Signatures of discrete symmetries in the scalar sector

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(Received 16 May 1994)

I discuss methods to identify the presence of discrete symmetries in the two-Higgs-doublet model by observing the masses and the cubic and quartic interactions of the scalars. The symmetries considered are a Z_2 symmetry under which $\phi_2 \to -\phi_2$, and a CP symmetry which enforces real coupling constants in the Higgs potential. Those symmetries are spontaneously broken, and the Z_2 symmetry may also be softly broken. I identify the signatures in the interactions of the scalars that these symmetries leave after their breaking.

PACS number(s): 11.30.Er, 11.30.Ly, 12.60.Fr, 14.80.Cp

Twenty-one years ago, Lee [1] pointed out that CP may be spontaneously broken. In his two-Higgs-doublet model, CP is a symmetry of the Lagrangian, which is broken by the relative phase between the vacuum expectation values (VEV's) of the two Higgs doublets. Other models of spontaneous CP violation have been suggested since then [2,3], and spontaneous CP violation has been used as an ingredient in the building of many models [4,5]. However, no one has yet attempted to answer the following basic questions: how can we experimentally distinguish between spontaneous and explicit CP violation? If CP violation is spontaneous, does that fact lead to some relationships among the coefficients of the various interaction terms in the Lagrangian, relationships which might be experimentally tested for? (At least in principle, even if the practical measurements might be too difficult.)

In the context of the two-Higgs-doublet model, it is usual to assume the existence of a discrete symmetry Z_2, under which one of the two doublets changes sign, while the other doublet remains unaffected. That symmetry is softly broken in some models. How can we assert experimentally whether or not such a symmetry exists, and whether or not it is softly broken?

After discrete symmetries in the scalar sector are spontaneously or softly broken, do they still leave traces of their presence in the fundamental Lagrangian?

I present in this Brief Report a partial answer to these questions.

For definiteness, I concentrate on the two-Higgs-doublet model. That model is now very popular, partly because two doublets is the Higgs structure of the minimal supersymmetric standard model. If there are more than two doublets the algebra involved becomes extremely heavy. I consider a SU(2)⊗U(1) gauge model with two scalar doublets ϕ_1 and ϕ_2. The most general Higgs potential consistent with renormalizability is

$$\begin{aligned} V = {} & m_1\phi_1^\dagger\phi_1 + m_2\phi_2^\dagger\phi_2 + (m_3\phi_1^\dagger\phi_2 + \text{H.c.}) + a_1(\phi_1^\dagger\phi_1)^2 + a_2(\phi_2^\dagger\phi_2)^2 + a_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + a_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ & + \left[a_5(\phi_1^\dagger\phi_2)^2 + a_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + a_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \text{H.c.}\right] . \end{aligned} \tag{1}$$

All the coupling constants, except m_3, a_5, a_6, and a_7, are real because of Hermiticity. I assume that the VEV's of ϕ_1 and ϕ_2 are aligned, in the sense that they preserve the U(1) of electromagnetism.[1] Those VEV's have a relative phase: the VEV of ϕ_1^0 is v_1, and the VEV of ϕ_2^0 is $v_2\exp(i\alpha)$, v_1 and v_2 being real and positive.[2] $v = \sqrt{v_1^2+v_2^2}$ is a measurable quantity, $v = 174$ GeV.

Instead of working with ϕ_1 and ϕ_2, it is convenient to work in the Georgi [6] basis of doublets H_1 and H_2, with the following defining features: H_1 has real and positive VEV v, while H_2 has vanishing VEV. The Georgi basis is reached by means of the transformation

$$\begin{aligned} \phi_1 &= (v_1H_1 + v_2H_2)/v \,, \\ \phi_2 &= e^{i\alpha}(v_2H_1 - v_1H_2)/v \,. \end{aligned} \tag{2}$$

In the Georgi basis, the Higgs potential reads

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[1] T. D. Lee [1] has shown that this happens if one inequality is satisfied by the coupling constants of the potential.

