Bounding anomalous gauge-boson couplings

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We investigate indirect phenomenological bounds on anomalous three-gauge-boson couplings. We do so by systematically determining their one-loop implications for precision electroweak experiments. We find that these loop-induced effects cannot be parametrized purely in terms of the parameters S, T, and U. Like some other authors, we find many cancellations among the loop-induced effects, and we show how to cast the low-energy effective theory into a form which makes these cancellations manifest at the outset. In a simultaneous fit of all CP-conserving anomalous three-gauge-boson couplings, our analysis finds only weak phenomenological constraints.

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I. INTRODUCTION

The experimental couplings of the electroweak gauge bosons to light fermions have now been quite well explored, particularly using low-energy lepton-scattering experiments and precision measurements at the Z resonance. However, accurate experimental information is not available for the self-interactions among the gauge bosons. This situation is likely to be partially alleviated once the center-of-mass energies at the CERN e^+e^- collider LEP are raised above the threshold for $W^\pm$ pair production (LEP 200). Given a sufficiently large sample of $W^\pm$ pairs, direct information becomes available concerning the nature of the $WW\gamma$ and WWZ couplings. It is expected that a deviation (of 10% or more) of the three-gauge-boson vertices (TGV's) from their standard model (SM) values can be detected in this way [1–3].

The key question is whether there is any kind of new, nonstandard physics that can give rise to this large a deviation from SM predictions for the TGV's, and yet which might not have been detected elsewhere in other low-energy experiments. As might have been expected, a great deal of effort has been expended on researching this subject, leading to a dauntingly large literature [4].

Two approaches to answering this question may be taken, depending on one's theoretical prejudices.

Theoretically motivated bounds. The first approach is to use (sometimes fairly benign) assumptions about the nature of the new physics in order to obtain an estimate of how large the anomalous TGV's might be. The strength of this type of conclusion is then inversely related to the restrictiveness of the assumptions that are required in order to derive it. The thrust of this line of thinking is usually to (reasonably convincingly) argue that induced anomalous TGV's are unlikely to be larger than ~1% of their SM counterparts. If true, this would make their detection at LEP improbable.

There are two broad classes of new physics for which this conclusion is probably true. First, if the new physics is perturbatively coupled to the electroweak gauge bosons, then its contributions to TGV's are of order $(g/4\pi)^2 \sim 10^{-3}$, where g is an electroweak coupling constant. Since the transverse gauge bosons couple with a universal strength, this estimate includes a great many models. Any couplings between the new physics and the longitudinal gauge bosons need not be so small, however, and so a strongly coupled symmetry-breaking sector might be considered.

In this case, a second line of reasoning leads to a similar conclusion concerning the detectability of anomalous TGV's [5]. To the extent that the low-energy $W^\pm$ physics is dominated purely by the couplings of the longitudinal modes, it may be parametrized using familiar techniques of chiral perturbation theory [6]. It is therefore quite plausible that the size of these effective interactions depends on the weak scale and the unknown scale, M, of the new physics in the same way as do the corresponding couplings in the low-energy chiral limit of QCD. This dependence may be succinctly summarized by the rules of "naive dimensional analysis" [7], which indicate that the relative size of anomalous and SM TGV's should be $O(M_W^2/M^2)$. Again, for M as large as a few TeV, as might be expected for a strongly interacting Higgs sector, for example, we are led to expect anomalous TGV's of $\sim 1\%$. In either case, these are too small to be observed.

A complementary line of argument is based on naturalness [8]. One way of phrasing this argument states that anomalous TGV's should be of the same size as other new-physics contributions to the purely gauge sector (e.g., "oblique" corrections) [9,10], since there are no symmetries that could naturally enforce a relative size difference. Since these oblique corrections are bounded by precision electroweak measurements to be $\lesssim 1\%$, so the argument goes, anomalous TGV's can also be expected to be at most this large.

Purely phenomenological bounds. The second, more conservative tack that may be taken in determining the potential size of anomalous TGV's is to put theoretical prejudices aside and ask for purely phenomenological bounds. In this case we ask whether anomalous couplings that are large enough to be detected can be already excluded based on other low-energy measurements. The potential bounds of this sort arise either due to the direct probing of nonstandard gauge-boson couplings in hadron colliders [3,11–13], or from their indirect influence on precisely measured quantities, through loops. Although there is agreement that existing hadron machines cannot rule out detectable nonstandard TGV's at LEP 200 [12], the extraction of bounds from loops has been more controversial [14].

Here again the most recent analyses may be classified according to the assumptions that are made. Some have entailed a weakly coupled framework within which the standard-model gauge group is linearly realized at low energies by including an explicit light physical Higgs particle [8,15,16]. In all of these studies it is found that detectably large anomalous TGV's cannot be ruled out purely by phenomenology. Other workers [17,18] have instead not assumed the existence of a light Higgs boson in an effort to extract bounds that are less constrained by assumptions concerning how the gauge symmetry is realized. Again the detectable TGV's are not ruled out on purely phenomenological grounds.

It is natural to ask why purely phenomenological bounds should be pursued at all, given that reasonably persuasive theoretical arguments indicate that detectable anomalous TGV's are unlikely. Our own point of view is that neither a purely theoretical estimate, nor a purely phenomenological analysis is sufficient in itself. We can only hope to learn anything if *both* types of investigations are performed, since it is only through the comparison of both, and their subsequent confrontation with the direct measurements at LEP, that we learn something about the nature of any new physics.

The present paper is intended as a contribution to the purely phenomenological line of thought. We wish here to determine the constraints on CP-preserving anomalous TGV's with an absolute minimum of assumptions about the nature of the new physics from which they are generated. As with the previous analyses of Refs. [8,16–18], we assume that the scale M that is associated with the new physics is high enough in comparison with the weak scale M_W to justify an effective-Lagrangian treatment that is controlled by powers of $1/M$. Integrating out the physics at scale M generates a host of effective interactions, including anomalous TGV's among others:

$$\mathcal{L}_{\text{eff}}(\mu^2 = M^2) = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{TGV}} + \mathcal{L}_{\text{rest}} \ . \qquad (1)$$

Here $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian, $\mathcal{L}_{\text{TGV}}$ represents the anomalous TGV interactions, and $\mathcal{L}_{\text{rest}}$ denotes all of the other effective operators. Only some of the effective interactions in $\mathcal{L}_{\text{rest}}$ contribute at the tree level to well-measured low-energy observables, and so only these are presently well constrained by the data. All other operators, including TGV's in particular, do not contribute in this way and so are only bounded to the extent that they generate the better-constrained operators as the effective theory is run down from $\mu = M$ to the much lower scales where the low-energy measurements are ultimately performed. Our purpose here is to compute which operators are generated by TGV's in this way, and so to indirectly bound their coefficients.

Before describing our conclusions, it is useful to orient our calculation in relation to the others that have recently been performed. Our calculation differs from those of Refs. [8,15,16] in that we assume that the dominant degrees of freedom that govern the loop contribution of TGV's to low-energy observables are *only* the presently known particles (including the top quark). In particular, we do *not* assume a light Higgs boson and do *not* linearly realize the electroweak gauge group. As was emphasized in Ref. [14], one can choose to realize this gauge symmetry nonlinearly, using a chiral Lagrangian to describe the longitudinal gauge bosons, or one can ignore it completely [apart from its $U_{\text{em}}(1)$-invariant subgroup]. In both cases one is led to precisely the same low-energy effective Lagrangian [19] (we will elaborate on this point below). The price to be paid is that the new-physics scale cannot be arbitrarily large, $M_W/M \gtrsim g/4\pi$, or else perturbative unitarity is lost.

Our analysis also differs in important ways from those of Refs. [17,18]. Perhaps the most basic difference lies in the number of effective operators that are considered. We use the most general effective interactions of Ref. [1] that are consistent with CP conservation. Our calculations therefore include the five effective couplings $\Delta\kappa_\gamma$, $\Delta\kappa_Z$, Δg_{1Z}, λ_Z, and λ_γ (see the following section for detailed definitions). Our results may be compared to those of Ref. [18] by taking $\Delta g_{1Z} = 0$, and to those of Ref. [17] by choosing $\lambda_Z = \lambda_\gamma = 0$. It should be pointed out that the neglect of λ_Z and λ_γ in Ref. [17] is what would be expected if the new physics were to produce an effective theory that satisfies the power-counting rules [7] that have been found from experience with chiral perturbation theory in QCD. We do not make this assumption here, however, since it is not a generic feature of all underlying theories at scale M.[1]

A further difference with other workers arises because the authors of Refs. [8,17] also make explicit uses of quadratic divergences in arriving at their bounds. In the present language, their calculation corresponds to an estimate of the size of direct contributions of new physics to $\mathcal{L}_{\text{rest}}$ at scale $\mu = M$, rather than of the induced effects at low energies due to the TGV's defined at this scale [14]. This distinction is less important in the linearly realized case, where the low-energy divergences are less severe.

