Quark-lepton unification and rare meson decays

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We study meson decays mediated by the heavy gauge bosons of the Pati-Salam model of quark-lepton unification. We consider the scenarios in which the τ lepton is associated with the third, second, and first generation of quarks. The most sensitive probes, depending on the scenario, are rare K, π, and B decays.

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I. INTRODUCTION

One of the unexplained features of the standard model of the strong and electroweak interactions is why some fermions, the quarks, experience the strong interaction while others, the leptons, do not. Experience has taught us to look for symmetry even when it is not apparent, and this leads one to speculate that, at some deeper level, quarks and leptons are identical. Perhaps there exists a symmetry between quarks and leptons which is broken at high-energy, in much the same way that the electroweak symmetry is broken at an energy scale of $(\sqrt{2}G_F)^{-1/2} \approx$ 250 GeV.

If we further speculate that the quark-lepton symmetry is a local gauge symmetry, we are led to predict a new force of nature which mediates transitions between leptons and quarks. The simplest model which incorporates this idea is the Pati-Salam model [1], based on the group $SU(4)_c$. The subgroup $SU(3)_c$ is the ordinary strong interaction, and lepton number is the fourth "color." At some high-energy scale, the group $SU(4)_c$ is spontaneously broken to $SU(3)_c$, liberating the leptons from the influence of the strong interaction and breaking the symmetry between quarks and leptons.

In this paper we explore signals for quark-lepton unification in the Pati-Salam model. We show that rare K, π, and B decays are the most sensitive probes of the presence of quark-lepton transitions mediated by heavy Pati-Salam bosons. A new feature of our analysis is that we do not restrict ourselves to the assumption that the τ lepton is associated with the third generation of quarks, but also consider the possibility that it is associated with the second, or even first, generation. A recent paper on Pati-Salam bosons also considers this possibility [2]. Our analyses overlap for $K_L \to \mu^{\pm}e^{\mp}$ and $\Gamma(\pi^+ \to e^+\nu)/\Gamma(\pi^+ \to \mu^+\nu)$, and agree. We further show that $\Gamma(K^+ \to e^+\nu)/\Gamma(K^+ \to \mu^+\nu)$ and rare B decays are the most sensitive probes in the scenario in which the τ lepton is associated with the first generation of quarks.

Pati-Salam bosons are members of a class of bosons called "leptoquarks," since they mediate transitions between leptons and quarks. They are spin one, and have nonchiral couplings to quarks and leptons. There are several recent model-independent analyses of bounds on leptoquarks. Reference [3] concentrates on spin-zero leptoquarks with chiral couplings, and Ref. [4] on spin-one leptoquarks with chiral couplings. Reference [5] considers both spin-zero and spin-one leptoquarks, with chiral and nonchiral couplings, and with the leptons associated with the quark generations in all six permutations.

In Sec. II we review the Pati-Salam model. In Secs. III–V we discuss rare decays mediated by Pati-Salam bosons in the scenarios where the τ lepton is associated with the third, second, and first generation, respectively. The bounds on the Pati-Salam-boson mass from rare K, π, and B decays are summarized in Table I. Section VI contains our conclusions.

II. PATI-SALAM MODEL

Pati and Salam proposed a class of unified models which incorporate quark-lepton unification [1].[1] A common feature of these models is the group $SU(4)_c$, with the subgroup $SU(3)_c$ corresponding to the strong interaction, and with lepton number identified as the fourth "color." In this section we discuss the minimal model which embodies quark-lepton unification via the $SU(4)_c$ Pati-Salam group.

Because quarks and leptons with the same $SU(2)_L$ quantum number have different hypercharge, the Pati-Salam group $SU(4)_c$ cannot commute with hypercharge. Furthermore, although $SU(4)_c$ can break to $SU(3)_c \times U(1)$, this $U(1)$ is not hypercharge, but rather the difference of baryon number and lepton number; we henceforth refer to it as $U(1)_{B-L}$, as is standard. Another group is needed to replace hypercharge. The simplest possibility is to introduce another $U(1)$ group, called $U(1)_{T_{3R}}$ (notation to be explained shortly), such that $U(1)_{B-L} \times U(1)_{T_{3R}}$ breaks spontaneously to $U(1)_Y$.

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[1]For a review, see Ref. [6].

