Gluon bremsstrahlung from massive quarks in high energy collisions of polarized electrons and positrons

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(Received 28 March 1994)

The gluon bremsstrahlung cross section $e^+e^- \to q\bar{q}g$, including effects of finite quark and antiquark masses, is calculated for arbitrarily spin-polarized electron-positron beams. Mass effects and polarization effects are given and are shown to have a sizable influence on the cross section. It is shown, however, that for the left-right asymmetry A_{LR} the mass corrections and radiative corrections vanish at the Z^0 pole. The use of longitudinal polarized electrons in measurements of the forward-backward asymmetry A_{FB} may give sizeable enhancements.

PACS number(s): 13.65.+i, 13.87.−a, 13.88.+e

I. INTRODUCTION

Technique of obtaining spin-polarized high-energy electron beams have improved over the last years. Linear polarizations with an average value of 22.4% have recently been obtained at the SLAC Linear Collider [1] and at the DESY electron storage ring HERA transverse electron polarizations up to nearly 60% have been obtained [2]. Calculations of the gluon bremsstrahlung from massless quarks in high-energy electron-positron annihilation for arbitrary electron and positron polarizations [3] show that beam polarizations affect cross sections and asymmetries in distinct ways. It has been proposed [4] that flavor separation may be obtained by means of transverse electron and positron beam polarizations. Further, it has been shown that gluon linear and circular polarizations are influenced by electron-positron beam polarizations [4,5].

In the present paper we take into account the finite mass of the quark and antiquark. A calculation of gluon bremsstrahlung from massive quarks for unpolarized beams was made by Grunberg, Ng, and Tye [6] (photon exchange only) and by Jersák, Laermann, and Zerwas [7] who included Z^0 exchange. Recent calculations of cross sections and asymmetries for unpolarized electrons and positrons are given by Djouadi [8], Djouadi, Kühn, and Zerwas [9], and Arbuzov, Bardin and Leike [10]. Related QED processes are μ-$\bar{\mu}$ creation processes for massive μ particles with emission of photons in collisions of polarized electrons and positrons [11].

II. THE GLUON BREMSSTRAHLUNG CROSS SECTION

The cross section for the process

$$e^+ + e^- \to \gamma\ , Z^0 \to q + \bar{q} + (g)\ ,$$

where a quark q, an antiquark $\bar{q}$, and a gluon g are created in the collision of an electron e^- and a positron e^+, with a photon or a Z^0 boson in the intermediate state, is given by

$$\frac{d^5\sigma}{d\Omega\, d\chi\, dx\, d\bar{x}} = \frac{1}{64}\frac{1}{(2\pi)^5} \sum_{\text{color},s_q,s_{\bar{q}},e} |M^f|^2\ . \quad (2.1)$$

Here the matrix elements for flavor f is

$$M^f = -\frac{ie^2 g_s T_a}{s}[-Q_f L^\mu_\gamma H^f_{\mu\gamma} + f(s) L^\mu_Z H^f_{\mu Z}]\ , \quad (2.2)$$

with the electron and quark charges $-e$ and eQ_f, respectively, and g_s the strong-coupling constant. T_a is the color matrix normalized such that

$$\sum_{a,b} \mathrm{Tr}(T_a T_b) = 4\ .$$

The leptonic currents for γ exchange, L^μ_γ, and for Z^0 exchange, L^μ_Z, are given by

$$L^\mu_\gamma = \bar{v}(p_+, s_+)\gamma^\mu u(p_-, s_-)\ ,$$
$$L^\mu_Z = \bar{v}(p_+, s_+)\gamma^\mu (v - a\gamma_5) u(p_-, s_-)\ , \quad (2.3)$$

for specified momenta p_+ and p_- and polarizations s_+ and s_-.

The hadronic matrix elements including emission of a gluon with polarization e_μ are similarly given by

$$H^f_{\mu\gamma} = \bar{u}_f(q, s_q)\left[\not{e}\frac{\not{q} + \not{g} + m_f}{2qg}\gamma_\mu - \gamma_\mu \frac{\bar{\not{q}} + \not{g} + m_f}{2\bar{q}g}\not{e}\right] v_f(\bar{q}, s_{\bar{q}})\ ,$$

$$H^f_{\mu Z} = \bar{u}_f(q, s_q)\left[\not{e}\frac{\not{q} + \not{g} + m_f}{2qg}\gamma_\mu (v_f - a_f\gamma_5) - \gamma_\mu (v_f - a_f\gamma_5) \frac{\bar{\not{q}} + \not{g} + m_f}{2\bar{q}g}\not{e}\right] v_f(\bar{q}, s_{\bar{q}})\ , \quad (2.4)$$

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for specified quark, q, s_q, and antiquark, $\bar{q}, s_{\bar{q}}$, momenta and polarizations, respectively. In Eq. (2.2) m_f is the mass of the quark of flavor f, and $f(s)$ is proportional to the ratio of Z_0 and the photon propagators:

$$f(s) = \frac{1}{4\sin^2 2\theta_W} \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z^{\rm tot}} ,$$

where θ_W is the weak mixing angle, M_Z is the mass and $\Gamma_Z^{\rm tot}$ the total width of the Z^0. The standard model coupling constants are

$$\begin{aligned}&\text{for } e^-, \quad v = -1 + 4\sin^2\theta_W, \quad a = -1 ,\\ &\text{for } u,c,t, \quad v_f = 1 - \tfrac{8}{3}\sin^2\theta_W, \quad a_f = -1, \quad Q_f = \tfrac{2}{3} ,\\ &\text{for } d,s,b, \quad v_f = -1 + \tfrac{4}{3}\sin^2\theta_W, \quad a_f = -1, \quad Q_f = -\tfrac{1}{3} .\end{aligned}$$

