

## High-energy vector-boson scattering with nonstandard interactions and the role of a scalar sector

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The high-energy behavior of vector-boson scattering amplitudes is examined within an effective theory for nonstandard self-interactions of electroweak vector bosons. Irrespective of whether this theory is brought into a gauge-invariant form by including nonstandard interactions of a Higgs particle, I find that terms that grow particularly strongly with increasing scattering energy are absent. Different theories are compared concerning their high-energy behavior and the appearance of divergences at the one-loop level.

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### I. INTRODUCTION

The standard electroweak theory [1] has been the most promising candidate to describe the electroweak interactions ever since it was proven that this theory is renormalizable [2] and that tree-level scattering amplitudes do not exceed the unitarity bounds at high energies if the mass of the Higgs boson is not too large [3–5]. The inclusion of a scalar (Higgs) sector and the generation of the vector boson masses by the spontaneous breakdown of an underlying linearly realized local  $SU(2)_L \times U(1)_Y$  symmetry appears to be a necessary ingredient for a unitary and renormalizable theory in which massive vector particles interact with fermions.

In fact, the standard theory consistently describes all currently known experimental data and the agreement between theory and experiment is particularly remarkable for the recent precision data from the CERN  $e^+e^-$  collider LEP 1, as these measurements test the theory at the level of one-loop radiative corrections.

However, the scalar sector of the theory has been accessed *only* via its indirect, i.e., loop-induced, effects on the current observables. Direct measurements of the vector-boson self-couplings have up to now been performed only with large uncertainties [6]. In view of near-future measurements of the process  $e^+e^- \rightarrow W^+W^-$  at LEP 2 which will improve the sensitivity to the vector-boson self-couplings [7], various models that can incorporate nonstandard vector-boson self-interactions but coincide with the standard model in the well-established sector of the interactions of the vector bosons with the leptons and quarks have been recently under consideration. It is not clear whether such a theory is also, as in the case with standard self-interactions, theoretically favored if it incorporates a scalar sector and is gauge invariant.<sup>1</sup>

Restricting ourselves to theories with the same particle content as of the standard model, we are dealing with nonrenormalizable effective theories that must be regularized by some ultraviolet cutoff  $\Lambda$ . At the tree level these models give rise to some four-point amplitude that will at high energies eventually exceed the unitarity bound [8]. Above some scale, which is usually taken to be the cutoff  $\Lambda$ , these theories have to be embedded into some higher theory which would again be renormalizable and unitary. Different approaches for constructing such effective theories exist.

(1) Assuming that nonstandard physics already appears not far above the weak scale the scalar sector may be omitted from the theory. These theories exhibit either *no*  $SU(2)_L \times U(1)_Y$  symmetry [7,9–12] or this symmetry is realized in a nonlinear way [13,14]. The two cases are equivalent which can be seen by applying a Stueckelberg transformation [15].

(2) The standard theory is adopted as the correct theory for electroweak interactions up to a certain scale  $\Lambda$  at which new degrees of freedom occur and which is large compared to the mass of a Higgs particle. The nonstandard interactions may manifest themselves at energies not too far above the weak scale as small deviations from the standard interactions. The Lagrangian for such a theory is an expansion in powers of  $1/\Lambda$  around the standard theory with gauge-invariant additional operators [16]. Nonstandard vector-boson self-interactions have been recently discussed in such theories [17–23].

An example of the first approach is the Kuroda-Maalampi-Schildknecht-Schwarzer (KMSS) model [9] in which trilinear and quadrilinear vector-boson self-interactions are parametrized under a minimal set of symmetry assumptions. A two-parameter reduction of the KMSS model can be embedded<sup>2</sup> [23] into a gauge-

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<sup>1</sup>In this paper, the term “gauge invariant” means “exhibiting a linearly realized local  $SU(2)_L \times U(1)_Y$  symmetry.”

<sup>2</sup>If not explicitly stated otherwise, statements made in this paper are only valid as far as terms linear in the deviations from the standard model are concerned.

invariant framework. In this way one obtains an example of the second approach. The addition of a dimension-six single-parameter quadrupole interaction (which contains no scalar particles and is gauge invariant itself) to this model leads to a model with nonstandard vector-boson self-interactions, which has been obtained by adding gauge-invariant dimension-six terms to the standard Lagrangian. This model is a general one provided some reasonable additional assumptions on the new physics are made.<sup>3</sup> I call this model the gauge-invariant dimension-six (GIDS) model.

Here, I examine the behavior of vector-boson scattering-amplitudes for large values of the center-of-mass scattering energy  $s$  in the GIDS model. My calculation evidences that the amplitudes show a particularly mild growth with increasing  $s$ . I further find that this behavior is only due to the form of the vector-boson self-couplings while the scalar sector plays no role.

One obvious requirement that a theory beyond the standard model has to fulfill is that it agrees *at the one-loop level* with the present precision data. If a particular model shows a dependence of one-loop radiative corrections on large positive powers of the cutoff, then it can presumably not incorporate large deviations from the

standard model since these would bring the model (in the absence of cancellations) in conflict with present data.

## II. THE MODELS

The KMSS model [9] describes nonstandard vector-boson self-couplings by four free parameters. It has been derived assuming that a global SU(2) weak isospin symmetry is broken only by electromagnetic interactions. In particular, the symmetry is broken by a term that causes mixing between the neutral vector bosons. In addition, only dimensionless couplings and interactions that are  $P$  and  $C$  even have been incorporated. The trilinear self-couplings are described by two parameters,  $\kappa_\gamma$ , and  $\hat{g}$ . Two other parameters,  $\hat{g}$  and  $\tilde{g}$ , describe the quadrilinear interactions. A two parameter reduction of the KMSS model, which I will henceforth simply call “the KMSS model,” is obtained by imposing the conditions [23]

$$\begin{aligned}\hat{g} &= \hat{g}^2, \\ \tilde{g} &= 0.\end{aligned}\tag{1}$$

The Lagrangian of the KMSS model is given by

$$\begin{aligned}\mathcal{L}_{\text{KMSS}} &= -ieA_\mu(W^{-\mu\nu}W_\nu^+ - W^{+\mu\nu}W_\nu^-) - ie\kappa_\gamma A_{\mu\nu}W^{+\mu}W^{-\nu} \\ &+ i\left(e\frac{\sin\theta_W}{\cos\theta_W} - \frac{\hat{g}}{\cos\theta_W}\right)Z_\mu(W^{-\mu\nu}W_\nu^+ - W^{+\mu\nu}W_\nu^-) \\ &+ i\left(e\kappa_\gamma\frac{\sin\theta_W}{\cos\theta_W} - \frac{\hat{g}}{\cos\theta_W}\right)Z_{\mu\nu}W^{+\mu}W^{-\nu} - e^2(A_\mu A^\nu W_\mu^+ W_\nu^- - A_\mu A_\nu W^{+\mu}W^{-\nu}) \\ &+ 2e\left(e\frac{\sin\theta_W}{\cos\theta_W} - \frac{\hat{g}}{\cos\theta_W}\right)\left[A_\mu Z^\mu W_\nu^+ W^{-\nu} - \frac{1}{2}A_\mu Z_\nu(W^{+\mu}W^{-\nu} + W^{-\mu}W^{+\nu})\right] \\ &- \frac{(\hat{g} - e\sin\theta_W)^2}{\cos^2\theta_W}(Z_\mu Z^\mu W_\nu^+ W^{-\nu} - Z_\mu Z_\nu W^{+\mu}W^{-\nu}) \\ &+ \frac{1}{2}\hat{g}^2(W_\mu^+ W_\nu^- W^{+\mu}W^{-\nu} - W_\mu^+ W_\nu^- W^{-\mu}W^{+\nu}).\end{aligned}\tag{2}$$

In (2),  $W_{\mu\nu}^+ = \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+$ , etc.,  $e$  is the electron charge, and  $\theta_W$  is the weak mixing angle.

