Test of factorization in Cabibbo-favored two-body hadronic decays of D mesons

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We propose a method that involves quantities independent of strong interaction phases to test the factorization model in Cabibbo-favored two-body decays of D mesons. The method tests if the factorization model correctly predicts the magnitudes of the isospin amplitudes. We have applied this method to the Cabibbo-favored decays $D \to \bar{K}\pi$, $\bar{K}\rho$, and $\bar{K}^*\pi$. The test, used as a diagnostic tool, allows us to pinpoint the isospin amplitudes for which the factorization model fails. We discuss the roles of annihilation terms and inelastic final state interactions in resolving the differences between the factorization model predictions and experiments.

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I. INTRODUCTION

With the availability of precise data [1] on some of the hadronic two-body and semileptonic decays of D mesons, we claim [2] it is now possible to test the factorization model purely from two-body hadronic decay measurements using the form factors extracted from the data on semileptonic decays [3]. The advantage of the tests we are proposing is that the quantities to be tested against experiment are independent of the strong interaction phases; in contrast, a comparison of two-body hadronic decays with semileptonic decays is often marred by strong interference effects [1,4,5]. A feature of our proposal is that it shifts the emphasis from the decay amplitudes for particular decay modes to the decay amplitudes in particular isospin states. Reference [1] has similarly discussed tests of factorization after the final state interaction phases are removed.

Precise semileptonic decay measurements have determined certain form factors rather well. For example, from the measurements of CLEO II [6], E-687 [7], and E-691 [8], one can extract [1]

$$f_+^{DK}(0) = 0.77 \pm 0.04 \ . \tag{1}$$

We remind the reader that $f_+(q^2) = F_0(q^2)$ of Ref. [9], a notation we use in this paper. For a comparison of this measurement with calculated values [10] of $f_+(0)$ we refer the reader to [1]. This experimental determination removes the dependence on theoretical models, at least for $f_+^{DK}(0)$. Experiments [1] are also narrowing down the freedom in $f_+^{D\pi}(0)$, a harder quantity to measure:

$$f_+^{D\pi}(0)/f_+^{DK}(0) = \begin{cases} 1.0 \pm^{0.6}_{0.3} \pm 0.1 & \text{(Mark III [11])} \ , \\ 1.29 \pm 0.21 \pm 0.11 & \text{(CLEO[12])} \ . \end{cases} \tag{2}$$

CLEO data, in particular, suggest that $f_+^{D\pi}(0)$ could be larger than $f_+^{DK}(0)$, contrary to the theoretical model calculation of Ref. [9] and many others [1,10]. There is, however, a calculation [13] based on heavy quark symmetry and chiral perturbation theory that yields $f_+^{D\pi}(0)/f_+^{DK}(0) = 1.18$, though this prediction is based on a monopole extrapolation from $q^2 = q^2_{\rm max}$ to $q^2 = 0$.

It has recently been inferred by Chau and Cheng [14] that $f_+^{D\pi}(0) \approx 0.83$ is needed to understand the ratio of the rates $\Gamma(D^+ \to \pi^0\pi^+)/\Gamma(D^+ \to \bar{K}^0\pi^+)$. Hence there is strong evidence that $f_+^{D\pi}(0) > f_+^{DK}(0)$. We show that this narrowing down of $f_+^{DK}(0)$ and $f_+^{D\pi}(0)$ helps us to test the factorization model in $D \to \bar{K}\pi$ decays. The tests for the Cabibbo-suppressed $D \to \pi\pi$ and $K\bar{K}$ decays have been discussed in Ref. [2].

Unfortunately, the same cannot be said about the form factors involved in $D \to K^*$ and $D \to \rho$ transitions. The determination of the form factors in $D \to K^*$ transitions is in a state of flux; the E-653 [15] and E-687 [16] results are significantly different from the older E-691 [17] determination. We refer the reader to Ref. [1] for a summary of the present status. As for the $D \to \rho$ transition form factors, measurements do not exist at the moment. As a consequence the test for $D \to \bar{K}\rho$ decays are based on theoretical arguments such as SU(3) symmetry and are not as independent of theoretical input as those involving $D \to \bar{K}\pi$ (and $D \to \pi\pi$ and $K\bar{K}$).

This paper is organized as follows. In Sec. II, we state the problem and define the quantities we propose to test against experiment for the Cabibbo-favored $D \to \bar{K}\pi, \bar{K}\rho$, and $\bar{K}^*\pi$ decays. We perform the test and discuss the missing elements in the factorization model where the tests fail. We summarize our findings in Sec. III.

