Top quark radiative corrections in nonminimal standard models

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We derive the one-loop effective action induced by a heavy top quark in models with an extended Higgs sector. We use the effective action to analyze the top quark corrections to the ρ parameter and to the Higgs-boson–gauge-boson couplings. We show that in models with $\rho \neq 1$ at the tree level, one does not lose generally the bound on m_t from the ρ parameter.

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I. INTRODUCTION

Recent precision measurements at the CERN e^+e^- collider LEP allow us to strongly constrain the top quark mass in the standard model (SM) [1]. These constraints are obtained by analyzing the radiative corrections induced by the top quark to measurable quantities. From the ρ parameter, $\rho = m_W^2/(m_Z^2\cos^2\theta_W)$, we get the strongest constraint on m_t, since the top radiative corrections to ρ grow quadratically with the top mass.

In the minimal SM in which the Higgs sector consists of one Higgs doublet, the value of ρ at the tree level, ρ_{tree}, is equal to unity so radiative corrections must be finite. Nevertheless, when an extended Higgs sector is considered (nonminimal SM's), one can have $\rho_{\text{tree}} \neq 1$. Since the experimental value of ρ is very close to unity, one expects that, in such nonminimal SM's, a simultaneous expansion in $(\rho_{\text{tree}}-1)$ and $g^2m_t^2/m_W^2$ can be carried out such that the top corrections to ρ are the same as that in the SM. It has been recently claimed [2], however, that such an expansion is meaningless; i.e., the limit $\rho_{\text{tree}} \to 1$ is not continuous. It has been argued that in these models ρ is a free parameter, so it cannot be computed, but must be extracted from the experiments. The explicit calculation of the top corrections to ρ was carried out in Ref. [2], and it was claimed not to be finite. It implies that one loses the bounds on m_t.

In this paper we show, using two different methods, that in nonminimal standard models the radiative corrections to ρ are finite and meaningful, even for large values of $\rho_{\text{tree}} - 1$. In the particular model considered in Ref. [2], we find that the bound on m_t is as strong as that in the SM. This has also been stressed in Refs. [1,3]. In Sec. II, we compute the top corrections to ρ following the effective action approach [4]. Such an approach is suited to computing radiative corrections to relations that depend on the vacuum expectation values (VEV's) of the scalar fields.[1] Neither tadpole diagrams nor counterterms for the VEV's of the scalars need to be considered, since the one-loop effective action is computed in the symmetric phase, before the electroweak symmetry breaking (ESB). Furthermore, the effective action approach allows one to relate the top corrections of different low-energy processes. In Sec. III, we reinforce our statement by computing the top corrections to ρ following the usual counterterm approach.

II. EFFECTIVE ACTION APPROACH

The effective action $\Gamma(\phi)$ is defined as the generator of the one-particle-irreducible (1PI) n-point Green's functions $\Gamma^{(n)}$:

$$\Gamma(\phi) = \sum_n \frac{1}{n!}\int d^4x_1\cdots d^4x_n \Gamma^{(n)}(x_1,\ldots,x_n) \times \phi(x_1)\cdots\phi(x_n) \, . \quad (1)$$

An alternative expansion of the effective action is in powers of momentum about the point where all external momenta vanish,

$$\Gamma(\phi) = \int d^4x \left(-V(\phi) + \frac{Z(\phi)}{2}\partial_\mu\phi\partial^\mu\phi + \cdots\right) , \quad (2)$$

where $V(\phi)$ is the so-called effective potential [6]:

$$V(\phi) = -\sum_n \frac{1}{n!}\Gamma^{(n)}(p_i=0)\phi^n \, . \quad (3)$$

Let us now consider the model of Ref. [2]. The Higgs sector consists of a Higgs doublet with $Y=1$ and a real Higgs triplet with $Y=0$:

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(\phi + iG^0) \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} \Sigma_+ \\ \Sigma_0 \\ \Sigma_- \end{pmatrix} , \quad (4)$$

respectively. Our phase convention is such that $\Sigma_- \equiv -(\Sigma_+)^*$. We want to analyze the one-loop effects of a heavy top quark. Since our model is $SU(2)_L\times U(1)_Y$ invariant, the one-loop effective action before the ESB is given by [following an expansion as in Eq. (2)]

$$\Gamma = \int d^4x\{ -V(\Phi,\Sigma) + [1 + A(\Phi^\dagger\Phi)](D_\mu\Phi)^\dagger(D^\mu\Phi) + \tfrac{1}{2}(D_\mu\Sigma)^\dagger(D^\mu\Sigma) + B(\Phi^\dagger\Phi)(\Phi^\dagger D_\mu\Phi)[(D^\mu\Phi)^\dagger\Phi] + \tfrac{1}{2}C(\Phi^\dagger\Phi)[(\Phi^\dagger D_\mu\Phi)(\Phi^\dagger D^\mu\Phi) + \text{H.c.}] + \cdots\} \, , \quad (5)$$

[1] See Ref. [5], for an example in which the effective potential is used to compute the top radiative corrections to the Higgs mass in the minimal supersymmetric model.

where we have only kept terms with a maximum of two covariant derivatives, which are the only terms relevant to our analysis. Note that the operators $A(\Phi^\dagger\Phi)$, $B(\Phi^\dagger\Phi)$, and $C(\Phi^\dagger\Phi)$ that arise at the one-loop level only depend on Φ because Σ does not couple to the quarks. When the neutral Higgs bosons develop VEV's, $\langle\phi\rangle \equiv v$ and $\langle\Sigma_0\rangle \equiv v_3$, the operators in Eq. (5) induce mass terms for the gauge bosons. The last three terms in Eq. (5) contribute differently to the W and Z masses; i.e., they break the custodial SU(2) symmetry [7]. The Higgs triplet kinetic term only contributes to the W mass, while $B(\Phi^\dagger\Phi)(\Phi^\dagger D_\mu\Phi)[(D^\mu\Phi)^\dagger\Phi]$ and $C(\Phi^\dagger\Phi)(\Phi^\dagger D_\mu\Phi)(\Phi^\dagger D^\mu\Phi)$ contribute only to the Z mass [4]. Notice that these two terms are finite, since they correspond to operators of dimension higher than 4. The first two terms in Eq. (5), however, are not finite. The effective potential $V(\Phi,\Sigma)$ can be renormalized following Ref. [6]. The kinetic term for the Higgs doublet can be made finite by a field redefinition

$$\Phi \to (1-A)^{1/2}|_{\Phi=\langle\Phi\rangle}\Phi\ . \tag{6}$$

After the rescaling (6) and the renormalization of the effective potential, the one-loop effective action is finite.

As a function of the neutral Higgs and gauge bosons, the effective action (5) before the redefinition (6) is given by

$$\Gamma = \int d^4x\{-V(\phi,\Sigma_0) + [Z(\phi^2)/2]\partial_\mu\phi\partial^\mu\phi + \tfrac{1}{2}\partial_\mu\Sigma_0\partial^\mu\Sigma_0 + \Pi_W(\Sigma_0^2,\phi^2)W_\mu W^\mu + \tfrac{1}{2}\Pi_Z(\phi^2)Z_\mu Z^\mu\}\ , \tag{7}$$

where

$$\begin{aligned} Z(\phi^2) &= 1 + A(\phi^2) + (\phi^2/2)[B(\phi^2)+C(\phi^2)]\ ,\\ \Pi_W(\Sigma_0^2,\phi^2) &= g^2\Sigma_0^2 + \frac{g^2\phi^2}{4}[1+A(\phi^2)]\ ,\\ \Pi_Z(\phi^2) &= \frac{g^2\phi^2}{4\cos^2\theta_W}[1+A(\phi^2)] + (g^2\phi^4/8\cos^2\theta_W)[B(\phi^2)-C(\phi^2)]\ , \end{aligned} \tag{8}$$