[2] The VEV of ϕ_1^0 is made real and positive by a gauge transformation. This represents no loss of generality.

$$V = \mu_1 H_1^\dagger H_1 + \mu_2 H_2^\dagger H_2 + (\mu_3 H_1^\dagger H_2 + \text{H.c.}) + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \left[\lambda_5 (H_1^\dagger H_2)^2 + \lambda_6 (H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7 (H_2^\dagger H_2)(H_1^\dagger H_2) + \text{H.c.}\right] , \tag{3}$$

in which all the coupling constants, except μ_3, λ_5, λ_6, and λ_7, are real by Hermiticity. Because only H_1 has a nonzero VEV v, which is real, the stationarity conditions of the vacuum read

$$\mu_1 = -2\lambda_1 v^2 , \tag{4}$$

$$\mu_3 = -\lambda_6 v^2 . \tag{5}$$

I use these conditions to eliminate μ_1 and μ_3 as independent variables from V. Because μ_3 is complex while μ_1 is real, Eqs. (4) and (5) constitute three real equations. They correspond to the three real equations which, in the basis of ϕ_1 and ϕ_2, determine the stability of the vacuum by fixing the partial derivatives of the vacuum potential with respect to v_1, v_2, and α, to be zero.

The Georgi basis is useful because the Goldstone modes are perfectly identified when one uses it. Writing

$$H_1 = \begin{pmatrix} G^+ \\ v + (H^0 + iG^0)/\sqrt{2} \end{pmatrix} , \tag{6}$$

$$H_2 = \begin{pmatrix} H^+ \\ (R + iI)/\sqrt{2} \end{pmatrix} , \tag{7}$$

G^+ and G^0 are the Goldstone bosons which, in the unitary gauge, become the longitudinal components of the W^+ and of the Z^0. H^0, R, and I are real neutral fields, which are linear combinations of the three physical scalars X_k:

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = T \begin{pmatrix} H^0 \\ R \\ I \end{pmatrix} , \tag{8}$$

T being an orthogonal matrix. H^+ is the physical charged scalar. As a consequence of this, it is easy to write down the mass terms [7] and the cubic and quartic interactions of the physical scalars as functions of the coupling constants of the potential in the Georgi basis, Eq. (3). By observation of those masses and interactions, μ_2 and the λ_i (i from 1 to 7) can be measured.[3] Because λ_5, λ_6, and λ_7 are complex, this corresponds to a total of 11 real quantities.

However, the Georgi basis is not totally well defined: H_2 can suffer a U(1) rephasing, while preserving its defining property of having a zero VEV. When this is done, the phases of λ_6 and of λ_7 get changed by the arbitrary phase φ, while the phase of λ_5 gets changed by 2φ. Only rephasing-invariant combinations are measurable. Therefore, there are only ten real measurable quantities in the scalar masses and interactions. Two of these are phases connected to the presence of CP violation [7]. For instance, the phases of $\lambda_6^* \lambda_7$ and of $\lambda_5^* \lambda_6 \lambda_7$ are two independent measurable phases, while the phase of $\lambda_5^* \lambda_7^3$ is not measurable. The other eight measurable quantities are the moduli of μ_2 and of the λ_i.

Let us consider exactly how the various parameters of the potential in the Georgi basis might be measured. Let us denote by A_k (k from 1 to 3) the squared masses of the three neutral scalars X_k. Those squared masses are the eigenvalues of the mass matrix M in the basis (H^0, R, I), which mass matrix has been explicitly written down in [7]. I denote by A_+ the squared mass of the charged scalar H^+. The only observables in the matrix T are the matrix elements of its first column [7]. For instance, the Z_μ^0 couples to the current

$$\frac{g}{2\cos\theta_W} \left[\sum_{k=1}^{3} T_{k1} \left(X_k \partial^\mu G^0 - G^0 \partial^\mu X_k \right) + \sum_{k,l,m} \epsilon_{klm} T_{k1} X_l \partial^\mu X_m \right] , \tag{9}$$

ϵ_{klm} being the totally antisymmetric tensor with[4] $\epsilon_{123} = 1$. One should remember that $T_{11}^2 + T_{21}^2 + T_{31}^2 = 1$ because of the orthogonality of T, therefore only two of the three T_{k1} are independent. Once one knows A_1, A_2, A_3, A_+, and the three T_{k1}, one can find the values of six parameters of the potential via the equations

$$A_+ = \mu_2 + \lambda_3 v^2 , \tag{10}$$

$$M_{11} = 4\lambda_1 v^2 = \sum_{k=1}^{3} A_k T_{k1}^2 , \tag{11}$$

$$M_{22} + M_{33} = 2\left(\frac{\mu_2}{v^2} + \lambda_3 + \lambda_4\right) v^2 = \sum_{k=1}^{3} A_k - M_{11} , \tag{12}$$

$$M_{22} M_{33} - M_{23}^2 = \left[\left(\frac{\mu_2}{v^2} + \lambda_3 + \lambda_4\right)^2 + 4|\lambda_5|^2 \right] v^4 = A_1 A_2 T_{31}^2 + A_1 A_3 T_{21}^2 + A_2 A_3 T_{11}^2 , \tag{13}$$

