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Landau gauge within the Gribov horizon

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We consider a model which effectively restricts the functional integral of Yang-Mills theories to the fundamental modular region. Using algebraic arguments, we prove that this theory has the same divergences as the ordinary Yang-Mills theory in the Landau gauge and that it is unitary. The restriction of the functional integral is interpreted as a kind of spontaneous breakdown of the BRS symmetry.

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I. INTRODUCTION

A perturbative expansion of gauge field theories requires gauge fixing. In 1978 Gribov [1] pointed out that covariant gauges (and most others that can be cast in the form of a local effective action) do not uniquely specify a single configuration on a gauge orbit in non-Abelian theories. This ambiguity can be disregarded when discussing high-energy processes and does not affect the well-known perturbative results of asymptotic freedom, or equivalently, asymptotic scaling. At low energies, the presence of additional gauge copies can however no longer be overlooked [1], and this is at least one reason why a perturbative analysis is found to fail in this case.

Gribov [1] suggested to restrict the functional integration to the space of configurations A which are transverse,

$$\partial A = 0 \ , \tag{1.1}$$

and such that

$$\partial D(A) \leq 0 \ , \tag{1.2}$$

where $D(A)$ is the covariant derivative. The boundary of the region so defined is called the Gribov horizon and lies inside a certain ellipsoid [2].

The restriction to the region defined by (1.1) and (1.2) would already imply that the gluon propagator differs from the usual one at low momenta [1],

$$D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{k^2}{k^4 + g^2 N\gamma^2} \ , \tag{1.3}$$

and depends on a dimensionful parameter γ.

The fundamental modular region (FMR), where the Hilbert norm of the connection

$$F_A(U) \equiv \|A^U\|^2 = \|U^\dagger A U + U^\dagger dU\|^2 \tag{1.4}$$

attains its absolute minimum with respect to gauge transformations U [3,4] ,

$$\text{FMR} \equiv \{A : F_A(1) \leq F_A(U)\} \ , \tag{1.5}$$

is a proper subset of the region defined by Gribov, since (1.1) and (1.2) characterize any relative minimum of the Hilbert norm (1.4), and such relative minima have been shown to exist [5]. The FMR is therefore a refinement of Gribov's region and in particular of the definition of the Landau gauge (1.1) and has for this reason been called the "minimal" Landau gauge [4]. Zwanziger proposed a local action which concretely implements the restriction to the FMR when a certain nonperturbative "horizon condition" [6,7] is satisfied. He studied this mechanism in the continuum [6] as well as in the critical limit of lattice gauge theory [7,8]. Quite remarkably this version of the SU(N) Yang-Mills (YM) theory naturally gives a gluon propagator of the Gribov type (1.3) where the parameter γ is self-consistently determined by the "horizon condition" [6,7].

The required locality of the classical action immediately raises the question of renormalizability: whether new divergences or anomalies not present in the original YM theory have been introduced, which would imply that the proposed gauge fixing is not renormalizable. In Ref. [9] an analysis based on the Becchi-Rouet-Stora (BRS) symmetry of the model indicated that radiative corrections develop at most four divergences. Arguments were put forward that only two of these should occur in a perturbative expansion.

The Landau gauges, the restriction of the FMR being the "minimal" one, have well-known nonrenormalization properties [10]. In this paper we algebraically recover these properties for the new gauge model, namely, that only two independent renormalization constants are needed, and thus confirm also from the renormalization point of view that the model belongs to the class of Landau gauge theories. We also prove that the restriction to the FMR does not spoil the unitarity of the theory.

The paper is organized as follows. In Sec. II, we present the symmetries of the model, which include a Ward iden-

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tity typical for the Landau gauge [11]. The full algebraic structure allows us to give in Sec. III a formal proof that indeed only two divergences are present. At the same time we demonstrate that the whole algebra is free of anomalies. Section IV is devoted to an interpretation of the model at the nonvanishing physical value for the external sources. We show that this is equivalent to a kind of spontaneous breakdown of the BRS symmetry. The results are then summarized in Sec. V.

II. THE MODEL AND ITS ALGEBRAIC STRUCTURE

To constrain the functional integral to the FMR, additional fields $(\omega_i^a, \varphi_i^a, \overline{\omega}^{ai}, \overline{\varphi}^{ai})$ and external sources $(U_\mu^{ai}, V_{\mu i}^a, M_\mu^{ai}, N_{\mu i}^a)$ were introduced [9] into the original Yang-Mills theory. These new fields transform under a global U(f) symmetry on the composite index $i=(\mu,a)$, with $f=4(N^2-1)$.

The model in Euclidean space-time is described by the action [9]

$$S = S_{\rm LYM} + s\int d^4x[(\partial_\mu\overline{\omega}^{ai})(D_\mu\varphi_i)^a + U_\mu^{ai}(D_\mu\varphi_i)^a + V_{\mu i}^a(D_\mu\overline{\omega}^i)^a + U_\mu^{ai}V_{\mu i}^a] , \tag{2.1}$$

where $S_{\rm LYM}$ is the ordinary Yang-Mills action in the Landau gauge,

$$S_{\rm LYM} = \frac{1}{4g^2}\int d^4x F_{\mu\nu}^a F_{\mu\nu}^a - s\int d^4x(\partial_\mu\overline{c}^a)A_\mu^a , \tag{2.2}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc}A_\mu^b A_\nu^c , \tag{2.3}$$

and the covariant derivative is defined as

$$(D_\mu X)^a = \partial_\mu X^a + f^{abc}A_\mu^b X^c , \tag{2.4}$$

with f^{abc} being the structure constants of the gauge group. Finally, the operator s is the ordinary BRS transformation extended to the additional fields

$$\begin{aligned} sA_\mu^a &= -(D_\mu c)^a , \\ sc^a &= \frac{1}{2}f^{abc}c^b c^c , \\ s\overline{c}^a &= b^a , \quad sb^a = 0 , \\ s\overline{\omega}^{ai} &= \overline{\varphi}^{ai} , \quad s\overline{\varphi}^{ai} = 0 , \\ s\varphi_i^a &= \omega_i^a , \quad s\omega_i^a = 0 , \end{aligned} \tag{2.5}$$

and to the sources by

$$\begin{aligned} sU_\mu^{ai} &= M_\mu^{ai} , \quad sM_\mu^{ai} = 0, \\ sV_{\mu i}^a &= N_{\mu i}^a , \quad sN_{\mu i}^a = 0 . \end{aligned} \tag{2.6}$$

One can easily verify the nilpotency of the BRS operator (2.5) and (2.6):

$$s^2 = 0 . \tag{2.7}$$

In Ref. [9] it is argued that the physical value for the sources is

$$\begin{aligned} N_{\mu\nu}^{ab} &= U_{\mu\nu}^{ab} = 0 , \\ M_{\mu\nu}^{ab} &= -V_{\mu\nu}^{ab} = \gamma\delta_{\mu\nu}\delta^{ab} , \end{aligned} \tag{2.8}$$

where γ is a parameter of dimension [mass]^2, whose value is determined by a self-consistency condition that will be discussed in Sec. IV. The BRS symmetry (2.5) and (2.6) is the simplest which is cohomologically equivalent to the ordinary one, because the additional fields transform as doublets [12]. With the BRS transformation (2.5) and (2.6), the action corresponding to (2.1) explicitly is[1]

$$\begin{aligned} S = \int d^4x\bigg(&\frac{1}{4g^2}F_{\mu\nu}^a F_{\mu\nu}^a - (\partial_\mu b^a)A_\mu^a - (\partial_\mu\overline{c}^a)(D_\mu c)^a + (\partial_\mu\overline{\varphi}^{ai})(D_\mu\varphi_i)^a - (\partial_\mu\overline{\omega}^{ai})(D_\mu\omega_i)^a \\ &+ f^{abc}(\partial_\mu\overline{\omega}^{ai})(D_\mu c)^b\varphi_i^c + M_\mu^{ai}(D_\mu\varphi_i)^a - U_\mu^{ai}(D_\mu\omega_i)^a + f^{abc}U_\mu^{ai}(D_\mu c)^b\varphi_i^c \\ &+ N_{\mu i}^a(D_\mu\overline{\omega}^i)^a + V_{\mu i}^a(D_\mu\overline{\varphi}^i)^a - f^{abc}V_{\mu i}^a(D_\mu c)^b\overline{\omega}^{ci} + M_\mu^{ai}V_{\mu i}^a - U_\mu^{ai}N_{\mu i}^a\bigg) , \end{aligned} \tag{2.9}$$

where the dimension and ghost charge assignments of the fields are summarized in Table I.

