 1$ for $A > 50$. Thus for $A > 50$ there is no nuclear structure enhancement of the spin-dependent event rate as compared to the spin-independent one. The next step is an estimation of the SUSY factor $\eta^{p(n)}_{\rm SUSY}$.

III. SPECIFIC SUSY-MODEL PREDICTIONS

To estimate the factor $\eta_{\rm SUSY}$ in Eqs. (18) and (20) one should calculate the parameters $\mathcal{A}_q$, and $\mathcal{C}_q$ of the effective Lagrangian (1) in the specific SUSY model. We will follow the MSSM. This model is specified by the standard SU(3)×SU(2)×U(1) gauge couplings as well as by the low-energy superpotential and "soft" SUSY-breaking terms [4].

The effective low-energy superpotential is

$$\tilde{W} = \sum_{\rm generations} (h_L\hat{H}_1\hat{L}\hat{E} + h_D\hat{H}_1\hat{Q}\hat{D} - h_U\hat{H}_2\hat{Q}\hat{U}) \\ -\mu\hat{H}_1\hat{H}_2 \,. \tag{21}$$

Here $\hat{L},\hat{E}$ are lepton doublets and singlets; $\hat{Q}$ are quark doublets, $\hat{U},\hat{D}$ are *up* and *down* quark singlets; $\hat{H}_1$ and $\hat{H}_2$ are the Higgs doublets with weak hypercharge $Y = -1,+1$, respectively.

The effect of soft supersymmetric breaking can be parametrized at the Fermi scale as a part of the scalar potential,

$$V_{\rm soft} = \sum_{i=\rm scalars} m_i^2|\phi_i|^2 + h_LA_LH_1\tilde{L}\tilde{E} + h_DA_DH_1\tilde{Q}\tilde{D} \\ -h_uA_UH_2\tilde{Q}\tilde{U} - (\mu BH_1H_2 + \text{H.c.}) \tag{22}$$

and a soft gaugino mass term

$$\mathcal{L}_{FM} = -\tfrac{1}{2}[M_1\tilde{B}\tilde{B} + M_2\tilde{W}^k\tilde{W}^k + M_3\tilde{g}^a\tilde{g}^a] - \text{H.c.} \tag{23}$$

As usual, $M_{1,2,3}$ are the masses of the SU(3) ×SU(2)×U(1) gauginos $\tilde{g}$, $\tilde{W}$, $\tilde{B}$, and m_i are the masses of scalar fields.

To reduce the number of free parameters we use the following unification conditions at the grand unified theory (GUT) scale M_X:

$$A_U = A_D = A_L = A_0 \ , \tag{24}$$

$$m_L = m_E = m_Q = m_U = m_D = m_0 \ , \tag{25}$$

$$M_1 = M_2 = M_3 = m_{1/2} \ , \tag{26}$$

$$g_1(M_X) = g_2(M_X) = g_3(M_X) = g_{\rm GUT} \ , \tag{27}$$

where g_3, g_2, g_1 are the SU(3)×SU(2)×U(1) gauge coupling constants equal to $g_{\rm GUT}$ at the unification scale M_X.

At the Fermi scale $Q \sim M_W$ these parameters can be evaluated on the basis of the renormalization-group equations (RGE's) [35,36]. The equation (26) implies, at $Q \sim M_W$,

$$M_1 = \tfrac{5}{3}\tan^2\theta_W M_2, \quad M_2 = 0.3 m_{\tilde{g}} \ . \tag{28}$$

Here $m_{\tilde{g}} = M_3$ is the gluino mass. One can see from (24)–(27) that we do not exploit the complete set of GUT unification conditions for the soft supersymmetry-breaking parameters, which leads to the supergravity scenario with radiative electroweak gauge symmetry breaking. Specifically, we do not unify Higgs boson soft masses m_{H_1}, m_{H_2} with the others in Eq. (25). Otherwise, strong correlations in the supersymmetric particle spectrum would emerge, essentially attaching the analysis to a particular supersymmetric scenario.

The neutralino mass matrix in the MSSM has the form [4]

$$M_\chi = \begin{pmatrix} M_2 & 0 & -M_Z c_W s_\beta & M_Z c_W c_\beta \\ 0 & M_1 & M_Z s_W s_\beta & -M_Z s_W c_\beta \\ -M_Z c_W s_\beta & M_Z s_W s_\beta & 0 & -\mu \\ M_Z c_W c_\beta & -M_Z s_W c_\beta & -\mu & 0 \end{pmatrix} , \tag{29}$$

where $c_W = \cos\theta_W$, $s_W = \sin\theta_W$, $t_W = \tan\theta_W$, $s_\beta = \sin\beta$, $c_\beta = \cos\beta$. The matrix is written in the basis of fields $(\tilde{W}^3, \tilde{B}, \tilde{H}_2^0, \tilde{H}_1^0)$.

The angle β is defined by the vacuum expectation values of the neutral components of the Higgs fields: $\tan\beta = \langle H_2^0\rangle/\langle H_1^0\rangle$. By diagonalizing the mass matrix (29) one can obtain the lightest neutralino of the mass M_χ with the field content

$$\chi = \mathcal{N}_{11}\tilde{W}^3 + \mathcal{N}_{12}\tilde{B} + \mathcal{N}_{13}\tilde{H}_2^0 + \mathcal{N}_{14}\tilde{H}_1^0 \ . \tag{30}$$

We apply a diagonalization by means of a real orthogonal matrix $\mathcal{N}$. Therefore the coefficients $\mathcal{N}_{ij}$ are real and the mass M_χ is positive or negative. The low-energy neutralino-quark interactions also depend on the spectrum of squarks $\tilde{q}$ and Higgs particles at the Fermi scale.

The mass matrices of squarks in the basis $\tilde{q}_L - \tilde{q}_R$ can be written in the form

$$\mathcal{M}_{\tilde{q}}^2 = \begin{pmatrix} m^2_{(\tilde{q})LL} & m^2_{(\tilde{q})LR} \\ m^2_{(\tilde{q})LR} & m^2_{(\tilde{q})RR} \end{pmatrix} \tag{31}$$

The matrix elements are different for *up* and *down* squarks $\tilde{u}, \tilde{d}$ and depend on generation. One can write them as [4,37,38]

$$m^2_{(\tilde{u})LL} = m_u^2 + m_Q^2 + (\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W) D \ , \tag{32}$$

$$m^2_{(\tilde{u})RR} = m_u^2 + m_U^2 + \tfrac{2}{3}\sin^2\theta_W D \ , \tag{33}$$

$$m^2_{(\tilde{u})LR} = m_u (A_U - \mu\cot\beta) \ , \tag{34}$$

$$m^2_{(\tilde{d})LL} = m_d^2 + m_Q^2 - (\tfrac{1}{2} - \tfrac{1}{3}\sin^2\theta_W) D \ , \tag{35}$$

$$m^2_{(\tilde{d})RR} = m_d^2 + m_D^2 - \tfrac{1}{3}\sin^2\theta_W D \ , \tag{36}$$

$$m^2_{(\tilde{d})LR} = m_d (A_D - \mu\tan\beta) \ . \tag{37}$$

Here,

$$D_Z = M_Z^2 \cos 2\beta \ .$$

The squark mass eigenstates $\tilde{q}_i$ are then mixtures of the $\tilde{q}_L - \tilde{q}_R$ states