[1]For example, λ_Z and $\Delta\kappa_Z$ are the same size in a linearly realized effective theory.

A calculation of the purely low-energy loop effects of TGV's has recently been made without the assumption of a light Higgs boson in Ref. [18]. These authors use the formalism of oblique corrections, as parametrized by Peskin and Takeuchi's S, T, and U [9], to obtain their low-energy bounds on TGV's. One of the points of the present paper, however, is that such an analysis, based on S, T, and U is not sufficiently general for inferring the complete low-energy effects of anomalous TGV's. As we have emphasized elsewhere [20], the Peskin-Takeuchi formalism applies only to the extent that new-physics contributions to self-energies for gauge bosons can be approximated as expansions in q^2/M^2, truncated at the linear term. The calculations of Ref. [18], as well as our own, indicate that this is not what is obtained through loops from anomalous TGV's. Instead we obtain self-energies of the form

$$\delta\Pi(q^2) = \omega M_W^2 + \alpha q^2 + \frac{\beta q^4}{M_W^2} + \frac{\gamma q^6}{M_W^4} \qquad (2)$$

with ω, α, β, and γ all of the same order of magnitude. The Peskin-Takeuchi parametrization is therefore insufficient, and must be replaced by the more general formalism, recently derived in Ref. [20], which involves three additional parameters, denoted V, W, and X. Our analysis here also differs from that of Ref. [18] in that we compute not only the oblique corrections resulting from loop integration, but also corrections to fermion-gauge-boson vertices. These vertex corrections can contribute significantly to low-energy observables.

Note that all our loop calculations are performed in unitary gauge. At first sight, this might cause some concern for two reasons. First, in the SM, the computations of oblique corrections (and of vertex corrections) are typically dependent on the gauge parameter ξ [21]. If the same were true here, then one might question whether we have verified that the final results are indeed gauge invariant. Second, in the same vein, one might wonder whether, for example, the q^4 and q^6 terms which appear in Eq. (2) might be simply unitary-gauge artifacts. In fact, neither of these points is a concern in the present calculation. In all our computations it is possible to make explicit the nonlinearly realized $\mathrm{SU}_L(2)\times\mathrm{U}_Y(1)$ symmetry by working with a chiral Lagrangian containing explicit would-be Goldstone bosons. In this case we could compute using a more conventional ξ gauge, at the expense of having to evaluate a great many more graphs. As discussed in Ref. [14], the key point here is that, in contrast with the usual linearly realized SM, the would-be Goldstone bosons in this chiral Lagrangian always couple derivatively to all other particles (rather than via Yukawa couplings to fermions, for example). This has the consequence that, in any of our loop calculations, the ξ dependence due to the contribution of a W in a loop is *canceled* by the (ξ-dependent) contribution of the corresponding would-be Goldstone boson. That is, in this formulation the sum of all one-loop graphs contributing to the n-point functions of interest is *ξ independent.* What is left over is nothing other than the result of the same loop calculation performed in unitary gauge, so that the results of a calculation in unitary gauge are *identical* to those obtained in any other gauge. Therefore, all our results, including the q^4 and q^6 terms of Eq. (2), are independent of the gauge used to perform the computation.

Our procedure consists of computing how the anomalous TGV's appear in the six observable parameters S-X that, in practice, completely parametrize all precision low-energy electroweak measurements. We can then perform a fit, including all charged- and neutral-current data at low energy and at the Z peak. The strength of the conclusions we can draw from such a fit depends crucially on our assumptions regarding which terms we include in our effective Lagrangian. If we follow the fairly common practice of fitting for one anomalous TGV at a time, setting the others to zero, we find that that some of the anomalous couplings can be constrained to be too small to detect at LEP 200. However, this is rather unrealistic—real models of underlying physics do not generate just one anomalous TGV at a time. A simultaneous fit for the general case in which expressions involving all five anomalous TGV's are fitted to data finds that the contraints are weak, in fact no stronger than those from direct measurements by UA2 [11].

It should be noted that even this is not the least restrictive assumption one can make. In addition to the contributions to the parameters S through X that are generated by low-energy loops involving the anomalous TGV's defined at the scale of new physics, $\mu = M$, there are also typically direct contributions to S-X that are generated from $\mathcal{L}_{\mathrm{rest}}$. Obviously, if cancellations are allowed among these two types of new-physics contributions, any constraints on anomalous TGV's are lost. In this sense, current precision electroweak data can never rule out the possibility of anomalous TGV's large enough to be detected at LEP 200. Nevertheless, it is a useful exercise to fit for one anomalous TGV at a time, since it does indicate the degree to which the discovery of anomalous TGV's at LEP 200 would require cancellations among the new-physics effects for some low-energy observables.

The paper is organized in the following way. In Sec. II, we define what we mean by an anomalous TGV Lagrangian sector. We imagine that the entire effective Lagrangian, defined just below the threshold for new physics, $\mu = M$, consists of the standard model Lagrangian plus a supplementary TGV sector. In Sec. III, we begin to estimate the indirect effects of this anomalous TGV vector on electroweak observables, by integrating out the physics between $\mu = M$, and the electroweak scale, $\mu \simeq 100$ GeV. We do so by using the renormalization group (RG), in the modified minimal subtraction ($\overline{\mathrm{MS}}$) renormalization scheme, to run our effective Lagrangian between these two scales. We follow in particular how the effective TGV interactions at $\mu = M$ induce other effective interactions at the weak scale that are detectable in precision electroweak measurements. We call the resulting weak-scale theory the "grown" Lagrangian, and present expressions for the coefficients of the "grown" operators in terms of the anomalous TGV's. Superficially, this "grown" Lagrangian appears to be complicated, consisting of some 29 different types of terms.

In Sec. IV, however, we show that this awkwardness is illusory, since the freedom to redefine fields can be used to greatly reduce the number of terms and to conceptually simplify the calculation of observables. Concretely, this amounts to the use of the SM equations of motion to reexpress the effective interactions [22]. Exploiting the leeway offered by this technique, we cast the "grown" Lagrangian into a particularly simple form, consisting of oblique vacuum-polarization corrections only, which necessarily involve higher-derivative interactions. These make a direct application of the Peskin-Takeuchi STU formalism invalid, requiring instead the more general analysis in terms of the parameters S through X of Ref. [20]. For completeness of presentation, we give here a very brief summary of the $STUVWX$ parametrization of new physics, as required by the present analysis.

In Sec. V, we derive formulas for electroweak observables in terms of the familiar S, T, U and the new parameters V, W, X. Knowing the relationship between the parameters $S-X$ and the anomalous TGV's, these formulas allow us to place phenomenological bounds on the TGV couplings. We then present the results of fits to these expressions. We first consider the limit in which each TGV is separately turned on at $\mu = M$, with all other effective interactions being zero. We find that, with this somewhat unphysical assumption, the data place strong phenomenological bounds on the TGV couplings that are of order of several percent. However, when we fit for the five anomalous TGV's simultaneously, we find that the bounds become considerably weaker, of order 1.

Finally, in Sec. VI, we discuss the results, commenting in particular on some rather startling cancellations in our final expressions. These cancellations decrease the sensitivity of some observables to certain anomalous TGV's.

II. THE ANOMALOUS TGV'S

Let us start by considering the effective Lagrangian, as defined at the scale of new physics, $\mu = M$, after the lightest of the heavy new particles has been integrated out. In general many effective interactions appear in this Lagrangian, but we wish to focus in this paper only on a few of these:[2]

$$\mathcal{L} = \mathcal{L}_{\rm SM}(\tilde{e}_i) + \mathcal{L}_{\rm TGV} \ , \tag{3}$$

where $\mathcal{L}_{\rm SM}(\tilde{e}_i)$ is the standard model (SM) Lagrangian, in which the as-yet-undetected particles, in particular the Higgs boson, have been integrated out. We place tildes on the three electroweak parameters $[\tilde{e}_i \equiv \{\tilde{e}, \tilde{s}_W(\equiv \sin\tilde{\theta}_W), \tilde{m}_Z\}]$ that appear in this Lagrangian as a reminder that their values have been shifted from their "standard" values, which we denote without tildes: e, s_W, m_Z. These "standard" values are the ones that are obtained by fitting radiatively corrected SM expressions for observables to precise data.