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II. TESTS OF FACTORIZATION IN CABIBBO-FAVORED DECAYS

The test of the factorization model we are proposing is to compare quantities that are independent of the strong interaction phases with experiments, as we show below (see also Ref. [18]). Such quantities are the sum $B(D^0 \to K^-\pi^+) + B(D^0 \to \bar{K}^0\pi^0)$ and $B(D^+ \to \bar{K}^0\pi^+)$ for $D \to \bar{K}\pi$ and $B(D^0 \to K^-\rho^+) + B(D^0 \to \bar{K}^0\rho^0)$ and $B(D^+ \to \bar{K}^0\rho^+)$ for the $D \to \bar{K}\rho$ decays, etc. We compute these quantities in the factorization model using experimentally measured form factors whenever possible, and compare them with experiments. If there is disagreement we search for the missing physics.

A. $D \to \bar{K}\pi$ decays

In terms of isospin amplitudes and strong interaction phases,

$$\begin{aligned} A(D^0 \to K^-\pi^+) &= \frac{1}{\sqrt{3}}[A_{3/2}\exp(i\delta_{3/2}) \\ &\quad +\sqrt{2}A_{1/2}\exp(i\delta_{1/2})] , \\ A(D^0 \to \bar{K}^0\pi^0) &= \frac{1}{\sqrt{3}}[\sqrt{2}A_{3/2}\exp(i\delta_{3/2}) \\ &\quad -A_{1/2}\exp(i\delta_{1/2})] , \\ A(D^+ \to \bar{K}^0\pi^+) &= \sqrt{3}A_{3/2}\exp(i\delta_{3/2}) . \end{aligned} \tag{3}$$

The phase-independent quantities are

$$|A(D^0 \to K^-\pi^+)|^2 + |A(D^0 \to \bar{K}^0\pi^0)|^2 = |A_{1/2}|^2 + |A_{3/2}|^2 \tag{4}$$

and

$$|A(D^+ \to \bar{K}^0\pi^+)|^2 = 3|A_{3/2}|^2 . \tag{5}$$

These equations imply that the sum

$$\sum B(D^0 \to \bar{K}\pi) \equiv B(D^0 \to K^-\pi^+) + B(D^0 \to \bar{K}^0\pi^0) \tag{6}$$

and $B(D^+ \to \bar{K}^0\pi^+)$ are independent of the strong interaction phases. Hence, one might as well evaluate them *without* the phases in the factorization model and test them against experiments. We emphasize one other point: Since $A_{3/2}/A_{1/2} \approx 1/3$ [19], the sum in Eq. (4) is not very sensitive to changes in $A_{3/2}$, while Eq. (5) is.

In evaluating the branching ratios in the factorization model we use the following weak Hamiltonian for Cabibbo-favored decays:

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ud}V_{cs}^*[a_1(\bar{u}d)_H(\bar{s}c)_H + a_2(\bar{u}c)_H(\bar{s}d)_H] , \tag{7}$$

where V_{ud} and V_{cs} are the Cabibbo-Kobayashi-Maskawa (CKM) angles and the Dirac bilinears $(\bar{u}d)_H$, etc. stand for $(V-A)$ color singlets, the subscript H instructs us to treat these bilinears as interpolating fields for hadrons (no further Fierz reordering in color is needed). a_1 and a_2 are related to the Wilson coefficients c_+ and c_- through the equations

$$\begin{aligned} a_1 &= c_1 + \frac{1}{N}c_2, \quad a_2 = c_2 + \frac{1}{N}c_1, \\ c_\pm &= c_1 \pm c_2 , \end{aligned} \tag{8}$$

and

$$c_\pm(\mu) = \left[\frac{\alpha_s(m_W^2)}{\alpha_s(\mu^2)}\right]^{-6\gamma_\pm/(33-2n_f)} , \tag{9}$$

where $\gamma_- = -2\gamma_+ = 2$ and n_f, the number of "active" flavors, 3 for D decays. N is the number of colors. Phenomenological evidence [9] is that, for charm decays, the limit $N \to \infty$ works well though for obscure reasons. We treat a_1 and a_2 as phenomenological parameters.