Nonstandard Higgs interactions can be added to the KMSS Lagrangian in such a way that a gauge invariant model results [23]. Adding to this model a dimension-six quadrupole interaction term  $\mathcal{L}_W$ , introduced in [17], which is gauge invariant itself, one obtains the GIDS model which has been thoroughly discussed in [23]. The Lagrangian of the GIDS model is given by

$$\begin{aligned}\mathcal{L}_{\text{GIDS}} &= \mathcal{L}_{\text{SM}} + \epsilon_{W\Phi}\frac{g}{M_W^2}\mathcal{L}_{W\Phi} \\ &+ \epsilon_{B\Phi}\frac{g'}{M_W^2}\mathcal{L}_{B\Phi} + \epsilon_W\frac{g}{M_W^2}\mathcal{L}_W,\end{aligned}\tag{3}$$

where  $\mathcal{L}_{\text{SM}}$  is the standard Lagrangian. The Lagrangian (3) contains the three gauge invariant dimension-six terms

$$\mathcal{L}_{W\Phi} = i\text{tr}[(D_\mu\Phi)^\dagger W^{\mu\nu}(D_\nu\Phi)],\tag{4}$$

$$\mathcal{L}_{B\Phi} = -\frac{1}{2}i\text{tr}[\tau_3(D_\mu\Phi)^\dagger(D_\nu\Phi)]B^{\mu\nu},\tag{5}$$

and

$$\mathcal{L}_W = -\frac{2}{3}i\text{tr}(W_\mu^\nu W_\nu^\lambda W_\lambda^\mu).\tag{6}$$

Here,  $\Phi$  denotes the standard complex scalar Higgs doublet field:

$$\Phi = \frac{1}{\sqrt{2}}[(v + H)\mathbf{1} + i\phi_i\tau_i],$$

<sup>3</sup>This will be discussed in Sec. II.

where  $H$  is a physical Higgs field,  $(v/\sqrt{2})\mathbf{1}$  is the vacuum expectation value of the Higgs doublet, the  $\phi_i$  are the would-be Goldstone bosons,  $\mathbf{1}$  is the unity matrix in two dimensions, and the  $\tau_i$  are the Pauli matrices. The covariant derivative of  $\Phi$  is given by

$$D_\mu \Phi = \partial_\mu \Phi + igW_\mu \Phi - \frac{i}{2}g'\Phi\tau_3 B_\mu, \quad (7)$$

where

$$W_\mu = \frac{1}{2}W_{\mu i}\tau_i, \quad (8)$$

denotes the non-Abelian vector field, and

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu], \quad (9)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

are the field strength tensors. As usual,  $g$  denotes the  $SU(2)_L$  and  $g'$  the  $U(1)_Y$  coupling,  $g' = e/\cos\theta_W$ ,  $M_W$  is the mass of the charged vector bosons, and  $B_\mu$  is the  $U(1)_Y$  gauge field. The traces are taken over the  $2 \times 2$  matrices. The nonstandard couplings are described by the parameters  $\epsilon_{W\Phi}$ ,  $\epsilon_{B\Phi}$ , and  $\epsilon_W$ . In this paper, I call these parameters collectively “the  $\epsilon_i$ .” In the context of an effective field theory, their order of magnitude is  $\epsilon_i = O(M_W^2/\Lambda^2)$  [16].

The vector-boson self-couplings of the GIDS model with  $\epsilon_W = 0$  coincide with the ones of the KMSS model (2) with the identifications

$$\hat{g} = \frac{e}{\sin\theta_W}(1 + \epsilon_{W\Phi}), \quad (10)$$

$$\kappa_\gamma = 1 + \epsilon_{W\Phi} + \epsilon_{B\Phi}.$$

The GIDS model is the general model for vector-boson self-interactions that can be constructed from gauge invariant dimension-six terms, provided interactions that violate the  $C$ ,  $P$ , or  $CP$  symmetry or contain higher derivatives are not considered. Also, terms that modify the well-tested couplings of the vector-bosons to fermions are not taken into account.<sup>4</sup> Under these constraints, the three terms (4)–(6) are the only ones which one reads off from a general listing [16] of gauge-invariant dimension-six operators. The phenomenologically relevant parts of the GIDS Lagrangian in terms of the physical fields have been given in [23].

<sup>4</sup>One particular term,  $\mathcal{L}_{WB} = -\frac{1}{4}\text{tr}(\Phi^\dagger W_{\mu\nu}\Phi\tau_3)B^{\mu\nu}$ , must be omitted, because it causes mixing between the neutral gauge bosons  $W_3$  and  $B$  and thus, after a suitable diagonalization procedure, modifies the vector-boson-fermion couplings. This has been thoroughly discussed in [23].

### III. VECTOR-BOSON SCATTERING

I examine the tree-level scattering amplitudes for all processes involving only *massive* vector bosons:

$$\begin{array}{l} W^-W^+ \rightarrow W^-W^+ \\ W^+W^+ \rightarrow W^+W^+ \quad (W^-W^- \rightarrow W^-W^-) \\ W^-W^+ \rightarrow ZZ \quad (ZZ \rightarrow W^-W^+) \\ W^-Z \rightarrow W^-Z \quad (W^+Z \rightarrow W^+Z) \\ ZZ \rightarrow ZZ \end{array} \quad (11)$$

as well as for the following processes involving photons:

$$\begin{array}{l} \gamma\gamma \rightarrow W^-W^+ \quad (W^-W^+ \rightarrow \gamma\gamma) \\ \gamma W^- \rightarrow \gamma W^- \quad (\gamma W^+ \rightarrow \gamma W^+) \\ \gamma Z \rightarrow W^-W^+ \quad (W^-W^+ \rightarrow \gamma Z) \\ W^- \gamma \rightarrow W^-Z \quad (W^+ \gamma \rightarrow W^+Z) \end{array} \quad (12)$$

The amplitudes for the processes appearing in parentheses are related to the corresponding process to the left of them as discussed in Appendix B. The relevant vertices following from the GIDS Lagrangian (3) are given in Appendix A. The Feynman diagrams can be classified according to the following scheme: diagrams with a virtual vector boson (denoted by  $V$  in Fig. 1); diagrams with a four-boson vertex (denoted by  $C$ ); diagrams with a virtual Higgs boson (denoted by  $H$ ).

Since the calculations are performed in the unitary gauge, there are no diagrams with would be Goldstone bosons. For the process  $W^-W^+ \rightarrow W^-W^+$ , the diagrams are shown in Fig. 1. There and in the general process  $V_1V_2 \rightarrow V_3V_4$ ,  $p_i$  is the four-momentum of particle  $V_i$  ( $i = 1, 2, 3, 4$ ) and I use the Mandelstam variables:

$$s \equiv (p_1 + p_2)^2, \quad t \equiv (p_1 - p_3)^2, \quad u \equiv (p_1 - p_4)^2. \quad (13)$$

In Fig. 1, incoming particles are to the left and outgoing particles to the right.