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_q & \sin\theta_q \\ -\sin\theta_q & \cos\theta_q \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix} , \tag{38}$$

with the mixing angle

$$\sin 2\theta_q = \frac{2 m^2_{LR}}{m^2_{\tilde{q}_1} - m^2_{\tilde{q}_2}} \ . \tag{39}$$

Their masses are given by the formula [4]

$$m^2_{\tilde{q}1,2} = \tfrac{1}{2}[m^2_{LL} + m^2_{RR} \mp \sqrt{(m^2_{LL} - m^2_{RR})^2 + 4 m^4_{LR}}] \ . \tag{40}$$

We calculate the soft supersymmetry-breaking parameters m_Q, m_U, m_D, A_U, A_D in terms of the parameters $m_0, m_{1/2}, A_0, \mu$ on the basis of the well-known approximate solutions [35,39] of the one-loop renormalization-group equations. In this case the approximation of the top Yukawa coupling dominance is implied and means that our analysis is limited to values of $\tan\beta$ well below $m_t/m_b \approx 35$.

We analyze the Higgs sector of the MSSM at the one-loop level [40], taking into account $\tilde{t}_L - \tilde{t}_R$, $\tilde{b}_L - \tilde{b}_R$ mixing between the third-generation squarks. Diagonalization of the Higgs boson mass matrix leads to three neutral mass-eigenstates: two CP-even states, h, H, with the masses m_h, m_H and the relevant mixing angle α_H, as well as one CP-odd state A with the mass m_A. We take the mass m_A as an independent free parameter. A complete list of the essential free parameters we use in the analysis is

$$\tan\beta, A_0, \mu, M_2, m_A, m_0, m_t \ . \tag{41}$$

In view of recently reported events by the Collider Detector at Fermilab (CDF) Collaboration [41], which may correspond to the top quark with a mass $m_t = 174 \pm 10^{+13}_{-12}$ GeV, we fix further for definiteness $m_t = 174$ GeV.

Having a particle spectrum one can derive the effective Lagrangian $L_{\rm eff}$ of low-energy neutralino-quark interactions. In the MSSM the first term of $L_{\rm eff}$ in Eq. (1) is induced by the Z boson and $\tilde{q}$ exchange [42], whereas the second one is due to the Higgs boson [43] and $\tilde{q}$ exchange [8,44].

Direct calculation of the relevant set of Feynman diagrams gives the following formulas for the coefficients of the effective Lagrangian $L_{\rm eff}$ in terms of the MSSM parameters:

$$\begin{aligned}
\mathcal{A}_q = -\frac{g_2^2}{4M_W^2}\Bigg[&\frac{\mathcal{N}_{13}^2-\mathcal{N}_{14}^2}{2}T_3 - \frac{M_W^2}{m_{\tilde{q}1}^2-M_\chi^2}(\cos^2\theta_q\phi_{qL}^2+\sin^2\theta_q\phi_{qR}^2)\\
&-\frac{M_W^2}{m_{\tilde{q}2}^2-M_\chi^2}(\sin^2\theta_q\phi_{qL}^2+\cos^2\theta_q\phi_{qR}^2) - \frac{m_q^2}{4}P_q^2\left(\frac{1}{m_{\tilde{q}1}^2-M_\chi^2}+\frac{1}{m_{\tilde{q}2}^2-M_\chi^2}\right)\\
&-\frac{m_q}{2}M_W P_q\sin 2\theta_q T_3(\mathcal{N}_{11}-\tan\theta_W\mathcal{N}_{12})\left(\frac{1}{m_{\tilde{q}1}^2-M_\chi^2}-\frac{1}{m_{\tilde{q}2}^2-M_\chi^2}\right)\Bigg] ,
\end{aligned} \tag{42}$$

$$\begin{aligned}
\mathcal{C}_q = -\frac{g_2^2}{4}\Bigg[&\frac{F_h}{m_h^2}h_q+\frac{F_H}{m_H^2}H_q + P_q\left(\frac{\cos^2\theta_q\phi_{qL}-\sin^2\theta_q\phi_{qR}}{m_{\tilde{q}1}^2-(1-\delta_{qt})M_\chi^2} - \frac{\cos^2\theta_q\phi_{qR}-\sin^2\theta_q\phi_{qL}}{m_{\tilde{q}2}^2-(1-\delta_{qt})M_\chi^2}\right)\\
&+\sin 2\theta_q\left(\frac{m_q}{4M_W}P_q^2-\frac{M_W}{m_q}\phi_{qL}\phi_{qR}\right)\left(\frac{1}{m_{\tilde{q}1}^2-(1-\delta_{qt})M_\chi^2}-\frac{1}{m_{\tilde{q}2}^2-(1-\delta_{qt})M_\chi^2}\right)\Bigg] .
\end{aligned}$$

Here $\delta_{qt} = 1$ for $q = t$ and is zero otherwise. The other coefficients are

$$\begin{aligned}
F_h &= (\mathcal{N}_{11}-\mathcal{N}_{12}\tan\theta_W)(\mathcal{N}_{13}\cos\alpha_H+\mathcal{N}_{14}\sin\alpha_H) ,\\
F_H &= (\mathcal{N}_{11}-\mathcal{N}_{12}\tan\theta_W)(\mathcal{N}_{13}\sin\alpha_H-\mathcal{N}_{14}\cos\alpha_H) ,\\
h_q &= (\tfrac{1}{2}+T_3)\frac{\cos\alpha_H}{\sin\beta} - (\tfrac{1}{2}-T_3)\frac{\sin\alpha_H}{\cos\beta} ,
\end{aligned} \tag{43}$$

$$H_q = (\tfrac{1}{2}+T_3)\frac{\sin\alpha_H}{\sin\beta} + (\tfrac{1}{2}-T_3)\frac{\cos\alpha_H}{\cos\beta} ,$$

$$\begin{aligned}
\phi_{qL} &= \mathcal{N}_{11}T_3+\mathcal{N}_{12}(Q-T_3)\tan\theta_W ,\\
\phi_{qR} &= \tan\theta_W Q\mathcal{N}_{12} ,\\
P_q &= (\tfrac{1}{2}+T_3)\frac{\mathcal{N}_{13}}{\sin\beta}+(\tfrac{1}{2}-T_3)\frac{\mathcal{N}_{14}}{\cos\beta} .
\end{aligned} \tag{44}$$

Formulas (42) take into account squark mixing $\tilde{q}_L-\tilde{q}_R$ and the contribution of both CP-even Higgs bosons h, H. As pointed out in Ref. [45], the contribution of the heavier Higgs boson H can be important in certain cases. It is seen from Eqs. (43) that at some values of the angles α_H, β and the neutralino composition coefficients $\mathcal{N}_{13}, \mathcal{N}_{14}$ the contribution of the heavier Higgs boson H to the coefficients $\mathcal{C}_q$ can be larger than the contribution of the lightest Higgs boson h. The above formulas coincide with the relevant formulas in Ref. [13], neglecting terms $\sim 1/m_{\tilde{q}}^4$ and higher. As stated in Sec. II we adopt the approximate treatment proposed in [13]. It allows us to take into account these terms "effectively" by introducing an "effective" top squark $\tilde{t}$ propagator of the form $1/m_{\tilde{q}}^2$ in the coefficient $\mathcal{C}_t$. This is accomplished in formulas (42) by introducing the Kronecker symbol δ_{qt} in the coefficients $\mathcal{C}_q$. In the limit $\theta_q \to 0$, where θ_q is the $\tilde{q}_L$-$\tilde{q}_R$ mixing angle (39) the above formulas (42) agree with [12] except the relative sign between the Z and $\tilde{q}$ exchange terms in the coefficients $\mathcal{A}_q$ and up to the overall sign in the coefficients $\mathcal{C}_q$. These errors in [12] were also observed in [13]. Now we are ready to estimate the $\eta_{\rm SUSY}$ factor (20) numerically.