In the factorization model without the annihilation term [we are omitting an overall factor of $(G_F/\sqrt{2})V_{ud}V_{cs}^*$], the $D \to \bar{K}\pi$ decay amplitudes are

$$\begin{aligned} A(D^0 \to K^-\pi^+) &= a_1 f_\pi F_0^{DK}(m_\pi^2)(m_D^2 - m_K^2) , \\ A(D^0 \to \bar{K}^0\pi^0) &= \frac{a_2}{\sqrt{2}} f_K F_0^{D\pi}(m_K^2)(m_D^2 - m_\pi^2) , \\ A(D^+ \to \bar{K}^0\pi^+) &= a_1 f_\pi F_0^{DK}(m_\pi^2)(m_D^2 - m_K^2) \\ &\quad + a_2 f_K F_0^{D\pi}(m_K^2)(m_D^2 - m_\pi^2) . \end{aligned} \tag{10}$$

We emphasize once more that factorization model would admit a factorized annihilation term which would have entered $A(D^0 \to K^-\pi^+)$ and $A(D^0 \to \bar{K}^0\pi^0)$.

As the sum $\sum B(D^0 \to \bar{K}\pi)$ is independent of the strong interaction phases, we calculate this sum from Eq. (10) with phases set to zero. After a straightforward calculation we find

$$\sum B(D^0 \to \bar{K}\pi) = (7.5 \pm 0.9)\% . \tag{11}$$

The numerical input in this calculation is taken from Refs. [2,20] and [21]:

$$\begin{aligned} &a_1 = 1.26 \pm 0.05 , \quad a_2 = -0.51 \pm 0.05 , \\ &F_0^{DK}(0) = 0.76 \pm 0.04 , \quad F_0^{D\pi}(0) = 0.83 \pm 0.08 , \\ &f_\pi = 132 \text{ MeV} , \quad f_K = 161 \text{ MeV} , \\ &f_\rho = (212 \pm 4) \text{ MeV} , \quad f_{K^*} = (221 \pm 15) \text{ MeV} , \\ &V_{ud} = 0.975 , \quad V_{cd} \approx -V_{us} = 0.220 , \\ &\tau_{D^0} = 4.15 \times 10^{-13} \text{ s} , \quad \tau_{D^+} = 10.57 \times 10^{-13} \text{ s} . \end{aligned} \tag{12}$$

The central values of a_1 and a_2 are taken from Ref. [20] as are also the errors in f_ρ and f_{K^*}. f_π and f_K are quoted without errors, as they are much smaller than the errors in other parameters. The reason for assigning only a 10% error to $F_0^{D\pi}(0)$, and a similar 10% error to a_2 and a less than 5% error to a_1 is given in Ref. [2] where it was shown that experiments, together with factorization, admit only a 15% error in the product $(a_1 + a_2)F_0^{D\pi}(0)$. Hence no single parameter of the three can have an error larger than 15%. This was the rationale in distributing the errors among a_1, a_2, and $F_0^{D\pi}(0)$. From [21], the sum of the branching ratios is

$$\sum B(D^0 \to \bar{K}\pi) = (6.06 \pm 0.43)\% \ . \quad (13)$$

Thus within experimental and theoretical errors, theory and experiment could just barely be in agreement. On the other hand, comparing the central values one might argue that the prediction of the factorization model without the annihilation term is higher than experiment by $\approx 1.5\%$. To determine which of the two amplitudes $A_{1/2}$ and $A_{3/2}$ is being overestimated, we check, in the following, how well the factorization model predicts $B(D^+ \to \bar{K}^0\pi^+)$. With the parameters of Eq. (12) and the decay amplitude in Eq. (10) we obtain

$$B(D^+ \to \bar{K}^0\pi^+) = (2.5 \pm 1.3)\% \quad (14)$$

and

$$|A_{3/2}^{\bar{K}\pi}|/|A_{1/2}^{\bar{K}\pi}| = (0.22 \pm 0.11) \ . \quad (15)$$

Though the errors are large, the prediction for $B(D^+ \to \bar{K}^0\pi^+)$ is in agreement with experiment [21]:

$$B(D^+ \to \bar{K}^0\pi^+) = (2.74 \pm 0.29)\% \ . \quad (16)$$

The agreement with experiment is also good for the ratio of the isospin amplitude. From [21] we obtain

$$\begin{aligned} |A_{1/2}^{\bar{K}\pi}| &= (3.029 \pm 0.115) \times 10^{-6} \text{ GeV} \ , \\ |A_{3/2}^{\bar{K}\pi}| &= (0.761 \pm 0.04) \times 10^{-6} \text{ GeV} \ , \\ |A_{3/2}^{\bar{K}\pi}|/|A_{1/2}^{\bar{K}\pi}| &= (0.251 \pm 0.023) \ , \\ \delta_{3/2}^{K\pi} - \delta_{1/2}^{K\pi} &= (86 \pm 8)^\circ \ . \end{aligned} \quad (17)$$

We emphasize that the choice of $F_0^{D\pi}(0)$ in [14] not only fits $B(D^+ \to \bar{K}^0\pi^+)$ as above, but it also fits $B(D^+ \to \pi^+\pi^0)$, as shown in [2].