As customary, we couple external sources to the nonlinear BRS variations in (2.5) of the quantum fields. The full classical action

$$\Sigma = S + S_{\rm ext} , \tag{2.10}$$

with

$$S_{\rm ext} = \int d^4x[K_\mu^a(sA_\mu^a) + L^a(sc^a)] , \tag{2.11}$$

then satisfies the Slavnov identity

$$\mathcal{S}(\Sigma) = 0 , \tag{2.12}$$

where

[1] Our conventions differ from those of [9] and the φ-$\overline{\varphi}$ propagator of (2.9) is positive.

$$\mathcal{S}(\Sigma)=\int d^4x\left(\frac{\delta\Sigma}{\delta K^a_\mu}\frac{\delta\Sigma}{\delta A^a_\mu}+\frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^a}+b^a\frac{\delta\Sigma}{\delta\bar c^a}+\omega^a_i\frac{\delta\Sigma}{\delta\varphi^a_i}+\overline{\varphi}^{ai}\frac{\delta\Sigma}{\delta\overline{\omega}^{ai}}+M^{ai}_\mu\frac{\delta\Sigma}{\delta U^{ai}_\mu}+N^a_{\mu i}\frac{\delta\Sigma}{\delta V^a_{\mu i}}\right), \tag{2.13}$$

and the corresponding linearized operator

$$\mathcal{B}_\Sigma=\int d^4x\left(\frac{\delta\Sigma}{\delta K^a_\mu}\frac{\delta}{\delta A^a_\mu}+\frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta K^a_\mu}+\frac{\delta\Sigma}{\delta L^a}\frac{\delta}{\delta c^a}+\frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta L^a}+b^a\frac{\delta}{\delta\bar c^a}+\omega^a_i\frac{\delta}{\delta\varphi^a_i}+\overline{\varphi}^{ai}\frac{\delta}{\delta\overline{\omega}^{ai}}+M^{ai}_\mu\frac{\delta}{\delta U^{ai}_\mu}+N^a_{\mu i}\frac{\delta}{\delta V^a_{\mu i}}\right), \tag{2.14}$$

is nilpotent:

$$\mathcal{B}_\Sigma\mathcal{B}_\Sigma=0\ . \tag{2.15}$$

For theories in the Landau gauge, the integrated ghost equation of motion gives a Ward identity [11], which in our case is

$$\mathcal{G}^a\Sigma=\Delta^a\ , \tag{2.16}$$

where

$$\mathcal{G}^a=\int d^4x\left[\frac{\delta}{\delta c^a}+f^{abc}\left(\bar c^b\frac{\delta}{\delta b^c}+\varphi^b_i\frac{\delta}{\delta\omega^c_i}+\overline{\omega}^{bi}\frac{\delta}{\delta\overline{\varphi}^{ci}}+V^b_{\mu i}\frac{\delta}{\delta N^c_{\mu i}}+U^{bi}_\mu\frac{\delta}{\delta M^{ci}_\mu}\right)\right] \tag{2.17}$$

and

$$\Delta^a=\int d^4x f^{abc}(K^b_\mu A^c_\mu-L^bc^c) \tag{2.18}$$

is a linear breaking in the quantum fields, and, thus, only present at the classical level.

The anticommutator between the ghost equation (2.16) and the Slavnov identity (2.12) is known to give the ward identity of rigid gauge invariance [11] [see (2.33)]:

$$\mathcal{H}^a_{\rm rig}\Sigma=0\ , \tag{2.19}$$

where

$$\mathcal{H}^a_{\rm rig}=\sum_{(\text{all fields }\Phi)}\int d^4x\, f^{abc}\Phi^b\frac{\delta}{\delta\Phi^c}\ . \tag{2.20}$$

We can also write the Ward identity

$$\mathcal{F}^i\Sigma=0\ , \tag{2.21}$$

with

$$\mathcal{F}^i=\int d^4x\left(c^a\frac{\delta}{\delta\omega^a_i}-\overline{\omega}^{ai}\frac{\delta}{\delta\bar c^a}-U^{ai}_\mu\frac{\delta}{\delta K^a_\mu}\right)\ . \tag{2.22}$$

Commuting $\mathcal{F}^i$ with the Slavnov operator (2.13), another nonlinear symmetry emerges [see (2.33)]:

$$\mathcal{J}^i(\Sigma)=0\ , \tag{2.23}$$

with

$$\mathcal{J}^i(\Sigma)=\int d^4x\left(c^a\frac{\delta\Sigma}{\delta\varphi^a_i}-\frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta\omega^a_i}+\overline{\varphi}^{ai}\frac{\delta\Sigma}{\delta\bar c^a}+M^{ai}_\mu\frac{\delta\Sigma}{\delta K^a_\mu}\right)\ . \tag{2.24}$$

In addition we have the symmetry

$$\mathcal{R}^j_i\Sigma=0\ , \tag{2.25}$$

where

$$\mathcal{R}^j_i=\int d^4x\left(\varphi^a_i\frac{\delta}{\delta\omega^a_j}+V^a_{\mu i}\frac{\delta}{\delta N^a_{\mu j}}-\overline{\omega}^{aj}\frac{\delta}{\delta\overline{\varphi}^{ai}}-U^{aj}_\mu\frac{\delta}{\delta M^{ai}_\mu}\right)\ . \tag{2.26}$$

Anticommuting the symmetry (2.25) with the Slavnov identity (2.12) we get the Ward identity of the U(f) symmetry [see (2.33)]:

$$\mathcal{U}^j_i\Sigma=0\ , \tag{2.27}$$

where

$$\mathcal{U}^j_i=\int d^4x\left(\varphi^a_i\frac{\delta}{\delta\varphi^a_j}+\omega^a_i\frac{\delta}{\delta\omega^a_j}+V^a_{\mu i}\frac{\delta}{\delta V^a_{\mu j}}+N^a_{\mu i}\frac{\delta}{\delta N^a_{\mu j}}-\overline{\varphi}^{aj}\frac{\delta}{\delta\overline{\varphi}^{ai}}-\overline{\omega}^{aj}\frac{\delta}{\delta\overline{\omega}^{ai}}-U^{ai}_\mu\frac{\delta}{\delta U^{ai}_\mu}-M^{aj}_\mu\frac{\delta}{\delta M^{ai}_\mu}\right)\ . \tag{2.28}$$

By means of the diagonal operator $Q_f=\mathcal{U}^i_i$ the i-valued fields are assigned an additional quantum number. In Table I we summarize the quantum numbers of the fields and sources. Apart from the familiar gauge condition and antighost equation

$$\frac{\delta\Sigma}{\delta b^a}-\partial_\mu A^a_\mu=0\ , \tag{2.29}$$

$$\partial^\mu\frac{\delta\Sigma}{\delta K^{a\mu}}+\frac{\delta\Sigma}{\delta\bar c^a}=0\ , \tag{2.30}$$

this model is also characterized by the set of local equations

TABLE I. Dimensions, Faddeev-Popov charges, and Q_f numbers of the fields.