$\mathcal{L}_{\rm TGV}$ contains the anomalous CP-conserving TGV's which are the focus of this study. Following Refs. [1,23] we write

$$\begin{aligned}\mathcal{L}_{\rm TGV} = {} & ig_Z \Delta g_{1Z}(W_{\alpha\beta}W^{*\beta} - W^*_{\alpha\beta}W^{\beta})Z^{\alpha} \\ & + i\sum_{V=Z,\gamma}\Bigg[g_V \Delta\kappa_V W^*_{\alpha} W_{\beta} V^{\alpha\beta} \\ & + g_V \frac{\lambda_V}{M_W^2} W^*_{\rho\mu} W^{\mu}{}_{\sigma} V^{\sigma\rho}\Bigg] \ .\end{aligned} \tag{4}$$

Here $g_\gamma = e$ and $g_Z = ec_W/s_W$ denote the SM couplings. Electromagnetic gauge invariance requires that $\Delta g_{1\gamma} = 0$. A gauge field having two Lorentz indices, such as $W_{\alpha\beta}$, denotes the Abelian curl of the corresponding gauge potential, defined using the appropriate electromagnetic gauge-covariant derivative: $W_{\alpha\beta} = D_\alpha W_\beta - D_\beta W_\alpha$. Here $D_\alpha W_\beta = \partial_\alpha W_\beta + ieA_\alpha W_\beta$. Following the convention established in the literature, we scale the λ_V term to M_W^2, despite the fact that it is a dimension-six operator and should more correctly have a factor M^2 in the denominator. Because of the appearance of the $U_{\rm em}(1)$-covariant derivatives, $\mathcal{L}_{\rm TGV}$ also contains four- and five-point gauge-boson vertices:

$$\mathcal{L}_{\rm TGV} = \mathcal{L}_{3B} + \mathcal{L}_{4B} + \mathcal{L}_{5B} \ , \tag{5}$$

where $\mathcal{L}_{3B}$ is simply $\mathcal{L}_{\rm TGV}$ with the replacement $D_\mu \to \partial_\mu$, and $\mathcal{L}_{4B}$ is given by

$$\begin{aligned}\mathcal{L}_{4B} = {} & eg_Z\Delta g_{1Z}[W^*_\alpha W_\beta(Z^\alpha A^\beta + Z^\beta A^\alpha) \\ & -2W^*_\alpha W^\alpha A_\beta Z^\beta] + (\lambda_V \text{ terms}) \ .\end{aligned} \tag{6}$$

The "λ_V terms" here consist of those four-point interactions that are generated from the λ_V terms in $\mathcal{L}_{\rm TGV}$. We do not list them explicitly because, for present purposes, all graphs which use these rules turn out to vanish. As for $\mathcal{L}_{5B}$, such terms contribute only in two-loop diagrams and are ignored here.

III. LOOP INTEGRATION AND THE LOW-ENERGY LAGRANGIAN

We next calculate the loop effects of the anomalous TGV's of the previous section. We do so by running the effective interactions of the Lagrangian defined at the new-physics scale, $\mu \simeq M$, down to the weak scale, $\mu \simeq M_Z$, where we can extract their consequences for precision experiments. We compute here the coefficients of the effective interactions at the weak scale that are generated through this renormalization group (RG) mixing with the operators in $\mathcal{L}_{\rm TGV}$.

A. Vacuum polarization and vertex corrections

The required diagrams fall into two categories: contributions to the gauge-boson vacuum polarization, i.e., "oblique," corrections, as shown in Fig. 1, and fermion–gauge-boson vertex corrections, as shown in Fig. 2. We define the running of our operators using the $\overline{\rm MS}$ renormalization scheme. Here we simply quote the results, pre-

[2]Note that the terms we ignore include direct new-physics contributions to S, T, and U [9], as well as to gauge-boson-fermion-fermion and other vertices.

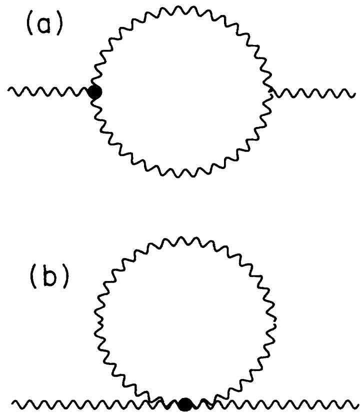

FIG. 1. The Feynman graphs through which the anomalous three- and four-point gauge-boson vertices contribute to the gauge-boson vacuum polarization. The solid circles represent anomalous couplings, and all other interactions are standard.

senting only the coefficient of $[2/(4-n)-\gamma_E+\ln(4\pi\mu^2)]$, where n is the dimension of spacetime.

Evaluating the graphs of Fig. 1, we find the following TGV-induced contributions to the gauge-boson vacuum polarization tensors. With the definition

$$\delta\Pi_{ab}^{\mu\nu}(q) = \eta^{\mu\nu}\delta\Pi_{ab}(q^2) + q^\mu q^\nu \text{ terms} , \tag{7}$$

where $a, b = W$, Z and γ, the coefficient of $[2/(4-n) - \gamma_E + \ln(4\pi\mu^2)]$ in $\delta\Pi_{ab}(q^2)$ is

$$\begin{aligned}
\delta\Pi_{\gamma\gamma}(q^2) &= \alpha_\gamma q^2 + \beta_\gamma\frac{q^4}{M_Z^2} + \gamma_\gamma\frac{q^6}{M_Z^4} ,\\
\delta\Pi_{Z\gamma}(q^2) &= \alpha_{Z\gamma} q^2 + \beta_{Z\gamma}\frac{q^4}{M_Z^2} + \gamma_{Z\gamma}\frac{q^6}{M_Z^4} ,\\
\delta\Pi_{WW}(q^2) &= \omega_W M_W^2 + \alpha_W q^2 + \beta_W\frac{q^4}{M_W^2} + \gamma_W\frac{q^6}{M_W^4} ,\\
\delta\Pi_{ZZ}(q^2) &= \omega_Z M_Z^2 + \alpha_Z q^2 + \beta_Z\frac{q^4}{M_Z^2} + \gamma_Z\frac{q^6}{M_Z^4} ,
\end{aligned} \tag{8}$$

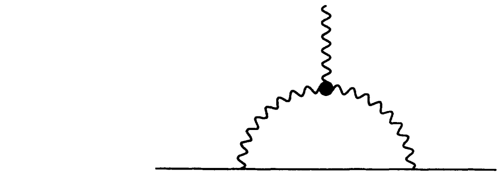

FIG. 2. The Feynman graph through which the anomalous TGV's contribute to the gauge-boson-fermion vertex corrections. The solid circle represents the anomalous TGV, and all other interactions are standard.

where α_Z, β_γ, etc., are given as functions of the couplings Δg_{1Z}, $\Delta\kappa_V$, and λ_V in Table I. Anticipating that these anomalous couplings are small, we drop all terms past linear order in the expressions in the table. The vacuum polarization diagrams have also been calculated by the authors of Ref. [18], and our results are in agreement with theirs for diagrams involving $\Delta\kappa_V$ and λ_V. They did not calculate the diagrams with Δg_{1Z}.

As for the vertex graphs of Fig. 2, we obtain the following expressions for the $[2/(4-n)-\gamma_E+\ln(4\pi\mu^2)]$ coefficient in the fermion–fermion–gauge-boson vertex corrections:

$$\begin{aligned}
\delta\Lambda_{\rm em}(q^2) &= \left(p_{\rm em}^{(2)}\frac{q^2}{M_Z^2} + p_{\rm em}^{(4)}\frac{q^4}{M_Z^4}\right) T_{3f}\gamma_L ,\\
\delta\Lambda_{\rm CC}(q^2) &= \left(p_{\rm CC}^{(0)} + p_{\rm CC}^{(2)}\frac{q^2}{M_W^2} + p_{\rm CC}^{(4)}\frac{q^4}{M_W^4}\right) V_{ff'}\gamma_L ,\\
\delta\Lambda_{\rm NC}(q^2) &= \left(p_{\rm NC}^{(2)}\frac{q^2}{M_Z^2} + p_{\rm NC}^{(4)}\frac{q^4}{M_Z^4}\right) T_{3f}\gamma_L ,
\end{aligned} \tag{9}$$

where we have normalized the vertex corrections such that standard model tree-level vertices are corrected in the following manner:

$$\begin{aligned}
i\Lambda_{\rm em}^\mu(q^2) &= -ie\gamma^\mu\left[Q_f + \frac{1}{s_W}\delta\Lambda_{\rm em}(q^2)\right] ,\\
i\Lambda_{\rm CC}^\mu(q^2) &= -i\frac{e}{\sqrt{2}s_W}\gamma^\mu[V_{ff'}\gamma_L + \delta\Lambda_{\rm CC}(q^2)] ,\\
i\Lambda_{\rm NC}^\mu(q^2) &= -i\frac{e}{s_W c_W}\gamma^\mu[T_{3f}\gamma_L - Q_f s_W^2 + c_W\delta\Lambda_{\rm NC}(q^2)] ,
\end{aligned} \tag{10}$$

TABLE I. One-loop results for the coefficients in the gauge-boson vacuum polarization in terms of the various TGV couplings, where the TGV couplings are defined at scale μ'.