In view of the agreement between theory and experiment for $B(D^+ \to \bar{K}^0\pi^+)$, we conclude that the source of the possible discrepancy between theory and experiment in $\sum B(D^0 \to \bar{K}\pi)$ is that the factorization model overestimates $A_{1/2}$. This conclusion is valid despite the fact that the error in Eq. (14) is of the same size as the possible overestimate of 1.5% in $\sum B(D^0 \to \bar{K}\pi)$, the reason being that the role of $|A_{3/2}|^2$ in $\sum B(D^0 \to \bar{K}\pi)$ is diluted by a factor of 3 compared to its role in $B(D^+ \to \bar{K}^0\pi^+)$ [see Eqs. (4) and (5)]. As penguin diagrams do not contribute to Cabibbo-favored decays, this difference could be attributed to a rather small inelasticity in the I=1/2 channel or to an annihilation term, or, indeed, to both.

If we ascribe this difference of about 1.5% between the factorization model prediction for $\sum B(D^0 \to \bar{K}\pi)$ and experiment, to the annihilation process, we can set an upper limit on its contribution as follows.

Inclusion of an annihilation term to the decay amplitudes can be parametrized as [an overall factor of $(G_F/\sqrt{2})V_{ud}V_{cs}^*$ is suppressed]

$$\begin{aligned} A(D^0 \to K^-\pi^+) &= a_1 f_\pi F_0^{DK}(m_\pi^2)(m_D^2 - m_K^2) \\ &\quad + a_2(m_K^2 - m_\pi^2)\xi \ , \\ A(D^0 \to \bar{K}^0\pi^0) &= \frac{a_2}{\sqrt{2}}[f_K F_0^{D\pi}(m_K^2)(m_D^2 - m_\pi^2) \\ &\quad - (m_K^2 - m_\pi^2)\xi] \ . \end{aligned} \quad (18)$$

In the factorization approximation,

$$\xi = f_D F_0^{K\pi}(m_D^2) \ . \quad (19)$$

ξ given in Eq. (19) would of necessity be complex being proportional to the form factor $F_0^{K\pi}$ at $q^2 = m_D^2$ which is above the $\bar{K}\pi$ threshold. However, if we treat it as essentially real and demand that it suppress $\sum B(D^0 \to \bar{K}\pi)$ by 1.5%, we find

$$\xi \approx 0.35 \text{ GeV} \ . \quad (20)$$

This value of ξ implies that the upper limit of the annihilation term in $A(D^0 \to K^-\pi^+)$ is a little under 10% of the term calculated in Eq. (10), while in $A(D^0 \to \bar{K}^0\pi^0)$ it is $\leq 23\%$ of the term in Eq. (10). The presence of a strange, $J^P = 0^+$ resonance $K_0^*(1430)$ with a width of $\approx$300 MeV [21], which decays almost exclusively to the $\bar{K}\pi$ channel, lends credence to the presence of the annihilation term.

Summarizing this section, the factorization model does very well in predicting $B(D^+ \to \bar{K}^0\pi^+)$. There are no annihilation terms here. This implies that the rescattering in I=3/2 state is weak and very likely elastic with a scattering phase sufficiently small as not to cause the magnitude of the amplitude to change significantly [22]. Evidence in support of this statement comes from partial wave analysis of $\bar{K}\pi$ scattering [23] where it is found that the S-wave scattering in I=3/2 state is elastic up to 1.4 GeV, the maximum energy involved in their analysis.