IV. NUMERICAL ANALYSIS

In our numerical analysis we scan the MSSM parameter space within a broad domain

$$\begin{aligned}
20\ \text{GeV} < M_2 < 1\ \text{TeV},\ \ |\mu| < 1\ \text{TeV} ,\\
1 < \tan\beta < 20,\ \ |A_0| < 1\ \text{TeV} ,\\
0 < m_0 < 1\ \text{TeV},\ \ 50\ \text{GeV} < m_A < 1\ \text{TeV} .
\end{aligned} \tag{45}$$

The upper limit $\tan\beta < 20$ is taken for definiteness and to be well below $m_t/m_b \approx 35$ for consistency of the top Yukawa dominance approximation we use in the RGE. Other upper limits are inspired by the well-known "naturalness" arguments for soft SUSY-breaking parameters.

Further limitations on the parameter space are imposed by the experimental lower bounds on supersymmetric particle and Higgs boson masses from measurements at the CERN e^+e^- collider LEP [46] and Fermilab Tevatron [47] (see also [48–50]). With these constraints the neutralino mass varies within the interval 20 GeV$\leq M_\chi \leq$ 500 GeV.

The additional constraint we use in the analysis is a realistic sensitivity of a DM detector. In terms of the total event rate R we choose the sensitivity to be not better than

$$R > 0.01 \frac{\text{events}}{\text{kg day}} . \tag{46}$$

We do not expect DM detectors to go below this lower bound in near future [22]. Therefore, the constraint (46) reflects the realistic capacities of the present and near-future set-ups. It excludes the region in the parameter space corresponding to low rate DM signals inaccessible to these detectors.

The neutralino relic density Ω_χ is also under control in our analysis. We calculate it following the standard procedure on the basis of the approximate formula [42,51,52]:

$$\Omega_\chi h_0^2 = 2.13 \times 10^{-11} \left(\frac{T_\chi}{T_\gamma}\right)^3 \left(\frac{T_\gamma}{2.7K}\right)^3 \times N_F^{1/2} \left(\frac{\text{GeV}^{-2}}{a x_F + b x_F^2/2}\right) . \tag{47}$$

Here T_γ is the present day photon temperature, T_χ/T_γ is the reheating factor, $x_F = T_F/M_\chi \approx 1/20$, T_F is the neutralino freeze-out temperature, and N_F is the total number of relativistic degrees of freedom at T_F The coefficients a, b are determined from the expansion

$$\langle \sigma_{\text{ann}} v \rangle \approx a + bx \tag{48}$$

of the thermally averaged cross section $\langle \sigma_{\text{ann}} v \rangle$ of neutralino annihilation. We use an approximate treatment not taking into account complications, which occur when expansion (48) fails [53–55]. We take into account all possible channels of the χ-χ annihilation. The relevant formulas for the coefficients a, b and numerical values for the other parameters in Eqs. (47) and (48) can be found in the literature [52–58].

It is well known that cosmologically acceptable neutralinos should produce a relic density in the interval

$$0.025 < \Omega_\chi h_0^2 < 1 . \tag{49}$$

In this case neutralinos do not overclose the Universe and account for a significant fraction of the halo DM. However, we do not restrict our analysis to this domain of $\Omega_\chi h_0^2$ but survey all possible values of η_{SUSY} within region (45). We use the quoted cosmological criterion at the final stage to discriminate special points of the parameter space.

We have performed a complete numerical analysis of the MSSM parameter space within the above-defined constraints. The following upper bound for the SUSY factor in Eq. (18) was found

$$\eta_{\text{SUSY}} \lesssim 2 , \tag{50}$$

except for the narrow domain of negative values of the parameter μ, where η_{SUSY} exceeds this bound and sometimes approaches the order of magnitude of $\sim 10^3$. However in this region of the MSSM parameter space the event rate R is yet rather small, $R \lesssim 0.1$, and the neutralino relic density is well below the lower bound in (49), $\Omega_\chi h_0^2 < 0.025$. Therefore, one can safely disregard this region either on the cosmological grounds or as corresponding to a low rate DM signal. We consider the cosmological cut to be well motivated and accept it in our further analysis.

To represent our results we adopt the scatter plot approach, which is most suitable for our purposes. In this approach one can treat on equal footing all points of the MSSM parameter space within the domain (45), randomly generating their images in the space of observable variables such as the event rate and the neutralino mass. It is suitable for recognizing upper and/or lower bounds of the relevant observables from the shape of the domains obtained with this procedure.

In Figs. 2–10 we show our results in this form obtained by random point generation within the constrained MSSM parameter space.

Figure 2 shows the distribution of the points in the R-η_{SUSY} plane. Recall $\eta_{\text{SUSY}} = \eta_A^{-1}(R_{\text{SD}}/R_{\text{SI}})$ is the supersymmetric model contribution to the ratio of the spin-dependent (R_{SD}) to spin-independent (R_{SI}) parts of the total event rate R, as defined in (12)–(20). The nuclear structure factor η_A is presented in Fig. 1. Plots are given for two representative nuclei, with an unpaired proton (p-like) ^{71}Ga, and with an unpaired neutron (n-like) ^{73}Ge. The nuclei are taken for convenience near the point $A = 50$ (see Fig. 1). For heavier nuclei we have obtained basically the same picture, and our further conclusions correspond to all nuclei with $A > 50$. For comparison we display results obtained with Δq extracted from the EMC [Figs. 2(a) and 2(b)] and from the SMC [Figs. 2(c) and 2(d)] measurements (see Table I). In the SMC case we take values corresponding to the variant of $\Delta\Sigma + 1\sigma$ (last column of Table I). One can see the above-quoted (50) upper bound $\eta_{\text{SUSY}} \lesssim 2$ in all four cases presented in Fig. 2 at the level of the total event rate $R \gtrsim 0.01$. The plots also show a clear depletion of the large η_{SUSY} region in the SMC case as compared with the EMC one. This effect reflects a reduction of the spin-dependent neutralino-nucleus cross section. The tendency is much stronger for nuclei with an unpaired neutron [n-like nuclei in Figs. 2(b) and 2(d)]. It is compatible with the observation recently reported in Ref. [34], where the same effect was found for this sort of nuclei when the SMC preliminary data are used. We do not see another effect of a moderate, by about a factor of 2, enhancement for nuclei with an unpaired proton also quoted in Ref. [34]. However, this effect may be hidden in scatter plots such as Fig. 2, which give the most transparent information about correlations near the borders of the domains in the R-η_{SUSY} plane.