In Eq. (2.1) we sum over quark, antiquark, and gluon polarizations, while the electron and positron polarizations are specified by the invariants

$$S_\pm^\mu S_{\pm,\mu} = -\mathbf{P}_\pm^2 = -(\mathbf{P}_\pm^{\|2} + \mathbf{P}_\pm^{\perp 2}) ,$$

where $\mathbf{P}_\pm^\|$ and $\mathbf{P}_\pm^\perp$ are the positron and/or electron longitudinal and transverse polarizations in the rest system of the particles, respectively. In the laboratory system the polarization four-vectors are

$$S_\pm = (S_0, \mathbf{S})_\pm = \left(P_\pm^\| \frac{|\mathbf{p}_\pm|}{m}, \mathbf{P}_\pm^\perp + \frac{E_\pm}{m}\mathbf{P}_\pm^\| \right) , \quad (2.5)$$

satisfying the invariant relation

$$S_\pm p_\pm = 0 .$$

For a pure spin state $S_\mu S^\mu = -\mathbf{P}_\pm^2 = -1$. Partial polarized states $\mathbf{P}_\pm^2 < 1$ are described by the density matrices

$$\rho(p_\pm, S_\pm) = \tfrac{1}{2}(1 + \gamma_5 \not{S}_\pm)(\not{p}_\pm \mp m_e) ,$$

with $S_\pm$ given by Eq. (2.5). For high electron-positron energies ρ simplifies to

$$\rho(p_\pm, S_\pm) \simeq \tfrac{1}{2}[1 + \gamma_5(\not{S}_\pm \mp P_\pm^\|)]\not{p}_\pm . \quad (2.6)$$

The cross section Eq. (2.1) is obtained from the matrix element, Eq. (2.2):

$$\frac{d^5\sigma}{d\Omega\, d\chi\, dx\, d\bar{x}} = \frac{\alpha^2}{(2\pi)^2}\frac{\alpha_s}{s}\sum_f \{ L_{\gamma\gamma}^{\mu\mu} H_{\gamma\gamma\mu\nu}^f + 2\,{\rm Re} f(s) L_{\gamma Z}^{\mu\nu} H_{\gamma Z\mu\nu}^f + |f(s)|^2 L_{ZZ}^{\mu\nu} H_{ZZ\mu\nu}^f \} , \quad (2.7)$$

where χ is the azimuthal angle of $\mathbf{p}_-$ in the coordinate system with the z axis along $\mathbf{q}$, and the leptonic tensors are given by

$$\begin{aligned} L_{\gamma\gamma}^{\mu\nu} &= 4L_\gamma^\mu L_\gamma^{\nu *} = 4{\rm Tr}\gamma^\mu \rho(p_-, s_-)\gamma^\nu \rho(p_+, s_+) = \Xi L_1^{\mu\nu} + \xi L_2^{\mu\nu} - L_3^{\mu\nu} ,\\ L_{\gamma Z}^{\mu\nu} &= 4L_\gamma^\mu L_Z^{\nu *} = -(v\Xi - a\xi)L_1^{\mu\nu} - (v\xi - a\Xi)L_2^{\mu\nu} + vL_3^{\mu\nu} + aL_4^{\mu\nu} ,\\ L_{ZZ}^{\mu\nu} &= 4L_Z^\mu L_Z^{\nu *} = [(v^2 + a^2)\Xi - 2va\xi]L_1^{\mu\nu} + [(v^2 + a^2)\xi - 2va\Xi]L_2^{\mu\nu} - (v^2 - a^2)L_3^{\mu\nu} , \end{aligned} \quad (2.8)$$

with

$$\begin{aligned} L_1^{\mu\nu} &= 4(p_+^\mu p_-^\nu + p_-^\mu p_+^\nu - g^{\mu\nu} p_+ p_-) ,\\ L_2^{\mu\nu} &= -4i\varepsilon^{\mu\nu}{}_{\alpha\beta} p_+^\alpha p_-^\beta , \end{aligned} \quad (2.9)$$

$$\begin{aligned} L_3^{\mu\nu} &= 4(p_+ p_-)(P_+^{\perp\mu} P_-^{\perp\nu} + P_-^{\perp\mu} P_+^{\perp\nu}) + (\mathbf{P}_+^\perp \mathbf{P}_-^\perp) L_1^{\mu\nu} ,\\ L_4^{\mu\nu} &= 4i\varepsilon_{\alpha\beta\gamma\delta}[P_+^{\perp\alpha} p_+^\beta g^{\gamma\mu}(P_-^{\perp\delta} p_-^\nu - P_-^{\perp\nu} p_-^\delta) - (p_+, P_+^\perp \Leftrightarrow p_-, P_-^\perp)] . \end{aligned} \quad (2.10)$$