We also find that the factorization model without the annihilation term possibly overestimates $\sum B(D^0 \to \bar{K}\pi)$ by about 1.5%. If this discrepancy is attributed to an annihilation term in the I=1/2 amplitude then it amounts to a little under 10% of the factorization model amplitude without the annihilation terms in $D^0 \to K^-\pi^+$ decay and $\approx 23\%$ in $D^0 \to \bar{K}^0\pi^0$ decay. An alternative mechanism to bring about the same effect would be to assume that the S-wave scattering in I=1/2 state has a small inelasticity. In fact there is supporting evidence for this statement. In [23] it was found that the S-wave $K\pi$ scattering in I=1/2 state becomes inelastic above 1.3 GeV. We emphasize, in view of the known inelasticity in I=1/2 channel, that our analysis should not be interpreted as positive evidence for the annihilation term in $D \to \bar{K}\pi$ decays. Our estimate of the annihilation term is only an upper limit.

B. $D \to \bar{K}\rho$ decays

The isospin structure of the decay amplitudes for $D \to \bar{K}\rho$ is exactly as in Eq. (3) and therefore the analogs of Eqs. (4) and (5) and their consequences follow. A test of the factorization model can, therefore, be made in $D \to \bar{K}\rho$ decays also. In the factorization model without the annihilation term the decay amplitudes are given by [an overall factor $(G_F/\sqrt{2})V_{ud}V_{cs}^*$ is being suppressed]

$$\begin{aligned} A(D^0 \to K^-\rho^+) &= 2a_1 m_\rho f_\rho F_1^{DK}(m_\rho^2)\epsilon^* \cdot p\ , \\ A(D^0 \to \bar{K}^0\rho^0) &= 2\frac{a_2}{\sqrt{2}} m_\rho f_k A_0^{D\rho}(m_k^2)\epsilon^* \cdot p\ , \\ A(D^+ \to \bar{K}^0\rho^+) &= 2m_\rho \Big[a_1 f_\rho F_1^{DK}(m_\rho^2) \\ &\quad + a_2 f_K A_0^{D\rho}(m_K^2)\Big]\epsilon^* \cdot p\ . \end{aligned} \tag{21}$$

Here p is the D meson four-momentum and the form factors are defined in Ref. [9]. We evaluate $F_1^{DK}(m_\rho^2)$ using both a monopole and a dipole formula with mass 2.11 GeV (D_s^* pole) and $F_1^{DK}(0)$ given in Eq. (12). For $A_0^{D\rho}(0)$, unknown experimentally, we use the value 0.669 [9] with 10% error, and extrapolate it with a monopole formula with mass 1.865 GeV (D pole). Although the model of Ref. [9] cannot be trusted for $D \to$ light vector meson transitions, we show later that it correctly predicts $A_0^{DK^*}(0)$ despite the fact that its estimates of $A_1^{DK^*}$ and $A_2^{DK^*}$ are individually wrong. A simple calculation then results in ("monopole" and "dipole" refer to the extrapolation of F_1^{DK})

$$\sum B(D^0 \to \bar{K}\rho) = \begin{cases} (10.9 \pm 1.5)\% & \text{(monopole)}\ , \\ (14.5 \pm 1.9)\% & \text{(dipole)}\ , \end{cases} \tag{22}$$

with

$$\sum B(D^0 \to \bar{K}\rho) \equiv B(D^0 \to K^-\rho^+) + B(D^0 \to \bar{K}^0\rho^0)\ . \tag{23}$$

Experimentally [21]

$$\sum B(D^0 \to \bar{K}\rho) = (11.5 \pm 1.3)\%\ . \tag{24}$$

A comparison of Eqs. (22) and (24) shows that the theoretical prediction for $\sum B(D^0 \to \bar{K}\rho)$ with a monopole extrapolation of $F_1^{DK}(m_\rho^2)$ is in excellent agreement with experiment. Within errors the prediction using a dipole form for $F_1^{DK}(m_\rho^2)$ also agrees with experiment.

Let us now check to see how well the factorization model fares in predicting $B(D^+ \to \bar{K}^0\rho^+)$ and hence $|A_{3/2}|$. A simple calculation yields

$$B(D^+ \to \bar{K}^0\rho^+) = \begin{cases} (15.0 \pm 2.7)\% & \text{(monopole)}\ , \\ (20.7 \pm 4.0)\% & \text{(dipole)}\ , \end{cases} \tag{25}$$

and for the ratio of the isospin amplitude we find

$$\begin{aligned} |A_{3/2}^{\bar{K}\rho}|/|A_{1/2}^{\bar{K}\rho}| &= (0.47 \pm 0.08)\ \ \text{(monopole)}\ , \\ |A_{3/2}^{\bar{K}\rho}|/|A_{1/2}^{\bar{K}\rho}| &= (0.48 \pm 0.08)\ \ \text{(dipole)}\ . \end{aligned} \tag{26}$$