In Fig. 3 we present scatter plots of the ratio $\eta_{\text{SUSY}} = \eta_A^{-1}(R_{\text{SD}}/R_{\text{SI}})$ versus the neutralino mass M_χ. Again we show two variants corresponding to the EMC and to the SMC data used for extraction of the parameters Δq. It can be seen from the figures that η_{SUSY} exceeds 1 only within the interval 30 GeV $\lesssim M_\chi \lesssim$ 150 GeV. The reduction of the maximal values of η_{SUSY} at fixed M_χ for the SMC case as compared with the EMC one is obvious

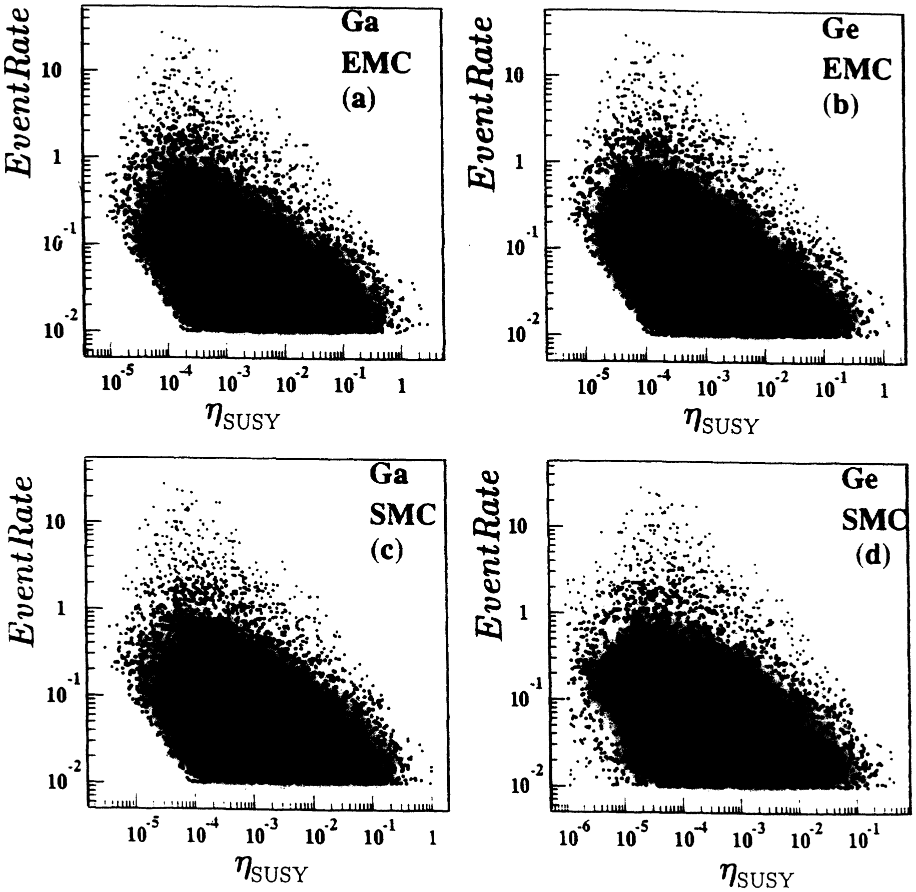

FIG. 2. Scatter plots of the total event rate $R = R_{\rm SD} + R_{\rm SI}$ vs the ratio $\eta_{\rm SUSY} = \eta_A^{-1}(R_{\rm SD}/R_{\rm SI})$ for the nucleon spin content Δq taken from the EMC and SMC data (see Table I). $R_{\rm SD}$ and $R_{\rm SI}$ are the spin-dependent and spin-independent contributions to R; for the nuclear factor η_A see Fig. 1. Two representative nuclei with an unpaired proton (^{71}Ga) and an unpaired neutron (^{73}Ge) are presented. All free parameters are randomly varying: the MSSM parameters in domain (45) and the strange quark matrix element in the range $0.07 \le f_s \le 0.3$. The same pattern of a point distribution holds for all nuclei with $A > 50$.

and has no noticeable M_χ dependence.

The scatter plots in Figs. 2 and 3 have been obtained for values of the strange quark matrix element f_s randomly varied within the interval $0.07 \le f_s \le 0.3$ in order to simulate uncertainties in its definition as discussed in Sec. II. To display explicitly the dependence of the maximal values of $\eta_{\rm SUSY}$ and of the total event rate R on this parameter we present in Fig. 4 scatter plots in the $\eta_{\rm SUSY} - M_\chi$ plane for the end point values $f_s = 0.07$ and 0.3. Plots are given for a nucleus with an unpaired neutron. As a representative we use ^{73}Ge. The figures are almost the same for nuclei with an unpaired proton because the f_s dependence comes into play only through the spin-independent part $R_{\rm SI}$ of the event rate. The values of Δq are taken from the EMC data. It is seen that the largest $\eta_{\rm SUSY}$ values, lying in the interval 30 GeV $\lesssim M_\chi \lesssim$ 150 GeV, are strongly enhanced by about a factor of 5 when f_s is changed from 0.3 to 0.07. The maximal $\eta_{\rm SUSY}$ values outside this interval are almost independent of f_s. An essential dependence of $\eta_{\rm SUSY}$ and of the total event rate on the mass m_A of the *CP*-odd Higgs boson A and on $\tan\beta$ is naturally expected. To illustrate this dependence explicitly we give, in Fig. 5, $\eta_{\rm SUSY}$-M_χ scatter plots for nuclei with an unpaired neutron at different fixed values of these parameters. In fact, the plots demonstrate a strong and rather specific dependence of $\eta_{\rm SUSY}$ on both m_A and $\tan\beta$. The top and bottom branches of the plots correspond to negative and positive values of the parameter μ, respectively. The general tendency is that $\eta_{\rm SUSY}$ increases with m_A. The dependence on $\tan\beta$ is more peculiar. When $\tan\beta$ increases, the bottom ($\mu > 0$) branch goes up, whereas the top ($\mu < 0$) branch goes down. A similar picture holds for nuclei with an unpaired proton. We also report in Fig. 6 the "integrated" dependence of $\eta_{\rm SUSY}$ and R on m_A when other free parameters are randomly varying in the above defined domain.

After this discussion we may combine the bound (50) with the values of the nuclear factor η_A represented in Fig. 1. Then we obtain the conservative estimate

$$\eta = R_{\rm SD}/R_{\rm SI} = \eta_A \eta_{\rm SUSY}^{p(n)} \lesssim 1.6 \quad \text{for nuclei with } A > 50 \tag{51}$$

at a detector sensitivity up to $R > 0.01$ events/(kg day). However, as is seen in Fig. 2, the majority of points generated in the domain (45) of the MSSM parameter space concentrates at $\eta \le 1$. The tendency is that at higher sensitivities (lower R accessible) we get $\eta \le 1$ for heavier nuclei and vice versa.

