Errata

Erratum: Analytic solution of the relativistic Coulomb problem for a spinless Salpeter equation [Phys. Rev. D 28, 396 (1983)]

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PACS number(s): 03.65.Pm, 03.65.Ge, 99.10.+g

The supposedly exact formula for the bound-state spectrum of the Hamiltonian

$$H = 2\sqrt{-\nabla^2 + m^2} - \frac{\alpha}{r} \tag{1}$$

given in Eq. (66) of this paper is incorrect, as has been pointed out by a number of people [1–3]. The perturbative check which we had made unfortunately contained an error such that the result fortuitously agreed with that in Eq. (66) to $O(\alpha^4)$, the highest order checked. The argument given preceding the equation works for the nonrelativistic problem. In that case, $\mathrm{disc}\Phi_1(p)$ reduces to a constant multiple of $[(p_0-p)/(p_0+p)]^{-i\eta}$, and $\Phi_1(p)$ has the same form, $\Phi_1 \propto [(p-p_0)/(p+p_0)]^{-i\eta}$ for general complex p. The condition $i\eta = 1, 2, \ldots$ removes the branch point at $p = -i|p_0|$ in the bound-state problem, and the first of Eqs. (61) gives the usual hydrogenic wave functions. The argument fails here because of the different analytic structure involved; i.e., $\Phi_1(p)$ as defined by the integral in Eq. (54) is not simply a continuation of $\mathrm{disc}\Phi_1$. We have not obtained a corrected spectrum. However, a systematic expansion in powers of α, based in part on a transformation introduced in our paper, has been developed recently by LeYaouanc, Oliver, and Raynal [4].

The remaining principal results in the paper—the solution of the one-dimensional problem, the expression for the critical coupling in Eq. (96), and the expansion of the wave function near the origin— have been checked by other methods [5] and other authors [2,6], and are unaffected by the error in Eq. (66).

We regret that we did not point out the problem with the spectrum earlier.

[1] The error has been pointed out to us by T. Imbo, J. Sucher, W.H.E. Schwarz, J.C. Raynal, and A. LeYaouanc (private communications). An equivalent question was raised about the limiting behavior of the formula as α approaches its critical value by V. Singh.

[2] G. Hardekopf and J. Sucher, Phys. Rev. A **31**, 2020 (1985), explicitly note the error in the spectrum and our agreement with their conclusion.

[3] W. Lucha and F.F. Schöberl, Phys. Rev. D **50**, 5443 (1994).

[4] A. LeYaouanc, L. Oliver, and J.-C. Raynal, Orsay Report No. LPTHE 93/43 (unpublished).

[5] L.J. Nickisch, L. Durand, and B. Durand, Phys. Rev. D **30**, 660 (1984), especially the appendix to the paper.

[6] P. Castorina, P. Cea, G. Nardulli, and G. Paiano, Phys. Rev. D **29**, 2660 (1984).

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Erratum: Effective potential and first-order phase transitions: Beyond leading order [Phys. Rev. D 47, 3546 (1993)]

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We made a number of misprints and omissions when transcribing our formulas into Appendix A, and the following changes should be made.

The formula for λ_{bare} in (A11) should read

$$\lambda_{\mathrm{bare}} = \left[\lambda + \left(\frac{27}{8}g_2^4 + \frac{9}{4}g_2^2 g_1^2 + \frac{9}{8}g_1^4 - 18 g_y^4\right)\frac{1}{(4\pi)^2\epsilon} + \cdots\right]\mu^{2\epsilon}.$$

In (A19l), $g_s^2 - \frac{1}{12}g_1^2$ should be replaced by $g_s^2 + \frac{1}{12}g_1^2$.

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In the last line of (A19m), $\mathcal{D}_{LLT}(M_{LW},0,M_W)$ should be replaced by $\mathcal{D}_{LLT}(M_W,0,M_W)$.

In the last line of (A19o), $M_W\Delta\mathcal{M}_2$ should be replaced by

$$M_W[\Delta\mathcal{M}_2 + (\cos\theta_W - 1)(M_{LW} - M_W)]\,.$$

In the last line of (A25), $\mathcal{D}_{LLT}(M_{L1},M_{L2},M_{L3})$ should be replaced by $\mathcal{D}_{LLT}(M_{L1},M_{L2},M_3)$.

In (A45), $-MT^3/(4\pi)$ should be added to the right-hand side.

In addition, in the main text, the equation $\sigma = \bar{M}_W/g$ in the sentence after (8.3) should read $\sigma = 2\bar{M}_W/g$. The last term of (4.19), $-(e^4\phi^2/4\pi)(2M^{-1} + M_L^{-1})T$, should be multiplied by the Kronecker delta δ_{i1}.

None of the above errors affect the numerical results because they were errors we made in presenting our formulas rather than original errors in the calculation. However, there was a minor error in our original evaluation of (A19o) that did affect our numerical work, where we mistakenly used $\tilde{\theta}$ in one instance where we should have used θ_W. Correcting our error turns out to have no noticeable impact on our numerical results (the results change by 1 to 2%) because $U(1)$ effects are not very significant.

We should comment that the sort of analysis we made in (3.33), of the order of diagrammatic contributions, is dangerous and invalid. The analysis of the diagrams, and of the effects of resummation, is really only justified by the method described in footnote 11 on that page. The expansion and argument by dimensional regularization we presented in (3.33) was given because it seemed to simplify our presentation in the paper, but it is incorrect. In general, dimensionally regularized integration does not commute with series expansions unless there is some single dimension in which the integrals of all the terms converge. [As a simple explicit example, try the one-dimensional integral

$$\int \frac{dp}{(p+m)(p+T)}$$

by first expanding $(p+T)^{-1}$ as a series in p/T.] In particular, the integral (3.33) turns out to be $O(g^4T^4)$ and is evaluated in Ref. [1] below. However, none of this affects the other formulas in the paper, such as (3.35), or our final results.

Finally, we take this opportunity to note that the numerical constant $c_H \approx 5.3025$ of (3.18) has since been found analytically [2]:

$$c_H = 2\left(\gamma_E - \frac{\zeta'(2)}{\zeta(2)} + \ln\frac{9}{2}\right)\,.$$

We are grateful to Zoltan Fodor, Arthur Hebecker, and Misha Shaposhnikov for pointing out most of the above corrections. Fodor and Hebecker have recently computed the scalar $O(\lambda^2)$ pieces of the potential in Ref. [3] below.

[1] K. Farakos, K. Kajantie, K. Rummukainen, and M. Shaposhnikov, CERN Report No. CERN-TH.6973/94 (unpublished).
[2] P. Arnold and C. Zhai, Univ. of Washington Report No. UW/PT-94-03 (unpublished).
[3] Z. Fodor and A. Hebecker, DESY Report No. DESY 94-025 (unpublished).