

## QCD accurately predicts the induced pseudoscalar coupling constant

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(Received 24 March 1994)

Using chiral Ward identities of QCD, we derive a relation for the induced pseudoscalar coupling constant which is accurate within a few percent,  $g_P = 8.44 \pm 0.23$ .

PACS number(s): 14.20.Dh, 11.30.Rd, 12.38.Aw

The structure of the nucleon as probed by weak charged currents is encoded in two form factors: the axial and the induced pseudoscalar ones. While much attention has been focused on the first, the latter is generally believed to be understood well in terms of pion pole dominance as indicated from ordinary muon capture experiments,  $\mu^- + p \rightarrow \nu_\mu + n$  (see, e.g., Refs. [1, 2]). However, it now seems feasible to measure the induced pseudoscalar coupling constant (the form factor evaluated at  $t = -0.88M_\mu^2$ ) within a few percent accuracy via new techniques which allow one to minimize the uncertainty in the neutron detection [3]. We will demonstrate here that one is also able to calculate this fundamental quantity within a few percent accuracy by making use of the chiral Ward identities of QCD.

To be specific, consider the matrix element of the isovector axial-vector quark current  $A_\mu^\alpha = q\gamma_\mu\gamma_5(\tau^\alpha/2)q$  between nucleon states [4]:

$$\langle N(p') | A_\mu^\alpha | N(p) \rangle = \bar{u}(p') \left[ \gamma_\mu G_A(t) + \frac{(p' - p)_\mu}{2m} G_P(t) \right] \times \gamma_5 \frac{\tau^\alpha}{2} u(p), \quad (1)$$

with  $t = (p' - p)^2$  the invariant momentum transfer squared and  $m$  the nucleon mass. The form of Eq. (1) follows from Lorentz invariance, isospin conservation, and the discrete symmetries  $C$ ,  $P$ , and  $T$ .  $G_A(t)$  is called the nucleon axial form factor and  $G_P(t)$  the induced pseudoscalar form factor. Here, we are interested in the pseudoscalar coupling constant

$$g_P = \frac{M_\mu}{2m} G_P(t = -0.88M_\mu^2), \quad (2)$$

as can be measured in ordinary muon capture. Our aim is to give an accurate prediction for  $g_P$  in terms of well-known physical parameters. For doing that, we exploit the chiral Ward identity of QCD,

$$\partial^\mu \left[ \bar{q} \gamma_\mu \gamma_5 \frac{\tau^\alpha}{2} q \right] = \hat{m} \bar{q} i \gamma_5 \tau^\alpha q, \quad (3)$$

with  $\hat{m}$  the average light quark mass [5]. Sandwiching Eq. (3) between nucleon states, one obtains [6]

$$mG_A(t) + \frac{t}{4m} G_P(t) = 2\hat{m} B m^0 g_A^0 \frac{1 + h(t)}{M_\pi^2 - t}, \quad (4)$$

where the superscript zero denotes quantities in the chiral limit,  $\mathcal{Q} = \mathcal{Q}^0[1 + O(\hat{m})]$ . Here,  $B = -\langle 0 | \bar{u}u | 0 \rangle / F_\pi^2$  is the order parameter of the spontaneous chiral symmetry breaking and  $F_\pi$  the weak pion decay constant determined from the decay  $\pi^+ \rightarrow \mu^+ + \nu_\mu$ . The pion pole in Eq. (4) originates from the direct coupling of the pseudoscalar density to the pion,  $\langle 0 | \bar{q} i \gamma_5 \tau^\alpha q | \pi^b \rangle = \delta^{ab} G_\pi$  [7]. The residue at the pion pole  $t = M_\pi^2$  is [6, 7]

$$\hat{m} G_\pi g_{\pi N} = g_{\pi N} F_\pi M_\pi^2, \quad (5)$$

with  $g_{\pi N}$  the strong pion-nucleon coupling constant. To go further, we make use of heavy baryon chiral perturbation theory (HBCHPT) as formulated in Ref. [8] and detailed Ref. [9]. To order  $q^4$ , we have

$$G_A(t) = g_A \left( 1 + \frac{r_A^2}{6} t \right), \quad (6)$$

$$h(t) = \text{const} - \frac{2b'_{11}}{F_\pi^2} t, \quad (7)$$

with  $g_A = G_A(0)$  the axial-vector coupling constant,  $r_A^2$  the mean square axial radius of the nucleon, and  $b'_{11}$  a low-energy constant [10]. The reason for the linear dependence in Eqs. (6) and (7) is the following. The corresponding form factors  $G_A(t)$  and  $h(t)$  have a cut starting

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at  $t = (3M_\pi)^2$  which in the chiral expansion first shows up at two-loop order  $O(q^5)$  ( $q$  denotes a small external momentum or a meson mass). Therefore, the contribution to order  $q^4$  must be polynomial in  $t$ . Furthermore, from chiral counting it follows that the  $t^2$  terms are related to order  $q^5$  of the full matrix elements. Putting the pieces together, we arrive at

$$m g_A + m g_A \frac{r_A^2}{6} t + \frac{t}{4m} G_P(t) = \frac{g_{\pi N} F_\pi}{M_\pi^2 - t} t + g_{\pi N} F_\pi + \frac{2b'_{11} M_\pi^2 g_{\pi N}}{F_\pi}, \quad (8)$$

where we have used  $2\hat{m} B g_A^0 m^0 = M_\pi^2 [g_{\pi N} F_\pi + O(M_\pi^2)]$ . At  $t = 0$ , Eq. (8) reduces to the Goldberger-Treiman discrepancy [6, 9]

$$g_A m = g_{\pi N} F_\pi \left( 1 + \frac{2b'_{11}}{F_\pi} M_\pi^2 \right). \quad (9)$$