As a by product of our analysis in Figs. 7–9 we give plots of the event rate for some target materials (CaF_2, NaI, ^{73}Ge, and ^{129}Xe) of special interest in the DM search (see [22] and references therein). In Fig. 10 we also present plots of the ratio $r(A) = R_{\rm SD}(A)/R_{\rm SD}(^{73}\text{Ge})$ of

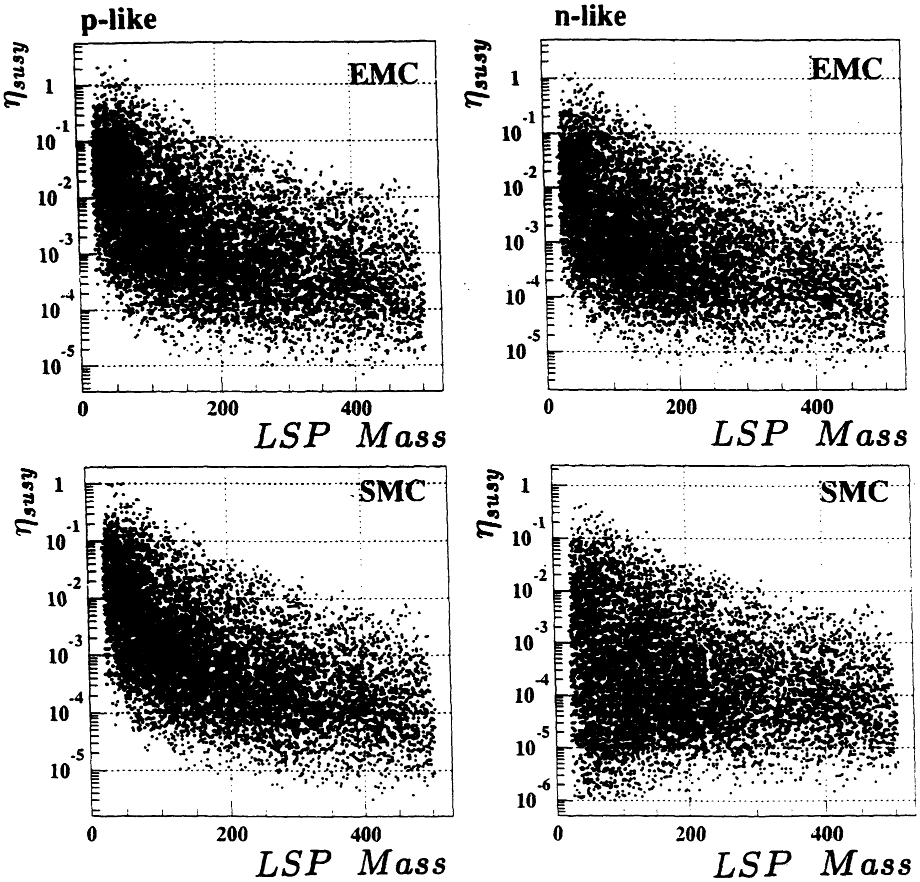

FIG. 3. Scatter plots of the ratio $\eta_{\rm SUSY} = \eta_A^{-1}(R_{\rm SD}/R_{\rm SI})$ vs the neutralino (LSP) mass M_χ. The left panel for nuclei with an unpaired proton (p like) and the right panel for nuclei with an unpaired neutron (n like). Other conditions as in Fig. 2.

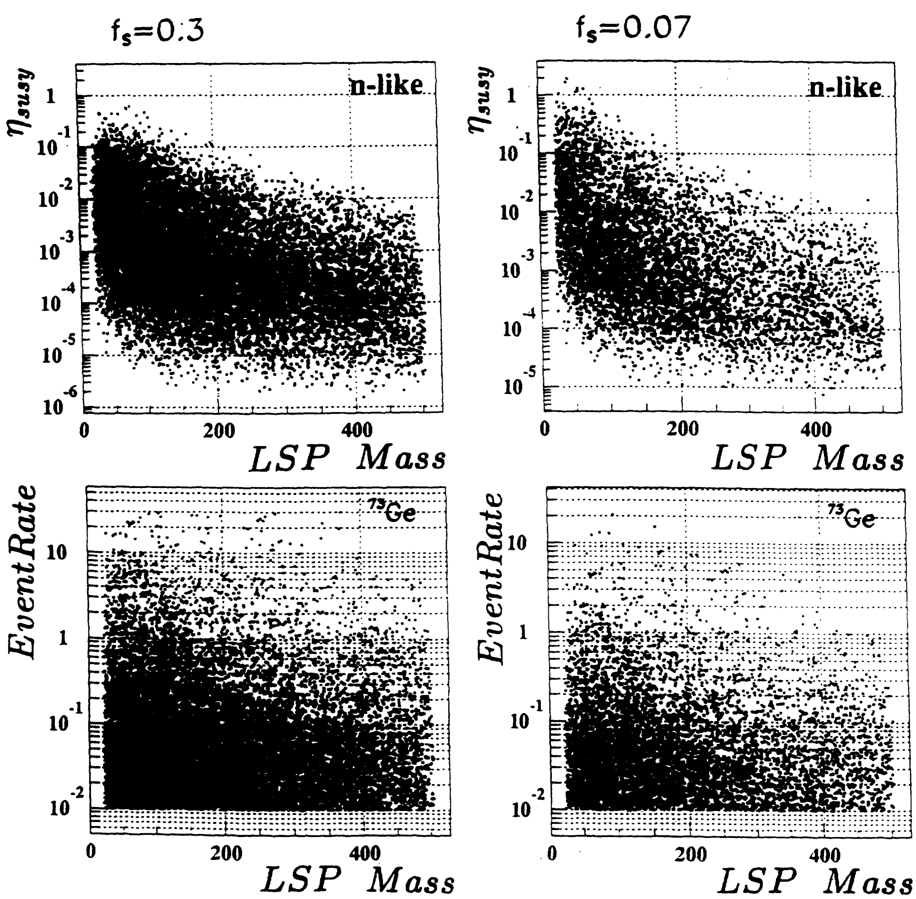

FIG. 4. Scatter plots of the ratio $\eta_{\rm SUSY} = \eta_A^{-1}(R_{\rm SD}/R_{\rm SI})$ and of the total event rate $R = R_{\rm SD} + R_{\rm SI}$ as functions of the neutralino (LSP) mass M_χ for the strange quark matrix element f_s at the endpoint values 0.07 and 0.3 of the interval analyzed. Δq parameters are taken from the EMC data. ^{73}Ge is presented as an example of a nucleus with an unpaired neutron (n like).

the spin-dependent part $R_{\rm SD}$of the event rate for some materials (A) to that for ^{73}Ge. We do not plot this ratio for ^{129}Xe because $r(^{129}{\rm Xe}) \approx 1.2$ is almost independent of the neutralino mass M_χ. This is the case because both ^{73}Ge and ^{129}Xe are nuclei with an unpaired neutron. As seen from Eq. (12), for nuclei having the same type of unpaired neutron (p or n), one can write

$$\frac{R_{\rm SD}(A_1)}{R_{\rm SD}(A_2)} = \frac{\zeta(M_\chi, M_{A_1}, r^{(1)}_{\rm spin})\lambda_1^2 J_1(J_1+1)}{\zeta(M_\chi, M_{A_2}, r^{(2)}_{\rm spin})\lambda_2^2 J_2(J_2+1)} . \qquad (52)$$

This ratio does not depend on the details of the neutralino-quark interactions and is determined completely by the nuclear structure. It is approximately a constant independent of the neutralino mass M_χ [8].