Coefficient	One-loop result $\times(\alpha/4\pi s_W^2)\ln(\mu'^2/\mu^2)$
$\alpha_\gamma(\mu^2)$	$s_W^2(6\Delta\kappa_\gamma - 12\lambda_\gamma)$
$\beta_\gamma(\mu^2)$	$s_W^2(-\frac{2}{3}\Delta\kappa_\gamma + 2\lambda_\gamma)/c_W^2$
$\gamma_\gamma(\mu^2)$	$-\frac{1}{6}s_W^2\Delta\kappa_\gamma/c_W^4$
$\alpha_{Z\gamma}(\mu^2)$	$c_W s_W[3\Delta\kappa_Z + 3\Delta\kappa_\gamma - 6\lambda_Z - 6\lambda_\gamma + 4\Delta g_{1Z}]$
$\beta_{Z\gamma}(\mu^2)$	$(s_W/c_W)[-\frac{1}{3}(\Delta\kappa_Z + \Delta\kappa_\gamma) + \lambda_Z + \lambda_\gamma - \frac{5}{6}\Delta g_{1Z}]$
$\gamma_{Z\gamma}(\mu^2)$	$-(s_W/12c_W^3)[\Delta\kappa_Z + \Delta\kappa_\gamma]$
$\omega_W(\mu^2)$	$3[s_W^2\Delta\kappa_\gamma + (1 + c_W^2 - 1/(2c_W^2))\Delta\kappa_Z + (1 + c_W^2 - \frac{1}{2}c_W^4)\Delta g_{1Z}]$
$\alpha_W(\mu^2)$	$\frac{5}{3}s_W^2\Delta\kappa_\gamma + \frac{1}{3}(7 + 5c_W^2)\Delta\kappa_Z - 6s_W^2\lambda_\gamma - 6(1 + c_W^2)\lambda_Z + \frac{1}{2}(1 + \frac{40}{3}c_W^2 + \frac{17}{3}c_W^4)\Delta g_{1Z}$
$\beta_W(\mu^2)$	$[-\frac{5}{6}(s_W^2\Delta\kappa_\gamma + c_W^2\Delta\kappa_Z) + 2(s_W^2\lambda_\gamma + c_W^2\lambda_Z) - \frac{1}{6}(2c_W^2 + 7c_W^4)\Delta g_{1Z}]$
$\gamma_W(\mu^2)$	$-\frac{1}{6}c_W^4\Delta g_{1Z}$
$\omega_Z(\mu^2)$	$9c_W^4\Delta g_{1Z}$
$\alpha_Z(\mu^2)$	$c_W^2[6\Delta\kappa_Z - 12\lambda_Z + 8\Delta g_{1Z}]$
$\beta_Z(\mu^2)$	$[-\frac{2}{3}\Delta\kappa_Z + 2\lambda_Z - \frac{5}{3}\Delta g_{1Z}]$
$\gamma_Z(\mu^2)$	$-(1/6c_W^2)\Delta\kappa_Z$

where f, f' denote fermion type, T_{3f} is the weak isospin, and Q_f the electric charge. In the charged-current expression, $V_{ff'}$ represents the usual Cabibbo-Kobayashi-Maskawa (CKM) matrix in generation space when the external fermions are quarks, and is given by $\delta_{ff'}$ when they are leptons.

We give expressions for the $p^{(n)}$ coefficients as linear combinations of the anomalous TGV couplings in Table II. Note that since we have calculated in the approximation that all fermions are massless, we have "grown" only left-handed corrections to the standard model fermion–fermion–gauge-boson couplings.

The above expressions are universal corrections because of our neglect of all fermion masses. However, the one situation for which this assumption is inadequate is the coupling of the down-type quarks to the photon and to the Z boson, since these involve virtual top quarks, whose mass is not small.[3] Only the $Zb\bar{b}$ vertex is of practical importance, though, because the only process in which these interactions are probed is in the decay of the Z into $b\bar{b}$ pairs. For this observable the vertex correction has the form

$$\delta\Lambda^{b\bar{b}}_{\rm NC}(q^2) = \delta\Lambda^{\rm univ}_{\rm NC}(q^2) + \frac{1}{64\pi^2}|V_{tb}|^2\frac{e^2 c_W}{s_W^2}\left(\frac{m_t^2}{M_W^2}\right)\ln\left(\frac{\mu'^2}{\mu^2}\right)\left[3\Delta g_{1Z} + \frac{1}{2c_W^2}\Delta\kappa_Z\left(\frac{q^2}{M_Z^2}\right)\right]\gamma_L\ , \qquad (11)$$

where $\delta\Lambda^{\rm univ}_{\rm NC}(q^2)$ is the result given in Eq. (9) and Table II.

B. The weak-scale effective Lagrangian

These expressions may be interpreted as contributions to the effective Lagrangian at lower-energy scales. In the $\overline{\rm MS}$ scheme the resulting expressions for the induced couplings at scale μ may be obtained from the table by simply multiplying the results of Tables I and II by

$$y \equiv \ln[(\mu'/\mu)^2]\ , \qquad (12)$$

where the TGV couplings $\Delta\kappa_z$, $\Delta\kappa_\gamma$, etc., are taken to be defined at scale μ'.

We may therefore write out those terms that are "grown" in the low-energy Lagrangian at scale μ, due to the appearance of $\mathcal{L}_{\rm TGV}$ at the higher scale $\mu' = M$:

$$\frac{1}{y}\mathcal{L}_{\rm grown} = \mathcal{L}_{\rm vac\ pol} + \mathcal{L}_{\rm ver\ corr} + \mathcal{L}_{\rm nonuniv} \qquad (13)$$

with $\mathcal{L}_{\rm nonuniv}$ containing the nonuniversal m_t-dependent contributions of Eq. (11), while

$$\begin{aligned}\mathcal{L}_{\rm vac\ pol} = {} & \frac{\omega_Z M_Z^2}{2} Z_\mu Z^\mu + \frac{\alpha_Z}{4} Z_{\mu\nu}Z^{\mu\nu} - \frac{\beta_Z}{4M_Z^2} Z_{\mu\nu}\,\Box\, Z^{\mu\nu} + \frac{\gamma_Z}{4M_Z^4} Z_{\mu\nu}\,\Box^2\, Z^{\mu\nu} \\ & + \frac{\alpha_{Z\gamma}}{2} Z_{\mu\nu}F^{\mu\nu} - \frac{\beta_{Z\gamma}}{2M_Z^2} Z_{\mu\nu}\,\Box\, F^{\mu\nu} + \frac{\gamma_{Z\gamma}}{2M_Z^4} Z_{\mu\nu}\,\Box^2\, F^{\mu\nu} + \frac{\alpha_\gamma}{4} F_{\mu\nu}F^{\mu\nu} - \frac{\beta_\gamma}{4M_Z^2} F_{\mu\nu}\,\Box\, F^{\mu\nu} + \frac{\gamma_\gamma}{4M_Z^4} F_{\mu\nu}\,\Box^2\, F^{\mu\nu} \\ & + \omega_W M_W^2 W^*_\mu W^\mu + \frac{\alpha_W}{2} W^*_{\mu\nu} W^{\mu\nu} - \frac{\beta_W}{2M_W^2} W^*_{\mu\nu}\,\Box\, W^{\mu\nu} + \frac{\gamma_W}{2M_W^4} W^*_{\mu\nu}\,\Box^2\, W^{\mu\nu}\end{aligned} \qquad (14)$$

and

$$\begin{aligned}\mathcal{L}_{\rm ver\ corr} = {} & (s_W j^\mu + c_W N^\mu)\left(-p^{(2)}_{\rm NC}\frac{\Box}{M_Z^2} + p^{(4)}_{\rm NC}\frac{\Box^2}{M_Z^4}\right) Z_\mu + (s_W j^\mu + c_W N^\mu)\left(-p^{(2)}_{\rm em}\frac{\Box}{M_Z^2} + p^{(4)}_{\rm em}\frac{\Box^2}{M_Z^4}\right) A_\mu \\ & + \left[J^\mu\left(p^{(0)}_{\rm CC} - p^{(2)}_{\rm CC}\frac{\Box}{M_W^2} + p^{(4)}_{\rm CC}\frac{\Box^2}{M_W^4}\right) W^*_\mu + \text{H.c.}\right]\ .\end{aligned} \qquad (15)$$

Here j^μ, N^μ, and J^μ are, respectively, the total SM electromagnetic, neutral, and charged currents:

$$j^\mu = -e\sum_f \bar{\Psi}_f\gamma^\mu Q_f\Psi_f\ , \qquad (16)$$

$$J^\mu = -\frac{e}{\sqrt{2}s_W}\sum_{ff'}\bar{\Psi}_f\gamma^\mu V_{ff'}\gamma_L\Psi_{f'}\ , \qquad (17)$$

$$N^\mu = -\frac{e}{s_W c_W}\sum_f \bar{\Psi}_f\gamma^\mu[T_{3f}\gamma_L - Q_f s_W^2]\Psi_f\ , \qquad (18)$$

so that $\mathcal{L}_{\rm SM} = j^\mu A_\mu + N^\mu Z_\mu + (J^\mu W^*_\mu + \text{H.c.}) + \cdots$.