Here $\Xi = 1 - P_+^\| P_-^\|$ and $\xi = P_-^\| - P_+^\|$ with the four-vectors $P_\pm^\perp = (0, \mathbf{P}_\pm^\perp)$. The hadronic tensors are similarly given by

$$\begin{aligned} H_{\gamma\gamma\mu\nu}^f &= \sum_{{\rm colors}, S_q, S_{\bar{q}}, e} H_{\mu\gamma}^f H_{\nu\gamma}^{f *} = 8sQ_f^2 H_{V\,\mu\nu}^f ,\\ H_{\gamma Z\mu\nu}^f &= \sum H_{\mu\gamma}^f H_{\nu Z}^{f *} = 8sQ_f[v_f H_{V\,\mu\nu}^f - a_f H_{A\,\mu\nu}^f] ,\\ H_{ZZ\mu\nu}^f &= \sum H_{\mu Z}^f H_{\nu Z}^{f *} = 8s[(v_f^2 + a_f^2)H_{V\,\mu\nu}^f - 2a_f v_f H_{A\,\mu\nu}^f + 2a_f^2 m_f^2 H_{V\,\mu\nu}^{Zf}] , \end{aligned} \quad (2.11)$$

where

$$H^{f}_{V\,\mu\nu} = \frac{4}{(qg)(\overline{q}g)}\left[\left(Qq - m_f^2\frac{Qg}{\overline{q}g}\right)[Q_\mu q_\nu + Q_\nu q_\mu - g_{\mu\nu}(Qq)]\right.$$
$$\left. -\left(Q^2 - 2m_f^2\frac{Qg}{\overline{q}g}\right)q_\mu\overline{q}_\nu + m_f^2[(Qg)g_{\mu\nu} - g_\mu g_\nu] + (q \Leftrightarrow \overline{q})\right] ,$$
$$H^{f}_{A\,\mu\nu} = \frac{-4i}{(qg)(\overline{q}g)}\varepsilon_{\mu\nu\alpha\beta}\left[\left(Qq - m_f^2\frac{Qg}{\overline{q}g}\right)q^\alpha Q^\beta - (q \Leftrightarrow \overline{q})\right) ,$$
$$H^{Z_f}_{V\,\mu\nu} = \frac{4}{(qg)(\overline{q}g)}\left[\left(q\overline{q} - m_f^2\frac{qg}{\overline{q}g}\right)g_{\mu\nu} + g_\mu g_\nu + (q \Leftrightarrow \overline{q})\right] . \tag{2.12}$$

The cross section may then be written in the form

$$\frac{d^5\sigma}{d\Omega\, d\chi\, dx\, d\overline{x}} = \frac{\alpha^2}{(2\pi)^2}\frac{\alpha_s}{s}\frac{1}{(1-x)(1-\overline{x})}$$
$$\times \sum_f \{h_f^{(1)}(s,P_-^{\parallel}P_+^{\parallel})X_0 + h_f^{(2)}(s,P_-^{\parallel}P_+^{\parallel})Y_0 + h_f^{(5)}(s,P_-^{\parallel}P_+^{\parallel})Z_0$$
$$+h_f^{(3)}(s)X_s + h_f^{(6)}(s)Y_s + h_f^{(4)}(s)Z_s\} , \tag{2.13}$$

where the coupling functions, depending on energy, flavor and linear polarization are given by

$$h_f^{(1)}(s,P_-^{\parallel}P_+^{\parallel}) = Q_f^2\Xi - 2Q_f\,\mathrm{Re}f(s)(v\Xi - a\xi)v_f + |f(s)|^2[(v^2+a^2)\Xi - 2va\xi](v_f^2+a_f^2) ,$$
$$h_f^{(2)}(s,P_-^{\parallel}P_+^{\parallel}) = -2Q_f\,\mathrm{Re}f(s)(a\Xi - v\xi)a_f - 2|f(s)|^2[(v^2+a^2)\xi - 2va\Xi]v_f a_f ,$$
$$h_f^{(3)}(s) = Q_f^2 - 2Q_f\,\mathrm{Re}f(s)vv_f + |f(s)|^2(v^2-a^2)(v_f^2+a_f^2), \tag{2.14}$$
$$h_f^{(4)}(s) = 2Q_f\,\mathrm{Im}f(s)av_f ,$$
$$h_f^{(5)}(s,P_-^{\parallel}P_+^{\parallel}) = 2|f(s)|^2[(v^2+a^2)\Xi - 2va\xi]a_f^2 ,$$
$$h_f^{(6)}(s) = -2|f(s)|^2(v^2-a^2)a_f^2 .$$

It is convenient to define $h_f^{(1)+} = h_f^{(1)}$, $h_f^{(1)-} = h_f^{(1)} - h_f^{(5)}$,

$$h_f^{(1)\pm}(s,P_-^{\parallel}P_+^{\parallel}) = Q_f^2\Xi - 2Q_f\,\mathrm{Re}f(s)(v\Xi - a\xi)v_f + |f(s)|^2[(v^2+a^2)\Xi - 2va\xi](v_f^2 \pm a_f^2) ,$$

and in the same way, $h_f^{(3)+} = h_f^{(3)}$, $h_f^{(3)-} = h_f^{(3)} - h_f^{(6)}$,

$$h_f^{(3)\pm}(s) = Q_f^2 - 2Q_f\,\mathrm{Re}f(s)vv_f + |f(s)|^2(v^2-a^2)(v_f^2 \pm a_f^2) . \tag{2.15}$$