The points in Figs. 7–10 are obtained for one fixed value of $m_A = 50$ GeV and for two fixed values of $\tan\beta = 2$ and 8. Other MSSM parameters are varied randomly within the domain (45). Parameters of nucleonic matrix elements are taken as follows: $f_s = 0.14$ and Δq corresponding to the EMC data (see Table I).

It follows from Figs. 7–9 that the maximal values of the total event rate for NaI, ^{73}Ge, and ^{129}Xe are typically the same, while for CaF_2 they are lower by about a factor of 5. On the other hand, Fig. 10 shows that the sensitivity of CaF_2 to the spin-dependent part of the neutralino-nucleus interaction is by about a factor of 10 larger than that of NaI, ^{73}Ge, and ^{129}Xe. The last three materials have approximately an equal spin sensitivity.

We do not take into account a possible rescaling of the local neutralino density ρ, which may occur in the region of the MSSM parameter space, where $\Omega h^2 < 0.05$ [10]. This effect, if it took place, would essentially modify the event rate R [14]. Of course, it has no influence on the ratio η in the formula (17) and on our conclusion about the role of nuclear spin. The plots in Figs. 7–9 correspond to a situation when neutralinos constitute a dominant component of the DM halo of our Galaxy with a density $\rho = 0.3$ GeV cm^{-3} in the solar vicinity.

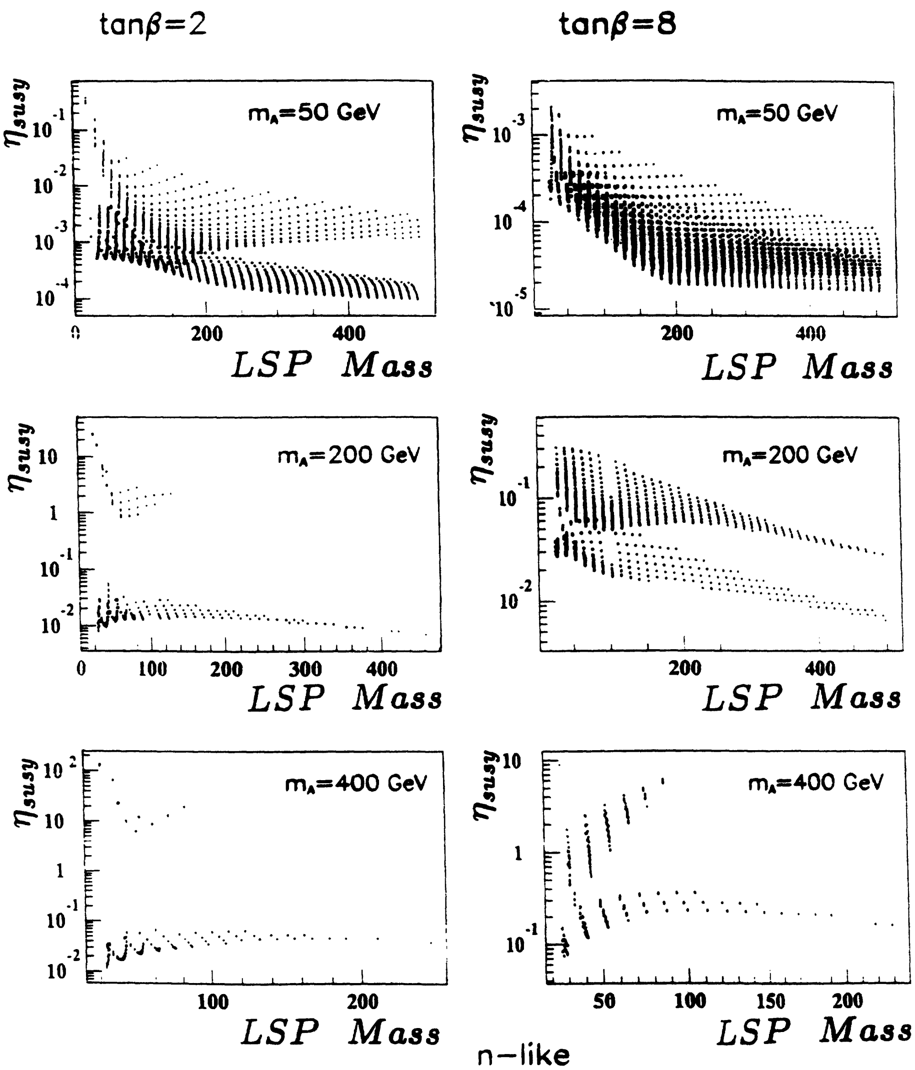

FIG. 5. Scatter plots of the ratio $\eta_{\rm SUSY} = \eta_A^{-1}(R_{\rm SD}/R_{\rm SI})$ for nuclei with an unpaired neutron (n like) vs the neutralino (LSP) mass M_χ. Points are generated at the representative values $\tan\beta = 2$ and 8 and the axial Higgs boson mass $m_A = 50$, 200, and 400 GeV. Δq parameters are taken from the EMC data.

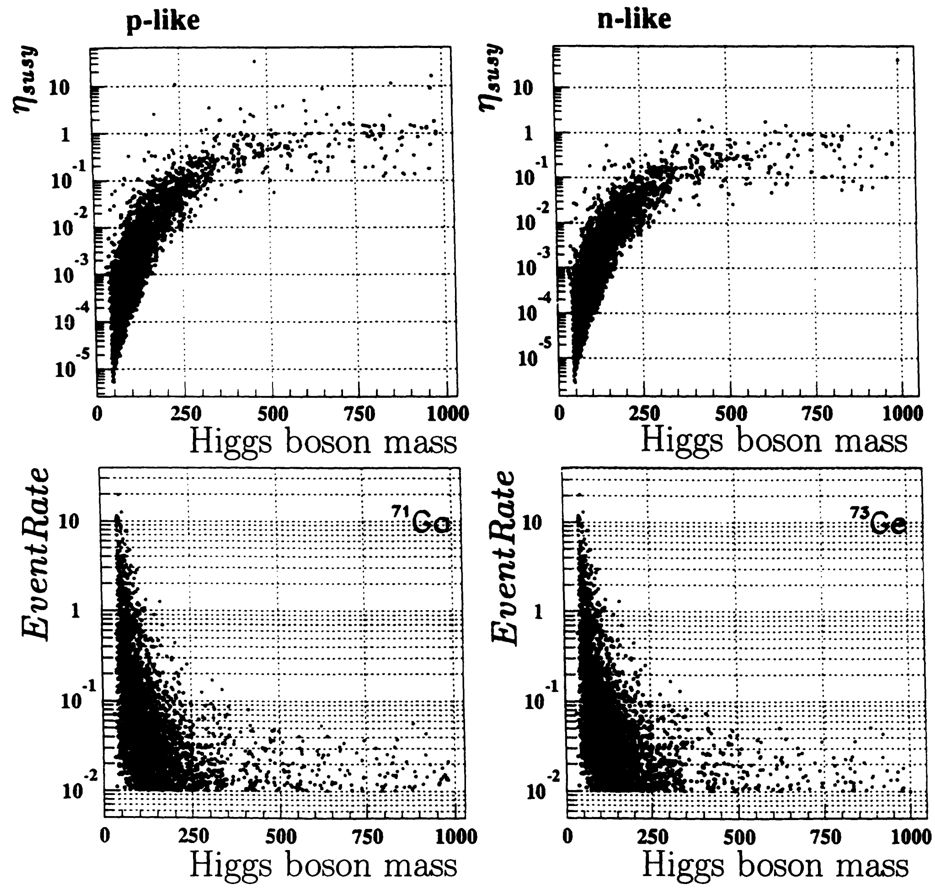

FIG. 6. Scatter plots of the ratio $\eta_{\rm SUSY} = \eta_A^{-1}(R_{\rm SD}/R_{\rm SI})$ and the total event rate $R = R_{\rm SD} + R_{\rm SI}$ as a function of the axial Higgs boson mass m_A (p like and n like correspond to nuclei with unpaired proton and neutron). All free parameters are randomly varying as in Fig. 2. Δq parameters are taken from the EMC data.