To the extent that the loop effects of anomalous TGV's are universal in their coupling to fermion generations, we see that it is possible to express the "grown" fermion-

[3]This necessity to keep track of the top-quark mass is one of the differences between our calculation and that of the authors of Ref. [16], who find that the fermion masses do not affect the $\overline{\rm MS}$ RG evolution at the one-loop level in the linearly realized effective theory.

TABLE II. One-loop results for the coefficients in the gauge-boson–fermion vertex corrections in terms of the various TGV couplings.

Coefficient	One-loop result $\times(\alpha/4\pi s_W^2)\ln(\mu'^2/\mu^2)$
$p_{\rm NC}^{(2)}(\mu^2)$	$(1/2c_W)[\Delta\kappa_Z - 2\lambda_Z + \frac{5}{3}\Delta g_{1Z}]$
$p_{\rm NC}^{(4)}(\mu^2)$	$(1/12c_W^3)\Delta\kappa_Z$
$p_{\rm CC}^{(0)}(\mu^2)$	$\frac{3}{4}[(c_W^2-1)\Delta\kappa_Z + s_W^2\Delta\kappa_\gamma + s_W^2c_W^2\Delta g_{1Z}]$
$p_{\rm CC}^{(2)}(\mu^2)$	$\frac{1}{4}[\frac{5}{3}(c_W^2\Delta\kappa_Z + s_W^2\Delta\kappa_\gamma) - 4(c_W^2\lambda_Z + s_W^2\lambda_\gamma) + (c_W^2 + \frac{8}{3}c_W^4)\Delta g_{1Z}]$
$p_{\rm CC}^{(4)}(\mu^2)$	$\frac{1}{12}c_W^4\Delta g_{1Z}$
$p_{\rm em}^{(2)}(\mu^2)$	$(s_W/2c_W^2)[\Delta\kappa_\gamma - 2\lambda_\gamma]$
$p_{\rm em}^{(4)}(\mu^2)$	$(s_W/12c_W^4)\Delta\kappa_\gamma$

gauge couplings in terms of linear combinations of total SM currents. This is important, since it in turn allows, through the use of equations of motion, a significant simplification of the effective Lagrangian. The exception to this universal form is in the m_t-dependent interactions of $\mathcal{L}_{\rm nonuniv}$, in which the Z boson and the photon do not couple with the same strength to all generations of down quarks. This particular case must be treated separately, but does not affect the arguments of the subsequent sections.

IV. SIMPLIFYING THE EFFECTIVE THEORY

At this point one might be daunted by the fact that there are no fewer than 29 types of terms in this "grown" Lagrangian. This large number of terms seems to render any further analysis very cumbersome. The complicated form for $\mathcal{L}_{\rm grown}$ is illusory, however, since not all of the effective interactions displayed in Eqs. (14) and (15) are independent of one another. The SM equations of motion can be used to reduce the number of operators in $\mathcal{L}_{\rm grown}$ and to thereby reveal its essentially simple form. In this analysis, we opt to transform the vertex corrections into corrections to the gauge-boson propagators. Beyond simply reducing the number of operators that must be considered, this particular choice has the advantage of allowing a direct application of the $STUVWX$ formalism [20], which is an extension of the STU formalism of Peskin and Takeuchi [9].

A. $STUVWX$ formalism

The STU formalism of Peskin and Takeuchi provides an elegant means of parametrizing oblique corrections due to new physics in electroweak phenomenology. This formalism allows one to write a wide range of observables as a standard model prediction plus some linear combination of the three oblique parameters S, T, and U. The STU parametrization is based on the assumption that new physics contributions to gauge-boson self-energies are linear functions of q^2, i.e., of the form $\delta\Pi(q^2) = A + Bq^2$.

However, in the present analysis the total effective self-energies have higher-order q^2 terms as well, and the STU formulation is therefore insufficient. For the case in which the $\delta\Pi(q^2)$ are not linear functions of q^2 but are instead general functions of q^2, an extension of STU is required. Such an extension is presented in [20], where it is shown that in practice, all oblique effects can be parametrized in terms of the six parameters S, T, U, V, W, and X. These are defined in terms of the new-physics gauge-boson self-energies:

$$\alpha S = -4s_W^2c_W^2\delta\hat{\Pi}_{\gamma\gamma}(0) + \frac{4s_W^2c_W^2}{M_Z^2}[\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)] - 4(c_W^2 - s_W^2)s_Wc_W\delta\hat{\Pi}_{Z\gamma}(0)\ ,$$

$$\alpha T = \frac{\delta\Pi_{WW}(0)}{M_W^2} - \frac{\delta\Pi_{ZZ}(0)}{M_Z^2}\ ,$$

$$\alpha U = -4s_W^4\delta\hat{\Pi}_{\gamma\gamma}(0) + \frac{4s_W^2}{M_W^2}[\delta\Pi_{WW}(M_W^2) - \delta\Pi_{WW}(0)] - \frac{4s_W^2c_W^2}{M_Z^2}[\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)] - 8c_Ws_W^3\delta\hat{\Pi}_{Z\gamma}(0)\ , \quad (19)$$

$$\alpha V = \delta\Pi'_{ZZ}(M_Z^2) - \left[\frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}\right]\ ,$$

$$\alpha W = \delta\Pi'_{WW}(M_W^2) - \left[\frac{\delta\Pi_{WW}(M_W^2) - \delta\Pi_{WW}(0)}{M_W^2}\right]\ ,$$

$$\alpha X = -s_Wc_W[\delta\hat{\Pi}_{Z\gamma}(M_Z^2) - \delta\hat{\Pi}_{Z\gamma}(0)]\ ,$$

where $\delta\hat{\Pi}(q^2) \equiv \delta\Pi(q^2)/q^2$, and where $\delta\Pi'(q^2)$ denotes the ordinary derivative with respect to q^2.

As is explained in Refs. [9,20,24], oblique corrections are incorporated into Feynman rules for fermion–fermion–gauge-boson vertices in the following simple manner. At low energies, for the charged and neutral currents, we have

$$i\Lambda^\mu_{\rm CC}(q^2 \sim 0) = -i\frac{e}{s_W\sqrt{2}}\gamma^\mu\gamma_L\left(1 - \frac{\alpha S}{4(c_W^2 - s_W^2)} + \frac{c_W^2\alpha T}{2(c_W^2 - s_W^2)} + \frac{\alpha U}{8s_W^2}\right) , \tag{20}$$

$$i\Lambda^\mu_{\rm NC}(q^2 \sim 0) = -i\frac{e}{s_W c_W}(1 + \tfrac{1}{2}\alpha T)\gamma^\mu\left[T_{3f}\gamma_L - Q_f\left(s_W^2 + \frac{\alpha S}{4(c_W^2 - s_W^2)} - \frac{c_W^2 s_W^2\alpha T}{c_W^2 - s_W^2}\right)\right] . \tag{21}$$

On the other hand, at the W and Z poles, respectively, we have

$$i\Lambda^\mu_{\rm CC}(M_W^2) = -i\frac{e}{s_W\sqrt{2}}\gamma^\mu\gamma_L\left(1 - \frac{\alpha S}{4(c_W^2 - s_W^2)} + \frac{c_W^2\alpha T}{2(c_W^2 - s_W^2)} + \frac{\alpha U}{8s_W^2} + \frac{1}{2}\alpha W\right) , \tag{22}$$

$$i\Lambda^\mu_{\rm NC}(M_Z^2) = -i\frac{e}{s_W c_W}\left(1 + \frac{1}{2}\alpha T + \frac{1}{2}\alpha V\right)\gamma^\mu\left[T_{3f}\gamma_L - Q_f\left(s_W^2 + \frac{\alpha S}{4(c_W^2 - s_W^2)} - \frac{c_W^2 s_W^2\alpha T}{c_W^2 - s_W^2} + \alpha X\right)\right] . \tag{23}$$

From the above Feynman rules, we can readily infer expressions for various electroweak observables in terms of S–X. For example, from Eq. (21), we see that the effective weak mixing angle measured in low-energy neutral current processes is given by

$$(s_W^2)_{\rm eff}(0) = (s_W^2)^{\rm SM}_{\rm eff}(0) + \frac{\alpha S}{4(c_W^2 - s_W^2)} - \frac{c_W^2 s_W^2\alpha T}{c_W^2 - s_W^2} , \tag{24}$$

where $(s_W^2)^{\rm SM}_{\rm eff}(0)$ is the SM prediction.

