Quantum state during and after O(4)-symmetric bubble nucleation with gravitational effects

Takahiro Tanaka and Misao Sasaki
Department of Physics, Kyoto University, Kyoto 606-01, Japan
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We extend our previous analysis of the quantum state during and after $O(4)$-symmetric bubble nucleation to the case including gravitational effects. We find that there exists a simple relationship between the case with and without gravitational effects. In a special case of a conformally coupled scalar field which is massless except on the bubble wall, the state is found to be conformally equivalent to the case without gravity.

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I. INTRODUCTION

Field-theoretical quantum tunneling phenomena such as false vacuum decay are considered to have played important roles in the dynamics of the Universe in its early stage. One good example is the so-called extended inflation [1], in which the inflationary stage of the Universe ends with nucleation of true vacuum bubbles and thermalization of the Universe by collisions of these nucleated bubbles.

As another interesting possibility, we have recently proposed a simple one-bubble scenario of the inflationary Universe by considering the particle creation during and after the false vacuum decay [2]. Provided enough entropy is produced, it is possible to have our Universe inside one nucleated bubble. However, since our knowledge of the quantum state after the false vacuum decay is far from sufficient, we are unable to argue further for or against this possibility at present.

In addition to the inflationary universe scenario, what happens after bubble nucleation is an interesting issue as a fundamental process relating to the quantum matter production or the quantification of quantum effects in general in the early Universe.

Toward a clear understanding of the issue, various attempts have been made. Among them, Rubakov developed a method of nonunitary Bogoliubov transformation to treat particle production during tunneling [3]. Then Vachaspati and Vilenkin investigated general features of the quantum state during and after nucleation of an O(4)-symmetric bubble, paying particular attention to the symmetry of the state [4]. Meanwhile, we have developed a method to analyze the quantum state during and after field-theoretical quantum tunneling by constructing a multidimensional wave function in a covariant manner [5] (hereafter paper I), which was originally developed by Vega, Gervais, and Sakita [6]. Then we have applied it to the O(4)-symmetric bubble nucleation and investigated the properties of the quantum state of fluctuating degrees of freedom in detail by constructing an analytically soluble model [7] (hereafter paper II).

However, all of these previous analyses were based on several nontrivial assumptions or simplification of models, the validity of which is not clear. To mention one such simplification, the effect of gravity was neglected in all of them. In this paper we focus on this point and tackle the problem of incorporating the effect of gravity. More precisely, as a first step, we take into account the background spacetime curvature induced by the tunneling field solution and investigate its effect on the quantum state of fluctuating degrees of freedom. Hence, in particular, the false vacuum is de Sitter space.

This paper is organized as follows. In Sec. II we extend our method developed in paper I to the case when the initial state is excited with respect to the false vacuum, which is necessary to incorporate the gravitational effect. In Sec. III we show that there is an elegant interrelation between the quantum states after tunneling with and without the effect of gravity. As a specific example, we then consider a conformally coupled scalar field, which interacts with the tunneling field on the bubble wall but is massless elsewhere, and we show that its quantum state is conformally equivalent to the case without gravity, i.e., the same result as for the Minkowski background obtained in paper II can be applied. However, we also point out a paradoxical situation we encounter when evaluating the regularized energy-momentum tensor. In Sec. IV we summarize our results.

II. MULTIDIMENSIONAL TUNNELING WAVE FUNCTION

In paper I we developed a method to construct the multidimensional tunneling wave function from the false vacuum ground state in a covariant manner. As we shall see in the next section, in order to construct the tunneling wave function in curved spacetime, it is necessary to extend the formalism given in paper I to the case when the initial state is in an excited state. Some basic parts of this extension have been recently given by Yamamoto [8].