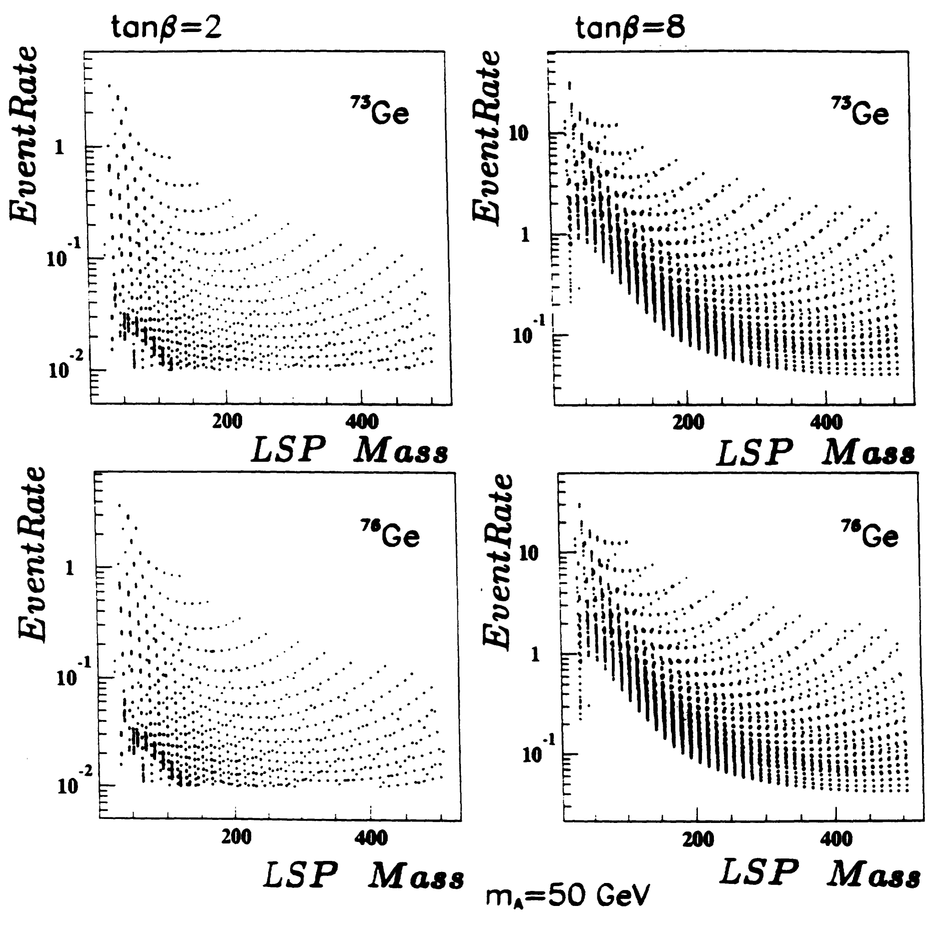

FIG. 7. Scatter plots of the total event rate R for ^{73}Ge and ^{76}Ge as a function of the neutralino (LSP) mass M_χ at one fixed value of the CP-odd Higgs boson mass $m_A = 50$ GeV. Two representative values of tanβ are taken: tan$\beta = 2$ (left panel) and tan$\beta = 8$ (right panel). Other free parameters are treated as in Fig. 6.

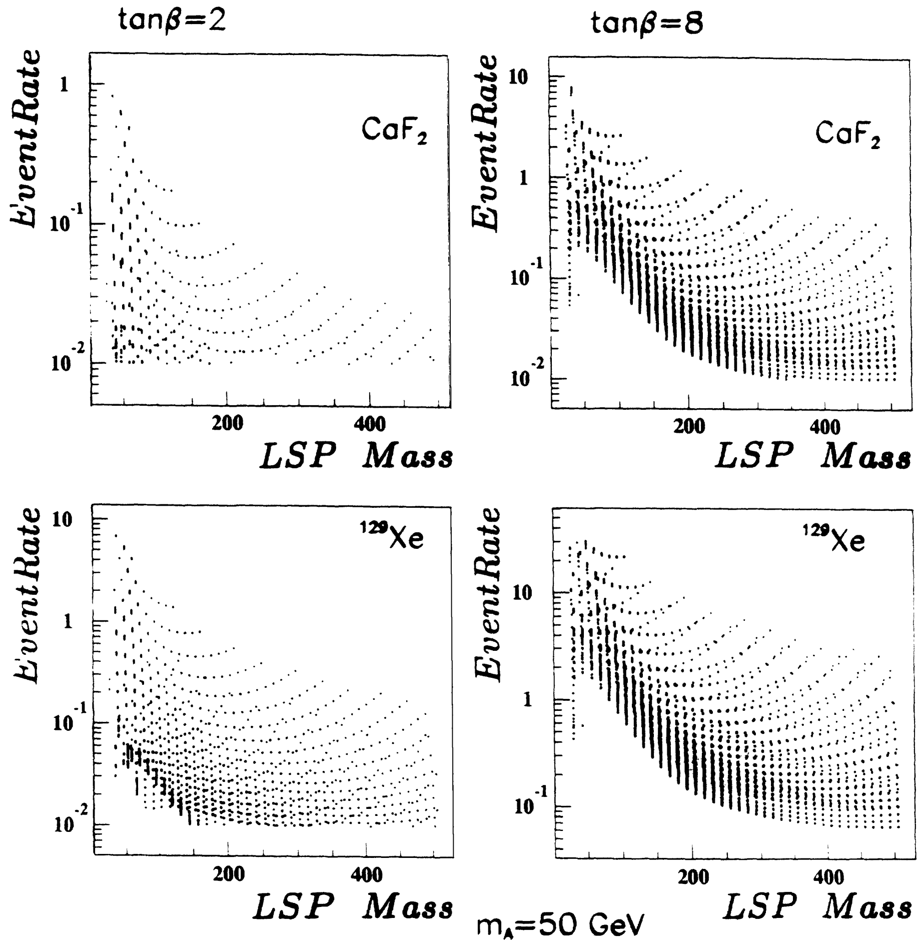

FIG. 8. Same as in Fig. 7, but for CaF_2 (calcium fluoride) and xenon as target material.