Experiments yield [21]

$$B(D^+ \to \bar{K}^0\rho^+) = (6.6 \pm 2.5)\% \tag{27}$$

and using the branching ratios from [21] we obtain

$$\begin{aligned} |A_{1/2}^{\bar{K}\rho}| &= (4.67 \pm 0.29) \times 10^{-6}\ \text{GeV}\ , \\ |A_{3/2}^{\bar{K}\rho}| &= (1.33 \pm 0.25) \times 10^{-6}\ \text{GeV}\ , \\ |A_{3/2}^{\bar{K}\rho}|/|A_{1/2}^{\bar{K}\rho}| &= (0.28 \pm 0.07)\ , \\ \delta_{3/2}^{K\rho} - \delta_{1/2}^{K\rho} &= (0.0 \pm 30)^\circ\ . \end{aligned} \tag{28}$$

Note that there are neither penguin contributions nor annihilation terms in $D^+ \to \bar{K}^0\rho^+$. The factorization model obviously overestimates the branching ratio by more than a factor of 2 [see Eqs. (25) and (27)]. Of the two terms in $A(D^+ \to \bar{K}^0\rho^+)$ in Eq. (21) the first is well determined experimentally; however, the second term proportional to $A_0^{D\rho}$ has to be calculated in a theoretical model. We used $A_0^{D\rho}(0)$=0.669 [9] with a 10% error. We would have to more than double this estimate to make theory agree with experiment. We contend that this is an unlikely scenario. A more interesting scenario is that there is considerable inelasticity in I=3/2 state, $\bar{K}\rho$ channel mixing with $\bar{K}^*\pi$ channel. In the following section we will provide further credibility to this contention.

C. $D \to \bar{K}^*\pi$ decays

The considerations of Sec. II A and II B also apply to $D \to \bar{K}^*\pi$ decays. The factorization model amplitudes without the annihilation terms are [an overall factor $(G_F/\sqrt{2})V_{ud}V_{cs}^*$ is being suppressed]

$$\begin{aligned} A(D^0 \to K^{*-}\pi^+) &= 2a_1 m_{K^*} f_\pi A_0^{DK^*}(m_\pi^2)\epsilon^* \cdot p\ , \\ A(D^0 \to \bar{K}^{*0}\pi^0) &= 2\frac{a_2}{\sqrt{2}} m_{K^*} f_{K^*} F_1^{D\pi}(m_{K^*}^2)\epsilon^* \cdot p\ , \\ A(D^+ \to \bar{K}^{*0}\pi^+) &= 2m_{K^*}[a_1 f_\pi A_0^{DK^*}(m_\pi^2) \\ &\quad + a_2 f_{K^*} F_1^{D\pi}(m_{K^*}^2)]\epsilon^* \cdot p\ . \end{aligned} \tag{29}$$

Among the form factors appearing in Eq. (29), we have good control over $F_1^{D\pi}(0) = F_0^{D\pi}(0)$ [see Eq. (12)]. To minimize having to rely on theory for $A_0^{DK^*}(0)$ [$= A_3^{DK^*}(0)$] we use [9]

$$2m_{K^*} A_3^{DK^*}(0) = (m_D + m_{K^*})A_1(0) - (m_D - m_{K^*})A_2(0)\ , \tag{30}$$

with experimental input [1] for $A_1^{DK^*}(0)$ and $A_1^{DK^*}(0)$. Theoretical models have so far failed in predicting correctly the form factor $A_1^{DK^*}(0)$ [1]. We use [1]

$$A_1^{DK^*}(0) = 0.61 \pm 0.05\ ,\quad A_2^{DK^*}(0) = 0.45 \pm 0.09\ , \tag{31}$$

to obtain, from Eq. (30),

$$A_3^{DK^*}(0) = 0.70 \pm 0.09\ . \qquad (32)$$

We note in passing that the model of [9] yields 0.73, which is in agreement with Eq. (32). A straightforward calculation then yields

$$\sum B(D^0 \to \bar{K}^*\pi) = \begin{cases} (4.4 \pm 0.9)\% & \text{(monopole)}\ , \\ (5.2 \pm 1.0)\% & \text{(dipole)}\ , \end{cases} \qquad (33)$$

with

$$\sum B(D^0 \to \bar{K}^*\pi) \equiv B(D^0 \to K^{*-}\pi^+) + B(D^0 \to \bar{K}^{*0}\pi^0)\ , \qquad (34)$$

where "monopole" and "dipole" refer to the extrapolation of $F_1^{D\pi}(m_{K^*}^2)$, using $F_1^{D\pi}(0) = F_0^{D\pi}(0)$ given in Eq. (12).