For the general process an amplitude  $\mathcal{M}$  is a sum of contributions from the three sets of graphs:

$$\mathcal{M} = \mathcal{M}_V + \mathcal{M}_C + \mathcal{M}_H. \quad (14)$$

The  $\mathcal{M}_i$ ,  $i = V, C$ , or  $H$ , can in turn be written as a product of polarization vectors and a part which is independent of the particles' polarizations:

$$\mathcal{M}_i = \mathcal{A}_{\alpha\beta\gamma\delta}^i(p_1, p_2, p_3, p_4)\epsilon_1^\alpha(\lambda_1)\epsilon_2^\beta(\lambda_2)\epsilon_3^{*\gamma}(\lambda_3)\epsilon_4^{*\delta}(\lambda_4). \quad (15)$$

In (15),  $\epsilon_j(\lambda)$  is a polarization vector for particle  $j$  with helicity  $\lambda$  and  $\alpha, \gamma, \beta, \delta$  are Lorentz indices. I use the phase conventions of Jacob and Wick [24] for the helicity eigenstates. The appropriate polarization vectors  $\epsilon_j(\lambda)$  can be found in [25].

The high-energy amplitudes are listed in Appendix C. Concerning the terms bilinear in the  $\epsilon_i$ , which are not

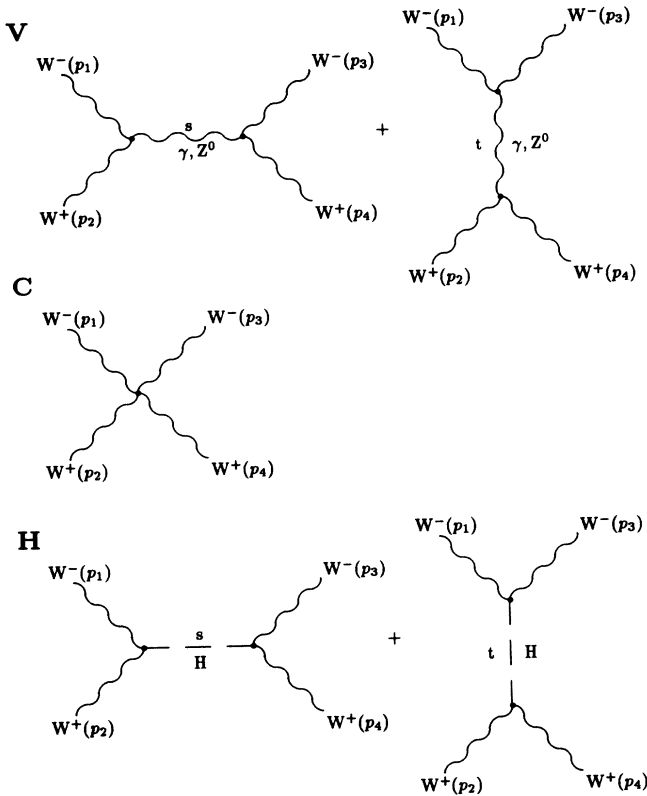


FIG. 1. Feynman diagrams for  $W^-W^+ \rightarrow W^-W^+$  in the tree approximation.

listed, one observes that they grow at most as  $O(s^2)$ .<sup>5</sup> To be specific, the following terms appear:

$$\mathcal{M} = O(s^2)(\epsilon_W^2, \epsilon_{W\Phi}^2, \epsilon_{B\Phi}^2, \epsilon_W \epsilon_{W\Phi}) + O(s^{3/2}),$$

while the combinations  $\epsilon_{B\Phi} \epsilon_W$  and  $\epsilon_{B\Phi} \epsilon_{W\Phi}$  do not contribute to the  $s^2$  terms.<sup>6</sup>

I turn to an investigation of the cancellations that take place among the sets of diagrams (14) (compare [4] for a similar analysis in the standard model). I have analyzed all amplitudes for the processes (11) and (12) and find that the sum of the vector-exchange diagrams for any amplitude grows at most as

$$\mathcal{M}_V = s^2 + \epsilon_{W\Phi} s^2 + \epsilon_{B\Phi} s + \epsilon_W s^2,$$

where  $s^2$  is to be understood as  $O(s^2)$  etc. All of these powers typically appear when all external particles are in the longitudinal polarization state.

The contact graphs get no contribution from  $\epsilon_{B\Phi}$ , since  $\mathcal{L}_{B\Phi}$  does not contain quartic interactions. I find a growth as

<sup>5</sup>The  $\epsilon_i \epsilon_j O(s^2/M_W^4)$  terms are suppressed by a factor of  $s/\Lambda^2$  with respect to the  $\epsilon_i O(s/M_W^2)$  terms.

<sup>6</sup>This behavior is unchanged if the Higgs diagrams are omitted.

$$\mathcal{M}_C = s^2 + \epsilon_{W\Phi} s^2 + \epsilon_W s^2,$$

or a more decent growth for particular amplitudes. The sum of vector-exchange and contact diagrams is found to grow as

$$\mathcal{M}_V + \mathcal{M}_C = s + \epsilon_{W\Phi} s + \epsilon_{B\Phi} s + \epsilon_W s,$$

or more decent. The most important result can be stated here: All the  $s^2$  powers vanish already in the sum of vector exchange and contact diagrams. This fact holds for all three  $\epsilon_i$  and in all amplitudes. If all external particles are in the longitudinal polarization state, there is for many amplitudes a cancellation in this sum of one power of  $s$  in the standard term (see [4]). Simultaneously there is a cancellation of one power of  $s$  in the  $\epsilon_{W\Phi}$  and the  $\epsilon_W$  terms, which demonstrates the special form of  $\mathcal{L}_{W\Phi}$  and  $\mathcal{L}_W$  as far as vector-boson self-couplings are concerned.

Finally, the Higgs diagrams do not depend on  $\epsilon_W$  and grow at most as

$$\mathcal{M}_H = s + \epsilon_{W\Phi} s + \epsilon_{B\Phi} s,$$

and frequently they are only  $O(s^0)$ .

Adding the Higgs graphs to  $\mathcal{M}_V + \mathcal{M}_C$  in order to obtain the complete amplitude, the residual positive powers of  $s$  in the standard terms are canceled, so that the unitarity limit for partial wave amplitudes is not exceeded in the standard theory at energies large compared to the Higgs boson mass. This cancellation only takes place for amplitudes in which all external particles are in the longitudinal polarization state, because in the other amplitudes the standard terms are already  $O(s^0)$ . All other effects of adding the Higgs contribution are nonsystematic: Sometimes powers of  $\epsilon_{W\Phi} s$ ,  $\epsilon_{B\Phi} s$ ,  $\epsilon_{W\Phi} \sqrt{s}$ , or  $\epsilon_{B\Phi} \sqrt{s}$  are introduced again, while sometimes the terms growing as  $\epsilon_{B\Phi} s$  disappear.

One thus obtains the result

$$\mathcal{M} = \epsilon_{W\Phi} s + \epsilon_{B\Phi} s + \epsilon_W s, \quad (16)$$

or a more decent growth. Concluding this analysis, the  $\epsilon_i O(s)$  behavior is entirely due to the form of the nonstandard vector-boson self-interactions. The nonstandard Higgs interactions yield terms of  $\epsilon_i O(s)$  but do not change the high-energy behavior. The inclusion of a scalar sector in nonstandard interactions is thus of no relevance as far as the high-energy behavior of the theory is concerned.