We consider a system of $D+1$ degrees of freedom whose Lagrangian is given by

$$L = \frac{1}{2} G_{\alpha\beta}(\phi)\dot{\phi}^\alpha \dot{\phi}^\beta - V(\phi)$$

$$(\alpha, \beta = 0, 1, \ldots, D;\ i, j = 1, \ldots, D), \quad (2.1)$$

where ϕ^α are the coordinates for the $(D+1)$-dimensional space of dynamical variables (i.e., superspace) and $G_{\alpha\beta}$ is the superspace metric. In this section, Greek and Latin indices run from 0 to D and from 1 to D, respectively. For simplicity, we assume the potential $V(\phi)$ of the form

$$V(\phi) = U(X) + \frac{1}{2} m_{ij}^2(X)\phi^i\phi^j, \quad (2.2)$$

where the tunneling degree of freedom is represented by $X = \phi^0$ as a collective coordinate and the fluctuating degrees of freedom by ϕ^i, respectively. Further we focus on the case when the superspace metric depends only on the tunneling degree of freedom, $G_{\alpha\beta} = G_{\alpha\beta}(X)$, and has no cross terms between X and ϕ^i (i.e., $G_{0i} = 0$), and we assume that the signature of the metric is positive definite. The potential $U(X)$ is supposed to have a local minimum at $X = X_F$, which is not the absolute minimum. We call the point $(X, \phi^i) = (X_F, 0)$ the false vacuum origin throughout this paper.

The Hamiltonian operator in the coordinate representation is obtained by replacing the conjugate momenta in the Hamiltonian with the corresponding differential operators. In general, there exists the operator ordering ambiguity. Here we fix it in such a way that the resulting Hamiltonian takes the form

$$\hat{H} = -\frac{\hbar^2}{2} G^{\alpha\beta}(X)\nabla_\alpha\nabla_\beta + U(X) + \frac{1}{2} m_{ij}^2(X)\phi^i\phi^j, \quad (2.3)$$

where $G^{\alpha\beta}(X)$ is the inverse matrix of $G_{\alpha\beta}(X)$.

In paper I we constructed the *quasi-ground-state* wave function using the WKB approximation. The *quasi-ground-state* wave function is the lowest eigenstate of the Hamiltonian sufficiently localized at the false vacuum. Let us briefly summarize the method without rigor. The detailed discussion is given in paper I.

(i) First, we impose the WKB ansatz on the wave function,

$$\Psi = e^{-\frac{1}{\hbar}(W^{(0)} + \hbar W^{(1)} + \cdots)}, \quad (2.4)$$

which should solve the time-independent Schrödinger equation,

$$\hat{H}\Psi = E\Psi. \quad (2.5)$$

We solve this equation order by order with respect to $\hbar$. The energy eigenvalue E is formally divided into two parts, E_0 and E_1, of $O(\hbar^0)$ and $O(\hbar^1)$, respectively. The equation in the lowest order of $\hbar$ becomes the Hamilton-Jacobi equation with the energy E_0:

$$-\frac{1}{2} G^{\alpha\beta}\nabla_\alpha W^{(0)}\nabla_\beta W^{(0)} + V(\phi) = E_0. \quad (2.6)$$

By setting $G^{\alpha\beta}\nabla_\beta W^{(0)} = \dot{\phi}^\alpha$, Eq. (2.6) gives the Euclidean equation of motion, i.e., with respect to the imaginary time $\tau = it$.

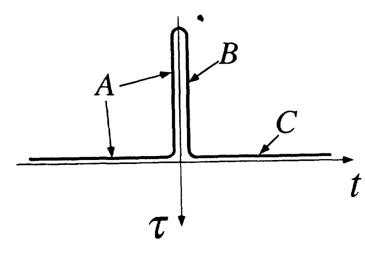

FIG. 1. A path on the complex plane of time, which represents a tunneling process. The segments A, B, and C correspond to the motion staying in the false vacuum, an instanton, and the motion after nucleation, respectively.

(ii) We consider a solution of the Euclidean equation of motion which starts from the false vacuum at $\tau = -\infty$ with zero kinetic energy [i.e., $E = E_0 := U(X_F)$] and arrives at the turning point at $\tau = 0$, which is the boundary of the classically allowed and forbidden regions. If there are several nontrivial solutions, we choose the one that gives the minimum Euclidean action. We call it the dominant escape path (DEP) and denote it by $\phi_{(0)}^\alpha(\tau)$. In the present case we have