TABLE I. Lower bound on the mass of the Pati-Salam boson (TeV) from rare K, π, and B decays. The first column indicates how the leptons are associated with the first, second, and third generation of quarks. The best bound for each scenario is enclosed in a box. The bounds assuming $B(B_d^0, B_s^0 \to \mu^\pm e^\mp) < 10^{-9}$ are shown in parentheses. A dash indicates the decay does not occur via the Pati-Salam interaction.

	$K_L \to \mu^\pm e^\mp$	$\frac{\pi^+ \to e^+ \nu}{\pi^+ \to \mu^+ \nu}$	$\frac{K^+ \to e^+ \nu}{K^+ \to \mu^+ \nu}$	$B_d^0 \to \mu^\pm e^\mp$	$B_s^0 \to \mu^\pm e^\mp$	$B^+ \to e^+ \nu$	$B^+ \to \mu^+ \nu$
$e\mu\tau$	1400	250	4.9	-	-	-	-
$\mu e\tau$	1400	76	130	-	-	-	-
$e\tau\mu$	-	250	-	16(140)	-	-	12
$\mu\tau e$	-	76	-	16(140)	-	13	-
$\tau\mu e$	-	-	4.9	-	(140)	13	-
$\tau e\mu$	-	-	130	-	(140)	-	12

The particle content of the $SU(4)_c \times SU(2)_L \times U(1)_{T_{3R}}$ model is

$$\begin{pmatrix} u_R \ u_G \ u_B \ \nu \\ d_R \ d_G \ d_B \ e \end{pmatrix}_L \quad (4, 2, 0),$$

$$\begin{pmatrix} u_R \ u_G \ u_B \ \nu \end{pmatrix}_R \quad (4, 1, +\tfrac{1}{2}),$$

$$\begin{pmatrix} d_R \ d_G \ d_B \ e \end{pmatrix}_R \quad (4, 1, -\tfrac{1}{2}),$$

where the subscripts on the quarks denote color (red, green, and blue), and the subscripts L, R denote chirality. The model is free of gauge and mixed gravitational anomalies. The $U(1)_{T_{3R}}$ quantum numbers of the $SU(2)_L$ singlet fields, $\pm\tfrac{1}{2}$, suggest that $U(1)_{T_{3R}}$ is a subgroup of an $SU(2)_R$ group; hence the notation. We will not make this additional assumption, since it does not affect our analysis. However, we remark that $SU(4)_c \times SU(2)_L \times SU(2)_R$ is a maximal subgroup of $SO(10)$, so another motivation for considering $SU(4)_c$ is $SO(10)$ grand unification [7, 8]. However, the $SU(4)_c$-breaking scale in the minimal model is very high, at least 10^{11} GeV, well out of reach of low-energy experiments [9].

Another motivation for considering the Pati-Salam group is provided by extended technicolor models. One can show that these models must incorporate gauged quark-lepton unification, or massless neutral Goldstone bosons (axions) and light (~ 5 GeV) charged pseudo-Goldstone bosons will result from electroweak symmetry breaking [10]. The simplest way to achieve this, often employed in model building [11–13], is to introduce a Pati-Salam group.

One canonically associates the τ lepton with the third generation of quarks, both for reasons of mass (they are the heaviest known fermions of their respective classes), and for historical reasons (the τ lepton and the b quark were the last fundamental fermions discovered; evidence for the top quark has recently been presented [14]). This is certainly a natural assumption. However, the flavor-symmetry-breaking mechanism, which is responsible for fermion-mass generation, is a mystery. One should keep an open mind to the possibility that the τ lepton is actually associated with the second or first generation of quarks.

Generically, one would expect that there is a mixing matrix, analogous to the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes the mixing of the lepton

generations with the quark generations [2]. We will make the assumption that this matrix is nearly diagonal, as is the CKM matrix, but consider the scenarios where the τ lepton is most closely associated with the third, second, or first generation in the following sections.

Because the Pati-Salam interaction conserves $B - L$ and fermion number, it cannot mediate nucleon decay. Purely leptonic transitions, such as $\mu \to e\gamma$ and $\mu N \to eN$, and meson-antimeson mixing, are induced only at one loop, and vanish in the limit of zero intergenerational mixing. The natural place to search for the Pati-Salam interaction is therefore in meson decays. These will be considered in the following sections.