where $\sin^2\theta_W = g'^2/(g'^2+g^2)$ with g and g' being the gauge coupling of $SU(2)_L$ and $U(1)_Y$, respectively. The effective action (7) is not finite, since it has not been yet renormalized and then it is given as a function of the bare parameters and fields of the model. The Π_W and Π_Z Green's functions can be easily calculated. They correspond to the 1PI Green's functions with two external W or Z and an arbitrary number of external ϕ. To one top-bottom loop order, they are given by

$$\Pi_W = g^2\Sigma_0^2 + \frac{g^2\phi^2}{4} + \frac{g^2N_c}{32\pi^2}\left[\sum_{i=t,b} m_i^2\left(\Delta - \ln\frac{m_i^2}{\mu^2} + \frac{1}{2}\right) - \frac{m_t^2m_b^2}{m_t^2-m_b^2}\ln\frac{m_t^2}{m_b^2}\right]\ , \tag{9}$$

$$\Pi_Z = \frac{g^2\phi^2}{4\cos^2\theta_W} + \frac{g^2N_c}{32\pi^2\cos^2\theta_W}\left[\sum_{i=t,b} m_i^2\left(\Delta - \ln\frac{m_i^2}{\mu^2}\right)\right]\ ,$$

where

$$m_{t,b} = (h_{t,b}/\sqrt{2})\phi\ , \tag{10}$$

N_c is the color number ($N_c = 3$ for quarks), μ is the renormalization constant, and $\Delta = \ln 4\pi - \gamma + 1/\epsilon$, where γ is the Euler constant and $\epsilon = (4-n)/2$ with n being the space-time dimension. From Eqs. (8) and (9), we can extract[2] $A(\phi^2)$ and $[B(\phi^2)-C(\phi^2)]$. We now rescale the neutral Higgs doublet as in Eq. (6), and we obtain

$$\begin{aligned} \Pi_W &= g^2\Sigma_0^2 + g^2\phi^2/4\ ,\\ \Pi_Z &= \frac{g^2\phi^2}{4\cos^2\theta_W} - (m_W^2/\cos^2\theta_W)\Delta\rho_{tb}\ , \end{aligned} \tag{11}$$

where

$$\begin{aligned} \Delta\rho_{tb} &\equiv -(g^2\phi^4/8m_W^2)[B(\phi^2)-C(\phi^2)]\\ &= \frac{g^2N_c}{32\pi^2m_W^2}\left[\frac{1}{2}(m_t^2+m_b^2) - \frac{m_t^2m_b^2}{m_t^2-m_b^2}\ln\frac{m_t^2}{m_b^2}\right]\ . \end{aligned} \tag{12}$$

Notice that we only need to renormalize (rescale) the Higgs doublet field to get a finite result; i.e., there is not renormalization of the gauge couplings. We can now compute the physical observables

$$\begin{aligned} m_W^2|_{\rm phy} &= \Pi_W(\Sigma_0^2,\phi^2)|_{\rm VEV} = g^2v_3^2 + g^2v^2/4\ ,\\ m_Z^2|_{\rm phy} &= \Pi_Z(\Sigma_0^2,\phi^2)|_{\rm VEV} = \frac{g^2v^2}{4\cos^2\theta_W} - \frac{m_W^2}{\cos^2\theta_W}\Delta\rho_{tb}\ , \end{aligned} \tag{13}$$

$$G_F/\sqrt{2} = g^2/8\Pi_W(\Sigma_0^2,\phi^2)\Big|_{\rm VEV} = 1/2[v^2+4v_3^2]\ ,$$

$$\alpha_{\rm EM} = g^2\sin^2\theta_W/4\pi\ ,$$

where $m_W^2|_{\rm phy}$ and $m_Z^2|_{\rm phy}$ are the W and Z physical masses, G_F is the Fermi constant measured from the μ decay, and $\alpha_{\rm EM}$ is the electromagnetic fine-structure constant. If we define the ρ parameter as the measurement of the relative strength of neutral to charged currents in neutrino deep-inelastic scattering, we have