	A	c	$\bar c$	b	ω	$\overline{\omega}$	φ	$\overline{\varphi}$	K	L	M	N	U	V
dim	1	0	2	2	1	1	1	1	3	4	2	2	2	2
ΦΠ	0	1	−1	0	1	−1	0	0	−1	−2	0	1	−1	0
Q_f	0	0	0	0	1	−1	1	−1	0	0	−1	1	−1	1

$$\mathcal{T}_{(\Phi)}\Sigma = \Delta_{(\Phi)} , \tag{2.31}$$

where

$$\begin{aligned}
\mathcal{T}^{ai}_{(\omega)} &= \frac{\delta}{\delta\omega^a_i} + \partial_\mu \frac{\delta}{\delta N^a_{\mu i}} + f^{abc}\overline{\omega}^{bi}\frac{\delta}{\delta b^c} , \\
\mathcal{T}^{ai}_{(\varphi)} &= \frac{\delta}{\delta\varphi^a_i} + \partial_\mu \frac{\delta}{\delta V^a_{\mu i}} \\
&\quad + f^{abc}\left(\overline{\varphi}^{bi}\frac{\delta}{\delta b^c} + \overline{\omega}^{bi}\frac{\delta}{\delta \overline{c}^c} + U^{bi}_\mu \frac{\delta}{\delta K^c_\mu}\right) , \\
\mathcal{T}^a_{(\overline{\varphi})i} &= \frac{\delta}{\delta\overline{\varphi}^{ai}} + \partial_\mu \frac{\delta}{\delta M^{ai}_\mu} , \\
\mathcal{T}^a_{(\overline{\omega})i} &= \frac{\delta}{\delta\overline{\omega}^{ai}} + \partial_\mu \frac{\delta}{\delta U^{ai}_\mu} + f^{abc}V^b_{\mu i}\frac{\delta}{\delta K^c_\mu} , \\
\Delta^{ai}_{(\omega)} &= f^{abc}U^{bi}_\mu A^c_\mu , \\
\Delta^{ai}_{(\varphi)} &= f^{abc}M^{bi}_\mu A^c_\mu , \\
\Delta^a_{(\overline{\varphi})i} &= f^{abc}V^b_{\mu i}A^c_\mu , \\
\Delta^a_{(\overline{\omega})i} &= -f^{abc}N^b_{\mu i}A^c_\mu .
\end{aligned} \tag{2.32}$$

In the proof of the renormalization of a model, the nonlinear algebra formed by the symmetry operators plays an important role because it yields consistency conditions on the counterterm and on the possible anomalies of the theory.

The only nontrivial algebraic relations are

$$\begin{aligned}
&\mathcal{B}_\Psi \mathcal{S}(\Psi) = 0 , \\
&\frac{\delta}{\delta b^a}\mathcal{S}(\Psi) - \mathcal{B}_\Psi\left(\frac{\delta\Psi}{\delta b^a} - \partial_\mu A^a_\mu\right) = \frac{\delta\Psi}{\delta\overline{c}^a} + \partial_\mu\frac{\delta\Psi}{\delta K^a_\mu} , \\
&\mathcal{G}^a\mathcal{S}(\Psi) + \mathcal{B}_\Psi(\mathcal{G}^a\Psi - \Delta^a) = \mathcal{H}^a_{\text{rig}}\Psi , \\
&\mathcal{F}^i\mathcal{S}(\Psi) - \mathcal{B}_\Psi\mathcal{F}^i\Psi = \mathcal{J}^i(\Psi) , \\
&\mathcal{T}^a_{(\overline{\varphi})i}\mathcal{S}(\Psi) - \mathcal{B}_\Psi(\mathcal{T}^a_{(\overline{\varphi})i}\Psi - \Delta^a_{(\overline{\varphi})i}) = \mathcal{T}^a_{(\overline{\omega})i}\Psi - \Delta^a_{(\overline{\omega})i} , \\
&\mathcal{T}^{ai}_{(\omega)}\mathcal{S}(\Psi) + \mathcal{B}_\Psi(\mathcal{T}^{ai}_{(\omega)}\Psi - \Delta^{ai}_{(\omega)}) = \mathcal{T}^{ai}_{(\varphi)}\Psi - \Delta^{ai}_{(\varphi)} , \\
&\mathcal{G}^a\mathcal{F}^i\Psi - \mathcal{F}^i(\mathcal{G}^a\Psi - \Delta^a) = \int d^4x(\mathcal{T}^{ai}_{(\omega)}\Psi - \Delta^{ai}_{(\omega)}) , \\
&\left(\frac{\delta}{\delta\overline{c}^a} + \partial_\mu\frac{\delta}{\delta K^a_\mu}\right)(\mathcal{G}^b\Psi - \Delta^b) + \mathcal{G}^b\left(\frac{\delta\Psi}{\delta\overline{c}^a} + \partial_\mu\frac{\delta\Psi}{\delta K^a_\mu}\right) \\
&\qquad = -f^{abc}\left(\frac{\delta\Psi}{\delta b^c} - \partial_\mu A^c_\mu\right) , \\
&\mathcal{T}^{ai}_{(\varphi)}(\mathcal{G}^b\Psi - \Delta^b) - \mathcal{G}^b(\mathcal{T}^{ai}_{(\varphi)}\Psi - \Delta^{ai}_{(\varphi)}) \\
&\qquad = -f^{abc}(\mathcal{T}^{ci}_{(\omega)}\Psi - \Delta^{ci}_{(\omega)}) , \\
&\mathcal{T}^a_{(\overline{\omega})i}(\mathcal{G}^b\Psi - \Delta^b) + \mathcal{G}^b(\mathcal{T}^a_{(\overline{\omega})i}\Psi - \Delta^a_{(\overline{\omega})i}) \\
&\qquad = -f^{abc}(\mathcal{T}^c_{(\overline{\varphi})i}\Psi - \Delta^c_{(\overline{\varphi})i}) , \\
&\mathcal{G}^a\mathcal{J}^i(\Psi) + \mathcal{J}^i_\Psi(\mathcal{G}^a\Psi - \Delta^a) = \int d^4x(\mathcal{T}^{ai}_{(\varphi)}\Psi - \Delta^{ai}_{(\varphi)}) , \\
&\mathcal{T}^a_{(\overline{\varphi})i}\mathcal{J}^j(\Psi) - \mathcal{J}^j_\Psi(\mathcal{T}^a_{(\overline{\varphi})i}\Psi - \Delta^a_{(\overline{\varphi})i}) = \delta^j_i\left(\frac{\delta\Psi}{\delta\overline{c}^a} + \partial_\mu\frac{\delta\Psi}{\delta K^a_\mu}\right) , \\
&\mathcal{T}^a_{(\overline{\omega})i}\mathcal{F}^j\Psi - \mathcal{F}^j(\mathcal{T}^a_{(\overline{\omega})i}\Psi - \Delta^a_{(\overline{\omega})i}) = -\delta^j_i\left(\frac{\delta\Psi}{\delta\overline{c}^a} + \partial_\mu\frac{\delta\Psi}{\delta K^a_\mu}\right) , \\
&\mathcal{R}^j_i\mathcal{S}(\Psi) + \mathcal{B}_\Psi\mathcal{R}^j_i\Psi = \mathcal{U}^j_i\Psi , \\
&\mathcal{R}^j_i\mathcal{J}^k(\Psi) + \mathcal{J}^k_\Psi\mathcal{R}^j_i\Psi = \delta^k_i\mathcal{F}^j\Psi , \\
&\mathcal{T}^{ai}_{(\varphi)}\mathcal{R}^j_k\Psi - \mathcal{R}^j_k(\mathcal{T}^{ai}_{(\varphi)}\Psi - \Delta^{ai}_{(\varphi)}) = \delta^i_k(\mathcal{T}^{aj}_{(\omega)}\Psi - \Delta^{aj}_{(\omega)}) , \\
&\mathcal{T}^a_{(\overline{\omega})i}\mathcal{R}^j_k\Psi + \mathcal{R}^j_k(\mathcal{T}^a_{(\overline{\omega})i}\Psi - \Delta^a_{(\overline{\omega})i}) = -\delta^j_i(\mathcal{T}^a_{(\overline{\varphi})k}\Psi - \Delta^a_{(\overline{\varphi})k}) ,
\end{aligned} \tag{2.33}$$

where Ψ is a generic functional of even ghost charge and $\mathcal{J}^i_\Psi$ is the linearized operator corresponding to (2.24).