$$(\phi_{(0)}^0(\tau), \phi_{(0)}^i(\tau)) = (X(\tau), 0). \quad (2.7)$$

(iii) Next, along DEP, we introduce an orthonormal basis, $\hat{e}_{\mathbf{a}}^i(\tau)$, lying in the hypersurface $\tilde{\Sigma}(\tau)$ orthogonal to it; $G_{ij}(X(\tau))\hat{e}_{\mathbf{a}}^i(\tau)\hat{e}_{\mathbf{b}}^j(\tau) = \delta_{\mathbf{ab}}$, where $\mathbf{a}$ runs through the range $1, 2, \ldots, D$. For convenience, we fix the orthonormal basis in such a way that $e_{\mathbf{a}}^i := \hat{e}_{\mathbf{a}}^i(\tau = -\infty)$ diagonalizes $\omega_{\mathbf{ab}} := \omega_{ij} e_{\mathbf{a}}^i e_{\mathbf{b}}^j$, i.e., $\omega_{\mathbf{ab}} = \omega_{\mathbf{a}}\delta_{\mathbf{ab}}$, where

$$\omega_{ij}^2 := \lim_{X \to X_F} m_{ij}^2(X). \quad (2.8)$$

Then the required orthonormal basis along DEP is constructed by solving the equation

$$\frac{\partial}{\partial\tau}\hat{e}_{\mathbf{a}}^i + \frac{1}{2} G^{ik}\dot{G}_{kj}\hat{e}_{\mathbf{a}}^j = 0, \quad (2.9)$$

which corresponds to a special case of Eq. (2.17) in paper I.

(iv) Assuming that $W^{(0)}(\phi)$ is known around DEP, we span the hypersurface orthogonal to DEP by the coordinates $\eta^{\mathbf{a}}$ with respect to the basis $e_{\mathbf{a}}^i$ and define

$$\tilde{\Omega}_{\mathbf{ab}} := W_{;ij}^{(0)}\Big|_{\eta^c = 0} \hat{e}_{\mathbf{a}}^i \hat{e}_{\mathbf{b}}^j, \quad (2.10)$$

where the semicolon represents the $(D+1)$-dimensional covariant differentiation with respect to $G_{\alpha\beta}$. We also introduce a matrix $K_{\mathbf{b}}^{\mathbf{a}}$, which is determined by solving

$$K_{\mathbf{b}}^{\mathbf{a}} = \hat{e}_\alpha^{\mathbf{a}} K_{\mathbf{b}}^\alpha,$$

$$\frac{D^2}{d\tau^2} K_{\mathbf{b}}^\alpha = [(U + \frac{1}{2} m_{ij}^2\phi^i\phi^j)^{;\alpha}{}_{;\beta} - \dot{X}(\tau)^2 R^\alpha{}_{0\beta 0}] K_{\mathbf{b}}^\beta, \quad (2.11)$$

with the boundary condition

$$K^{\mathbf{a}}_{\mathbf{b}} \to \left(e^{\omega\tau}\right)^{\mathbf{a}}_{\mathbf{b}} \quad (\tau \to -\infty), \tag{2.12}$$

where $D/d\tau$ and $R^{\alpha}_{\gamma\beta\rho}$ denote the Lie derivative along DEP and the Riemann tensor of the superspace metric $G_{\alpha\beta}$, respectively. One then finds $\tilde{\Omega}_{\mathbf{ab}}$ is expressed in terms of $K_{\mathbf{ab}}$ as

$$\tilde{\Omega}_{\mathbf{ab}} = \dot{K}^{\mathbf{c}}_{\mathbf{a}}\left(K^{-1}\right)_{\mathbf{cb}}. \tag{2.13}$$

(v) With these results in hand, the *quasi-ground-state* wave function [5] is found to be

$$\Psi = \frac{C\left(\det\omega/\pi\hbar\right)^{1/4} e^{\frac{1}{2}\omega_0\tau}}{\{2[U(X(\tau)) - E_0]\}^{1/4}\sqrt{|\det K^{\mathbf{a}}_{\mathbf{b}}(\tau)|}} \exp\Big(-\frac{1}{\hbar}\int_{-\infty}^{\tau} d\tau' 2[U(X(\tau)) - E_0] + \frac{1}{2}\mathrm{Tr}\omega\tau - \frac{1}{2\hbar}\tilde{\Omega}_{\mathbf{ab}}\eta^{\mathbf{a}}\eta^{\mathbf{b}}\Big), \tag{2.14}$$

where $\omega_0 := U''(X_F)$, $\mathrm{Tr}\omega = \sum_{\mathbf{a}=1}^{D}\omega_{\mathbf{a}}$, $\det\omega = \prod_{\mathbf{a}=1}^{D}\omega_{\mathbf{a}}$, and the normalization constant C is given by

$$C = \lim_{\tau\to-\infty} \frac{\{2[U(X(\tau)) - E_0]\}^{1/4}}{e^{\frac{1}{2}\omega_0\tau}}\left(\frac{\omega_0}{\pi\hbar}\right)^{1/4}. \tag{2.15}$$