[3] Once μ_2 and the λ_i have been measured, one can get from them the coupling constants of the potential in the original basis, Eq. (1), and also v_1/v_2 and α. But the quantities more directly measured are μ_2 and the λ_i.

[4] I omitted in the current in Eq. (9) the terms involving the charged scalars.

$$M_{12}^2 + M_{13}^2 = 4|\lambda_6|^2 v^4 = \sum_{k=1}^{3} A_k^2 T_{k1}^2 (1 - T_{k1}^2) - 2 \sum_{k<l} A_k A_l T_{k1}^2 T_{l1}^2 , \quad (14)$$

$$2M_{12}M_{13}M_{23} - M_{22}M_{13}^2 - M_{33}M_{12}^2 + M_{11}(M_{22} + M_{33}) = 4v^6 \left[2\mathrm{Re}\left(\lambda_5^* \lambda_6^2\right) - \left(\frac{\mu_2}{v^2} + \lambda_3 + \lambda_4\right) |\lambda_6|^2\right] + M_{11}(M_{22} + M_{33}) = A_1 A_2 A_3 . \quad (15)$$

The other four parameters of the potential might be determined in the following way. The following cubic interactions are present in the potential:

$$V = \sqrt{2} v H^- H^+ \left(\lambda_3 H^0 + R\mathrm{Re}\lambda_7 - I\mathrm{Im}\lambda_7\right) + \cdots . \quad (16)$$

By diagonalizing the mass matrix M we find that this interaction is written in terms of the eigenstates of mass X_1, X_2, and X_3 as

$$\sqrt{2} v H^- H^+ \left(\lambda_3 H^0 + R\mathrm{Re}\lambda_7 - I\mathrm{Im}\lambda_7\right) = \sqrt{2} v H^- H^+ \sum_{k=1}^{3} X_k \left[\lambda_3 T_{k1} + \frac{4v^4 \mathrm{Re}\left(\lambda_5^* \lambda_6 \lambda_7\right)}{T_{k1}(A_k - A_l)(A_k - A_m)} + \frac{v^2}{T_{k1}} \mathrm{Re}\left(\lambda_6 \lambda_7^*\right) \left(\frac{1 - T_{l1}^2}{A_k - A_m} + \frac{1 - T_{m1}^2}{A_k - A_l}\right)\right] , \quad (17)$$

with $l \neq k$ and $m \neq k$ and $l \neq m$. This allows us to find the values of λ_3, $\mathrm{Re}\left(\lambda_6 \lambda_7^*\right)$, and $\mathrm{Re}\left(\lambda_5^* \lambda_6 \lambda_7\right)$. Finally, λ_2 can be found, for instance, from the fact that it is the coefficient of the $(H^- H^+)^2$ quartic interaction.

Suppose that there is an exact symmetry Z_2 under which $\phi_1 \rightarrow \phi_1$ while $\phi_2 \rightarrow -\phi_2$. Then, $m_3 = 0$ and $a_6 = a_7 = 0$. Because there is then only one term in the potential which sees the relative phase of ϕ_1 and ϕ_2 [the term $a_5(\phi_1^\dagger \phi_2)^2$ + H.c.], there is no CP violation [3,8]. a_5 can be set real by a rephasing of ϕ_2. In the case of exact Z_2 symmetry there are therefore seven parameters in the Higgs potential: m_1, m_2, a_1, a_2, a_3, a_4, and a_5. These seven parameters determine v and the ten coefficients in the scalar masses and cubic and quartic interactions. We thus expect four predictions. Two of those predictions are connected to the absence of CP violation in this model: the two independent measurable phases vanish. The other two predictions can be derived by the following method. One first uses the stability conditions of the vacuum to write m_1 and m_2 as functions of v_1 and of v_2, or, equivalently, of v and the ratio v_1/v_2. One then uses the change of basis in Eq. (2) to write μ_2/v^2 and the seven λ_i (which in this case are all real) as functions of v_1/v_2 and of a_1, a_2, ..., a_5 (being dimensionless, they cannot depend on v). One finally inverts those equations to find v_1/v_2 and a_1, a_2, ..., a_5 as functions of μ_2/v^2 and of the λ_i, in this process obtaining the following two relationships among the measurable parameters:

$$\frac{\mu_2}{v^2}(\lambda_6 + \lambda_7) + 2(\lambda_1 \lambda_7 + \lambda_2 \lambda_6) = 0 , \quad (18)$$

$$\left(\frac{\mu_2}{v^2} + \lambda_3 + \lambda_4 + 2\lambda_5\right)(\lambda_2 - \lambda_1) + \lambda_6^2 - \lambda_7^2 = 0 . \quad (19)$$

These equations are the answer to the questions addressed in this Brief Report for the particular case of the model with nonsoftly broken Z_2 symmetry. After measuring μ_2/v^2 and the seven λ_i by observing the scalar masses and the scalar cubic and quartic interactions, one should check whether there is no CP violation, and whether Eqs. (18) and (19) are satisfied. If this applies, we are in the presence of a model with nonsoftly broken Z_2 symmetry.

I now address the case of softly broken Z_2 symmetry and spontaneously broken CP symmetry [9–11]. In this case, a_6 and a_7 are zero because of the Z_2 symmetry. m_3 is nonzero, breaking the Z_2 symmetry softly. However, there is CP invariance in the Higgs potential; m_3 and a_5 are real. CP is broken by the phase α between the VEV's of ϕ_1 and ϕ_2. We now have ten measurable quantities written as functions of seven parameters (a_1, a_2, ..., a_5, α and v_1/v_2). We expect three equations among the observable quantities to hold. After some work we find them to be

$$\mathrm{Im}\left[\lambda_5^* (\lambda_6 + \lambda_7)^2\right] + (\lambda_2 - \lambda_1)\mathrm{Im}(\lambda_6 \lambda_7^*) = 0 , \quad (20)$$

and

$$(\lambda_2 - \lambda_1)\mathrm{Im}\left[\lambda_5^*(\lambda_6^2 - \lambda_7^2)\right] - \left[\lambda_2^2 - \lambda_1^2 - (\lambda_2 - \lambda_1)(\lambda_3 + \lambda_4) + |\lambda_7|^2 - |\lambda_6|^2\right]\mathrm{Im}(\lambda_6 \lambda_7^*) = 0 , \quad (21)$$

and

$$\left[\left(\frac{\mu_2}{v^2}\right)^2 (\lambda_1 - \lambda_2) + 2\frac{\mu_2}{v^2}(\lambda_3 + \lambda_4)(\lambda_1 - \lambda_2) + \left(\frac{\mu_2}{v^2} + \lambda_1 + \lambda_2\right) f_2 + (2\lambda_3 + 2\lambda_4 - \lambda_1 - \lambda_2)(\lambda_1^2 - \lambda_2^2)\right](f_1)^2 + (\lambda_1 - \lambda_2)\left[f_2^2 - 4f_3^2 + 2(\lambda_1 - \lambda_2)(\lambda_3 + \lambda_4 - \lambda_1 - \lambda_2)\right] f_1 - (\lambda_1 - \lambda_2)^3 \left[(f_2)^2 + 4(f_3)^2\right] = 0 , \quad (22)$$

where

$$f_1 \equiv |\lambda_6|^2 + |\lambda_7|^2 + 2\mathrm{Re}(\lambda_6\lambda_7^*)\,, \quad (23)$$
$$f_2 \equiv |\lambda_7|^2 - |\lambda_6|^2\,, \quad (24)$$
$$f_3 \equiv \mathrm{Im}(\lambda_6\lambda_7^*)\,. \quad (25)$$

Let us now consider the Lee model of spontaneous CP violation. In that model, there is no Z_2 symmetry, and therefore a_6 and a_7 are nonzero, but there is CP invariance at the Lagrangian level, which means that m_3, a_5, a_6, and a_7 are real. Therefore, the difference from the previous model is the existence of two extra real couplings a_6 and a_7. However, ϕ_1 and ϕ_2, because there is now no symmetry Z_2 which distinguishes between them, are not uniquely defined. One may rotate ϕ_1 and ϕ_2 freely by means of an orthogonal transformation (orthogonal and not unitary, because we want to preserve a real scalar potential)