The model we are considering turns out to be completely determined by the gauge condition (2.29) and the local equations (2.31), the Slavnov identity (2.12), the ghost equation (2.16), the symmetry (2.21), and the quantum numbers listed in Table I.

III. RENORMALIZATION

We prove the renormalizability of the model by first finding the most general counterterm compatible with the algebraic structure described in the previous section and then by showing that the symmetries considered hold to all orders of perturbation theory, i.e., that they are not anomalous.

A. Counterterm

According to the quantum action principle (QAP) [13], the counterterm is the most general integrated local functional Σ_Δ of dimension four with vanishing ghost and Q_f numbers satisfying the identities

$$\frac{\delta\Sigma_\Delta}{\delta b^a} = 0 ,$$

$$\partial^\mu\frac{\delta\Sigma_\Delta}{\delta K^{a\mu}} + \frac{\delta\Sigma_\Delta}{\delta\overline{c}^a} = 0 , \tag{3.1}$$

$$\mathcal{T}_{(\varphi)}\Sigma_\Delta = \mathcal{T}_{(\overline{\varphi})}\Sigma_\Delta = \mathcal{T}_{(\omega)}\Sigma_\Delta = \mathcal{T}_{(\overline{\omega})}\Sigma_\Delta = 0 ,$$

and

$$B_\Sigma\Sigma_\Delta = 0 , \tag{3.2}$$

$$\mathcal{G}^a\Sigma_\Delta = 0 , \tag{3.3}$$

$$\mathcal{F}^i\Sigma_\Delta = 0 . \tag{3.4}$$

The relations (3.1) imply [9] that Σ_Δ is in fact only a functional of

$$\Sigma_\Delta[A, c, \tilde{K}, L, \tilde{M}, \tilde{N}, \tilde{U}, \tilde{V}] , \tag{3.5}$$

where

$$\begin{aligned}\tilde{K}^a_\mu &= K^a_\mu + \partial_\mu \bar{c}^a + f^{abc}(U^{bi}_\mu + \partial_\mu \overline{\omega}^{bi})\varphi^c_i + f^{abc}V^b_{\mu i}\overline{\omega}^{ci} \ , \\ \tilde{M}^{ai}_\mu &= M^{ai}_\mu + \partial_\mu \overline{\varphi}^{ai} \ , \\ \tilde{N}^a_{\mu i} &= N^a_{\mu i} + \partial_\mu \omega^a_i \ , \\ \tilde{U}^{ai}_\mu &= U^{ai}_\mu + \partial_\mu \overline{\omega}^{ai} \ , \\ \tilde{V}^a_{\mu i} &= V^a_{\mu i} + \partial_\mu \varphi^a_i \ .\end{aligned} \tag{3.6}$$

The most general counterterm satisfying the Slavnov condition (3.2) with ghost and Q_f numbers zero therefore is

$$\Sigma_\Delta = c_0 \int d^4x F^a_{\mu\nu} F^a_{\mu\nu} + B_\Sigma \int d^4x (c_1 \tilde{K}^a_\mu A^a_\mu + c_2 \tilde{L}^a c^a + c_3 \tilde{U}^{ai}_\mu \tilde{V}^a_{\mu i}) \ , \tag{3.7}$$

where c_0, c_1, c_2, c_3 are arbitrary constants. Exploiting the ghost condition (3.3), we obtain

$$c_2 = 0 \ , \tag{3.8}$$

a reduction of the possible divergences peculiar for the Landau gauge [11]. Finally the constraint (3.4) implies that

$$c_1 = -c_3 \ . \tag{3.9}$$

We have thus algebraically shown that the model defined by the classical action (2.9) and the symmetries (2.12), (2.16), (2.21) has two divergences,

$$\Sigma_\Delta = c_0 \int d^4x F^a_{\mu\nu} F^a_{\mu\nu} + c_1 B_\Sigma \int d^4x (\tilde{K}^a_\mu A^a_\mu - \tilde{U}^{ai}_\mu \tilde{V}^a_{\mu i}) \ , \tag{3.10}$$

that can be absorbed through two independent multiplicative renormalization constants, in complete agreement with ordinary Yang-Mills theory in the Landau gauge [10]. The degrees of freedom introduced to constrain the functional integral to the FMR [6,7] therefore do not lead to additional divergences.

B. Anomalies

Since the new fields introduced to constrain the functional integral to the FMR all appear as BRS doublets, it is obvious that they do not belong to the cohomology of the Slavnov operator [12], which therefore is cohomologically equivalent to the ordinary one. Algebraically one only finds the usual Adler-Bardeen anomaly, whose coefficient is known to vanish if all fields transform according to real representations of the gauge group [14]. One still has to show that the other symmetries used to obtain the result (3.10) are not anomalous as well.

Although the conventional procedure [15] of individually implementing the identities defining the model is quite straightforward in our case thanks to the absence of gauge anomalies, we adopt here the technique of collecting the symmetries into one nilpotent operator by introducing global ghosts [16]. This method is particularly convenient when proving that a whole algebra of operators is free of anomalies.

It is trivial to show that the identities (2.29)–(2.31) hold for the quantum vertex functional

$$\Gamma = \Sigma + \Gamma^{(\mathrm{qu})} \ , \tag{3.11}$$

where $\Gamma^{(\mathrm{qu})}$ is at least of order $\hbar$. In Sec. IV, we will exploit that $\Gamma^{(\mathrm{qu})}$ is consequently a functional of the combinations (3.6), c, and the connection A only.

To collect the symmetries (2.12), (2.16), (2.19), (2.21), and (2.23) into one operator, we first consider the transformation on the fields generated by

$$\begin{aligned}Q = s &+ \xi^a \mathcal{G}^a + \eta^a \mathcal{H}^a_{\mathrm{rig}} + \lambda_i \mathcal{F}^i_0 + \varrho_i \mathcal{J}^i_0 + \sigma^a_i \tilde{T}^{ai}_{(\omega)} + \tau^a_i \tilde{T}^{ai}_{(\varphi)} \\ &- (\xi^a - \tfrac{1}{2} f^{abc}\eta^b\eta^c)\frac{\partial}{\partial \eta^a} - \lambda_i \frac{\partial}{\partial \varrho_i} + f^{abc}\eta^b\xi^c \frac{\partial}{\partial \xi^a} \\ &+ [\xi^a \lambda_i + f^{abc}(\eta^b \sigma^c_i - \xi^b \tau^c_i)]\frac{\partial}{\partial \sigma^a_i} \\ &- (\sigma^a_i + \xi^a \varrho_i - f^{abc}\eta^b \tau^c_i)\frac{\partial}{\partial \tau^a_i} \ ,\end{aligned} \tag{3.12}$$