In the present case, the above result can be reexpressed in terms of the superspace coordinates (τ, ϕ^i), as follows. First, note that the hypersurface $\tilde{\Sigma}(\tau)$ is generally different from the τ =const hypersurface $\Sigma(\tau)$ off the DEP; the latter is warped if $\dot{G}_{ij} \neq 0$. Let a point $(\tilde{\tau}, \eta^{\mathbf{a}})$ on the hypersurface $\tilde{\Sigma}(\tilde{\tau})$ correspond to (τ, ϕ^i) in the original coordinates. Then from the fact $\tilde{Q}(\tilde{\tau}, \eta^{\mathbf{a}}) = Q(\tau, \phi^i)$ for any scalar function Q and the equation

$$\tilde{\Omega}_{ij} = \Omega_{ij} + \frac{1}{2}\dot{G}_{ij}, \tag{2.16}$$

which follows from the definition of the covariant derivative, where

$$\Omega_{ij} := \left.\frac{\partial^2}{\partial\phi^i\partial\phi^j}W^{(0)}\right|_{\phi^k=0}, \quad \tilde{\Omega}_{ij} := \hat{e}^{\mathbf{a}}_i\hat{e}^{\mathbf{b}}_j\tilde{\Omega}_{\mathbf{ab}}, \tag{2.17}$$

we find

$$\tilde{\tau} = \tau - \frac{1}{2}\left(\frac{dW^{(0)}}{d\tau}\right)^{-1}\dot{G}_{ij}\phi^i\phi^j. \tag{2.18}$$

Now, replacing τ in Eq. (2.14) with $\tilde{\tau}$ given by Eq. (2.18), expanding the result around τ, noting the fact that $dW^{(0)}/d\tau = 2[U(X) - E_0]$, and regarding $\eta^{\mathbf{a}}$ and ϕ^i as quantities of $O(\hbar^{1/2})$, we find the wave function to the first WKB order to have the form

$$\Psi(\phi^\alpha) = \Theta(X)\Phi(X, \phi^i), \tag{2.19}$$

where Θ is the lowest WKB part,

$$\Theta(X) = \frac{C}{\left[2\left(V(X(\tau)) - E_0\right)\right]^{1/4}} \times \exp\Big(-\frac{1}{\hbar}\int_{-\infty}^{\tau} d\tau' 2[U(X(\tau')) - E_0] + \frac{1}{2}\omega_0\tau\Big), \tag{2.20}$$

and Φ is the first WKB correction,

$$\Phi(X, \phi^i) = \frac{\left(\det\omega/\pi\hbar\right)^{1/4}}{\sqrt{|\det K^i_{\mathbf{a}}(\tau)\sqrt{G}|}} \times \exp\Big(\frac{1}{2}\mathrm{Tr}\omega\tau - \frac{1}{2\hbar}\Omega_{ij}\phi^i\phi^j\Big). \tag{2.21}$$

Further, from Eqs. (2.13) and (2.16) we obtain

$$\Omega_{ij} = \sum_{\mathbf{a}} G_{ik}\dot{K}^k_{\mathbf{a}}K^{-1}_{j\mathbf{a}}. \tag{2.22}$$

Also, from Eq. (2.11), we find $K^i_{\mathbf{a}}$ satisfies the equation

$$G_{ij}\ddot{K}^j_{\mathbf{a}} + \dot{G}_{ij}\dot{K}^j_{\mathbf{a}} = m^2_{ij}K^j_{\mathbf{a}}, \tag{2.23}$$

which is just the classical equation of motion for ϕ^i in Euclidean time. With the help of Eq. (2.23), it is then straightforward to show that to the first WKB order (2.21) satisfies

$$[\mathcal{D} - E'_1]\,\Phi = 0\cdot$$

$$\mathcal{D} := \left[G^{-1/4}\partial_\tau G^{1/4} - \frac{\hbar}{2}G^{ij}\partial_i\partial_j + \frac{1}{2\hbar}m^2_{ij}\phi^i\phi^j\right], \tag{2.24}$$

where $E'_1 = \hbar\,\mathrm{Tr}\omega$. One sees that this is just the Euclidean time Schrödinger equation for the fluctuating degrees of freedom.