Equation (9) clarifies the meaning of the low-energy constant  $b'_{11}$ . Finally,  $G_P(t)$  can be isolated from Eq. (8),

$$G_P(t) = \frac{4m g_{\pi N} F_\pi}{M_\pi^2 - t} - \frac{2}{3} g_A m^2 r_A^2 + O(t, M_\pi^2). \quad (10)$$

A few remarks are in order. First, notice that only physical and well-determined parameters enter in Eq. (10). Second, while the first term on the right-hand side of Eq. (10) is of order  $q^{-2}$ , the second one is  $O(q^0)$  and the corrections not calculated are of order  $q^2$ . For  $g_P$ , this leads to

$$g_P = \frac{2M_\mu g_{\pi N} F_\pi}{M_\pi^2 + 0.88M_\mu^2} - \frac{1}{3} g_A M_\mu m r_A^2. \quad (11)$$

Indeed, the relation Eq. (11) was derived a long time ago by Adler and Dothan [11] with the help of PCAC (partial conservation of axial-vector current) and by Wolfenstein [12] using a once-subtracted dispersion relation for the right-hand side of Eq. (4) (weak PCAC). It is gratifying that result of Refs. [11, 12] can be firmly based on the systematic chiral expansion of low-energy QCD Green functions. In chiral perturbation theory, one could in principle calculate the corrections to Eq. (11) by performing a two-loop calculation while in the methods of Adler and Dothan or Wolfenstein these either depend (completely) on the PCAC assumption or could only be estimated. It is important to realize the difference of the HBCHPT derivation from the one of Ref. [11]. In that paper, the pion field was expressed in terms of the divergence of the axial-vector current. In CHPT, the pion field (whose exact structure is unknown in QCD) is merely an integration variable and plays no role in any physical observable (i.e., quark current matrix elements). Of course, in certain cases these methods can lead to the same result, but this is not the case in general. As a particular example we quote the failure of the method of Ref. [11] to give the correct low-energy theorem for pion electroproduction at threshold [13]. Consequently, the procedure used here is fail-safe and only based on chiral

Ward identities. To stress it again, the main ingredient to arrive at Eq. (11) in HBCHPT is the linear  $t$  dependence in Eqs. (6), (7). Since we are interested here in a very small momentum transfer  $t = -0.88M_\mu^2 \simeq -0.5M_\pi^2$ , curvature terms of order  $t^2$  have to be negligible. If one uses for example the dipole parametrization for the axial form factor,  $G_A(t) = g_A (1 - t/M_A^2)^{-2}$ , the  $t^2$  term amounts to a 1.3% correction to the one linear in  $t$ . Consequently, our results can also be used in radiative muon capture off hydrogen where the four-momentum transfer varies as  $-M_\mu^2 \dots + M_\mu^2$ .

The masses  $m$ ,  $M_\mu$ , and  $M_\pi = M_{\pi^+}$  are accurately known and so are  $F_\pi = 92.5 \pm 0.2$  MeV and  $g_A = 1.2573 \pm 0.0028$  [14]. The situation concerning the strong pion-nucleon coupling constant is less favorable. The methodologically best determination based on dispersion theory gave  $g_{\pi N}^2/4\pi = 14.28 \pm 0.36$  [15]; more recent determinations seem to favor smaller values [16]. We use here  $g_{\pi N} = 13.31 \pm 0.34$  [17]. The most accurate determinations of  $r_A$  stem from (anti)neutrino-nucleon scattering, the world average being  $r_A = 0.65 \pm 0.03$  fm. This uncertainty plays, however, no role in the final result since the second term on the right-hand side of Eq. (11) is much smaller than the first one:

$$g_P = (8.89 \pm 0.23) - (0.45 \pm 0.04) = 8.44 \pm 0.23. \quad (12)$$

The uncertainties in Eq. (12) stem from the range of  $g_{\pi N}$  and from the one for  $r_A$  for the first and second terms, in order. For the final result on  $g_P$ , we have added these uncertainties in quadrature. A measurement with a 2% accuracy of  $g_P$  could therefore cleanly separate between the pion pole contribution and the improved CHPT result. This would mean a significant progress in our understanding of this fundamental low-energy parameter since the presently available determinations have too large error bars to disentangle these values (see, e.g., [1]). In fact, one might turn the argument around and eventually use a precise determination of  $g_P$  to get an additional determination of the strong pion-nucleon coupling constant which has been at the center of much controversy over the last years.

To summarize, we have shown that the chiral Ward identities allow one to predict the induced pseudoscalar coupling constant entirely in terms of well-determined physical parameters within a few percent accuracy. As already noted by Wolfenstein [12], an accurate empirical determination of this quantity therefore poses a stringent test of our understanding of the underlying dynamics which is believed to be realized in the effective low-energy field theory of QCD (i.e., chiral perturbation theory). It is also important to stress that in comparison to the original derivations in Refs. [11, 12], the uncertainties in many of the parameters entering Eq. (11) have decreased and an accurate theoretical prediction is of great importance for further experiments using ordinary or radiative muon capture [3, 18] to determine the induced pseudoscalar coupling constant.

We thank D. Taqqu for interesting us in this problem and Professor G. Höhler for instructive comments.

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