As for Z-pole observables, using Eq. (23) one obtains the tree-level Z-decay width:

$$\Gamma_{Zf} = \frac{M_Z}{24\pi}\frac{e^2}{c_W^2 s_W^2}(1 + \alpha T + \alpha V)[(T_{3f} - Q_f(s_W^2)_{\rm eff}(M_Z^2))^2 + (Q_f(s_W^2)_{\rm eff}(M_Z^2))^2] , \tag{25}$$

where

$$(s_W^2)_{\rm eff}(M_Z^2) = (s_W^2)^{\rm SM}_{\rm eff}(M_Z^2) + \frac{\alpha S}{4(c_W^2 - s_W^2)} - \frac{c_W^2 s_W^2\alpha T}{c_W^2 - s_W^2} + \alpha X . \tag{26}$$

From the above expressions, one can readily infer the correct way to incorporate oblique corrections into formulas for Γ_{Zf}, the end result consisting of a SM prediction plus a linear combination of S–X.

Similarly, using Eq. (22), one sees that the partial width for the decay $W \to l\nu_l$ is given by

$$\Gamma_W = (\Gamma_W)^{\rm SM}\left(1 - \frac{\alpha S}{2(c_W^2 - s_W^2)} + \frac{c_W^2\alpha T}{(c_W^2 - s_W^2)} + \frac{\alpha U}{4s_W^2} + \alpha W\right) . \tag{27}$$

Finally, as is shown in Refs. [9,20,24], the oblique-corrected expression for W mass is

$$M_W^2 = (M_W^2)^{\rm SM}\left[1 - \frac{\alpha S}{2(c_W^2 - s_W^2)} + \frac{c_W^2\alpha T}{c_W^2 - s_W^2} + \frac{\alpha U}{4s_W^2}\right] . \tag{28}$$

The above review of the $STUVWX$ formalism illustrates how one can express the new-physics oblique corrections to a wide variety of electroweak observables in terms of the six parameters S–X. In the present analysis of anomalous TGV's, it proves convenient to use this formalism. Thus, in the next subsection, one of our goals is to reexpress the effects of the anomalous TGV's [given in Eq. (13)] as oblique corrections. This is possible via an application of the classical equations of motion of the standard model, which can be used to transform the new terms in the effective Lagrangian.

B. Using equations of motion to transform the effective Lagrangian

As was mentioned earlier, the analysis presented in Sec. III seems somewhat cluttered, since $\mathcal{L}_{\rm grown}$ entails 29 types of terms, including both oblique corrections and vertex corrections up to dimension eight. To overcome this difficulty, we exploit the freedom to transform interactions using the Euler-Lagrange equations of motion derived from $\mathcal{L}_{\rm SM}(\tilde{e}_i)$,

$$\tilde{j}^\nu = -\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) ,$$

$$\tilde{N}^\nu = -\partial_\mu(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \tilde{m}_Z^2 Z^\nu , \quad (29)$$

$$\tilde{J}^\nu = -\partial_\mu(\partial^\mu W^\nu - \partial^\nu W^\mu) - \tilde{m}_W^2 W^\nu .$$

We also make liberal use of the freedom to integrate by parts.

In order to make use of the $STUVWX$ formalism, we choose to transform the grown vertex corrections into physically equivalent gauge-boson propagator corrections. We display the transformations that we use in Table III. The operators in the right column are obtained from those in the left column using Eqs. (29). Note that an operator in the right column of this table is meant to be equivalent to the operator on the left to within (i) total divergences, and (ii) terms (such as those whose Feynman rules are proportional to $q^\mu q^\nu$) which do not contribute significantly to the well-measured physical processes we later consider.

C. Final form of the "grown" effective Lagrangian

With the help of the transformations in Table III, $\mathcal{L}_{\text{grown}}$ can be recast as

$$\mathcal{L}'_{\text{grown}} = \mathcal{L}'_{STUVWX} + \mathcal{L}_{\text{nonuniv}} , \quad (30)$$

where $\mathcal{L}_{\text{nonuniv}}$ contains the m_t-dependent effects, unchanged from Eq. (13), and where

$$\frac{1}{y}\mathcal{L}'_{STUVWX} = \frac{A_1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{A_2}{4}F_{\mu\nu}\frac{\Box}{M_Z^2}F^{\mu\nu} + M_W^2 w W_\mu^* W^\mu + \frac{B_1}{2}W_{\mu\nu}^* W^{\mu\nu} - \frac{B_2}{2}W_{\mu\nu}^*\frac{\Box}{M_W^2}W^{\mu\nu}$$
$$+\frac{M_Z^2}{2}zZ_\mu Z^\mu + \frac{C_1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{C_2}{4}Z_{\mu\nu}\frac{\Box}{M_Z^2}Z^{\mu\nu} + \frac{G_1}{2}F_{\mu\nu}Z^{\mu\nu} - \frac{G_2}{2}F_{\mu\nu}\frac{\Box}{M_Z^2}Z^{\mu\nu} \quad (31)$$

with A_1, etc., defined in Table IV. Note that the effects of TGV's have been reduced from 29 types of operators in $\mathcal{L}_{\text{grown}}$ to 10, a considerable simplification. Part of this simplification consists of the remarkable cancellation of all of the dimension-8 operators that had appeared in intermediate steps. Also, for all practical purposes the A_2 operator in the above equation does not contribute to any observables. On the one hand, at low energies its effects are negligible due to the explicit momentum dependence. On the other hand, at the Z pole the SM photon-exchange diagram is already suppressed relative to Z exchange, so this operator, which represents a correction to the contribution from photon exchange, can be ignored.

The effective self-energies corresponding to the interactions given in Eq. (31) are

$$\frac{1}{y}\delta\Pi_{\gamma\gamma}^{\text{tot}}(q^2) = A_1 q^2 + A_2\frac{q^4}{M_Z^2} ,$$
$$\frac{1}{y}\delta\Pi_{Z\gamma}^{\text{tot}}(q^2) = G_1 q^2 + G_2\frac{q^4}{M_Z^2} ,$$
$$\frac{1}{y}\delta\Pi_{WW}^{\text{tot}}(q^2) = M_W^2 w + B_1 q^2 + B_2\frac{q^4}{M_W^2} ,$$
$$\frac{1}{y}\delta\Pi_{ZZ}^{\text{tot}}(q^2) = M_Z^2 z + C_1 q^2 + C_2\frac{q^4}{M_Z^2} , \quad (32)$$

where the $\delta\Pi^{\text{tot}}$ designate the total effective oblique effects due to the anomalous TGV loops.

We have thus succeeded in transforming all the universal corrections into the form of gauge-boson self-energies.

TABLE III. Operator transformations used to simplify the grown Lagrangian. These transformations were derived using the SM equations of motion [Eq. (29)].

Original operator	Transformed version of operator
$j_\mu \Box^n A^\mu$	$\frac{1}{2}F_{\mu\nu}\Box^n F^{\mu\nu}$
$N_\mu \Box A^\mu$	$\frac{1}{2}F_{\mu\nu}\Box Z^{\mu\nu} + \frac{1}{2}M_Z^2 F_{\mu\nu}Z^{\mu\nu}$
$N_\mu \Box^2 A^\mu$	$\frac{1}{2}F_{\mu\nu}\Box^2 Z^{\mu\nu} + \frac{1}{2}M_Z^2 F_{\mu\nu}\Box Z^{\mu\nu}$
$j_\mu \Box^n Z^\mu$	$\frac{1}{2}F_{\mu\nu}\Box^n Z^{\mu\nu}$
$N_\mu \Box Z^\mu$	$\frac{1}{2}Z_{\mu\nu}\Box Z^{\mu\nu} + \frac{1}{2}M_Z^2 Z_{\mu\nu}Z^{\mu\nu}$
$N_\mu \Box^2 Z^\mu$	$\frac{1}{2}Z_{\mu\nu}\Box^2 Z^{\mu\nu} + \frac{1}{2}M_Z^2 Z_{\mu\nu}\Box Z^{\mu\nu}$
$J_\mu^* W^\mu$ + H.c.	$W_{\mu\nu}^* W^{\mu\nu} - 2M_W^2 W_\mu^* W^\mu$
$J_\mu^* \Box W^\mu$ + H.c.	$W_{\mu\nu}^* \Box W^{\mu\nu} + M_W^2 W_{\mu\nu}^* W^{\mu\nu}$
$J_\mu^* \Box^2 W^\mu$ + H.c.	$W_{\mu\nu}^* \Box^2 W^{\mu\nu} + M_W^2 W_{\mu\nu}^* \Box W^{\mu\nu}$

TABLE IV. Definitions of parameters A_1, A_2, B_1, etc. appearing in final form of "grown" effective Lagrangian.