I compare my result to an analysis [10] of vector-boson scattering amplitudes in the four-parameter KMSS model in which the authors did not restrict themselves to small values of the  $\epsilon_i$ . It has been shown there that a one-parameter reduction of this model, the Bilchak-Kuroda-Schildknecht (BKS) model, exists, in which terms that grow with  $O(s^2)$  or stronger are absent in the amplitudes. The Lagrangian of the BKS model can be obtained from (2) by eliminating  $\hat{g}$  in favor of  $\kappa_\gamma$  by the relation

$$\hat{g} = \frac{e}{\sin \theta_W} \kappa_\gamma. \quad (17)$$

I note that from (10) and (17) one sees that the BKS model is equivalent to the GIDS model (3) with

$$\epsilon_{W\Phi} = \kappa_\gamma - 1 \quad \text{and} \quad \epsilon_{B\Phi} = \epsilon_W = 0, \quad (18)$$

as far as the vector-boson self-couplings are concerned. Thus, the model with only  $\epsilon_{W\Phi}$  should yield only amplitudes that grow at most as  $O(s)$  if the Higgs interactions are turned off. My calculations show that the terms  $\epsilon_{W\Phi}O(s)$  remain if the Higgs interactions are added. For the processes  $W^-W^+ \rightarrow ZZ$  and  $\gamma Z \rightarrow W^-W^+$ , I expected this latter fact from a result in [10]. The authors of this reference showed for these processes that in the BKS model unitarity is violated in partial waves with angular momentum  $J > 0$ . Since an additional neutral scalar particle can only be exchanged in the  $s$ -channel in these processes, only the  $J = 0$  partial waves can be affected so that the addition of a Higgs particle cannot be sufficient to restore unitarity. For other processes, also  $t$ - and  $u$ -channel exchange occurs. However, even here the addition of a Higgs particle does not cure the bad high-energy behavior.

Considering the equivalence of the KMSS model and the GIDS model without Higgs interactions and without  $\mathcal{L}_W$ ,<sup>7</sup> one expects that amplitudes for a model with only  $\mathcal{L}_{B\Phi}$  are in general  $\mathcal{M} = O(s^2)$ , since the BKS model (which is equivalent to taking only  $\mathcal{L}_{W\Phi}$  as far as only terms linear in the deviations from the standard model are concerned) is the *only* model that can be embedded into the KMSS model in which the amplitudes are only  $O(s)$  even and bilinear in the deviations from the standard couplings. In this context, I re-emphasize that the  $s^2$  terms are, however, absent also linear in  $\epsilon_{B\Phi}$  and not only in  $\epsilon_{W\Phi}$ .

#### IV. CONCLUSIONS AND REMARKS

It is well known that the inclusion of the Higgs extension in the standard model is crucial to ensure the perturbative unitarity as well as the renormalizability of the theory. Concerning the dependence of one-loop radiative corrections to four-fermion scattering amplitudes on the mass of the Higgs boson, the standard theory only shows a mild, logarithmic  $M_H$  dependence (screening theorem [26]).

The role of the Higgs sector in the standard theory can be studied by looking at the corresponding behavior in the nonlinear sigma model [27], which is obtained by integrating out the physical Higgs particle of the standard theory. The nonlinear sigma model in the unitary gauge corresponds to the case of *no* Higgs particle, or, equivalently, to the limit of an infinite mass of the Higgs particle in the standard model. In the nonlinear sigma model, tree amplitudes grow as  $O(s)$ . In contrast to the standard model, this model is nonrenormalizable. It has

a logarithmic cutoff dependence.

I have investigated the role of the Higgs extension in effective theories with nonstandard vector-boson self-couplings. These theories are nonrenormalizable even when a Higgs extension is included. Only terms at most linear in the deviations from the standard couplings have been considered. This restriction is also explicitly assumed in the following discussion, if not otherwise specified.

I start with the KMSS model, (2). This model is equivalent to the GIDS model with  $\epsilon_W = 0, \epsilon_{W\Phi} \neq 0, \epsilon_{B\Phi} \neq 0$  and no Higgs interactions. We saw that vector-boson scattering amplitudes grow at most as  $O(s)$ . The  $O(s)$  growth remains if the Higgs interactions are added. Thus, in distinction from the case of the models with *standard* vector-boson self-couplings, the omission of the Higgs extension in the nonstandard interactions does *not* change the high-energy behavior of tree amplitudes. If we add the quadrupole interaction  $\mathcal{L}_W$  we also find at most an  $O(s)$  growth.

As to loop effects, a complete analysis of one-loop corrections to four-fermion scattering amplitudes due to the dimension-six terms of the GIDS model has been presented in [20]. It is shown there that the effects of these terms can be described by cutoff-dependent (renormalized) coefficients of other dimension-six terms that have tree-level effects and by a renormalization of the standard-model parameters. I note that only the dependence of the coefficients on the scale  $\Lambda$  can be determined unless one knows the underlying renormalizable theory [28]. If the Higgs interactions are excluded (KMSS model), the renormalized coefficients are proportional to  $\Lambda^2$  and  $\ln\Lambda$ . When the Higgs sector is included, the quadratic  $\Lambda$  dependence disappears and only an  $\ln\Lambda$  dependence remains. In addition, a quadratic  $M_H$  dependence appears. This behavior is similar to the replacement rule  $M_H \rightarrow \Lambda$  when going from the standard model to the nonlinear sigma model, although for the effective theory this replacement does not quantitatively reproduce the heavy-Higgs-boson limit.

The behavior of the different models is summarized in Table I.

Finally, I note that in the four-parameter KMSS

$$\begin{aligned}
 & V_{VVW}(\alpha, \beta, \gamma, k_1, k_2, k_3, g_V, \kappa_V, y_V) \\
 & = ieg_V \left[ -g^{\alpha\beta} k_2^\gamma + g^{\beta\gamma} (k_2 - k_3)^\alpha + g^{\gamma\alpha} k_3^\beta \right. \\
 & \quad \left. + \kappa_V (g^{\alpha\beta} k_1^\gamma - g^{\gamma\alpha} k_1^\beta) \right] \\
 & + ie \frac{y_V}{M_W^2} \left[ k_1^\gamma k_2^\alpha k_3^\beta - k_1^\beta k_2^\gamma k_3^\alpha \right. \\
 & \quad + k_1 \cdot k_2 (k_3^\alpha g^{\beta\gamma} - k_3^\beta g^{\alpha\gamma}) \\
 & \quad + k_1 \cdot k_3 (k_2^\alpha g^{\beta\gamma} - k_2^\beta g^{\alpha\gamma}) \\
 & \quad \left. + k_2 \cdot k_3 (k_1^\alpha g^{\beta\gamma} - k_1^\beta g^{\alpha\gamma}) \right] \\
 & \text{with } V = \gamma \text{ or } Z \\
 & \text{and } g_\gamma = 1, \quad g_Z = \frac{c_W^2 + \epsilon_{W\Phi}}{s_W c_W} \\
 & \kappa_\gamma = 1 + \epsilon_{W\Phi} + \epsilon_{B\Phi}, \quad \kappa_Z = \frac{c_W^2}{c_W^2 + \epsilon_{W\Phi}} \left( 1 + \epsilon_{W\Phi} - \frac{s_W^2}{c_W^2} \epsilon_{B\Phi} \right) \\
 & y_\gamma = \epsilon_W, \quad y_Z = \frac{c_W}{s_W} \epsilon_W
 \end{aligned}$$

FIG. 2. Three-boson vertex.

<sup>7</sup>I remind the reader that this equivalence only holds linearly in the deviations from the standard model.

TABLE I. Growth of tree amplitudes, cutoff dependence, and  $M_H$  dependence in various theories.