III. τ LEPTON ASSOCIATED WITH THIRD-GENERATION QUARKS

The long-lived kaon, due to its longevity, is a sensitive probe of suppressed interactions which produce unusual decays. Pati and Salam observed that the decay $K_L \to \mu^\pm e^\mp$, shown in Fig. 1, provides the best bound on the mass of the Pati-Salam bosons [1]. This bound was later refined in Refs. [15,16], and leading-log QCD effects were included in Ref. [17]. Here we update this bound, based on the recent upper bound $B(K_L \to \mu^\pm e^\mp) < 3.9 \times 10^{-11}$ (90% C.L.) from Brookhaven E791 [18]. Combined with previous experiments, this yields

$$B(K_L \to \mu^\pm e^\mp) < 3.3 \times 10^{-11} \quad (90\% \text{ C.L.}) . \quad (1)$$

The diagram in Fig. 1 gives rise to an effective four-fermion interaction

$$\mathcal{L}_{\text{eff}} = \frac{g_4^2}{2M_c^2} \bar{d}\gamma^\mu e \bar{\mu}\gamma_\mu s + \text{H.c.}, \quad (2)$$

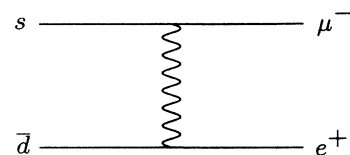where a sum on color is implicit. M_c is the mass of the

FIG. 1. Diagram for $\overline{K}^0 \to \mu^- e^+$, mediated by a heavy Pati-Salam boson.

Pati-Salam bosons, and g_4 is the Pati-Salam coupling at the scale M_c. Since $SU(4)_c$ breaks to $SU(3)_c$, this coupling is equal to the strong coupling at M_c. A Fierz rearrangement gives

$$\mathcal{L}_{\text{eff}} = \frac{g_4^2}{2M_c^2}\left[-\bar{d}s\bar{\mu}e + \frac{1}{2}\bar{d}\gamma^\mu s\bar{\mu}\gamma_\mu e + \frac{1}{2}\bar{d}\gamma^\mu\gamma_5 s\bar{\mu}\gamma_\mu\gamma_5 e \right.$$

$$\left. + \bar{d}\gamma_5 s\bar{\mu}\gamma_5 e + \text{H.c.}\right] . \tag{3}$$

For $K_L \to \mu^\pm e^\mp$, the required matrix elements are

$$\langle 0|\bar{d}\gamma^\mu\gamma_5 s|\overline{K}^0(p)\rangle = i\sqrt{2}F_K p^\mu \quad (F_K = 114 \text{ MeV}), \tag{4}$$

$$\langle 0|\bar{d}\gamma_5 s|\overline{K}^0(p)\rangle = -i\sqrt{2}B_0 F_K, \tag{5}$$

where [19]

$$B_0 = \frac{m_K^2}{m_s + m_d}, \tag{6}$$

and m_s, m_d are the running modified minimal subtraction scheme ($\overline{\text{MS}}$) quark masses evaluated at the Pati-Salam scale. These masses are evolved to low energy using leading-log QCD evolution [17]:

$$m(\mu) = m(M_c)\left(\frac{\alpha_s(m_t)}{\alpha_s(M_c)}\right)^{4/7}\left(\frac{\alpha_s(m_b)}{\alpha_s(m_t)}\right)^{12/23}$$

$$\times \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{12/25}\left(\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right)^{4/9} . \tag{7}$$

The light-quark $\overline{\text{MS}}$ masses are not well known, although their ratios are known from chiral perturbation theory: $m_u/m_d = 0.56$, $m_s/m_d = 20.1$, at leading order [20]. The absolute scale of the quark masses must be obtained from nonperturbative QCD. Lattice gauge theory provides a rough estimate of the light-quark $\overline{\text{MS}}$ masses.[2] From Ref. [21], we estimate $\hat{m} = (m_u + m_d)/2$ =2–5 MeV at $\mu = 1$ GeV. Since the quark masses enter in the denominator of B_0, we conservatively use the high values: $m_d = 7.7$ MeV, $m_s = 125$ MeV, at $\mu = 1$ GeV.