Parameter from Eq. (31)	Definition
A_1	α_γ
A_2	$\beta_\gamma + 2s_w p_{\text{em}}^{(2)}$
B_1	$\alpha w + 2p_{\text{CC}}^{(0)} - 2p_{\text{CC}}^{(2)}$
B_2	$\beta w + 2p_{\text{CC}}^{(2)} - 2p_{\text{CC}}^{(4)}$
C_1	$\alpha_Z - 2c_w p_{\text{NC}}^{(2)}$
C_2	$\beta_Z + 2c_w p_{\text{NC}}^{(2)} - 2c_w p_{\text{NC}}^{(4)}$
G_1	$\alpha_{Z\gamma} - c_w p_{\text{em}}^{(2)}$
G_2	$\beta_{Z\gamma} + s_w p_{\text{NC}}^{(2)} + c_w p_{\text{em}}^{(2)} - c_w p_{\text{em}}^{(4)}$
w	$\omega_W - 2p_{\text{CC}}$
z	ω_Z

Importantly, our procedure hinges upon the fact that the grown vertex corrections can be expressed as the interaction of a gauge boson with some linear combination of total standard model currents. Except for the m_t-dependent terms of $\mathcal{L}_{\rm nonuniv}$, this is a property shared by all vertex corrections involving TGV's be they standard or anomalous.

V. CONSTRAINTS ON THE ANOMALOUS TGV COUPLINGS

Having now transformed the effects of the anomalous TGV's into gauge-boson self-energies, we can directly apply the $STUVWX$ formalism to organize our calculation. Using Tables I, II, and IV, as well as Eqs. (12), (19), and (32), one can express observables in terms of the anomalous TGV coefficients, which will then allow a fit to precision data and the determination of phenomenological constraints.

A comprehensive list of expressions for the electroweak observables that we include in our analysis is given in Table V. These expressions consist of a radiatively corrected standard model prediction plus a linear combination of the six parameters S, T, U, V, W, and X. Γ_Z and $\Gamma_{b\bar{b}}$ are the total width of the Z and its partial width into $b\bar{b}$, respectively; $A_{\rm FB}(f)$ is the forward-backward asymmetry for $e^+e^- \to f\bar{f}$; $A_{\rm pol}(\tau)$, or P_τ, is the polarization asymmetry defined by $A_{\rm pol}(\tau) = (\sigma_R - \sigma_L)/(\sigma_R + \sigma_L)$, where $\sigma_{L,R}$ is the cross section for a correspondingly polarized τ lepton; $A_e(P_\tau)$ is the joint forward-backward–left-right asymmetry as normalized in Ref. [25]; and A_{LR} is the polarization asymmetry which has been measured by the SLD Collaboration at the SLAC Linear Collider (SLC) [26]. The low-energy observables g_L^2 and g_R^2 are measured in deep inelastic νN scattering, g_V^e and g_A^e are measured in $\nu e \to \nu e$ scattering, and $Q_W(\mathrm{Cs})$ is the weak charge measured in atomic parity violation in cesium. The expressions for the low-energy observables are derived in Refs. [9,24].

For the specific case of asymmetries in the decay $Z \to b\bar{b}$, the effects of m_t-dependent contributions must also be computed. Since these are not universal, they cannot be parametrized solely in terms of the variables S through X. Their effects on the $Z \to b\bar{b}$ asymmetries can be included by replacing X by $\hat{X}$, where

$$\hat{X} = X + \frac{c_W^2}{8\pi}|V_{tb}|^2 \left(\frac{m_t^2}{M_W^2}\right) \left[3\Delta g_{1Z} + \frac{1}{2c_W^2}\Delta\kappa_Z\right] \times \ln\left(\frac{\mu'^2}{\mu^2}\right) . \tag{33}$$

Similarly, the decay $Z \to b\bar{b}$ gets an additional contribution due to the m_t dependence of the TGV-induced Lagrangian. It may be incorporated by replacing X by $\hat{X}$, as defined in Eq. (33), in addition to making the replacement of V by

$$\hat{V} = V - \frac{c_W^2}{4\pi s_W^2}|V_{tb}|^2 \left(\frac{m_t^2}{M_W^2}\right) \times \left[3\Delta g_{1Z} + \frac{1}{2c_W^2}\Delta\kappa_Z\right] \ln\left(\frac{\mu'^2}{\mu^2}\right) . \tag{34}$$

$\hat{V}$ and $\hat{X}$ appear appropriately in the first two observables given in Table V.

There are several features in Table V worth pointing out. First, only the two parameters S and T contribute to the observables for which $q^2 \sim 0$. The parameter U appears only in M_W and Γ_W. Given the present uncertainty in Γ_W, the limit on U comes from the M_W measurement. The parameter W is weakly bounded, as it contributes only to Γ_W which is at present poorly measured. In addition to S and T, observables on the Z^0 resonance are also sensitive to V and X, which are expressly defined at $q^2 = M_Z^2$. Observables that are not explicitly given in Table V can be obtained using the given expressions. In particular the parameter R is defined as $R = \Gamma_{\rm had}/\Gamma_{l\bar{l}}$,

TABLE V. Summary of the dependence of electroweak observables on S, T, U, V, W, and X. In preparing this table we used the numerical values $\alpha(M_Z^2) = \frac{1}{128}$ and $s_W^2 = 0.23$.

Expressions for observables
$\Gamma_Z = (\Gamma_Z)_{\rm SM} - 0.009\,61S + 0.026\,3T + 0.016\,6V - 0.017\,0X + 0.002\,95\hat{V} - 0.003\,69\hat{X}$ (GeV)
$\Gamma_{b\bar{b}} = (\Gamma_{b\bar{b}})_{\rm SM} - 0.001\,71S + 0.004\,16T + 0.002\,95\hat{V} - 0.003\,69\hat{X}$ (GeV)
$A_{\rm FB}(\mu) = [A_{\rm FB}(\mu)]_{\rm SM} - 0.006\,77S + 0.004\,79T - 0.014\,6X$
$A_{\rm pol}(\tau) = [A_{\rm pol}(\tau)]_{\rm SM} - 0.028\,4S + 0.020\,1T - 0.061\,3X$
$A_e(P_\tau) = [A_e(P_\tau)]_{\rm SM} - 0.028\,4S + 0.020\,1T - 0.061\,3X$
$A_{\rm FB}(b) = [A_{\rm FB}(b)]_{\rm SM} - 0.018\,8S + 0.009\,84T - 0.040\,6\hat{X}$
$A_{\rm FB}(c) = [A_{\rm FB}(c)]_{\rm SM} - 0.014\,7S + 0.010\,4T - 0.031\,75X$
$A_{\rm LR} = (A_{\rm LR})_{\rm SM} - 0.028\,4S + 0.020\,1T - 0.061\,3X$
$M_W^2 = (M_W^2)_{\rm SM}(1 - 0.007\,23S + 0.011\,1T + 0.008\,49U)$
$\Gamma_W = (\Gamma_W)_{\rm SM}(1 - 0.007\,23S + 0.011\,1T + 0.008\,49I + 0.007\,81W)$
$g_L^2 = (g_L^2)_{\rm SM} - 0.002\,69S + 0.006\,63T$
$g_R^2 = (g_R^2)_{\rm SM} + 0.000\,937S - 0.000\,192T$
$g_V^e(\nu e \to \nu e) = (g_V^e)_{\rm SM} + 0.00723S - 0.005\,41T$
$g_A^e(\nu e \to \nu e) = (g_A^e)_{\rm SM} - 0.003\,95T$
$Q_W({}^{133}_{55}\mathrm{Cs}) = Q_W(\mathrm{Cs})\mathrm{SM} - 0.795S - 0.011\,6T$

TABLE VI. One-loop results for the induced parameters S, T, U, V, W, and X, defined at $\mu = 100$ GeV, in terms of the various TGV couplings defined at $M = 1$ TeV. We have used $\alpha(M_Z^2) = \frac{1}{128}$ and $s_W^2 = 0.23$.

Parameter	One-loop result
S	$2.63\Delta g_{1Z} - 2.98\Delta\kappa_\gamma + 2.38\Delta\kappa_Z + 5.97\lambda_\gamma - 4.50\lambda_Z$
T	$-1.82\Delta g_{1Z} + 0.550\Delta\kappa_\gamma + 5.83\Delta\kappa_Z$
U	$2.42\Delta g_{1Z} - 0.908\Delta\kappa_\gamma - 1.91\Delta\kappa_Z + 2.04\lambda_\gamma - 2.04\lambda_Z$
V	$0.183\Delta\kappa_Z$
W	$0.202\Delta g_{1Z}$
X	$-0.0213\Delta\kappa_\gamma - 0.0611\Delta\kappa_Z$
$\hat{V}$	$0.183\Delta\kappa_Z - (3.68\Delta g_{1Z} + 0.797\Delta\kappa_Z)(m_t^2/M_W^2)$
$\hat{X}$	$-0.0213\Delta\kappa_\gamma - 0.0611\Delta\kappa_Z + (0.423\Delta g_{1Z} + 0.0916\Delta\kappa_Z)(m_t^2/M_W^2)$

and $\sigma_p^h = 12\pi\Gamma_{e\bar{e}}\Gamma_{\rm had}/M_Z^2\Gamma_Z^2$ is the hadronic cross section at the Z pole.