Self-interactions	No Higgs boson			Linear Higgs sector			Cutoff
	Model	Amplitudes	Loops	Model	Amplitudes	$M_H$	
Standard	a	$O(s)$	$\ln\Lambda$	b	$O(s^0)$	$\ln M_H$	none
Nonstandard	c	$O(s)$	$\Lambda^2, \ln\Lambda$	d	$O(s)$	$M_H^2$	$\ln\Lambda$
	e	$O(s^2)$					

<sup>a</sup>Nonlinear  $\sigma$  model [27].

<sup>b</sup>Standard model [3–5,26].

<sup>c</sup>BKS model, two-parameter KMSS model [9,10,29].

<sup>d</sup>Linearly  $SU(2)_L \times U(1)_Y$  invariant dimension-six extension of the standard model (GIDS) [20,23].

<sup>e</sup>Four-parameter KMSS model [9,10].

$$\begin{aligned}
 &V_{\gamma\gamma W W}(\alpha, \beta, \gamma, \delta, k_1, k_2, k_3, k_4) \\
 &= ie^2 \left[ g^{\alpha\gamma} g^{\beta\delta} + g^{\alpha\delta} g^{\beta\gamma} - 2g^{\alpha\beta} g^{\gamma\delta} \right. \\
 &\quad \left. + \frac{\epsilon_W}{M_W^2} F^{\alpha\beta\gamma\delta}(k_1, k_2, k_3, k_4) \right] \\
 \\
 &V_{Z\gamma W W}(\alpha, \beta, \gamma, \delta, k_1, k_2, k_3, k_4) \\
 &= ie^2 \frac{\epsilon_W}{s_W} \left[ \left( 1 + \frac{\epsilon_W \Phi}{c_W^2} \right) \cdot (g^{\alpha\gamma} g^{\beta\delta} + g^{\alpha\delta} g^{\beta\gamma} - 2g^{\alpha\beta} g^{\gamma\delta}) \right. \\
 &\quad \left. + \frac{\epsilon_W}{M_W^2} F^{\alpha\beta\gamma\delta}(k_1, k_2, k_3, k_4) \right] \\
 \\
 &V_{ZZ W W}(\alpha, \beta, \gamma, \delta, k_1, k_2, k_3, k_4) \\
 &= ie^2 \frac{c_W^2}{s_W^2} \left[ \left( 1 + 2\frac{\epsilon_W \Phi}{c_W^2} \right) \cdot (g^{\alpha\gamma} g^{\beta\delta} + g^{\alpha\delta} g^{\beta\gamma} - 2g^{\alpha\beta} g^{\gamma\delta}) \right. \\
 &\quad \left. + \frac{\epsilon_W}{M_W^2} F^{\alpha\beta\gamma\delta}(k_1, k_2, k_3, k_4) \right] \\
 \\
 &V_{W W W W}(\alpha, \beta, \gamma, \delta, k_1, k_2, k_3, k_4) \\
 &= -ie^2 \frac{1}{s_W^4} \left[ (1 + 2\epsilon_W \Phi) \cdot (g^{\alpha\beta} g^{\gamma\delta} + g^{\alpha\delta} g^{\beta\gamma} - 2g^{\alpha\gamma} g^{\beta\delta}) \right. \\
 &\quad \left. + \frac{\epsilon_W}{M_W^2} F^{\alpha\beta\gamma\delta}(k_1, k_2, k_3, k_4) \right]
 \end{aligned}$$

where

$$\begin{aligned}
 F^{\alpha\beta\gamma\delta}(k_1, k_2, k_3, k_4) = & \\
 &g^{\alpha\beta} g^{\gamma\delta} ((k_1 \cdot k_3) + (k_1 \cdot k_4) + (k_2 \cdot k_3) + (k_2 \cdot k_4)) \\
 &- g^{\alpha\gamma} g^{\beta\delta} ((k_1 \cdot k_4) + (k_2 \cdot k_3)) \\
 &- g^{\alpha\delta} g^{\beta\gamma} ((k_1 \cdot k_3) + (k_2 \cdot k_4)) \\
 &- g^{\alpha\beta} (k_1^\gamma k_3^\delta + k_2^\gamma k_3^\delta + k_1^\delta k_4^\gamma + k_2^\delta k_4^\gamma) \\
 &- g^{\gamma\delta} (k_1^\beta k_3^\alpha + k_2^\beta k_3^\alpha + k_1^\alpha k_4^\beta + k_2^\alpha k_4^\beta) \\
 &+ g^{\alpha\gamma} (k_1^\beta k_3^\delta - k_1^\delta k_3^\beta + k_2^\beta k_3^\alpha + k_1^\alpha k_4^\beta) \\
 &+ g^{\alpha\delta} (k_1^\beta k_4^\gamma - k_1^\gamma k_4^\beta + k_2^\beta k_4^\alpha + k_1^\alpha k_4^\beta) \\
 &+ g^{\beta\gamma} (k_1^\delta k_3^\alpha + k_2^\delta k_3^\alpha - k_2^\delta k_3^\alpha + k_2^\delta k_4^\alpha) \\
 &+ g^{\beta\delta} (k_1^\gamma k_4^\alpha + k_2^\gamma k_4^\alpha - k_2^\gamma k_4^\alpha + k_2^\gamma k_3^\alpha)
 \end{aligned}$$

FIG. 3. Four-boson vertices.

model, which cannot be embedded into a gauge invariant framework without taking dimension-eight terms, we have an  $O(s^2)$  growth of amplitudes. The  $\rho$  parameter depends only quadratically on the cutoff [29], but the  $\Lambda$  dependence of the other one-loop contributions has not yet been investigated.

## ACKNOWLEDGMENTS

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## APPENDIX A: VERTICES

The vertices needed for the computation of the amplitudes can be classified according to vertices involving

$$\begin{aligned}
 &V_{H W W}(\beta, \gamma, k_1, k_2, k_3, ) \\
 &= ig \left[ M_W g^{\beta\gamma} \right. \\
 &\quad \left. + \frac{\epsilon_W \Phi}{M_W} \left( g^{\beta\gamma} k_1 \cdot (k_2 + k_3) - k_2^\gamma k_1^\beta - k_3^\beta k_1^\gamma \right) \right] \\
 \\
 &V_{H Z \gamma}(\beta, \gamma, k_1, k_2, k_3) \\
 &= i \frac{e}{c_W} \frac{1}{M_W} (\epsilon_W \Phi - \epsilon_{B\Phi}) \cdot (g^{\beta\gamma} k_1 \cdot k_3 - k_3^\beta k_1^\gamma) \\
 \\
 &V_{H Z Z}(\beta, \gamma, k_1, k_2, k_3, ) \\
 &= ig \left[ \frac{M_W}{c_W^2} g^{\beta\gamma} \right. \\
 &\quad \left. + \frac{1}{M_W} \left( \epsilon_W \Phi + \frac{s_W^2}{c_W} \epsilon_{B\Phi} \right) \cdot \left( g^{\beta\gamma} k_1 \cdot (k_2 + k_3) - k_2^\gamma k_1^\beta - k_3^\beta k_1^\gamma \right) \right]
 \end{aligned}$$

FIG. 4. Vertices with one Higgs boson and two vector bosons.

three vector bosons (Fig. 2), vertices involving four vector bosons (Fig. 3), and vertices involving one Higgs scalar and two vector bosons (Fig. 4). In Figs. 2–4, the vertices are explicitly given. All particles are understood to be entering the vertex. Vertex functions involving outgoing particles can easily be constructed by replacing an incoming particle by the outgoing antiparticle and simultaneously replacing the particle's four-momentum by its negative four-momentum.