Here $h_f^{(1)}(s)$-$h_f^{(4)}(s)$ are the same function as in Ref. [3]. The X, Y, and Z functions depending on angle and particle energies and momenta and on transverse electron and positron polarizations are obtained as

$$\begin{aligned}
X_0 &= \left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)[x^2(1+\beta_x^2\cos^2\theta)+\overline{m}_f^2]+\frac{\overline{m}_f^2}{4}[x_g^2(1+\cos^2\theta_g)-8x_g]+(x\Leftrightarrow\overline{x},\theta\Leftrightarrow\overline{\theta})\ ,\\
Y_0 &= 2\left\{\left(x-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)x\beta_x\cos\theta-(x\Leftrightarrow\overline{x},\theta\Leftrightarrow\overline{\theta})\right\}\ ,\\
Z_0 &= -\frac{\overline{m}_f^2}{4}\left\{4\left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)-x_g^2(1-\cos^2\theta_g)-4x_g\right\}\ ,\\
X_s &= P_-^\perp P_+^\perp\left\{\left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)x^2\beta_x^2\sin^2\theta\cos(2\phi-\phi_+-\phi_-)\right.\\
&\quad\left.+\frac{\overline{m}_f^2}{4}x_g^2\sin^2\theta_g\cos(2\phi_g-\phi_+-\phi_-)+(x\Leftrightarrow\overline{x},\theta\Leftrightarrow\overline{\theta},\phi\Leftrightarrow\overline{\phi})\right\}\ ,\\
Y_s &= P_-^\perp P_+^\perp\frac{\overline{m}_f^2}{4}x_g^2\sin^2\theta_g\cos(2\phi_g-\phi_+-\phi_-)\ ,\\
Z_s &= -P_-^\perp P_+^\perp\left\{\left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)x^2\beta_x^2\sin^2\theta\sin(2\phi-\phi_+-\phi_-)\right.\\
&\quad\left.+\frac{\overline{m}_f^2}{4}x_g^2\sin^2\theta_g\sin(2\phi_g-\phi_+-\phi_-)+(x\Leftrightarrow\overline{x},\theta\Leftrightarrow\overline{\theta},\phi\Leftrightarrow\overline{\phi})\right\}\ .
\end{aligned}\tag{2.16}$$

Here β_x and $\beta_{\overline{x}}$ are the quark and antiquark velocities, respectively, with $x\beta_x, \overline{x}\beta_{\overline{x}}$ the scaled momenta and $\overline{m}_f = m_f/E$ the scaled quark mass of flavor f. The polar angle θ_g is the angle between $\mathbf{p}_-$ and the gluon momentum $\mathbf{g}$; from the $\mathbf{q},\overline{\mathbf{q}},\mathbf{g}$ triangle it follows that

$$x_g\cos\theta_g = -x\beta_x\cos\theta-\overline{x}\beta_{\overline{x}}\cos\overline{\theta}\ .$$

The azimuthal angles are related to $\mathbf{p}_-$ as polar axis, defined in a right-handed sense; the $\mathbf{q},\overline{\mathbf{q}},\mathbf{g}$ triangle gives

$$\begin{aligned}
x_g\sin\theta_g\cos\phi_g &= -x\beta_x\sin\theta\cos\phi-\overline{x}\beta_{\overline{x}}\sin\overline{\theta}\cos\overline{\phi}\ ,\\
x_g\sin\theta_g\sin\phi_g &= -x\beta_x\sin\theta\sin\phi-\overline{x}\beta_{\overline{x}}\sin\overline{\theta}\sin\overline{\phi}\ .
\end{aligned}\tag{2.17}$$

The transverse polarizations are in the same way described in a plane perpendicular to $\mathbf{p}_-$:

$$\mathbf{P}_\pm = P_\pm(\cos\phi_\pm,\sin\phi_\pm,0)\ .$$

The cross-section differential in angles and energies for initially arbitrarily spin-polarized electrons and positrons may then be written in the form

$$\begin{aligned}
\frac{d^5\sigma}{d\Omega\, d\chi\, dx\, d\overline{x}} &= \frac{\alpha^2}{(2\pi)^2}\frac{\alpha_s}{s}\frac{1}{(1-x)(1-\overline{x})}\\
&\times\sum_f\left\{\left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)\{h_f^{(1)+}(s,P_-^{\|}P_+^{\|})x^2(1+\beta_x^2\cos^2\theta)+h_f^{(1)-}(s,P_-^{\|}P_+^{\|})\overline{m}_f^2\right.\\
&\quad+P_-^\perp P_+^\perp x^2\beta_x^2\sin^2\theta[h_f^{(3)+}(s)\cos(2\phi-\phi_+-\phi_-)-h_f^{(4)}(s)\sin(2\phi-\phi_+-\phi_-)]\}\\
&\quad+[2h_f^{(1)+}(s,P_-^{\|}P_+^{\|})-h_f^{(1)-}(s,P_-^{\|}P_+^{\|})]\frac{\overline{m}_f^2}{4}x_g^2+h_f^{(1)-}(s,P_-^{\|}P_+^{\|})\frac{\overline{m}_f^2}{4}x_g^2\cos^2\theta_g\\
&\quad-[h_f^{(1)+}(s,P_-^{\|}P_+^{\|})+h_f^{(1)-}(s,P_-^{\|}P_+^{\|})]\overline{m}_f^2x_g\\
&\quad+P_-^\perp P_+^\perp\frac{\overline{m}_f^2}{4}x_g^2\sin^2\theta_g[h_f^{(3)-}(s)\cos(2\phi_g-\phi_+-\phi_-)-h_f^{(4)}(s)\sin(2\phi_g-\phi_+-\phi_-)]\\
&\quad+(x\Leftrightarrow\overline{x},\beta_x\Leftrightarrow\beta_{\overline{x}},\theta\Leftrightarrow\overline{\theta},\phi\Leftrightarrow\overline{\phi})\\
&\quad\left.+2h_f^{(2)}(s,P_-^{\|}P_+^{\|})\left[\left(x-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)x\beta_x\cos\theta-(x\Leftrightarrow\overline{x},\theta\Leftrightarrow\overline{\theta})\right]\right\}\ .
\end{aligned}\tag{2.18}$$