$$\rho = \frac{\Pi_W(\Sigma_0^2,\phi^2)}{\Pi_Z(\phi^2)\cos^2\theta_W}\Bigg|_{\rm VEV} = \rho_0(1+\rho_0\Delta\rho_{tb})\ , \tag{14}$$

with

$$\rho_0 = (1 + 4v_3^2/v^2)\ . \tag{15}$$

Following Ref. [1], we can also define the $\hat\rho$ parameter as the ratio $\hat\rho = m_W^2|_{\rm phy}/(m_Z^2|_{\rm phy}\cos^2\hat\theta_W)$, where $\sin^2\hat\theta_W$ is the weak angle in the modified minimal-substraction scheme [8]. Using Eqs. (13) and the fact that $\sin^2\hat\theta_W = \sin^2\theta_W$, we have that $\hat\rho$ is equal to the ρ given in Eq. (14). In Eq. (15) v and v_3 are renormalized quantities (the values of ϕ and Σ_0 that minimize the renormalized effective potential), so the radiative corrections to the ρ parameter are finite and meaningful. In our particular nonminimal SM, we have, from the experimental value of the $\hat\rho$ parameter,[3] $\hat\rho = 1.005 \pm 0.0024$, a stronger upper

[2] To obtain the explicit form of $B(\phi^2)$ and $C(\phi^2)$ we need to calculate $Z(\phi^2)$. Nevertheless, only the difference $[B(\phi^2) - C(\phi^2)]$ is relevant to our analysis.

[3] We have taken the experimental value of $\hat\rho$ from Ref. [1]. Note that we can extract $\sin^2\hat\theta_W$ from the experimental data using the relation, $\sin^2\hat\theta_W = \pi\alpha_{\rm EM}/\sqrt{2}G_Fm_W^2|_{\rm phy}$, derived from Eqs. (13).

bound on m_t than that in the SM, since both contributions (from the Higgs triplet and the top) are positive. In the limit $v_3 \to 0$, we get the SM prediction. We can write Eq. (15) as a function of only v_3 using Eqs. (13):

$$\rho_0 = 1 + 4\sqrt{2} G_F v_3^2 \ , \tag{16}$$

which implies

$$v_3 < 7.8 \text{ GeV} \ . \tag{17}$$

In models with a non-minimal Higgs sector, large radiative corrections can also be induced by Higgs bosons [9]. In model (4), however, we have noted that, neglecting terms of $O(v_3/v) \sim 3 \times 10^{-2}$, the Higgs sector has an approximate global SU(2) custodial symmetry under which Σ transforms as a triplet. It follows that Higgs corrections to ρ are very small and the bound (17) holds. It is important to note that Eq. (14) is a result valid for any nonminimal SM. In a general case,

$$\rho_0 = \frac{\sum_i (T_i^2 - T_{3i}^2 + T_i)|\langle\phi_i\rangle|^2}{\sum_i 2T_{3i}^2 |\langle\phi_i\rangle|^2} \ , \tag{18}$$

where T_i and T_{3i} are the total and third component of the weak isospin of ϕ_i. As is well known [10], for a SM with an additional complex Higgs triplet with $Y = 2$, χ, we have $\rho_0 = 1 - 4\sqrt{2} G_F \langle\chi\rangle^2$ for small values of $\langle\chi\rangle$. Then, a partial cancellation can take place between the terms $4\sqrt{2} G_F \langle\chi\rangle^2$ and $\Delta\rho_{tb}$ so that a larger m_t is allowed in this model. For a very heavy top, however, a nonperturbative calculation of ρ is necessary. Such a calculation was carried out in Ref. [11] using a $1/N_c$ expansion.