## APPENDIX B: RELATIONS BETWEEN AMPLITUDES

Given the particle types in the initial and final states, there is in principle a number of 81 different amplitudes if all particles are massive. The number of distinct amplitudes can be significantly reduced, however, if one relates certain amplitudes to each other by using the fact that the  $S$  matrix is invariant under  $C$ ,  $P$ , and  $T$  transformations (e.g., [24,30,31]). Also, amplitudes for reactions involving different sets of particles can be related to each other.

I denote an amplitude for the reaction of the particles  $AB \rightarrow CD$  with helicities  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$  (in this order) and center-of-mass scattering angle  $\vartheta$  by  $\mathcal{M}(\lambda_1 \lambda_2 \lambda_3 \lambda_4)(AB \rightarrow CD)(\vartheta)$ . In the following, the frame axes in the center-of-mass system are defined in such a way that the reaction takes place in the  $\hat{x}$ - $\hat{z}$  plane. Particle  $A$  travels in the positive  $\hat{z}$  direction and particle  $C$  has momentum component  $p_x \geq 0$ . The scattering angle  $\vartheta$  is restricted to  $0 \leq \vartheta \leq \pi$ . In the relations I give here, I take into account the phase factors according to the Jacob and Wick phase convention. Derivations of the relations can be found in [25].

From the invariance of the  $S$  matrix under a *rotation* one obtains the relation

$$\begin{aligned} & \mathcal{M}(\lambda_1 \lambda_2 \lambda_3 \lambda_4)(AB \rightarrow CD)(\vartheta) \\ &= (-1)^{\lambda_1 - \lambda_2 + \lambda_4 - \lambda_3} \mathcal{M}(\lambda_2 \lambda_1 \lambda_4 \lambda_3)(BA \rightarrow DC)(\vartheta). \end{aligned} \quad (\text{B1})$$

This relation can also be obtained from exchanging the two particles in the initial states as well as the two particles in the final state.

A *parity* transformation  $P$  changes momentum  $\mathbf{p}$  and angular momentum  $\mathbf{J}$  according to  $\mathbf{p} \rightarrow -\mathbf{p}$  and  $\mathbf{J} \rightarrow \mathbf{J}$ . Consider Fig. 5. The small arrows symbolize the component of spin in the direction of flight; from them, the helicity can be read off. For example, an arrow perpendicular to the direction of flight designates a particle in the longitudinal polarization state. Rotating the figure obtained after applying  $P$  by an angle  $\pi$  about the  $y$  axis, one obtains the same physical situation as before  $P$  was applied; only the particles' helicities have changed sign. It is thus clear that  $\mathcal{M}(\lambda_1 \lambda_2 \lambda_3 \lambda_4)(AB \rightarrow CD)(\vartheta)$  is equal to  $\mathcal{M}(-\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)(AB \rightarrow CD)(\vartheta)$ , up to a possible phase factor. The phase factor is found to be  $(-1)^{\lambda_3 - \lambda_4 - \lambda_1 + \lambda_2}$ . Rotating further by an angle  $\pi$  about

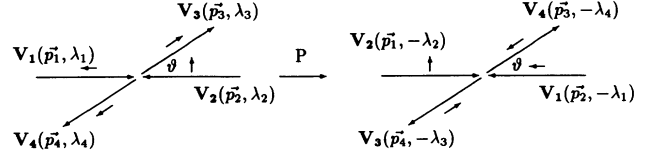


FIG. 5. Transformation of initial and final states under a parity transformation.

the  $z$  axis I obtain the relation

$$\begin{aligned} & \mathcal{M}(\lambda_1 \lambda_2 \lambda_3 \lambda_4)(AB \rightarrow CD)(\vartheta) \\ &= \mathcal{M}(-\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)(AB \rightarrow CD)(-\vartheta). \end{aligned} \quad (\text{B2})$$

When *charge conjugation* is applied to a state, particles are changed into their antiparticles, while their momenta and helicities remain unchanged. The invariance of the  $S$  matrix under charge conjugation implies the relation

$$\begin{aligned} & \mathcal{M}(\lambda_1 \lambda_2 \lambda_3 \lambda_4)(AB \rightarrow CD)(\vartheta) \\ &= \mathcal{M}(\lambda_1 \lambda_2 \lambda_3 \lambda_4)(\bar{A}\bar{B} \rightarrow \bar{C}\bar{D})(\vartheta), \end{aligned} \quad (\text{B3})$$

where  $\bar{A}$  is the antiparticle of particle  $A$  etc.

After a rotation by an angle  $\pi$  about the  $y$  axis, an exchange of particle labels in the two-particle states, and, successively, a rotation by an angle  $\pi$  about the  $z$  axis, one obtains the relation

$$\begin{aligned} & \mathcal{M}(\lambda_1 \lambda_2 \lambda_3 \lambda_4)(AB \rightarrow CD)(\vartheta) \\ &= \mathcal{M}(\lambda_2 \lambda_1 \lambda_4 \lambda_3)(\bar{B}\bar{A} \rightarrow \bar{D}\bar{C})(-\vartheta). \end{aligned} \quad (\text{B4})$$

*Time reversal* changes initial to final states, or, equivalently,

$$\left. \begin{array}{l} \mathbf{P} \rightarrow -\mathbf{P} \\ \mathbf{J} \rightarrow -\mathbf{J} \end{array} \right\} \lambda \rightarrow \lambda,$$

where  $\lambda$  denotes helicity (see Fig. 6). After a rotation by an angle  $\pi - \vartheta$  about the  $y$  axis one obtains the relation

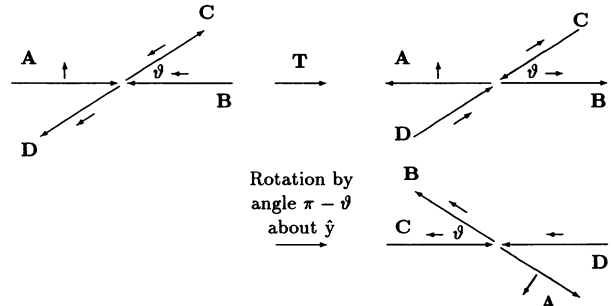


FIG. 6. Transformation of initial and final states under time reversal, succeeded by a rotation.

TABLE II. Relations among amplitudes for a given process  $AB \rightarrow CD$  using the Jacob and Wick phase conventions. A relation applies if the corresponding condition cited in the left column applies. The relation is due to the invariance under the transformation cited in the center column. Sample usage: For  $W^-W^+ \rightarrow W^-W^+$ , we can use the invariance of the  $S$  matrix under  $P$ ,  $C$ , and  $T$  transformations. One obtains relations due to  $P$ ,  $C$ , and  $T$  as well as relations due to the combined transformations  $CP$ ,  $CT$ ,  $PT$ , and  $CPT$ . For example, due to  $CPT$ ,  $\mathcal{M}_{+--0}(\vartheta) = \mathcal{M}_{0+-}(-\vartheta)$ , where the subscripts on  $\mathcal{M}$  denote the helicities.

Condition that is satisfied	Transformation	Relation $\mathcal{M}(\lambda_1\lambda_2\lambda_3\lambda_4)(\vartheta) =$
(always)	$P$ +rotations	$\mathcal{M}(-1-2-3-4)(-\vartheta)$
$A = \bar{B}$ $\wedge C = \bar{D}$	$C$ +rotations	$\mathcal{M}(2143)(-\vartheta)$
$AB = CD$	$T$ +rotations	$\mathcal{M}(3412)(-\vartheta)$
Identical initial particles, $A = B$	Exchange of labels+rotation	$(-1)^{\lambda_2-\lambda_3}\mathcal{M}(2134)(\vartheta \pm \pi)$
Identical final particles, $C = D$	Exchange of labels+rotation	$(-1)^{\lambda_4-\lambda_3}\mathcal{M}(1243)(\vartheta \pm \pi)$

$$\begin{aligned} &\mathcal{M}(\lambda_1\lambda_2\lambda_3\lambda_4)(AB \rightarrow CD)(\vartheta) \\ &= \mathcal{M}(\lambda_3\lambda_4\lambda_1\lambda_2)(CD \rightarrow AB)(-\vartheta). \quad (\text{B5}) \end{aligned}$$

The interactions of the GIDS model are actually invariant under  $P$ ,  $C$ , and  $T$  transformations. The relations (B1) to (B5) can therefore be applied as follows.