This way of presenting the gluon bremsstrahlung cross section shows clearly the relation to the cross section for creating $q\bar{q}$ pairs from annihilation of polarized electrons and positrons, $e^+e^- \to q\bar{q}$, which is easily obtained as

$$\frac{d^2\sigma}{d\Omega} = \frac{3}{4}\frac{\alpha^2}{s}\beta\sum_f\{h_f^{(1)+}(s,P_-^{\parallel}P_+^{\parallel})(1+\beta^2\cos^2\theta) + h_f^{(1)-}(s,P_-^{\parallel}P_+^{\parallel})\overline{m}_f^2$$
$$+P_-^{\perp}P_+^{\perp}\beta^2\sin^2\theta[h_f^{(3)}(s)\cos(2\phi-\phi_+-\phi_-) - h_f^{(4)}(s)\sin(2\phi-\phi_+-\phi_-)] + 2h_f^{(2)}(s,P_-^{\parallel}P_+^{\parallel})\beta\cos\theta\}\ , \qquad (2.19)$$

where β is the velocity of the quark or the antiquark. It should be noted that β_x, $\beta_{\bar{x}}$ and β in Eqs. (2.18) and (2.19) always appear in the momentum components $x\beta_x\cos\theta$, $x\beta_x\sin\theta$ as compared to the case of massless quarks of Ref. [3], where the β's are all unity.

The cross section as a function of the angle and energy of the quark and the energy of the antiquark is obtained from Eq. (2.18) by integrating over the azimuth angle χ of $\mathbf{p}_-$ with $\mathbf{q}$ the polar axis. The equations expressing the antiquark emission angles $\bar{\theta}$ and $\bar{\phi}$ in terms of the quark angles θ, ϕ, and χ and of the angle between the quark and antiquark momenta ϑ are

$$\begin{aligned}\sin\bar{\theta}\cos\bar{\phi} &= (-\cos\theta\cos\chi\sin\vartheta + \sin\theta\cos\vartheta)\cos\phi - \sin\vartheta\sin\chi\sin\phi\ ,\\ \sin\bar{\theta}\sin\bar{\phi} &= (-\cos\theta\cos\chi\sin\vartheta + \sin\theta\cos\vartheta)\sin\phi + \sin\vartheta\sin\chi\cos\phi\ ,\\ \cos\bar{\theta} &= \cos\vartheta\cos\theta + \sin\vartheta\sin\theta\cos\chi\ ,\end{aligned} \qquad (2.20)$$

where ϑ is given by

$$\begin{aligned} x^2\beta_x^2\bar{x}^2\beta_{\bar{x}}^2\sin^2\vartheta &= 4(1-x)(1-\bar{x})(1-x_g) - \overline{m}_f^2x_g^2\ ,\\ x\beta_x\bar{x}\beta_{\bar{x}}\cos\vartheta &= -x\bar{x} - 2(1-x-\bar{x}) - \overline{m}_f^2\ .\end{aligned} \qquad (2.21)$$

The cross-section differential in the angles θ and ϕ and in the energies x and $\bar{x}$ is found to be given by

$$\frac{d^4\sigma}{d\Omega\,dx\,d\bar{x}} = \frac{\alpha^2}{2\pi}\frac{\alpha_s}{s}\frac{1}{(1-x)(1-\bar{x})}\sum_f\Bigg(\mathcal{F}_1(x,\bar{x})\{h_f^{(1)+}(s,P_-^{\parallel}P_+^{\parallel})(1+\cos^2\theta)$$
$$+P_-^{\perp}P_+^{\perp}\sin^2\theta[h_f^{(3)+}(s)\cos(2\phi-\phi_+-\phi_-) - h_f^{(4)}(s)\sin(2\phi-\phi_+-\phi_-)]\}$$
$$+\frac{\overline{m}_f^2}{2}\mathcal{F}_2(x,\bar{x})\{h_f^{(1)-}(s,P_-^{\parallel}P_+^{\parallel})\cos^2\theta + P_-^{\perp}P_+^{\perp}\sin^2\theta[h_f^{(3)-}(s)\cos(2\phi-\phi_+-\phi_-)$$
$$-h_f^{(4)}(s)\sin(2\phi-\phi_+-\phi_-)]\} + 2h_f^{(2)}(s,P_-^{\parallel}P_+^{\parallel})\mathcal{F}_3(x,\bar{x})\cos\theta + h_f^{(1)+}(s,P_-^{\parallel}P_+^{\parallel})\mathcal{F}_4(x,\bar{x})$$
$$+\frac{\overline{m}_f^2}{4}h_f^{(1)-}(s,P_-^{\parallel}P_+^{\parallel})\mathcal{F}_5(x,\bar{x})\Bigg)\ , \qquad (2.22)$$