The Higgs effective potential, once renormalized, depend on m_t. Then, if v_3 is obtained from the minimization conditions of the effective potential, v_3 will depend on m_t. One would expect

$$v_3^2(m_t) = v_3^2(m_t = 0) + \Delta \ , \tag{19}$$

where Δ is of $O(g^2 m_t^2)$ or even of $O(g^2 m_t^4/m_W^2)$; i.e., the smallness of v_3 is not stable under radiative corrections of a heavy top. It is easy to see, however, that this cannot be the case. Consider the most general Higgs potential [10]

$$V(\Sigma_0, \phi) = a_1 \Sigma_0^2 + a_2 \Sigma_0^4 + a_3 \Sigma_0^2 \phi^2 + a_4 \Sigma_0 \phi^2 + V(\phi) \ . \tag{20}$$

From the minimization condition of Eq. (20), we have

$$v_3(m_t = 0) \simeq -a_4 v^2 / 2[a_1 + a_3 v^2] \ , \tag{21}$$

where v_3 has been assumed to be small. Because Σ does not couple to the top, there is no vertex correction to a_i. The only correction arises from the redefinition of the Higgs doublet (6). Thus,

$$\begin{aligned} V^{1\ \text{loop}}(\Sigma_0, \phi) = {} & a_1 \Sigma_0^2 + a_2 \Sigma_0^4 + a_3(1+\Delta)\Sigma_0^2 \phi^2 \\ & + a_4 (1+\Delta)\Sigma_0 \phi^2 + V(\phi) \ , \end{aligned} \tag{22}$$

with $\Delta = O(g^2 m_t^2)$. The explicit form of Δ depends on how we renormalize the effective potential, i.e., the definitions of the renormalized a_3 and a_4. From Eqs. (21) and (22), we have

$$v_3^2(m_t) = v_3^2(m_t = 0)\{1 + [a_1/(a_1 + a_3 v^2)]\Delta\} \ . \tag{23}$$

Therefore, v_3 has a weak dependence on the top mass and on the renormalization prescription of the effective potential.

The one-loop effective action (5) gives us more information than the top-bottom corrections to the gauge boson masses. From Eq. (5) one can also obtain the one-loop Higgs-boson–gauge-boson couplings. In the case of a neutral Higgs boson, the $\phi^n WW(\phi^n ZZ)$ coupling is given by the nth derivative of $\Pi_W(\Pi_Z)$ with respect to ϕ at $\phi = v$ [12]. For example, the one-loop ϕZZ vertex is given by

$$\begin{aligned} \Gamma_{\phi ZZ} = \left.\frac{\partial \Pi_Z}{\partial \phi}\right|_{\phi=v} = {} & \frac{g^2 v}{2\cos^2\theta_W} \\ & + g^2 N_c / 16\pi^2 v \cos^2\theta_W \\ & \times \left[\sum_{i=t,b} m_i^2 \left(\Delta - \ln\frac{m_i^2}{\mu^2} - 1 \right) \right] \ , \end{aligned} \tag{24}$$

in agreement with the explicit one-loop calculation [13]. In model (4), the H^+WZ coupling can also be obtained from Eq. (5). The H^+ is the orthogonal state to the charged Goldstone boson, i.e.,

$$H^+ = -\sin\beta\, \phi^+ + \cos\beta\, \Sigma_+ \ , \tag{25}$$

where $\tan\beta = 2v_3/v$. Equations (4)–(6), (12), and (25) yield

$$\mathcal{L}_{H^+WZ} = \left[\frac{g^2 v \sin\beta}{2\cos\theta_W} - \frac{g m_W^2 \sin\beta}{m_Z \cos^2\theta_W} \Delta\rho_{tb} \right] H^+ W_\mu Z^\mu \ . \tag{26}$$