Finally, in Table VI we present the expressions for observables in terms of the anomalous TGV coefficients. As stated previously, these expressions are derived using Tables I, II, IV, and V as well as Eqs. (32), (12), and (19). In regards to the numerical value of y [defined in Eq. (12)], we take $\mu' = M = 1$ TeV and $\mu = 100$ GeV. This signifies that we are running the operators from their initial conditions at scale M down to the electroweak scale μ.

To obtain constraints on the anomalous TGV's, we perform a global fit. The required expressions are obtained by substituting the results of Table VI into those of Table V. The TGV dependence of the nonuniversal, m_t-dependent terms is given by Eqs. (33) and (34).

The experimental values and standard model predictions of the observables used in our fit are given in Table VII. The standard model values have been calculated with $m_t = 150$ GeV and $M_H = 300$ GeV. The LEP observables in Table VII were chosen as they are closest to what is actually measured and are relatively weakly correlated. In our analysis we include the combined LEP values for the correlations [27].

As mentioned in the Introduction, the same new physics responsible for the anomalous TGV's will typ-

TABLE VII. Experimental values for electroweak observables included in global fit. The Z^0 measurements are the preliminary 1992 LEP results taken from Ref. [28]. The couplings extracted from neutrino scattering data are the current world averages taken from Ref. [25]. The standard model values are for $m_t = 150$ GeV and $M_H = 300$ GeV. We have not shown theoretical errors in the standard model values due to uncertainties in the radiative corrections, Δr, and due to errors in M_Z, as they are in general overwhelmed by the experimental errors. The exception is the error due to uncertainty in α_s, shown in square brackets. We include this error in quadrature in our fits. The error in square brackets for Q_W(Cs) reflects the theoretical uncertainty in the atomic wave functions [33] and is also included in quadrature with experimental error.

Quantity	Experimental value	Standard model prediction
M_Z (GeV)	91.187 ± 0.007 [28]	Input
Γ_Z (GeV)	2.488 ± 0.007 [28]	$2.490[\pm0.006]$
$R = \Gamma_{\rm had}/\Gamma_{l\bar{l}}$	20.830 ± 0.056 [28]	$20.78[\pm0.07]$
σ_p^h (nb)	41.45 ± 0.17 [28]	$41.42[\pm0.06]$
$\Gamma_{b\bar{b}}$ (MeV)	383 ± 6 [28]	$375.9[\pm1.3]$
$A_{\rm FB}(\mu)$	0.0165 ± 0.0021 [28]	0.0141
$A_{\rm pol}(\tau)$	0.142 ± 0.017 [28]	0.137
$A_e(P_\tau)$	$0.130\pm$ 0.025 [28]	0.137
$A_{\rm FB}(b)$	0.0984 ± 0.0086 [28]	0.096
$A_{\rm FB}(c)$	0.090 ± 0.019 [28]	0.068
A_{LR}	0.100 ± 0.044 [26]	0.137
M_W (GeV)	79.91 ± 0.39 [29]	80.18
M_W/M_Z	0.8798 ± 0.0028 [30]	0.8793
Γ_W (GeV)	2.12 ± 0.11 [31]	2.082
g_L^2	0.3003 ± 0.0039 [25]	0.3021
g_R^2	0.0323 ± 0.0033 [25]	0.0302
g_A^e	-0.508 ± 0.015 [25]	-0.506
g_V^e	-0.035 ± 0.017 [25]	-0.037
Q_W(Cs)	$-71.04 \pm 1.58 \pm [0.88]$ [32]	-73.20

ically also contribute directly to the various observables used in the fit. In our analysis we assume no cancellations between these "direct" contributions and those due to the TGV's. Although this may seem like a very strong assumption, we will see that in any case the constraints obtained are rather weak.

We first consider the case in which only one of the TGV couplings, $\Delta\kappa_V$, λ_V, and Δg_{1Z}, is nonzero at the scale M. In this case strong bounds on this parameter may be obtained, since there is no possibility of cancellations. Constraining one parameter at a time we obtain the following values with 1σ errors:

$$\begin{aligned} \Delta g_{1Z} &= -0.033 \pm 0.031 \ , \\ \Delta\kappa_\gamma &= 0.056 \pm 0.056 \ , \\ \Delta\kappa_Z &= -0.0019 \pm 0.044 \ , \\ \lambda_\gamma &= -0.036 \pm 0.034 \ , \\ \lambda_Z &= 0.049 \pm 0.045 \ . \end{aligned} \tag{35}$$

If taken at face value, these limits would imply that anomalous TGV's are too small to be seen at LEP 200.

Of course, although the bounds obtained in this way are the tightest bounds that are possible, they are somewhat artificial. After all, real underlying physics would produce more than just a single TGV. If we fit for all five anomalous TGV's simultaneously, the constraints virtually disappear, due to the possibility of cancellations. Several authors have fitted STU-corrected expressions for observables to electroweak data, and have concluded that the upper limit on these parameters is $\sim 0.1-1$. It is clear that if one were to do fit a using the $STUVWX$-corrected expressions displayed in Table V, then the limits on the six parameters would be looser still. We find that, at best, we can only conclude that the anomalous TGV couplings are less than order 1. TGV's of this size would, of course, be observable at LEP 200.

The bounds given in Eq. (35) are nevertheless interesting. These values can be interpreted as an indication of the sensitivity of the global fit of electroweak data to specific anomalous couplings. Once all of the couplings are allowed to vary simultaneously, no significant bound remains. This indicates that, in that part of the allowed region for which the TGV couplings are large, cancellations occur among the contribution of the various anomalous couplings to low-energy observables. Equation (35) allows one to gain a feel for the size of cancellations that would be required to account for the low-energy data, should an anomalous TGV at the 10% level ever be discovered at LEP 200.

Our results in this regard agree with those of Ref. [16], who similarly obtain no significant bounds for these couplings, subject to the somewhat stronger assumption that the effective theory be a linear realization of the electroweak gauge group. As is discussed in this reference, such a linear realization implies relationships among the various TGV parameters, and so would be expected to lead to tighter constraints than those obtained here. We here confirm this result within a more general phenomenological analysis, without theoretical biases.

VI. CONCLUSIONS

We have computed the bounds that may be obtained for CP-preserving anomalous TGV's from current low-energy phenomenology. These bounds arise due to the influence of these interactions, through loops, on well-measured electroweak observables. We compute this influence using an effective-Lagrangian description, in which TGV interactions are imagined to have been generated just below the scale for new physics, $\mu = M$, after all of the hitherto undiscovered heavy particles have been integrated out. Running this Lagrangian, using the $\overline{\rm MS}$ renormalization scheme, down to the weak scale, $\mu \simeq 100$ GeV, then generates a collection of secondary effective interactions. Unlike the TGV's, these new interactions contribute directly to low-energy observables, and so their couplings may be bounded by comparison to the data. We obtain limits on TGV's by requiring that the contributions to these couplings due to their RG mixing with the TGV's satisfy these experimental constraints. In so doing we are tacitly assuming that no cancellations arise between the induced values for these couplings, and their initial conditions at the new-physics scale, $\mu = M$.

In this analysis, upon calculating the loop diagrams involving the anomalous TGV's, one obtains two classes of effective interactions: corrections to fermion–fermion–gauge-boson vertices and corrections to gauge-boson propagators. We show how equations of motion may be used to transform the vertex corrections into physically equivalent propagator corrections. This allows us to directly apply the existing formalism for oblique corrections. We note that the STU formalism of Peskin and Takeuchi [9] is not sufficient for this analysis. The Peskin and Takeuchi parametrization is appropriate for the case in which new-physics self-energies are linear functions of q^2, i.e., $\delta\Pi(q^2) = A + Bq^2$. However, our analysis produces self-energies which include a q^4 term. This renders STU insufficient for present purposes, and an extension of the usual formalism [20], involving three new parameters V, W, and X must be applied.

We have found that the limits obtained cannot in themselves rule out TGV's that are large enough to be detectable once LEP runs at the threshold for $W^\pm$ pair production. Couplings of this size *would* have been ruled out if we had considered each TGV one at a time, with all of the others constrained to be zero. In this case the data would constrain the various TGV couplings to be 10% or less. This shows how (fairly mild) cancellations among the various TGV's would be required in low-energy observables should these TGV's be directly detected at LEP 200. Since the underlying theory of new physics is unlikely to produce only one TGV at a time, this also shows how misleading can be the practice of working with TGV's one by one. A simultaneous fit of all of the TGV's to the data does not yield useful bounds.

An interesting feature of the expressions in Table VI is the occurrence of some spectacular cancellations among anomalous TGV's. Although S, T, and U contain all of the combinations of the anomalous TGV's, such is not the case for V, W, and X. In particular, the λ_V couplings do not contribute to these parameters at all, V depends only on $\Delta\kappa_Z$, W only on Δg_{1Z}, and X depends only on $\Delta\kappa_V$. This implies that each of the TGV couplings tends to contribute only to particular kinds of observables at $q^2 = M_Z^2$ and M_W^2.

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