where the quark and antiquark energies are contained in the functions

$$\begin{aligned}\mathcal{F}_1(x,\bar{x}) &= \left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)x^2\beta_x^2 + \left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\bar{x}^2\beta_{\bar{x}}^2(1-\tfrac{3}{2}\sin^2\vartheta)\ ,\\ \mathcal{F}_2(x,\bar{x}) &= x^2\beta_x^2 + \bar{x}^2\beta_{\bar{x}}^2(1-\tfrac{3}{2}\sin^2\vartheta) + 2x\beta_x\bar{x}\beta_{\bar{x}}\cos\vartheta = x_g^2 - \tfrac{3}{2}\bar{x}^2\beta_{\bar{x}}^2\sin^2\vartheta\ ,\\ \mathcal{F}_3(x,\bar{x}) &= \left(x-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-x}\right)x\beta_x - \left(\bar{x}-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\bar{x}\beta_{\bar{x}}\cos\vartheta\ ,\\ \mathcal{F}_4(x,\bar{x}) &= 2\left(1-\frac{\overline{m}_f^2}{2}\frac{x_g}{1-\bar{x}}\right)\bar{x}^2\beta_{\bar{x}}^2\sin^2\vartheta + \overline{m}_f^2\left[2+x_g^2\left(1-\frac{\overline{m}_f^2}{2(1-x)(1-\bar{x})}\right)\right]\ ,\\ \mathcal{F}_5(x,\bar{x}) &= \bar{x}^2\beta_{\bar{x}}^2\sin^2\vartheta - 2(x_g^2+4x_g-4) - 2\overline{m}_f^2\frac{x_g^2}{(1-x)(1-\bar{x})}\ ,\end{aligned} \qquad (2.23)$$

with ϑ given by Eq. (2.21).

It should be noted that polarization and quark mass effects are contained in different functions: $h_f^{(1)\pm}(s,P_-^{\parallel}P_+^{\parallel})$ and $h_f^{(2)}(s,P_-^{\parallel}P_+^{\parallel})$ depend on s and the longitudinal polarizations only, while $\mathcal{F}_1(x,\bar{x}) - \mathcal{F}_5(x,\bar{x})$ depend on $x,\bar{x}$ only and contain the quark mass $\overline{m}_f$.

III. LONGITUDINALLY POLARIZED ELECTRON BEAM

Experimentally the most interesting case is at present one polarized beam. For longitudinally polarized electrons to a degree $P_-^{\|}$ and unpolarized positrons the cross sections, Eq. (2.22) simplifies to

$$\frac{d^4\sigma}{d\Omega\, dx\, d\overline{x}} = \frac{\alpha^2}{2\pi}\frac{\alpha_s}{s}\frac{1}{(1-x)(1-\overline{x})}$$
$$\times \sum_f \left\{ h_f^{(1)+}(s,P_-^{\|})\mathcal{F}_1(x,\overline{x})(1+\cos^2\theta) + h_f^{(1)-}(s,P_-^{\|})\frac{\overline{m}_f^2}{2}\mathcal{F}_2(x,\overline{x})\cos^2\theta \right.$$
$$\left. +2h_f^{(2)}(x,P_-^{\|})\mathcal{F}_3(x,\overline{x})\cos\theta + h_f^{(1)+}(s,P_-^{\|})\mathcal{F}_4(x,\overline{x}) + h_f^{(1)-}(s,P_-^{\|})\frac{\overline{m}_f^2}{4}\mathcal{F}_5(x,\overline{x}) \right\} , \tag{3.1}$$

with $\Xi = 1$ and $\xi = P_-^{\|}$:

$$\begin{aligned} h_f^{(1)\pm}(s,P_-^{\|}) &= Q_f^2 - 2Q_f \,\mathrm{Re} f(s)(v - aP_-^{\|})v_f + |f(s)|^2[(v^2+a^2) - 2vaP_-^{\|}](v_f^2 \pm a_f^2) , \\ h_f^{(2)}(s,P_-^{\|}) &= -2Q_f \,\mathrm{Re} f(s)(a - vP_-^{\|})a_f - 2|f(s)|^2[(v^2+a^2)P_-^{\|} - 2va]v_f a_f , \\ h_f^{(5)}(s,P_-^{\|}) &= 2|f(s)|^2[(v^2+a^2) - 2vaP_-^{\|}]a_f^2 . \end{aligned} \tag{3.2}$$

A measure of the importance of the polarization may be obtained from the quantities

$$R_f^{(i)}(E)P_-^{\|} = \frac{h_f^{(i)}(s,P_-^{\|}) - h_f^{(i)}(s,-P_-^{\|})}{h_f^{(1)}(s,P_-^{\|}) + h_f^{(1)}(s,-P_-^{\|})} , \quad i = 1,2,5 . \tag{3.3}$$

We show, in Fig. 1, $R_f^{(i)}(E)$ for a range of energies. We use $\sin^2\theta_W = 0.23$ and $M_Z = 91.2$ GeV, $\Gamma_Z = 2.48$ GeV.