V. CONCLUSION

The central result of this paper is that for sufficiently heavy nuclei with atomic weights $A > 50$ the spin-independent event rate R_{SI} is typically larger than the spin-dependent one R_{SD} if low rate DM signals with total event rates $R = R_{SD} + R_{SI} < 0.01$ events/kg day are ignored. This cutoff condition reflects the realistic sensitivities of the present and the near-future DM detectors. Even if the cosmological bound $0.025 < \Omega_\chi h_0^2$ (49) is disregarded, the same conclusion remains true for the cutoff condition $R < 0.1$, which also corresponds to very low rate DM signals probably hopelessly undetectable.

The main practical issue is that two different DM detectors with $(J = 0, A_1)$ and with $(J \neq 0, A_2)$ nuclei as target material have equal changes to discover DM

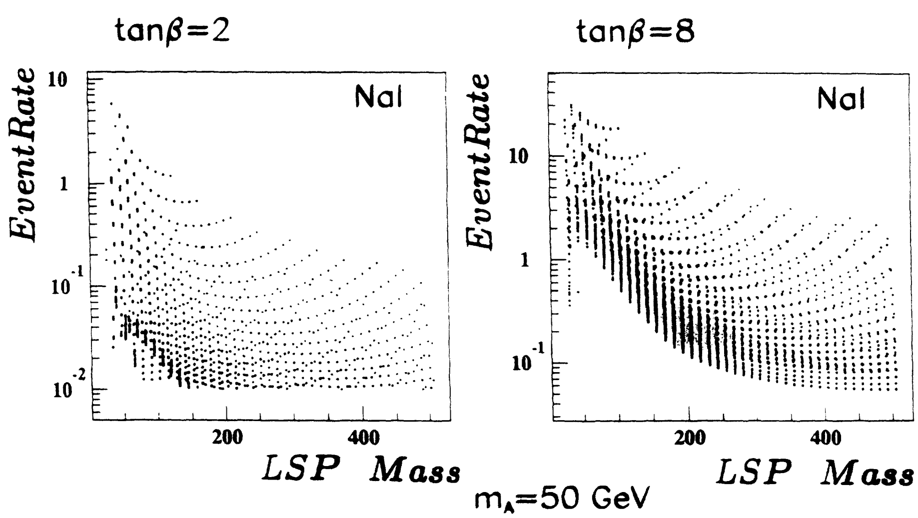

FIG. 9. Same as in Fig. 7, but for NaI (sodium iodine) as target material.

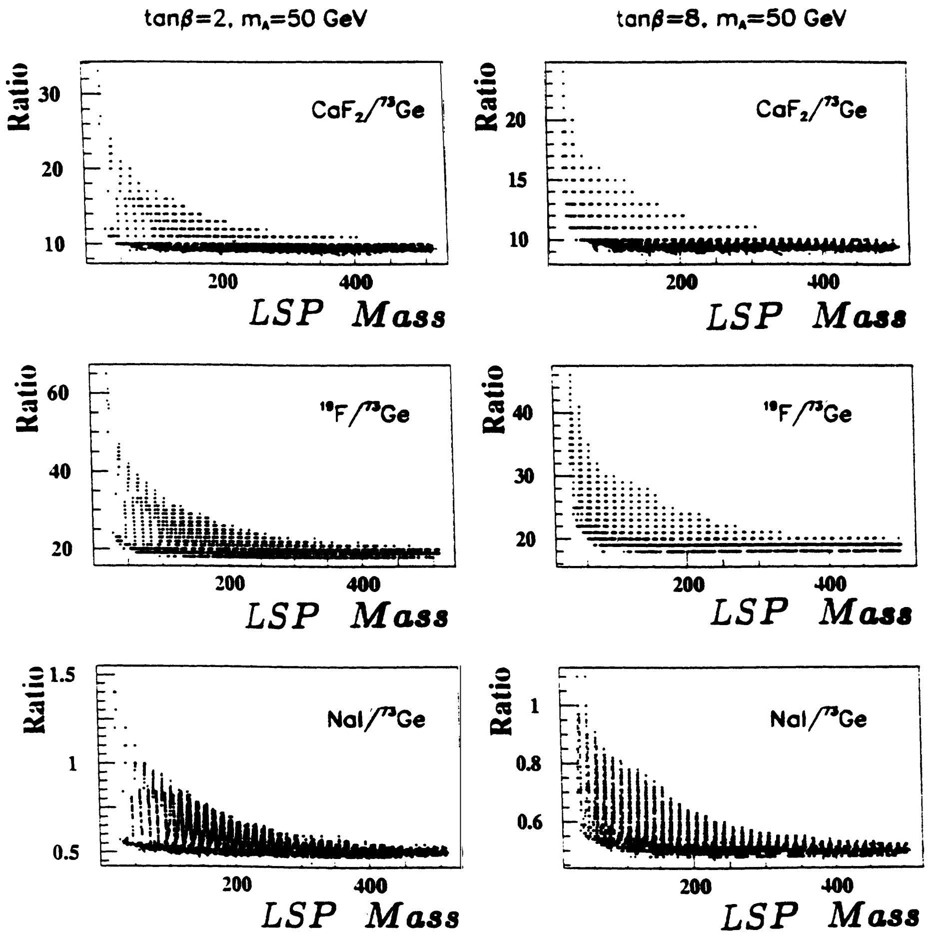

FIG. 10. Same as in Fig. 7, but for the ratio $r(A) = R_{SD}(A)/R_{SD}(^{73}Ge)$ of the spin-dependent event rate R_{SD} for nuclei CaF_2, ^{19}F and NaI to the spin-dependent event rate $R_{SD}(^{73}Ge)$ for ^{73}Ge.

events if $A_1 \sim A_2 > 50$. A similar conclusion has been recently made in [15] for some materials of interest in the DM search and in Ref. [13] for the particular case of ^{73}Ge, ^{76}Ge in some representative domains of the MSSM parameter space. A dominance of the coherent part of the event rate for several examples of nuclei was first observed in Ref. [12]. However, this effect has not been reproduced in this paper completely.[1]

Another aspect of the DM search is the investigation of the SUSY-model parameter space from nonobservation of DM events. For this purpose experiments both with $J = 0$ and $J \neq 0$ nuclei are equally important and complimentary (see also [59]).

We have compared several examples of popular (see for instance [22] and references therein) materials with nonzero-spin nuclei as a target in a DM detector. We have not found an essential difference between NaI, ^{73}Ge, and ^{129}Xe as a target material for DM detectors from the point of view of their total and spin sensitivity. We expect these materials to have a better prospect as compared with CaF_2 for discovering of DM events. The former materials have a total event rate by about a factor of 5 larger than the latter one. On the other hand CaF_2 can give a more stringent constraint on the spin-dependent part of the event rate, having a spin sensitivity by about a factor of 10 larger than NaI, ^{73}Ge, and ^{129}Xe.

The results presented above were obtained in the framework of rather general assumptions about the nuclear and nucleon structure. We have used the MSSM with general electroweak symmetry breaking, but imposing unification of the soft SUSY-breaking parameters, except Higgs boson mass, at the GUT scale. It is a natural question whether our basic conclusions hold in more general SUSY-model scenarios when these constraints are relaxed. We do not expect a dramatic change of the role of nuclear spin in this case, but we plan to investigate this question carefully in a subsequent paper.

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[1]The formulas in [12] for the effective neutralino-quark Lagrangian and for the event rate contain several mistakes.

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