Experiments [21] yield

$$\sum B(D^0 \to \bar{K}^*\pi) = (7.9 \pm 0.72)\%\ . \qquad (35)$$

Thus, the factorization model with a monopole or dipole extrapolation of $F_1^{D\pi}(m_{K^*}^2)$ underestimates $\sum B(D^0 \to \bar{K}^*\pi)$.

Coming now to $B(D^+ \to \bar{K}^{*0}\pi^+)$, from Eq. (29), due to large cancellations between the two terms proportional to a_1 and a_2, this branching ratio turns out to be rather small with the results

$$\begin{aligned} B(D^+ \to \bar{K}^{*0}\pi^+) &\leq 0.33\%\ \text{(monopole)}\ , \\ &= (0.5^{+1.3}_{-0.5})\%\ \text{(dipole)}\ , \end{aligned} \qquad (36)$$

and, for the ratio of the isospin amplitude,

$$\begin{aligned} |A_{3/2}^{\bar{K}^*\pi}|/|A_{1/2}^{\bar{K}^*\pi}| &= (0.01 \pm 0.13)\ \text{(monopole)}\ , \\ |A_{3/2}^{\bar{K}^*\pi}|/|A_{1/2}^{\bar{K}^*\pi}| &= (0.085 \pm 0.160)\ \text{(dipole)}\ . \end{aligned} \qquad (37)$$

Experiments [21] yield

$$B(D^+ \to \bar{K}^{*0}\pi^+) = (2.2 \pm 0.4)\% \qquad (38)$$

and an amplitude analysis of the branching ratios listed in [21] yields

$$\begin{aligned} |A_{1/2}^{\bar{K}^*\pi}| &= (3.85 \pm 0.18) \times 10^{-6}\ \text{GeV}\ , \\ |A_{3/2}^{\bar{K}^*\pi}| &= (0.75 \pm 0.07) \times 10^{-6}\ \text{GeV}\ , \\ |A_{3/2}^{\bar{K}^*\pi}|/|A_{1/2}^{\bar{K}^*\pi}| &= (0.195 \pm 0.027)\ , \\ \delta_{3/2}^{\bar{K}^*\pi} - \delta_{1/2}^{\bar{K}^*\pi} &= (103 \pm 17)^\circ\ . \end{aligned} \qquad (39)$$

Thus the monopole extrapolation of $F_1^{D\pi}(m_{K^*}^2)$ results in too low a prediction for $B(D^+ \to \bar{K}^{*0}\pi^+)$, while the dipole extrapolation yields an estimate consistent with the Particle Data Group listing [21]. Inspection of Eqs. (36) and (38) leads one to the conclusion that the factorization model underestimates $A_{3/2}^{\bar{K}^*\pi}$. This leads to a very interesting scenario considering that the factorization model had overestimated $B(D^+ \to \bar{K}^0\rho^+)$ [see Eqs. (25) and (27)]. It is possible that inelastic final state interactions could feed $\bar{K}^{*0}\pi^+$ channel at the expense of $\bar{K}^0\rho^+$ channel. As inelastic final state interactions, being absorptive, do not have to conserve branching ratios, they could lower $B(D^+ \to \bar{K}^0\rho^+)$ to agree with experiment and at the same time raise $B(D^+ \to \bar{K}^{*0}\pi^+)$ to agree with data. This would also help in bridging the gap between theory and experiment for $\sum B(D^0 \to \bar{K}^*\pi)$ [see Eqs. (33) and (35)].

Further, an inelastic coupling between the $\bar{K}^*\pi$ and $\bar{K}\rho$ channels in I=1/2 state could also help raise $\sum B(D^0 \to \bar{K}^*\pi)$ of Eq. (33) to agree with data, Eq. (35).

III. SUMMARY AND DISCUSSION

We have proposed a method to test the factorization model. This method uses quantities that are independent of the strong interaction phases. Thus, they depend only on the magnitudes of the isospin amplitudes. The method, in essence, allows us to test if the factorization model correctly predicts the magnitudes of the isospin amplitudes. In a previous publication [2] we have applied this method to $D \to \pi\pi$ and $D \to K\bar{K}$ decays.