The partial width for $K_L \to \mu^\pm e^\mp$ is[3]

$$\Gamma(K_L \to \mu^\pm e^\mp) = \pi\alpha_s^2(M_c)\frac{1}{M_c^4}F_K^2 m_K B_0^2\left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 . \tag{8}$$

We use $\alpha_s(M_Z)$ =0.115 ($\Lambda_4 = 0.275$ MeV), evolved to M_c via the two-loop renormalization group (with $m_t = 170$ GeV), assuming no other colored particles lie between m_t and M_c [such particles would increase $\alpha_s(M_c)$ and increase the lower bound on M_c]. Using the upper bound on $K_L \to \mu^\mp e^\pm$ of Eq. (1) we find

$$M_c > 1400 \text{ TeV}. \tag{9}$$

It is remarkable that physics at such a high scale can be probed by this decay. Future experiments may probe branching ratios as small as 10^{-14}, increasing the lower bound on M_c by a factor of about 7.

The Pati-Salam bosons also produce transitions between bottom quarks and τ leptons. If we replace the s quark and muon in Fig. 1 with a b quark and τ lepton, we obtain the diagram for the decay $\overline{B}_d^0 \to \tau^- e^+$. The partial width is

$$\Gamma(\overline{B}_d^0 \to \tau^- e^+) = \pi\alpha_s^2(M_c)\frac{1}{M_c^4}F_B^2 m_B^3$$

$$\times \left(R - \frac{1}{2}\frac{m_\tau}{m_B}\right)^2\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 , \tag{10}$$

where

$$R = \frac{m_B}{m_b}\left(\frac{\alpha_s(M_c)}{\alpha_s(m_t)}\right)^{4/7}\left(\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right)^{12/23} , \tag{11}$$

and m_b is the $\overline{\text{MS}}$ mass evaluated at $\mu = m_b$. This is known from lattice-QCD calculations of the Υ spectrum to be about $m_b(m_b) = 4.3$ GeV.

The experimental upper bound on this decay from CLEO is [22]

$$B(\overline{B}_d^0 \to \tau^\pm e^\mp) < 5.3 \times 10^{-4} \quad (90\% \text{ C.L.}) . \tag{12}$$

Using $F_B = 140$ MeV and $\tau_{B^0} = 1.3$ ps we find

$$M_c > 4.8 \text{ TeV} , \tag{13}$$

much less than the lower bound on M_c from $K_L \to \mu^\pm e^\mp$.

We have also considered all other meson decays mediated by Pati-Salam bosons: $\pi^+ \to e^+\nu$; $\pi^0 \to e^+e^-, \nu\bar{\nu}$; $K^+ \to \mu^+\nu$; $D^+ \to e^+\nu$; $D^0 \to \nu\bar{\nu}$; $D_s^+ \to \mu^+\nu$; $B^+ \to \tau^+\nu$; $B_c \to \tau^+\nu$; and $\overline{B}_s^0 \to \tau^-\mu^+$. None competes with $K_L \to \mu^\pm e^\mp$ in its sensitivity to the Pati-Salam interaction.

If we associate the muon with the first generation of quarks and the electron with the second, the relevant decays are $K_L \to \mu^\pm e^\mp$; $\overline{B}_d^0 \to \tau^-\mu^+$; etc. The best bound on M_c again comes from $K_L \to \mu^\pm e^\mp$.

IV. τ LEPTON ASSOCIATED WITH SECOND-GENERATION QUARKS

At first sight, associating the τ lepton with the second generation of quarks and, say, the muon with the third generation, seems unnatural. However, the τ lepton is comparable in mass to the second-generation charm quark. Although the muon is a factor of about 40 less massive than the bottom quark, the bottom quark is at least a factor of 30 less massive than the top quark ($m_t > 131$ GeV [23]), so large intragenerational mass ratios do occur.

Because the strange quark is associated with the τ lepton, the decay of K_L to leptons does not occur via the Pati-Salam interaction. Pati-Salam bosons also mediate transitions between up quarks and neutrinos, so if we replace the s quark and muon in Fig. 1 with an

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[2]Although we expect to eventually know the value of the light-quark $\overline{\text{MS}}$ masses from lattice calculations, at present these masses are not known with any accuracy.