where

$$\mathcal{F}^i_0 = \int d^4x \left(c^a \frac{\delta}{\delta \omega^a_i} - \overline{\omega}^{ai} \frac{\delta}{\delta \bar{c}^a} \right) \ , \tag{3.13}$$

$$\mathcal{J}^i_0 = \int d^4x \left(c^a \frac{\delta}{\delta \varphi^a_i} - (s c^a) \frac{\delta}{\delta \omega^a_i} + \overline{\varphi}^{ai} \frac{\delta}{\delta \bar{c}^a} \right) \ , \tag{3.14}$$

$$\tilde{T}^{ai}_{(\omega)} = \int d^4x \left(\frac{\delta}{\delta \omega^a_i} + f^{abc} \overline{\omega}^{bi} \frac{\delta}{\delta b^c} \right) \ , \tag{3.15}$$

$$\tilde{T}^{ai}_{(\varphi)} = \int d^4x \left[\frac{\delta}{\delta \varphi^a_i} + f^{abc} \left(\overline{\varphi}^{bi} \frac{\delta}{\delta b^c} + \overline{\omega}^{bi} \frac{\delta}{\delta \bar{c}^c} \right) \right] \ . \tag{3.16}$$

In definition (3.12) we introduced global ghost fields $(\xi, \eta, \lambda, \varrho, \sigma, \tau)$, whose quantum numbers are summarized in Table II. The operator Q is nilpotent:

$$Q^2 = 0 \ . \tag{3.17}$$

It does not describe a symmetry of the action S,

$$\begin{aligned}QS = \int d^4x [&(\varrho_i M^{ai}_\mu - \lambda_i U^{ai}_\mu + f^{abc}\tau^b_i U^{ci}_\mu)(D_\mu c)^a \\ &- \tau^a_i (D_\mu M^i_\mu)^a - \sigma^a_i (D_\mu U^i_\mu)^a] \ ,\end{aligned} \tag{3.18}$$

but the modified classical action

TABLE II. Dimensions, Faddeev-Popov charges, and Q_f numbers of the global ghosts.

	ξ	η	λ	ϱ	σ	τ
dim	0	0	1	1	1	1
ΦΠ	2	1	1	0	2	1
Q_f	0	0	1	1	−1	−1

$$I = S + S^{(Q)}_{\rm ext} \ , \tag{3.19}$$

where

$$S^{(Q)}_{\rm ext} = \int d^4x[K^a_\mu(QA^a_\mu) + L^a(Qc^a) + X^{ai}(Q\omega^a_i) + (\varrho_i M^{ai}_\mu - \lambda_i U^{ai}_\mu + f^{abc}\tau^b_i U^{ci}_\mu)A^a_\mu] \ , \tag{3.20}$$

satisfies the generalized Slavnov identity

$$\mathcal{D}(I) = 0 \ , \tag{3.21}$$

with

$$\begin{aligned}\mathcal{D}(I) = \int d^4x\Bigg(& \frac{\delta I}{\delta K^a_\mu}\frac{\delta I}{\delta A^a_\mu} + \frac{\delta I}{\delta L^a}\frac{\delta I}{\delta c^a} + \frac{\delta I}{\delta X^{ai}}\frac{\delta I}{\delta \omega^a_i} + (Qb^a)\frac{\delta I}{\delta b^a} + (Q\overline{c}^a)\frac{\delta I}{\delta \overline{c}^a} + (Q\varphi^a_i)\frac{\delta I}{\delta \varphi^a_i} + (Q\overline{\varphi}^{ai})\frac{\delta I}{\delta \overline{\varphi}^{ai}} \\ & + (Q\overline{\omega}^{ai})\frac{\delta I}{\delta \overline{\omega}^{ai}} + (QM^{ai}_\mu)\frac{\delta I}{\delta M^{ai}_\mu} + (QN^a_{\mu i})\frac{\delta I}{\delta N^a_{\mu i}} + (QU^{ai}_\mu)\frac{\delta I}{\delta U^{ai}_\mu} + (QV^a_{\mu i})\frac{\delta I}{\delta V^a_{\mu i}}\Bigg) \\ & - (\xi^a - \tfrac{1}{2}f^{abc}\eta^b\eta^c)\frac{\partial I}{\partial \eta^a} - \lambda_i\frac{\partial I}{\partial \varrho_i} + f^{abc}\eta^b\xi^c\frac{\partial I}{\partial \xi^a} \\ & + [\xi^a\lambda_i + f^{abc}(\eta^b\sigma^c_i - \xi^b\tau^c_i)]\frac{\partial I}{\partial \sigma^a_i} - (\sigma^a_i + \xi^a\varrho_i - f^{abc}\eta^b\tau^c_i)\frac{\partial I}{\partial \tau^a_i} \ . \end{aligned} \tag{3.22}$$

The corresponding linearized operator

$$\begin{aligned}\mathcal{D}_I = \int d^4x\Bigg(& \frac{\delta I}{\delta K^a_\mu}\frac{\delta}{\delta A^a_\mu} + \frac{\delta I}{\delta A^a_\mu}\frac{\delta}{\delta K^a_\mu} + \frac{\delta I}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta I}{\delta c^a}\frac{\delta}{\delta L^a} + \frac{\delta I}{\delta X^{ai}}\frac{\delta}{\delta \omega^a_i} + \frac{\delta I}{\delta \omega^a_i}\frac{\delta}{\delta X^{ai}} \\ & + (Qb^a)\frac{\delta}{\delta b^a} + (Q\overline{c}^a)\frac{\delta}{\delta \overline{c}^a} + (Q\varphi^a_i)\frac{\delta}{\delta \varphi^a_i} + (Q\overline{\varphi}^{ai})\frac{\delta}{\delta \overline{\varphi}^{ai}} + (Q\overline{\omega}^{ai})\frac{\delta}{\delta \overline{\omega}^{ai}} \\ & + (QM^{ai}_\mu)\frac{\delta}{\delta M^{ai}_\mu} + (QN^a_{\mu i})\frac{\delta}{\delta N^a_{\mu i}} + (QU^{ai}_\mu)\frac{\delta}{\delta U^{ai}_\mu} + (QV^a_{\mu i})\frac{\delta}{\delta V^a_{\mu i}}\Bigg) \\ & - (\xi^a - \tfrac{1}{2}f^{abc}\eta^b\eta^c)\frac{\partial}{\partial \eta^a} - \lambda_i\frac{\partial}{\partial \varrho_i} + f^{abc}\eta^b\xi^c\frac{\partial}{\partial \xi^a} \\ & + [\xi^a\lambda_i + f^{abc}(\eta^b\sigma^c_i - \xi^b\tau^c_i)]\frac{\partial}{\partial \sigma^a_i} - (\sigma^a_i + \xi^a\varrho_i - f^{abc}\eta^b\tau^c_i)\frac{\partial}{\partial \tau^a_i} \end{aligned} \tag{3.23}$$

is nilpotent:

$$\mathcal{D}_I\mathcal{D}_I = 0 \ . \tag{3.24}$$

The introduction of the global ghosts leads to the following identities for the action I:

$$\begin{aligned} & \frac{\partial I}{\partial \xi^a} = \Delta^a_{(\xi)} \ , \quad \frac{\partial I}{\partial \eta^a} = \Delta^a_{(\eta)} \ , \quad \frac{\partial I}{\partial \lambda_i} = \Delta^i_{(\lambda)} \ , \quad \frac{\partial I}{\partial \sigma^a_i} = \Delta^{ai}_{(\sigma)} \ , \quad \frac{\partial I}{\partial \tau^a_i} = \Delta^{ai}_{(\tau)} \ , \\ & \mathcal{T}^i_{(\varrho)}I \equiv \frac{\partial I}{\partial \varrho_i} + \int d^4x X^{ai}\frac{\delta I}{\delta L^a} = \Delta^i_{(\varrho)} \ , \end{aligned} \tag{3.25}$$