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}\,, \quad (26)$$

in such a way as to eliminate one of the parameters of the potential, m_3 for instance [1]. There is thus one parameter in the potential which is spurious. Therefore, the two extra parameters a_6 and a_7 really correspond to only one extra degree of freedom. We thus expect two relations (instead of three as in the previous model) to hold among the measurable quantities. Indeed, Eqs. (20) and (21) do not hold any more. However, one linear combination of them still holds when a_6 and a_7 are not zero:

$$\begin{aligned} &\left[(\lambda_2 - \lambda_1)\left(\lambda_3 + \lambda_4 + \frac{\mu_2}{v^2}\right) + |\lambda_6|^2 - |\lambda_7|^2\right] \\ &\times \mathrm{Im}(\lambda_6\lambda_7^*) + \frac{\mu_2}{v^2}\mathrm{Im}\left[\lambda_5^*(\lambda_6+\lambda_7)^2\right] \\ &+ 2\lambda_2 \mathrm{Im}\left[\lambda_5^*\lambda_6(\lambda_6+\lambda_7)\right] + 2\lambda_1 \mathrm{Im}\left[\lambda_5^*\lambda_7(\lambda_6+\lambda_7)\right] = 0\,. \end{aligned} \quad (27)$$

Equation (22) does not hold in the Lee model either. I expect one further constraint among the physical observables to hold in the Lee model, which should somehow involve the quantity in the left-hand side of Eq. (22). Unfortunately, I have been unable to find this extra condition characteristic of the Lee model.

As a final model, let us now assume that CP violation is explicit, not spontaneous, but that there is a softly broken Z_2 symmetry. That is, $a_6 = a_7 = 0$, while m_3 and a_5 are complex. However, because we are now free to rephase ϕ_2, it is only the phase of $m_3^* a_5^2$ which is relevant. Therefore, this model has only one more degree of freedom than the model with softly broken Z_2 symmetry and spontaneous CP violation. We therefore expect two conditions among the physical observables. Indeed, one finds that in this case Eqs. (20) and (21) hold, but Eq. (22) does not.

I summarize my results. Any two-Higgs-doublet model can conveniently be written in the Georgi basis. The parameters of the potential in that basis constitute a set of ten independent quantities, which can be directly measured by considering the coefficients of the various scalar cubic and quartic interactions, and the scalar masses. Those ten quantities are μ_2, λ_1, λ_2, λ_3, λ_4, $|\lambda_5|$, $|\lambda_6|$, $|\lambda_7|$, and two independent CP-violating phases, the phases of $\lambda_6\lambda_7^*$ and of $\lambda_5^*\lambda_6\lambda_7$, for instance. If some discrete symmetries, like a Z_2 symmetry or CP symmetry, are imposed on the potential in a non-Georgi basis, then some equations will hold among these ten otherwise independent quantities. In a model with nonsoftly broken (but spontaneously broken) Z_2 symmetry, there is CP conservation (which means that the two physical phases vanish), and the conditions of Eqs. (18) and (19) hold. If Z_2 is softly broken, but CP is a spontaneously broken symmetry of the potential, Eqs. (20)–(22) hold. If Z_2 is softly broken, and CP is explicitly broken, Eqs. (20) and (21) hold, but Eq. (22) does not. It is interesting to observe that the difference between these two cases is only whether or not Eq. (22) holds; as Eq. (22) is extremely complicated, in practice it will certainly be impossible to distinguish between the two models in this way. Finally, in the general Lee model of spontaneous CP violation, without any Z_2 symmetry, Eq. (27) holds and, presumably, another much more complicated condition will also hold.

It is fair to say that all the conditions found are rather complicated and should be very difficult to test. This unfortunate result means that, possibly, much theoretical speculation on the existence of discrete symmetries in the scalar scetor may be untestable in practice. Also note that those conditions are just tree-level ones and, because the symmetries on which they depend are broken, should receive finite radiative corrections.

I thank João P. Silva for useful discussions, and for collaboration in the early stages of this research. Lincoln Wolfenstein and Ling-Fong Li read and criticized the manuscript. This work was supported by the United States Department of Energy, under Contract No. DE-FG02-91ER-40682.

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