Note that the H^+WZ vertex at the tree level [first term of Eq. (26)] is very small because it is proportional to $\sin\beta \sim v_3/v$. Such a proportionality to $\sin\beta$ is maintained at one-loop level [second term of Eq. (26)], so top corrections to H^+WZ are also small. In models with two Higgs doublets, $\Phi_\alpha = (\phi_\alpha^+, 1/\sqrt{2}[\phi_\alpha + iI_\alpha])^T$, $\alpha = 1, 2$ (such as the minimal supersymmetric model), the H^+WZ coupling is zero at the tree level [14] but can be induced to one-loop order [15]. In these models, if we now neglect m_b, the one-loop effective action is given by Eq. (5) replacing Φ by Φ_2, the Higgs doublet that couples to the top, and Σ by Φ_1. The H^+ is given by

$$H^+ = -\sin\beta \phi_1^+ + \cos\beta \phi_2^+ \ , \tag{27}$$

where now $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle$. From Eqs. (5), (6), (12), and (27) we obtain

$$\begin{aligned} \mathcal{L}_{H^+WZ} & = -\frac{g^2 \phi_2^3}{4\cos\theta_W} [B(\phi_2^2) - C(\phi_2^2)] \Big|_{\phi_2 = \langle\phi_2\rangle} \phi_2^+ W_\mu Z^\mu \\ & = \frac{g^3 N_c m_t^2 \cot\beta}{64\pi^2 m_W \cos\theta_W} H^+ W_\mu Z^\mu \ . \end{aligned} \tag{28}$$

Notice that the H^+WZ vertex arises only from the custodial breaking terms of Eq. (5) [16].

III. COUNTERTERM APPROACH

The gauge sector of the SM depends on only three independent parameters that we choose to be g, $\sin^2\theta_W$, and v. Three conditions have to be given to fix the counterterms δg, $\delta \sin^2\theta_W$ and δv:

(a) We define the Z mass to be the physical mass, i.e., $m_Z^2|_{\rm phy} \equiv m_Z^2 = g^2v^2/(4\cos^2\theta_W)$. It follows that

$$\delta m_Z^2 = \frac{1}{2\cos^2\theta_W}[v^2 g\delta g + g^2 v\delta v] - \frac{g^2v^2\delta\cos^2\theta_W}{4\cos^4\theta_W} = -A_Z \ , \tag{29}$$

where A_Z is the coefficient of $g^{\mu\nu}$ in the vacuum polarization tensor of the Z [it corresponds to $\Pi_Z(\phi^2 = v^2) - m_Z^2$ in Eq. (9)].

(b) We identify $G_F/\sqrt{2} \equiv g^2/8m_W^2$, which implies

$$v\delta v = -(2/g^2)A_W \ . \tag{30}$$

(c) We define $\sin^2\hat{\theta}_W \equiv \sin^2\theta_W$, which leads to

$$\delta \sin^2\theta_W = 0 \ , \tag{31}$$

since there are not $O(g^2m_t^2)$ corrections in the $\gamma - Z$ mixing.

If we now add a Higgs triplet to the SM, a new parameter, v_3, is introduced in the model. We fix δv_3 following the renormalization prescription of the Higgs sector of Ref. [17], i.e., the renormalized v_3 is defined to be the true VEV of Σ_0 at one loop. Neglecting the mixing between the Higgs doublet and the triplet, which is of $O(v_3/v)$, one finds $\delta v_3 = 0$. Thus, Eqs. (29), (30), and (31) still hold, and the physical W mass is given by

$$\begin{aligned} m_W^2|_{\rm phy} &= \cos^2\theta_W m_Z^2 + \cos^2\theta_W \delta m_Z^2 \\ &\quad + g^2v_3^2 + 2v_3^2 g\delta g + A_W \\ &= m_Z^2|_{\rm phy}\cos^2\hat{\theta}_W \left\{ \left[1 + \frac{A_W - \cos^2\theta_W A_Z}{\cos^2\theta_W m_Z^2}\right] \right. \\ &\quad \left. + \frac{4v_3^2}{v^2}\left[1 + \frac{A_W - \cos^2\theta_W A_Z}{\cos^2\theta_W m_Z^2}\right]\right\} \ , \end{aligned} \tag{32}$$

and Eq. (14) is recovered. As was noted in Sec. II, a change in the renormalization prescription of the effective potential (or a change in the experimental input) implies a shift $v_3^2 \to v_3^2[1+\Delta]$ with $\Delta = O(g^2m_t^2)$ and then a negligible change in Eq. (32).

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