[3]The notation indicates a sum over the μ^+e^- and μ^-e^+ final states.

up quark and electron neutrino, respectively, we obtain the diagram for $\pi^+ \to e^+\nu_e$. This process involves only first-generation quarks. Since the decay $\pi^+ \to e^+\nu_e$ also proceeds via the weak interaction, the presence of a contribution from the Pati-Salam interaction manifests itself as a violation of lepton universality in $R_{e/\mu} = \Gamma(\pi^+ \to e^+\nu)/\Gamma(\pi^+ \to \mu^+\nu)$ [24]. The theoretical prediction from the weak interaction is [25]

$$R_{e/\mu}^{\text{theory}} = (1.2352 \pm 0.0005) \times 10^{-4}, \tag{14}$$

while the current experimental measurements are

$$R_{e/\mu} = (1.2265 \pm 0.0034 \pm 0.0044) \times 10^{-4}$$

$$\text{(TRIUMF [26]),} \tag{15}$$

$$R_{e/\mu} = (1.2346 \pm 0.0035 \pm 0.0036) \times 10^{-4}$$

$$\text{(PSI [27]),} \tag{16}$$

which when combined give

$$R_{e/\mu} = (1.2310 \pm 0.0037) \times 10^{-4} . \tag{17}$$

The theoretical uncertainty is much less than the experimental uncertainty.

The contribution of the Pati-Salam interaction to $\pi^+ \to e^+\nu_e$ is obtained via the interference of the Pati-Salam and weak amplitudes. We find

$$\Delta\Gamma(\pi^+ \to e^+\nu_e) = -\alpha_s(M_c)\frac{G_F}{\sqrt{2}}F_\pi^2\frac{m_e m_\pi}{M_c^2}V_{ud}B_0 \tag{18}$$

to be compared with the tree-level weak decay width

$$\Gamma(\pi^+ \to e^+\nu_e) = \frac{1}{4\pi}G_F^2 F_\pi^2 m_e^2 m_\pi |V_{ud}|^2 . \tag{19}$$

The absence of a deviation of the theoretical prediction from the experimental measurements yields a lower bound on the mass of the Pati-Salam boson of

$$M_c > 250 \text{ TeV} . \tag{20}$$

This bound is a factor of about five less stringent than the bound from $K_L \to \mu^\pm e^\mp$ in the previous section. Nevertheless, it is the strongest bound for the scenario considered here.

If we replace the s quark in Fig. 1 with a b quark, we obtain the diagram for $\overline{B}_d^0 \to \mu^- e^+$. The partial width is obtained from Eq. (10):

$$\Gamma(\overline{B}_d^0 \to \mu^- e^+) = \pi\alpha_s^2(M_c)\frac{1}{M_c^4}F_B^2 m_B^3 R^2, \tag{21}$$

where we have neglected the lepton masses. The present upper bound on this decay from CLEO [22],

$$B(\overline{B}_d^0 \to \mu^\pm e^\mp) < 5.9 \times 10^{-6}, \tag{22}$$

places a lower bound on the Pati-Salam-boson mass of

$$M_c > 16 \text{ TeV} . \tag{23}$$

The upper bound on this decay can be significantly im-

proved with B_d^0 mesons produced in hadron colliders. An upper bound on the branching ratio of 10^{-9} translates into $M_c > 140$ TeV.

If we further replace the d antiquark in Fig. 1 with a u antiquark and the positron with an antineutrino, we obtain the (charge conjugate of the) diagram for $B^+ \to \mu^+\nu_e$. The partial width is the same as Eq. (21):

$$\Gamma(B^+ \to \mu^+\nu) = \pi\alpha_s^2(M_c)\frac{1}{M_c^4}F_B^2 m_B^3 R^2 . \tag{24}$$

The present upper bound on this decay from CLEO [28],

$$B(B^+ \to \mu^+\nu) < 2.0 \times 10^{-5}, \tag{25}$$

places a lower bound on the Pati-Salam-boson mass of

$$M_c > 12 \text{ TeV} , \tag{26}$$

comparable to the bound from $\overline{B}_d^0 \to \mu^- e^+$.

If we replace the muon in Fig. 1 with a τ lepton, the diagram no longer describes K_L decay, but rather $\tau^- \to$ "K"e^-, where "K" denotes a meson or mesons with strangeness -1. The effective interaction is the same as Eq. (3), but with the muon replaced by the τ lepton. The decay to the ground state, $\tau^- \to K_s e^-$, is

$$\Gamma(\tau^- \to K_s e^-) = \frac{\pi}{4}\alpha_s^2(M_c)\frac{1}{M_c^4}F_K^2 m_\tau$$

$$\times \left(B_0 - \frac{1}{2}m_\tau\right)^2 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 . \tag{27}$$