In our analysis we have used experimental information on form factors as far as possible, and the parameter set Eq. (12). In $D \to \bar{K}\pi$, we found that $B(D^+ \to \bar{K}^0\pi^+)$ is well reproduced by the factorization model. We emphasize that this provides support for our choice of $F_0^{D\pi}$. Recall [2] that this choice also yields $B(D^+ \to \pi^+\pi^0)$ correctly. As there are no annihilation terms involved in this decay and rescattering in S-wave $\bar{K}\pi$ system in I=3/2 state is known to be weak [23], this result confirms theoretical expectation. As a consequence a possible overestimate in $\sum B(D^0 \to \bar{K}\pi)$ by $\approx 1.5\%$ is attributed to an overestimate of $A_{1/2}$. We suggest that the amplitude could be reduced either by a small, <10% in $A(D^0 \to K^-\pi^+)$, annihilation term or inelastic final state interactions. The latter scenario is not only likely but *must* occur since S-wave scattering in $\bar{K}\pi$ system in I=1/2 state is known to be inelastic [23].

The test involving $D \to \bar{K}\rho$ decays is not so clean as we had to use a theoretical input for $A_0^{D\rho}$. One conclusion is very likely correct, namely, that the factorization model overestimates the I=3/2 amplitude [see Eqs. (25) and (27)]. We suggest that the $\bar{K}\rho$ channel couples with $\bar{K}^*\pi$ channel in I=3/2 state and that the excess could be drained away by inelasticity.

In testing the factorization model for $D \to \bar{K}^*\pi$ decays we were able to use experimental input for the form factors $A_0^{DK^*}$. We were able to conclude that the factorization model underestimates the I=3/2 amplitude in $D \to \bar{K}^*\pi$ decays. We suggest, as hinted at the end of the last paragraph, that an inelastic coupling between $\bar{K}\rho$ and $\bar{K}^*\pi$ channels in I=3/2 state could feed $\bar{K}^*\pi$ final state at the expense of the $\bar{K}\rho$ channel. We suggest that the same might also be happening in I=1/2 state which could result in a slight, but needed, raising of $\sum B(D^0 \to \bar{K}^*\pi)$. In addition, there could also be an annihilation contribution in $I{=}1/2$ channel.

In [2] we had tested factorization in Cabibbo-suppressed in $D \to \pi\pi$ and $D \to K\bar{K}$ decays. For completeness, we present our amplitude analysis of the branching ratios listing of Ref. [21] for $D \to \pi\pi$ and $D \to K\bar{K}$ decays:

$$\begin{aligned} |A_0^{\pi\pi}| &= (1.04 \pm 0.04) \times 10^{-6}\ \text{GeV} , \\ |A_2^{\pi\pi}| &= (0.63 \pm 0.09) \times 10^{-6}\ \text{GeV} , \\ |A_2^{\pi\pi}|/|A_0^{\pi\pi}| &= (0.60 \pm 0.10) , \\ \delta_0^{\pi\pi} - \delta_2^{\pi\pi} &= (82.1 \pm 9.8)^\circ , \end{aligned} \tag{40}$$

where $A_0^{\pi\pi}$ and $A_2^{\pi\pi}$ are $I{=}0$ and $I{=}2$ isospin amplitudes and $\delta_0^{\pi\pi}$ and $\delta_2^{\pi\pi}$ their phases. From the $D \to K\bar{K}$ data, we find

$$\begin{aligned} |A_0^{K\bar{K}}| &= (0.85 \pm 0.06) \times 10^{-6}\ \text{GeV} , \\ |A_1^{K\bar{K}}| &= (0.52 \pm 0.06) \times 10^{-6}\ \text{GeV} , \\ |A_1^{K\bar{K}}|/|A_0^{K\bar{K}}| &= (0.61 \pm 0.08) , \\ \delta_0^{K\bar{K}} - \delta_1^{K\bar{K}} &= (40.1 \pm 12.6)^\circ , \end{aligned} \tag{41}$$

where $A_0^{K\bar{K}}$ and $A_1^{K\bar{K}}$ are $I{=}0$ and $I{=}1$ isospin amplitudes and $\delta_0^{K\bar{K}}$ and $\delta_1^{K\bar{K}}$ their phases. The central values of the phases in Eqs. (40) and (41) are so chosen as to yield symmetric errors.

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