where

$$\begin{aligned} \Delta^a_{(\xi)} &= \int d^4x(L^a - f^{abc}X^{bi}\varphi^c_i) \ , \\ \Delta^a_{(\eta)} &= \int d^4x f^{abc}(K^b_\mu A^c_\mu - L^b c^c - X^{bi}\omega^c_i) \ , \\ \Delta^i_{(\lambda)} &= \int d^4x(X^{ai}c^a - U^{ai}_\mu A^a_\mu) \ , \\ \Delta^i_{(\varrho)} &= \int d^4x M^{ai}_\mu A^a_\mu \ , \end{aligned} \tag{3.26}$$

$$\Delta^{ai}_{(\sigma)} = \int d^4x X^{ai} \ ,$$
$$\Delta^{ai}_{(\tau)} = \int d^4x f^{abc} U^{bi}_\mu A^c_\mu$$

are linear breakings. The nonlinear algebra, valid for any even ghost charged functional Ψ,

$$\frac{\partial}{\partial \xi^a}\mathcal{D}(\Psi) - \mathcal{D}_\Psi\left(\frac{\partial\Psi}{\partial\xi^a} - \Delta^a_{(\xi)}\right) = (G^a\Psi - \Delta^a) - \left(\frac{\partial\Psi}{\partial\eta^a} - \Delta^a_{(\eta)}\right) - f^{abc}\eta^b\left(\frac{\partial\Psi}{\partial\xi^c} - \Delta^c_{(\xi)}\right)$$
$$+\lambda_i\left(\frac{\partial\Psi}{\partial\sigma^a_i} - \Delta^{ai}_{(\sigma)}\right) - \varrho_i\left(\frac{\partial\Psi}{\partial\tau^a_i} - \Delta^{ai}_{(\tau)}\right) - f^{abc}\tau^b_i\left(\frac{\partial\Psi}{\partial\sigma^c_i} - \Delta^{ci}_{(\sigma)}\right)$$
$$- \int d^4x[f^{abc}(\varrho_i X^{bi}c^c + \varrho_i U^{bi}_\mu A^c_\mu + \eta^b L^c - \varphi^b_i f^{cde} X^{di}\eta^e) - \lambda_i X^{ai}] \ , \tag{3.27}$$

$$\frac{\partial}{\partial \eta^a}\mathcal{D}(\Psi) + \mathcal{D}_\Psi\left(\frac{\partial\Psi}{\partial\eta^a} - \Delta^a_{(\eta)}\right) = \mathcal{H}^a_{\rm rig}\Psi + f^{abc}\eta^b\left(\frac{\partial\Psi}{\partial\eta^c} - \Delta^c_{(\eta)}\right) + f^{abc}\xi^b\left(\frac{\partial\Psi}{\partial\xi^c} - \Delta^c_{(\xi)}\right)$$
$$+f^{abc}\sigma^b_i\left(\frac{\partial\Psi}{\partial\sigma^c_i} - \Delta^{ci}_{(\sigma)}\right) + f^{abc}\tau^b_i\left(\frac{\partial\Psi}{\partial\tau^c_i} - \Delta^{ci}_{(\tau)}\right)$$
$$+f^{abc}(\eta^b\Delta^c_{(\eta)} + \xi^b\Delta^c_{(\xi)} + \sigma^b_i\Delta^{ci}_{(\sigma)} + \tau^b_i\Delta^{ci}_{(\tau)}) \ , \tag{3.28}$$

$$\frac{\partial}{\partial \lambda_i}\mathcal{D}(\Psi) + \mathcal{D}_\Psi\left(\frac{\partial\Psi}{\partial\lambda_i} - \Delta^i_{(\lambda)}\right) = \mathcal{F}^i\Psi - (\mathcal{T}^i_{(\varrho)}\Psi - \Delta^i_{(\varrho)}) + \xi^a\left(\frac{\partial\Psi}{\partial\sigma^a_i} - \Delta^{ai}_{(\sigma)}\right) + \int d^4x(\xi^a X^{ai} + f^{abc}\eta^a U^{bi}_\mu A^c_\mu) \ , \tag{3.29}$$

$$\mathcal{T}^i_{(\varrho)}\mathcal{D}(\Psi) - \mathcal{D}_\Psi(\mathcal{T}^i_{(\varrho)}\Psi - \Delta^i_{(\varrho)}) = \mathcal{J}^i(\Psi) - \xi^a\left(\frac{\partial\Psi}{\partial\tau^a_i} - \Delta^{ai}_{(\tau)}\right) + \int d^4x f^{abc}\eta^a M^{bi}_\mu A^c_\mu \ , \tag{3.30}$$

$$\frac{\partial}{\partial \sigma^a_i}\mathcal{D}(\Psi) - \mathcal{D}_\Psi\left(\frac{\partial\Psi}{\partial\sigma^a_i} - \Delta^{ai}_{(\sigma)}\right) = \int d^4x(T^{ai}_{(\omega)}\Psi - \Delta^{ai}_{(\omega)}) - \left(\frac{\partial\Psi}{\partial\tau^a_i} - \Delta^{ai}_{(\tau)}\right) - f^{abc}\eta^b\left(\frac{\partial\Psi}{\partial\sigma^c_i} - \Delta^{ci}_{(\sigma)}\right) - f^{abc}\eta^b\Delta^{ci}_{(\sigma)} \ , \tag{3.31}$$

$$\frac{\partial}{\partial \tau^a_i}\mathcal{D}(\Psi) + \mathcal{D}_\Psi\left(\frac{\partial\Psi}{\partial\tau^a_i} - \Delta^{ai}_{(\tau)}\right) = \int d^4x(T^{ai}_{(\varphi)}\Psi - \Delta^{ai}_{(\varphi)}) + f^{abc}\eta^b\left(\frac{\partial\Psi}{\partial\tau^c_i} - \Delta^{ci}_{(\tau)}\right)$$
$$+f^{abc}\xi^b\left(\frac{\partial\Psi}{\partial\sigma^c_i} - \Delta^{ci}_{(\sigma)}\right) + \int d^4x f^{abc}(\xi^b X^{ci} + U^{bi}_\mu f^{cde}\eta^d A^e_\mu) \ , \tag{3.32}$$

implies, for a functional $\Psi = \Gamma^{(Q)}$ satisfying

$$\frac{\partial}{\partial\xi^a}\Gamma^{(Q)} = \Delta^a_{(\xi)} \ , \quad \frac{\partial}{\partial\eta^a}\Gamma^{(Q)} = \Delta^a_{(\eta)} \ , \quad \frac{\partial}{\partial\lambda_i}\Gamma^{(Q)} = \Delta^i_{(\lambda)} \ , \tag{3.33}$$

$$\mathcal{T}^i_{(\varrho)}\Gamma^{(Q)} = \Delta^i_{(\varrho)} \ , \quad \frac{\partial}{\partial\sigma^a_i}\Gamma^{(Q)} = \Delta^{ai}_{(\sigma)} \ , \quad \frac{\partial}{\partial\tau^a_i}\Gamma^{(Q)} = \Delta^{ai}_{(\tau)} \ , \tag{3.34}$$

$$\mathcal{D}(\Gamma^{(Q)}) = 0 \ , \tag{3.35}$$

that the following identities hold:

$$
\begin{aligned}
G^a\Gamma^{(Q)} &= \Delta^a + \int d^4x[f^{abc}(\varrho_i X^{bi}c^c + \varrho_i U^{bi}_\mu A^c_\mu \\
&\quad +\eta^b L^c - \varphi^b_i f^{cde} X^{di}\eta^e) - \lambda_i X^{ai}] , \\
\mathcal{H}^a_{\rm rig}\Gamma^{(Q)} &= -f^{abc}(\eta^b \Delta^c_{(\eta)} + \xi^b \Delta^c_{(\xi)} \\
&\quad +\sigma^b_i \Delta^{ci}_{(\sigma)} + \tau^b_i \Delta^{ci}_{(\tau)}) , \\
\mathcal{F}^i\Gamma^{(Q)} &= -\int d^4x(\xi^a X^{ai} + f^{abc}\eta^a U^{bi}_\mu A^c_\mu) ,
\end{aligned}
\tag{3.36}
$$

$$
\begin{aligned}
\mathcal{J}^i(\Gamma^{(Q)}) &= -\int d^4x f^{abc}\eta^a M^{bi}_\mu A^c_\mu , \\
\int d^4x T^{ai}_{(\omega)}\Gamma^{(Q)} &= \int d^4x(\Delta^{ai}_{(\omega)} + f^{abc}\eta^b \Delta^{ci}_{(\sigma)}) , \\
\int d^4x T^{ai}_{(\varphi)}\Gamma^{(Q)} &= \int d^4x[\Delta^{ai}_{(\varphi)} \\
&\quad -f^{abc}(\xi^b X^{ci} + U^{bi}_\mu f^{cde}\eta^d A^e_\mu)] .
\end{aligned}
$$

From Eqs. (3.21) and (3.36) one sees that at vanishing global ghosts and source X the quantum vertex functional $\Gamma \equiv \Gamma^{(Q)}|_{\xi=\eta=\lambda=\rho=X=0}$ satisfies

$$
\begin{aligned}
\mathcal{S}(\Gamma) &= 0 , \\
\mathcal{G}^a\Gamma &= \Delta^a , \\
\mathcal{H}^a_{\rm rig}\Gamma &= 0 , \\
\mathcal{F}^i\Gamma &= 0 , \\
\mathcal{J}^i(\Gamma) &= 0 , \\
\int d^4x T^{ai}_{(\omega)}\Gamma &= \int d^4x \Delta^{ai}_{(\omega)} , \\
\int d^4x T^{ai}_{(\varphi)}\Gamma &= \int d^4x \Delta^{ai}_{(\varphi)} ,
\end{aligned}
\tag{3.37}
$$

which is the desired result.

It is quite straightforward to show that relations (3.33) and (3.34) hold and it is apparent from the nonlinear algebra (3.27)–(3.32) that proving the absence of anomalies has been reduced to showing that the generalized Slavnov identity (3.21) is not anomalous. We can now apply the mathematical tools developed for nilpotent operators [12].

From the QAP [13] we know that the generalized Slavnov identity could be broken at the quantum level,

$$\mathcal{D}(\Gamma^{(Q)}) = \mathcal{A}\Gamma^{(Q)} , \tag{3.38}$$

only by a quantum insertion $\mathcal{A}\Gamma^{(Q)}$, which to lowest order in $\hbar$ is an integrated local functional of dimension four, ghost charge $+1$ and Q_f counting number zero:

$$\mathcal{A}\Gamma^{(Q)} = A + O(\hbar A) . \tag{3.39}$$

This lowest-order breaking A must satisfy the Wess-Zumino consistency condition [17,18]

$$\mathcal{D}_I A = 0 . \tag{3.40}$$

Since $\mathcal{D}_I$ is a nilpotent operator, Eq. (3.40) is a cohomology problem that we solve by decomposing $\mathcal{D}_I$ with the filtration operator [12]

$$
\begin{aligned}
\mathcal{N} &= \xi^a \frac{\partial}{\partial \xi^a} + \eta^a \frac{\partial}{\partial \eta^a} + \lambda_i \frac{\partial}{\partial \lambda_i} + \varrho_i \frac{\partial}{\partial \varrho_i} \\
&\quad +\sigma^a_i \frac{\partial}{\partial \sigma^a_i} + \tau^a_i \frac{\partial}{\partial \tau^a_i}
\end{aligned}
\tag{3.41}
$$

into

$$\mathcal{D}_I = \mathcal{D}^{(0)} + \mathcal{D}^{(R)} , \tag{3.42}$$

where

$$D_{(0)} = B_\Sigma - \xi^a \frac{\partial}{\partial \eta^a} - \lambda_i \frac{\partial}{\partial \varrho_i} - \sigma^a_i \frac{\partial}{\partial \tau^a_i} . \tag{3.43}$$

Because of (2.15), the operator $D^{(0)}$ is nilpotent, and the result of [12] ensures that the cohomology of $\mathcal{D}_I$ is isomorphic to a subspace of that of $D^{(0)}$, which does not depend on the global ghosts $(\eta,\xi;\varrho,\lambda;\tau,\sigma)$ nor on the fields $(\varphi,\omega;\overline{\omega},\overline{\varrho};U,M;V,N)$ since they appear in (3.43) as BRS doublets [12]. We are therefore left to study the cohomology problem

$$B_\Sigma X = 0 , \tag{3.44}$$

where B_Σ is the linearized Slavnov operator of ordinary Yang-Mills theory. As discussed previously, the solution of (3.44) is a trivial cocycle since there is no Adler-Bardeen anomaly in this model [14], and consequently the cohomology of $\mathcal{D}_I$ is empty.

We have thus proved that the solution of the Wess-Zumino consistency condition (3.40) is

$$A = \mathcal{D}_I \hat{A} , \tag{3.45}$$

i.e., that the generalized Slavnov identity (3.21) is not anomalous, and that the symmetries (3.37) we considered are therefore valid to all orders of perturbation theory.

Along the same lines, it is straightforward to also prove that the symmetries (2.25) and (2.27) are anomaly-free by starting from the transformations generated by the nilpotent operator

$$
\begin{aligned}
Q' &= s + \lambda^j_i \mathcal{R}^i_j + \varrho^j_i \mathcal{U}^i_j - (\lambda^i_j + \varrho^i_k \varrho^k_j)\frac{\partial}{\partial \varrho^i_j} \\
&\quad -(\lambda^k_j \varrho^i_k - \lambda^i_k \varrho^k_j)\frac{\partial}{\partial \lambda^i_j} ,
\end{aligned}
\tag{3.46}
$$

where $(\lambda^i_j, \varrho^i_j)$ are again global ghosts. All the symmetries that form the algebra (2.33) are thus valid at the quantum level, and the unitarity of the model is ensured [18].

IV. THE MODEL AT PHYSICAL SOURCES

The analysis of the Gribov ambiguity made in [6,7], has demonstrated that the functional integration is effectively constrained to the FMR in a quantum theory defined by the classical action (2.9) for nonvanishing physical sources:

$$M^a_{\mu\nu b}|_{\rm ph} = -V^a_{\mu\nu b}|_{\rm ph} = \gamma\delta_{\mu\nu}\delta^a_b \ , \tag{4.1}$$

where the mass parameter is determined self-consistently by the horizon condition [9]

$$\left.\frac{\partial\Gamma}{\partial\gamma}\right|_{\rm ph} = 0 \ , \tag{4.2}$$

when the quantum fields $\Phi \in \{A, c, \bar{c}, b, \varphi, \overline{\varphi}, \omega, \overline{\omega}\}$ assume their vacuum values

$$\left.\frac{\delta\Gamma}{\delta\Phi}\right|_{\Phi=\Phi|_{\rm ph}} = 0 \ . \tag{4.3}$$

As shown in [7], Eq. (4.2) can only be satisfied at a nonvanishing value for γ. At their physical values (4.1) the sources do not appear as BRS doublets and the classical action is no longer BRS symmetric:

$$s(S|_{\rm ph}) = \gamma s(D_\mu\varphi_{\mu a})^a \ . \tag{4.4}$$

In the following we will give another interpretation of the quantum vertex functional at nonvanishing physical sources (4.1) which in the previous sections was shown to be renormalizable. Surprisingly it can also be looked upon as the quantum vertex functional of the BRS-symmetric model with vanishing sources where a certain spontaneous symmetry breakdown has occurred. We will show that a simple redefinition of the quantum fields in the BRS-symmetric quantum vertex functional gives the one for nonvanishing physical sources. The horizon condition (4.2) can be interpreted as the condition that the quantum vertex functional is stationary for vanishing quantum fields.