(i) Amplitudes for the processes which I did not calculate, listed in parentheses in (11) and (12), can be obtained from the ones which I calculated. For example, using  $C$  conjugation,

$$\begin{aligned} &\mathcal{M}(\lambda_1\lambda_2\lambda_3\lambda_4)(W^+W^+ \rightarrow W^+W^+)(\vartheta) \\ &= \mathcal{M}(\lambda_1\lambda_2\lambda_3\lambda_4)(W^-W^- \rightarrow W^-W^-)(\vartheta). \end{aligned}$$

(ii) Amplitudes for a given process,  $AB \rightarrow CD$ , but with different helicities, can be related to each other (cf. Table II).

Parity together with rotational invariance always gives a relation. In addition, the fulfillment of each of the conditions that (1) particle  $A$  is the antiparticle of  $B$  ( $A = \bar{B}$ ) and particle  $C$  is the antiparticle of  $D$  ( $C = \bar{D}$ ), (2) the initial state contains the same particles as the final state (in any order) ( $AB = CD$ ), (3) the initial state contains identical particles ( $A = B$ ), and (4) the final state contains identical particles ( $C = D$ ), gives one more relation, each of which follows from the invariance under a certain transformation, possibly accompanied by rotations. For the cases (3) and (4) one obtains relations among amplitudes in which identical particles have been exchanged. It is clear that the two amplitudes differ at most by a phase. Finally, all combinations of relations can also be applied to the considered process.

### APPENDIX C: LISTING OF AMPLITUDES

The amplitudes have been expanded in powers of  $M_W^2/s \ll 1$ . The expansion is valid for two cases: (1)  $s \gg M_H^2$  and (2) no Higgs particle ( $M_H \rightarrow \infty$ ). I list the terms that grow as  $O(s)$ . There are no terms that grow

with higher powers of  $s$ . The terms depending on  $\epsilon_W$  are in agreement with [32]. The high-energy approximation has been carried out at a fixed center-of-mass scattering angle  $\vartheta$ . The expansion breaks down in the collinear region. More precisely, it is invalid if  $(1 \pm \cos \vartheta)$  is so small that it is comparable in magnitude to  $M_{W,Z}^2/s$ . Terms bilinear in the  $\epsilon_i$  are not listed. I omit these terms for consistency, because taking into account bilinear terms one would also have to consider terms of dimension eight in the Lagrangian density, since these are, like the bilinear terms, proportional to  $\Lambda^{-4}$ .

Amplitudes that are not listed are either related to one of the listed amplitudes by one of the relations of Table II or do not have  $O(s)$  terms. I note that, in case (1), no  $O(s)$  terms are present whenever the standard amplitude does not approach zero in the limit  $s \rightarrow \infty$  except when all external bosons are in the longitudinal polarization state.<sup>8</sup> In the listing,

$$\begin{aligned} s_g &\equiv i \frac{g^2 s}{4M_W^2}, & s_e &\equiv i \frac{e^2 s}{4M_W^2}, & s_Z &\equiv i \frac{e^2 s}{4M_Z^2}, \\ s_W &\equiv \sin \theta_W, & c_W &\equiv \cos \theta_W, & t_W &\equiv \tan \theta_W. \end{aligned}$$

The meaning of the variables  $\tilde{s}$ ,  $\tilde{t}$ , and  $\tilde{u}$  is

$$\tilde{s} = \frac{1}{4} \frac{s}{s - M_H^2} - \frac{1}{4}, \quad \tilde{t} = \frac{1}{4} \frac{s(1 - \cos \vartheta)}{t - M_H^2} + \frac{1}{2},$$

$$\tilde{u} = \frac{1}{4} \frac{s(1 + \cos \vartheta)}{u - M_H^2} + \frac{1}{2}.$$

However, in the two cases for which the expansion is valid, these expressions are simplified: For case (1), the variables are  $O(s^{-1})$  and are multiplied by at most one power of  $s$  in the amplitudes. Therefore, one can set  $\tilde{s} = \tilde{t} = \tilde{u} = 0$  for case (1). Case (2) is obtained by setting  $\tilde{s} = -\frac{1}{4}$  and  $\tilde{t} = \tilde{u} = \frac{1}{2}$ .

<sup>8</sup>High-energy forms for the standard amplitudes with a light Higgs particle can be found in [32].



$W^-W^+ \rightarrow W^-W^+$

$$\begin{aligned}\mathcal{M}_{0000} &= -4(1 - 4\epsilon_{W\Phi})s_g\tilde{s} - (1 - 4\epsilon_{W\Phi})s_g\tilde{t}(1 - \cos\vartheta) + 3(\epsilon_{W\Phi} + t_W^2\epsilon_{B\Phi})s_g(1 + \cos\vartheta), \\ \mathcal{M}_{00++} &= -8\epsilon_{W\Phi}s_g\tilde{s} + \epsilon_{W\Phi}s_g\cos\vartheta - 2\epsilon_{W\Phi}s_g, \\ \mathcal{M}_{0+0-} &= 2\epsilon_{W\Phi}s_g\tilde{t}(1 - \cos\vartheta) + \left(\epsilon_{W\Phi} - \frac{1}{2}\epsilon_W\right)s_g\cos\vartheta - \left(\epsilon_{W\Phi} + \frac{3}{2}\epsilon_W\right)s_g, \\ \mathcal{M}_{++++} &= -2\epsilon_{W\Phi}s_g(1 + \cos\vartheta), \\ \mathcal{M}_{++--} &= -4\epsilon_{W\Phi}s_g(1 + \cos\vartheta); \end{aligned}$$

$W^+W^+ \rightarrow W^+W^+$

$$\begin{aligned}\mathcal{M}_{0000} &= -(1 - 4\epsilon_{W\Phi})s_g\tilde{t}(1 - \cos\vartheta) - (1 - 4\epsilon_{W\Phi})s_g\tilde{u}(1 + \cos\vartheta) - 6(\epsilon_{W\Phi} + t_W^2\epsilon_{B\Phi})s_g, \\ \mathcal{M}_{0+0-} &= 2\epsilon_{W\Phi}s_g\tilde{t}(1 - \cos\vartheta) + \left(\epsilon_{W\Phi} + \frac{1}{2}\epsilon_W\right)s_g\cos\vartheta - \left(\epsilon_{W\Phi} - \frac{3}{2}\epsilon_W\right)s_g, \\ \mathcal{M}_{++++} &= 4\epsilon_{W\Phi}s_g, \\ \mathcal{M}_{++--} &= 8\epsilon_{W\Phi}s_g; \end{aligned}$$