In order to indicate the effect of the quark mass we give in Fig. 2 the functions $\mathcal{F}_1(x,\overline{x})$ and $\mathcal{F}_3(x,\overline{x})$ for $x = \overline{x}$, i.e., $x_g = 2(1-x)$, $(1+m_f^2)/2 \le x \le 1$, for $\overline{m}_f = m_f/E = 0.0$, 0.05, and 0.5.

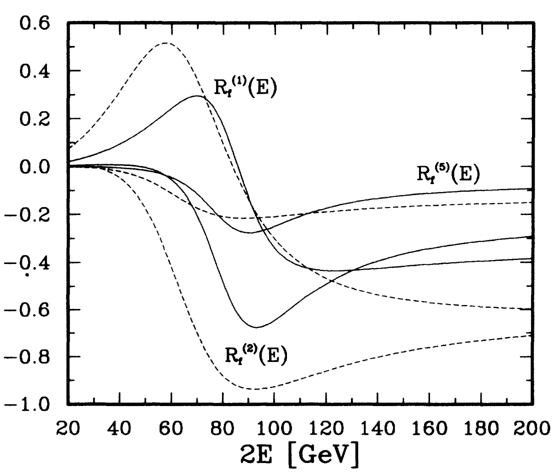

FIG. 1. The polarization asymmetries $R_f^{(i)}(E)$, Eq. (3.3), as function of the total e^+e^- energy. The solid curves: $Q_f = \frac{2}{3}$ quarks, u, c, t. Dashed curves: $Q_f = -\frac{1}{3}$ quarks, d, s, b.

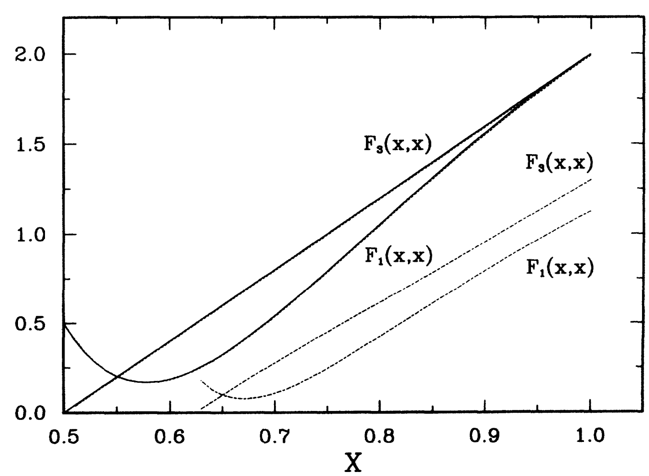

FIG. 2. The form factors $\mathcal{F}_1(x,\overline{x})$, and $\mathcal{F}_3(x,\overline{x})$ with $x = \overline{x}$ for $\overline{m}_f = m_f/E = 0.0$ (solid curve), $\overline{m}_f = 0.05$ (dashed curve), and $\overline{m}_f = 0.5$ (dot-dashed curve).

IV. TRANSVERSELY POLARIZED ELECTRONS AND POSITRONS

For electron and positron antiparallel transverse polarizations, $\phi_+ = \phi_- + \pi$, the cross section Eq. (2.22) becomes

$$\frac{d^4\sigma}{d\cos\theta d\phi_-\,dx\,d\overline{x}} = \frac{\alpha^2}{2\pi}\frac{\alpha_s}{s}\frac{1}{(1-x)(1-\overline{x})}$$
$$\times\sum_f\Bigg(\mathcal{F}_1(x,\overline{x})\{h_f^{(1)+}(s)(1+\cos^2\theta)$$
$$+P_-^{\perp}P_+^{\perp}\sin^2\theta[h_f^{(3)}(s)\cos 2(\phi-\phi_-) - h_f^{(4)}(s)\sin 2(\phi-\phi_-)]\}$$
$$+\frac{\overline{m}_f^2}{2}\mathcal{F}_2(x,\overline{x})\{h_f^{(1)-}(s)\cos^2\theta + P_-^{\perp}P_+^{\perp}\sin^2\theta[h_f^{(3)-}(s)\cos 2(\phi-\phi_-)$$
$$-h_f^{(4)}(s)\sin 2(\phi-\phi_-)]\}$$
$$+2h_f^{(2)}(s)\mathcal{F}_3(x,\overline{x})\cos\theta + h_f^{(1)+}(s)\mathcal{F}_4(x,\overline{x}) + \frac{\overline{m}_f^2}{4}h_f^{(1)-}(s)\mathcal{F}_5(x,\overline{x})\Bigg)\,, \tag{4.1}$$

where $h_f^{(n)}(s) = h_f^{(n)}(s,0,0)$ $(\Xi = 1, \xi = 0)$ for transverse polarizations, and where the spin azimuthal angle $\phi_- - \phi$ is measured from the $\mathbf{p}_- - \mathbf{q}$ plane.