The decay to the first excited state, $\tau^- \to \overline{K}^{*0}e^-$, involves only the vector current. Using

$$\langle 0 \mid \overline{d}\gamma^\mu s \mid \overline{K}^{*0}(p)\rangle = ig_{K^*}\epsilon^\mu(p) \quad (g_{K^*} = 0.133 \text{ GeV}^2), \tag{28}$$

we find

$$\Gamma(\tau^- \to \overline{K}^{*0}e^-) = \frac{\pi}{8}\alpha_s^2(M_c)\frac{1}{M_c^4}g_{K^*}^2 m_\tau$$

$$\times \left(1 + \frac{1}{2}\frac{m_\tau^2}{m_{K^*}^2}\right)\left(1 - \frac{m_{K^*}^2}{m_\tau^2}\right)^2 . \tag{29}$$

The two decay modes are of comparable sensitivity to Pati-Salam bosons. The upper bound on $\tau^- \to \overline{K}^{*0}e^-$ from CLEO [29],

$$B(\tau^- \to \overline{K}^{*0}e^-) < 1.1 \times 10^{-5}, \tag{30}$$

gives the best lower bound from τ decays on the Pati-Salam-boson mass. We find

$$M_c > 1.6 \text{ TeV} , \tag{31}$$

not nearly as strong as the lower bound from other decays.

If we associate the muon with the first generation of quarks and the electron with the third, the relevant decays are $\pi^+ \to \mu^+\nu_\mu$, $\overline{B}_d^0 \to \mu^+ e^-$, $B^+ \to e^+\nu$, $\tau^- \to K_s\mu^-$, $\tau^- \to \overline{K}^{*0}\mu^-$, and $\overline{B}_s^0 \to \tau^+ e^-$. The best lower bound on the mass of the Pati-Salam bosons again comes from $R_{e/\mu} = \Gamma(\pi^+ \to e^+\nu)/\Gamma(\pi^+ \to \mu^+\nu)$. Since the Pati-Salam interaction contributes to the un-

suppressed weak decay $\pi^+ \to \mu^+\nu$, rather than the suppressed decay $\pi^+ \to e^+\nu$ as in the previous case, the bound is not as strong as before. The interference of the Pati-Salam and weak amplitudes for $\pi^+ \to \mu^+\nu$ is given by

$$\Delta\Gamma(\pi^+ \to \mu^+\nu_\mu) = -\alpha_s(M_c)\frac{G_F}{\sqrt{2}}F_\pi^2 \frac{m_\mu m_\pi}{M_c^2}V_{ud}$$
$$\times \left(B_0 - \frac{1}{2}m_\mu\right)\left(1 - \frac{m_\mu}{m_\pi}\right)^2 \quad (32)$$

to be compared with the tree-level weak decay width

$$\Gamma(\pi^+ \to \mu^+\nu_\mu) = \frac{1}{4\pi}G_F^2 F_\pi^2 m_\mu^2 m_\pi |V_{ud}|^2 \left(1 - \frac{m_\mu}{m_\pi}\right)^2 .$$
$$(33)$$

The absence of a deviation of the theoretical prediction from the experimental measurements yields a lower bound on the mass of the Pati-Salam boson of

$$M_c > 76 \text{ TeV} . \quad (34)$$

The bound from $\overline{B}_d^0 \to \mu^+ e^-$ is the same as Eq. (23). This mode will ultimately place the best lower bound on the mass of the Pati-Salam boson using B_d^0 mesons produced in hadron colliders, as mentioned above. The bound from $B^+ \to e^+\nu$ is about the same as from $B^+ \to \mu^+\nu$, Eq. (26). The bound on the decay $\tau^- \to \overline{K}^{*0}\mu^-$ is similar to that with an electron in the final state [29],

$$R(\tau^- \to \overline{K}^{*0}\mu^-) < 8.7 \times 10^{-6}, \quad (35)$$

and yields approximately the same lower bound on the Pati-Salam-boson mass, Eq. (31).

V. τ LEPTON ASSOCIATED WITH FIRST-GENERATION QUARKS

In this section we discuss the case where the τ lepton is associated with the first generation of quarks. We first assume the muon is associated with the second generation and the electron with the third. One might imagine this scenario being realized by a "see-saw"-type mechanism for quark and lepton masses.