$W^-W^+ \rightarrow ZZ$

$$\begin{aligned}\mathcal{M}_{0000} &= -4(1 - 4\epsilon_{W\Phi} - 2t_W^2\epsilon_{B\Phi})s_g\tilde{s} + 6\epsilon_{W\Phi}s_g, \\ \mathcal{M}_{00++} &= -8(\epsilon_{W\Phi} + t_W^2\epsilon_{B\Phi})s_g\tilde{s} + 2[2s_W^2(\epsilon_{W\Phi} + \epsilon_{B\Phi}) - \epsilon_{W\Phi} - t_W^2\epsilon_{B\Phi}]s_g, \\ \mathcal{M}_{0+0-} &= -\frac{1}{2}(\epsilon_{W\Phi} + \epsilon_{B\Phi})s_g\frac{s_W^2}{c_W}(1 - \cos\vartheta) - \frac{1}{2}\epsilon_{W\Phi}s_gc_W(3 + \cos\vartheta), \\ \mathcal{M}_{++00} &= -8\epsilon_{W\Phi}s_g\tilde{s} - 2\epsilon_{W\Phi}s_g, \\ \mathcal{M}_{++++} &= \mathcal{M}_{-+-+} = -4\epsilon_{W\Phi}s_gc_W^2, \\ \mathcal{M}_{++--} &= -8\epsilon_{W\Phi}s_gc_W^2; \end{aligned}$$

$W^-Z \rightarrow W^-Z$

$$\begin{aligned}\mathcal{M}_{0000} &= -(1 + 4\epsilon_{W\Phi} - 2t_W^2\epsilon_{B\Phi})s_g\tilde{t}(1 - \cos\vartheta) - 3\epsilon_{W\Phi}s_g(1 - \cos\vartheta), \\ \mathcal{M}_{00++} &= -(\epsilon_{W\Phi} + \epsilon_{B\Phi})s_g\frac{s_W^2}{c_W} + \epsilon_{W\Phi}s_gc_W\cos\vartheta, \\ \mathcal{M}_{0+0-} &= 2(\epsilon_{W\Phi} + t_W^2\epsilon_{B\Phi})s_g\tilde{t}(1 - \cos\vartheta) - 2(\epsilon_{W\Phi} + \epsilon_{B\Phi})s_gc_W^2(1 - \cos\vartheta) \\ &\quad - \epsilon_{B\Phi}s_g\frac{1 - \cos\vartheta}{c_W^2} + (\epsilon_{W\Phi} + 3\epsilon_{B\Phi})s_g(1 - \cos\vartheta), \\ \mathcal{M}_{0+-0} &= \frac{1}{2}(\epsilon_{W\Phi} + \epsilon_{B\Phi})s_g\frac{s_W^2}{c_W}(1 + \cos\vartheta) - \frac{1}{2}\epsilon_{W\Phi}s_gc_W(3 - \cos\vartheta), \\ \mathcal{M}_{+0-0} &= 2\epsilon_{W\Phi}s_g\tilde{t}(1 - \cos\vartheta) - \epsilon_{W\Phi}s_g(1 - \cos\vartheta), \\ \mathcal{M}_{++++} &= \mathcal{M}_{-+-+} = 2\epsilon_{W\Phi}s_gc_W^2(1 - \cos\vartheta), \\ \mathcal{M}_{++--} &= 4\epsilon_{W\Phi}s_gc_W^2(1 - \cos\vartheta); \end{aligned}$$

$ZZ \rightarrow ZZ$

$$\begin{aligned}\mathcal{M}_{0000} &= (1 - 4c_W^2\epsilon_{W\Phi} - 4s_W^2\epsilon_{B\Phi})s_g[-4\tilde{s} - \tilde{t}(1 - \cos\vartheta) - \tilde{u}(1 + \cos\vartheta)], \\ \mathcal{M}_{00++} &= -8(\epsilon_{W\Phi} + t_W^2\epsilon_{B\Phi})s_g\tilde{s} - 2(\epsilon_{W\Phi} + t_W^2\epsilon_{B\Phi})s_g, \\ \mathcal{M}_{0+0-} &= 2(\epsilon_{W\Phi} + t_W^2\epsilon_{B\Phi})s_g\tilde{t}(1 - \cos\vartheta) - (\epsilon_{W\Phi} + t_W^2\epsilon_{B\Phi})s_g(1 - \cos\vartheta); \end{aligned}$$

$\gamma\gamma \rightarrow W^-W^+$

$$\begin{aligned}\mathcal{M}_{++00} &= -4(\epsilon_{W\Phi} + \epsilon_{B\Phi})s_e, \\ \mathcal{M}_{++++} &= \mathcal{M}_{-+-+} = -4\epsilon_{W\Phi}s_e, \\ \mathcal{M}_{++--} &= -8\epsilon_{W\Phi}s_e; \end{aligned}$$

$\gamma W^- \rightarrow \gamma W^-$

$$\begin{aligned}\mathcal{M}_{+0-0} &= -2(\epsilon_{W\Phi} + \epsilon_{B\Phi})s_e(1 - \cos\vartheta), \\ \mathcal{M}_{++++} &= \mathcal{M}_{++--} = 2\epsilon_W s_e(1 - \cos\vartheta), \\ \mathcal{M}_{+-+} &= 4\epsilon_W s_e(1 - \cos\vartheta); \end{aligned}$$

$\gamma Z \rightarrow W^- W^+$

$$\begin{aligned}\mathcal{M}_{+00-} &= -\frac{1}{2}(\epsilon_{W\Phi} + \epsilon_{B\Phi})\frac{s_e}{s_W}(1 + \cos\vartheta) - \frac{1}{2}\epsilon_W\frac{s_e}{s_W}(-3 + \cos\vartheta), \\ \mathcal{M}_{+0-0}(\vartheta) &= -\mathcal{M}_{+00-}(\pi - \vartheta), \\ \mathcal{M}_{++00} &= -4(\epsilon_{W\Phi} - \epsilon_{B\Phi})\frac{s_e\tilde{s}}{s_W c_W} + (\epsilon_{W\Phi} - \epsilon_{B\Phi})\frac{s_e}{s_W c_W} - 4(\epsilon_{W\Phi} - \tan^2\theta_W\epsilon_{B\Phi})s_e\frac{c_W}{s_W}, \\ \mathcal{M}_{++++} &= \mathcal{M}_{++--} = \mathcal{M}_{+-++} = \mathcal{M}_{+---} = -4\epsilon_W s_e\frac{c_W}{s_W}, \\ \mathcal{M}_{+-+} &= -8\epsilon_W s_e\frac{c_W}{s_W}; \end{aligned}$$

$W^- \gamma \rightarrow W^- Z$

$$\begin{aligned}\mathcal{M}_{0+0-} &= \left(\frac{1}{2}(\epsilon_{W\Phi} + 3\epsilon_{B\Phi}) - 2c_W^2(\epsilon_{W\Phi} + \epsilon_{B\Phi})\right)\frac{s_e}{c_W s_W}(1 - \cos\vartheta), \\ \mathcal{M}_{0+-0} &= -\frac{1}{2}(\epsilon_{W\Phi} + \epsilon_{B\Phi})\frac{s_e}{s_W}(1 + \cos\vartheta) + \frac{1}{2}\epsilon_W\frac{s_e}{s_W}(-3 + \cos\vartheta), \\ \mathcal{M}_{++00} &= (\epsilon_{W\Phi} + \epsilon_{B\Phi})\frac{s_e}{s_W} + \epsilon_W\frac{s_e}{s_W}\cos\vartheta, \\ \mathcal{M}_{++++} &= \mathcal{M}_{++--} = \mathcal{M}_{+-++} = \mathcal{M}_{+---} = \frac{1}{2}\mathcal{M}_{+---}, \\ &= 2\epsilon_W\frac{c_W}{s_W}s_e(1 - \cos\vartheta), \end{aligned}$$

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