We wish to emphasize that this is just an interpretation of the horizon condition and the quantum vertex functional at the physical sources. It is not a proof that a nontrivial solution to the horizon condition (4.2) exists. But if there is a solution with $\gamma \neq 0$, as the Gribov propagator (1.3) and the analysis of the FMR by Zwanziger [7] strongly suggest, then the horizon condition (4.2) is nothing else but a sort of gap equation for the (dynamical) spontaneous breakdown of some of the symmetries (including BRS) of the model.

To be specific, let us first consider the *symmetric* quantum vertex functional at vanishing sources:

$$\Gamma^{\rm sym} = \Gamma|_{M=N=U=V=0} \ . \tag{4.5}$$

In the previous sections we have shown that it is a finite functional of the renormalized fields and coupling constant. The replacements

$$\overline{\varphi}^a_{\mu b} \Rightarrow \overline{\varphi}'^a_{\mu b} + \gamma_M x_\mu \delta^a_b \ , \tag{4.6}$$

$$\varphi^a_{\mu b} \Rightarrow \varphi'^a_{\mu b} - \gamma_V x_\mu \delta^a_b \ , \tag{4.7}$$

$$\bar{c}^a \Rightarrow \bar{c}'^a - \gamma_V f^{abc} x_\mu \overline{\omega}^c_{\mu b} \ , \tag{4.8}$$

$$b^a \Rightarrow b'^a - \gamma_V f^{abc} \overline{\varphi}'^c_{\mu b} x_\mu \ , \tag{4.9}$$

lead to a quantum vertex functional of the shifted quantum fields, which is just the one for nonvanishing external sources[2]

$$\Gamma^{\rm sym}[\varphi, \overline{\varphi}, \omega, \overline{\omega}, c, \bar{c}, b, A] = \Gamma[\varphi', \overline{\varphi}', \omega, \overline{\omega}, c, \bar{c}', b', A; M', V'] \ , \tag{4.10}$$

with

$$M'^a_{\mu\nu b} = \gamma_M \delta_{\mu\nu}\delta^a_b \ , \quad V'^a_{\mu\nu b} = -\gamma_V \delta_{\mu\nu}\delta^a_b \ . \tag{4.11}$$

Relation (4.10) can easily be verified from the form of the classical action (2.9) and the fact that the radiative correction $\Gamma^{(\rm qu)}$ in (3.11) only depends on the combinations (3.6). That the radiative correction only depends on the combinations (3.6) is assured by our previous proof that the model is renormalizable (and in particular anomaly-free) for any value of the external sources. This proof is therefore crucial for (4.10) to hold.

We obtain the quantum vertex functional at the physical value of the sources (4.1) upon setting

$$\gamma_V = \gamma_M = \gamma \ . \tag{4.12}$$

The equality of γ_V and γ_M can always be achieved by suitably defining the normalizations of φ', $\overline{\varphi}'$, ω', and $\overline{\omega}'$. This freedom in the (relative) normalizations implies that correlation functions that do not involve these fields are only functions of the product $\gamma_V\gamma_M = \gamma^2$ and not of γ_V or γ_M separately. This is, in particular, true for the vacuum energy density and the gluon propagator.

It is remarkable that the explicit coordinate dependence of the shifts (4.6)–(4.9) is not reflected in Γ. This can be traced to the invariance of the symmetric quantum vertex functional under the global $U(f)$ group in addition to its $O(4)$ and $SU(N)$ symmetry under Euclidean coordinate, and rigid gauge, transformations. Each of these symmetries is individually broken spontaneously by the shifts (4.6)–(4.9) but a diagonal $SU(N) \times O(4)$ subgroup remains intact and assures coordinate and global color invariance also in the broken phase.

The analogy with spontaneous symmetry breakdown can be further pursued, because the shifts (4.6)–(4.9) also change the vacuum values of the quantum fields. Perturbation theory with physical values (4.1) of the sources corresponds to an expansion around nontrivial vacuum values in the symmetric theory. Furthermore the difference in the classical vacuum energy density between the trivial and nontrivial vacuum values (4.6)–(4.9),

$$\begin{aligned}\Delta\epsilon_{\rm vac} &= \frac{1}{V}(\Sigma^{\rm sym}|_{\overline{\varphi}'=\varphi'=\omega'=\bar{c}'=b'=\overline{\omega}=A=c=0} \\ &\quad -\Sigma^{\rm sym}|_{\overline{\varphi}=\varphi=\omega=\bar{c}=b=\overline{\omega}=A=c=0}) \\ &= -4(N^2-1)\gamma_V\gamma_M \ ,\end{aligned} \tag{4.13}$$

[2] Zwanziger observed that the shifts (4.6)–(4.9) also eliminate the BRS-breaking terms in the lattice regularized version of this model [8].

with $\Sigma^{\rm sym} = \Sigma|_{M=N=U=V=0}$, implies that the broken phase is energetically preferred. We can interpret the horizon equation (4.2) as a minimizing condition for the vacuum energy density in the presence of quantum fluctuations. These are in fact necessary to satisfy (4.2), because (4.13) only depends linearly on $\gamma_M\gamma_V = \gamma^2$.

Although surprising and perhaps even disturbing, a careful analysis of covariant gauge fixing on the lattice also indicated that the BRS symmetry of non-Abelian gauge theories could be spontaneously broken, in order to avoid that the summation over Gribov copies conspires to yield vanishing expectation values for gauge invariant observables [19].

V. CONCLUSIONS

The model defined by the classical action (2.9) was prosed [9] to effectively restrict the functional integration of Yang-Mills theories to the FMR (1.5) by means of additional fields and external sources which satisfy a self-consistency or "horizon" condition (4.1)–(4.3) at the physical point.

This restriction to the FMR is a refinement of the usual Landau gauge, a kind of *minimal* one without Gribov copies. We indeed recovered also for this model the property [11] of Landau gauges that the integrated ghost equation of motion yields a Ward identity [Eq. (2.16)]. The rich symmetry structure (2.33) allowed us to prove algebraically that only two independent divergences appear in a perturbative analysis, which means that the model has the same renormalization properties as ordinary Yang-Mills theory in Landau gauge [10] in spite of the additional fields and sources. The renormalization proof was completed by showing that the symmetries of the model do hold to all orders of perturbation theory, i.e., are not anomalous. Unitarity of the physical S matrix is then a consequence of the validity to all orders of perturbation theory of the Slavnov identity.

We believe that the algebraic structure of the enlarged theory effectively eliminates all the additional degrees of freedom introduced. This would be in the spirit which led to the construction of the model, namely, constraining the gauge field configurations to the FMR, without altering the physical content of the original Yang-Mills theory [6,7]. This conjecture is supported by the observation that the BRS-breaking term (4.4) at the physical point can be understood as resulting from the nonperturbative shifts (4.6)–(4.9) in the BRS-symmetric case and that the horizon condition extremizes the vacuum energy density.

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