The best current lower bound on the mass of the Pati-Salam boson comes from $B^+ \to e^+\nu$, which has the same partial width as $B^+ \to \mu^+\nu$, Eq. (24). The upper bound on this branching ratio from CLEO [28],

$$B(B^+ \to e^+\nu) < 1.3 \times 10^{-5}, \quad (36)$$

places a lower bound on the Pati-Salam-boson mass of

$$M_c > 13 \text{ TeV} . \quad (37)$$

The decay $\overline{B}_s^0 \to e^-\mu^+$ also occurs via the Pati-Salam interaction. There is currently no bound on this decay, but the large number of B_s^0 mesons produced in hadron collisions can potentially be used to probe branching ratios as small as 10^{-9}. This translates into $M_c > 140$ TeV.

In this scenario, as well as the scenarios in the preceding section, the Pati-Salam boson mediates charmless semileptonic B decay. This process is very suppressed in the standard model due to the small value of V_{ub}. The Pati-Salam and weak-decay amplitudes do not interfere because the neutrinos are different types. The ratio of the Pati-Salam and weak-partial widths in the spectator model is

$$\frac{\Gamma_{\rm PS}(b \to ue^-\bar{\nu}_\tau)}{\Gamma(b \to ue^-\bar{\nu}_e)} = \frac{2\pi^2\alpha_s^2}{G_F^2 M_c^4 |V_{ub}|^2} . \quad (38)$$

Using $|V_{ub}| > 0.002$ and $M_c > 13$ TeV [from Eq. (37)] yields a ratio less than 1%, which is too small to observe.

Now consider the scenario in which the electron is associated with the second generation, and the muon with the third. In this case the best bound on the mass of the Pati-Salam boson comes from its contribution to $K^+ \to e^+\nu$. This manifests itself as a violation of lepton universality in $R_{e/\mu} = \Gamma(K^+ \to e^+\nu)/\Gamma(K^+ \to \mu^+\nu)$. The theoretical prediction from the weak interaction is[4]

$$R_{e/\mu}^{\rm theory} = 2.57 \times 10^{-5} , \quad (39)$$

while the experimental measurement is [31]:

$$R_{e/\mu} = (2.45 \pm 0.11) \times 10^{-5} . \quad (40)$$

Unlike the case of $\pi^+ \to e^+\nu$, the Pati-Salam and weak amplitudes for $K^+ \to e^+\nu$ do not interfere, because the neutrinos are different types. The partial width for $K^+ \to e^+\nu_\tau$ via the Pati-Salam interaction is

$$\Gamma(K^+ \to e^+\nu_\tau) = \pi\alpha_s^2(M_c)\frac{1}{M_c^4}F_K^2 m_K B_0^2 \quad (41)$$

to be compared with the tree-level weak decay width

$$\Gamma(K^+ \to e^+\nu_e) = \frac{1}{4\pi}G_F^2 F_K^2 m_e^2 m_K |V_{us}|^2 . \quad (42)$$

The absence of a deviation of the theoretical prediction from the experimental measurement yields a lower bound on the mass of the Pati-Salam boson of

$$M_c > 130 \text{ TeV} . \quad (43)$$

The bound from the decay $\overline{B}_s^0 \to e^+\mu^-$, discussed above, can potentially approach this bound. The bound from $B^+ \to \mu^+\nu$ is the same as Eq. (26).

VI. CONCLUSIONS

In this paper we have studied rare meson decays induced by the heavy gauge bosons of the Pati-Salam model of quark-lepton unification. We have considered the scenarios in which the leptons are associated with the quark generations in all six permutations. The lower bounds obtained on the mass of the Pati-Salam bosons

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[4]This is the leading-order prediction with no electromagnetic radiative correction. This correction depends on the manner in which bremsstrahlung photons are dealt with experimentally [30].

are given in Table I. Bounds from $K_L \to \mu^{\pm} e^{\mp}$ and lepton universality in charged-pions decays are well known, and we have updated them. We have shown that in the two scenarios in which the τ lepton is associated with the first generation of quarks, the best bounds come from $B^+ \to e^+ \nu$ and lepton universality in charged-kaon decays. All of these measurements have the potential for improvement.

At present, the bounds from $B_d^0, B_s^0 \to \mu^{\pm} e^{\mp}$ are not the strongest in any of the scenarios. However, the large number of these mesons which are produced in hadron colliders can potentially be used to probe branching ratios as small as 10^{-9}. The resulting bound on the Pati-Salam-boson mass would be the best for three of the scenarios. A high-resolution silicon vertex detector is essential for such a measurement.

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