V. THE CROSS SECTION AT THE Z^0 POLE WITH LONGITUDINALLY POLARIZED ELECTRONS

We consider here specificly the cross section at the Z^0 pole. From Eq. (2.19) we obtain for the two jet cross section, $e^+e^- \to q\overline{q}$, for electron polarization $P_-^{\parallel}$ and unpolarized positrons $P_+ = 0$,

$$\frac{d^2\sigma}{d\Omega} = \frac{3}{4}\frac{\alpha^2}{s}|f(s)|^2\beta\Bigg\{(v^2+a^2-2vaP_-^{\parallel})\sum_f[1+\beta^2\cos^2\theta)(v_f^2+a_f^2) + \overline{m}_f^2(v_f^2-a_f^2)]$$
$$-4[(v^2+a^2)P_-^{\parallel} - 2va]\beta\cos\theta\sum_f v_f a_f\Bigg\}\,. \tag{5.1}$$

Correspondingly, for three jets, $e^+e^- \to q\overline{q}g$, from Eq. (3.1),

$$\frac{d^4\sigma}{d\Omega\,dx\,d\overline{x}} = \frac{\alpha^2}{2\pi}\frac{\alpha_s}{s}\frac{|f(s)|^2}{(1-x)(1-\overline{x})}$$
$$\times\Bigg\{(v^2+a^2-2vaP_-^{\parallel})\sum_f\Bigg[(1+\cos^2\theta)(v_f^2+a_f^2)\mathcal{F}_1(x,\overline{x}) + \overline{m}_f^2\cos^2\theta(v_f^2-a_f^2)\mathcal{F}_2(x,\overline{x})$$
$$+(v_f^2+a_f^2)\mathcal{F}_4(x,\overline{x}) + \frac{\overline{m}_f^2}{4}(v_f^2-a_f^2)\mathcal{F}_5(x,\overline{x})\Bigg]$$
$$-2[(v^2+a^2)P_-^{\parallel} - 2va]\beta\cos\theta\sum_f v_f a_f\mathcal{F}_3(x,\overline{x})\Bigg\}\,. \tag{5.2}$$

VI. ASYMMETRIES WITH LONGITUDINALLY POLARIZED ELECTRONS AT THE Z^0 POLE

We discuss here the left-right, A^{LR} and the forward-backward A^{FB} asymmetries with longitudinally polarized electrons. From Eqs. (5.1) and (5.2) follows that at the Z_0 pole the *left-right asymmetry*, A^{LR}, for two and three jets are equal:

$$A^{\mathrm{LR}} = \frac{1}{P_-^{\parallel}}\frac{\sigma(-P_-^{\parallel}) - \sigma(P_-^{\parallel})}{\sigma(-P_-^{\parallel}) + \sigma(P_-^{\parallel})}_{\text{2 jets,3 jets}}$$
$$= \frac{2va}{a^2+v^2} = \frac{2(1-4\sin^2\theta_W)}{1+(1-4\sin^2\theta_W)^2}\,, \tag{6.1}$$

where $\sigma(\pm P_-^{\parallel})$ is the total cross section for two or three jets. This implies that A^{LR} is without gluon radiative and quark mass correction to first order in α_s. The independence of the radiative corrections is a consequence of the fact that the decay probability of the Z_0 particle, integrated over all final states, is independent of the polarization of the created Z_0, the left-right asymmetry depends on the initial state only. The independence of radiative corrections is therefore true to all orders in α_s. Note, however, that also the asymmetry for *differential* three-jets cross sections is given by the same result:

$$\frac{1}{P_-^{\|}} \frac{\frac{d^2\sigma(-P_-^{\|})}{dx\,d\bar{x}} - \frac{d^2\sigma(P_-^{\|})}{dx\,d\bar{x}}}{\frac{d^2\sigma(-P_-^{\|})}{dx\,d\bar{x}} + \frac{d^2\sigma(P_-^{\|})}{dx\,d\bar{x}}} = \frac{2va}{a^2+v^2} . \qquad (6.2)$$

For the *forward backward asymmetry* $A^{\rm FB}$ an enhancement of the cross section may be obtained with the use of polarized electrons. From Eq. (5.1) and (5.2) one finds that, for $A^{\rm FB}$ at the Z^0 pole,

$$A^{\rm FB} = \frac{\sigma_F(P_-^{\|}) - \sigma_B(P_-^{\|})}{\sigma_F(P_-^{\|}) + \sigma_B(P_-^{\|})} = \frac{1}{A^{\rm LR}} \frac{A^{\rm LR} - P_-^{\|}}{1 - P_-^{\|} A^{\rm LR}} \times A^{\rm FB}(P_-^{\|} = 0) , \qquad (6.3)$$

where, as given in Eq. (6.1),

$$A^{\rm LR} = \frac{2va}{a^2+v^2} , \qquad (6.4)$$

and $A^{\rm FB}(P_-^{\|} = 0)$ is the usual forward-backward asymmetry for unpolarized electrons.

Equation (6.3) shows that considerable enhancement of $A^{\rm FB}$ may be obtained with polarized electrons with polarization $|P_-^{\|}| \gg A^{\rm LR}$. Note that Eq. (6.3) also is correct for pure leptonic forward-backward asymmetries, in which case $A^{\rm FB}(P_-^{\|} = 0)$ is particularly small. The use of polarized electrons may therefore in this case be particularly useful.

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