Following the procedure taken in [8], we now construct a set of generalized creation and annihilation operators, $A^\dagger_{\mathbf{a}}$ and $A_{\mathbf{a}}$ whose action on some eigenstate of the Hamiltonian produces another eigenstate. In other words, look for operators that correspond to the usual creation and annihilation operators at the false vacuum origin, i.e., $[\mathcal{D}, A_{\mathbf{a}}] = \omega_{\mathbf{a}}A_{\mathbf{a}}$ and $[\mathcal{D}, A^\dagger_{\mathbf{a}}] = -\omega_{\mathbf{a}}A^\dagger_{\mathbf{a}}$. To do so, first, we make a set of operators $a_{\mathbf{a}}$ and $a^\dagger_{\mathbf{a}}$ which commute with the differential operator $\mathcal{D}$. If we assume the forms of $a_{\mathbf{a}}$ and $a^\dagger_{\mathbf{a}}$ as

$$\hbar a_{\mathbf{a}} = \sqrt{\frac{\hbar}{2\omega_{\mathbf{a}}}}K^i_{\mathbf{a}}\hbar\frac{\partial}{\partial\phi^i} + G_{ij}\sqrt{\frac{\hbar}{2\omega_{\mathbf{a}}}}\dot{K}^i_{\mathbf{a}}\phi^j,$$
$$\hbar a^\dagger_{\mathbf{a}} = -\sqrt{\frac{\hbar}{2\omega_{\mathbf{a}}}}Q^i_{\mathbf{a}}\hbar\frac{\partial}{\partial\phi^i} - G_{ij}\sqrt{\frac{\hbar}{2\omega_{\mathbf{a}}}}\dot{Q}^i_{\mathbf{a}}\phi^j, \tag{2.25}$$

it is easy to see that $a_{\mathbf{a}}$ commute with $\mathcal{D}$, and the necessary condition that $a^\dagger_{\mathbf{a}}$ commute with $\mathcal{D}$ is that $Q^i_{\mathbf{a}}$ satisfies the same equation as Eq. (2.23) for $K^i_{\mathbf{a}}$. Then, as we have adopted the orthonormal basis $\hat{e}^i_{\mathbf{a}}$, which diagonalizes $\omega_{\mathbf{ab}}$ at $\tau \to -\infty$, if we set the boundary condition of $Q^i_{\mathbf{a}}$ as

$$Q^i_{\mathbf{a}} \to e^{-\omega_{\mathbf{a}}\tau} e^i_{\mathbf{a}} \quad \text{for } \tau \to -\infty\,, \tag{2.26}$$

we find $[a^\dagger_{\mathbf{a}}, a_{\mathbf{b}}] = \delta_{\mathbf{ab}}$. Hence the relevant creation and annihilation operators $A^\dagger_{\mathbf{a}}$ and $A_{\mathbf{a}}$ are found to be

$$\begin{aligned} A_{\mathbf{a}} &= e^{-\omega_{\mathbf{a}}\tau} a_{\mathbf{a}}, \\ A^\dagger_{\mathbf{a}} &= e^{\omega_{\mathbf{a}}\tau} a^\dagger_{\mathbf{a}}. \end{aligned} \tag{2.27}$$

An excited-state wave function with respect to the fluctuating degrees of freedom can be obtained by operating these creation operators, $A^\dagger_{\mathbf{a}}$, to the *quasi-ground-state* wave function (2.21).

Given these results, it is convenient to reformulate the method to construct the wave function in the following way. We consider a path in the complex plane of time as shown in Fig. 1. Along this path, we construct a solution $X(t)$ of the classical equation of motion with the initial condition $X(-\infty) = X_F$. In the segment A of the path, the solution stays at the false vacuum origin $X(t) = X_F$ which is certainly a solution. This solution can be smoothly connected to the DEP solution at a sufficiently large negative τ $(= it)$. The segment B corresponds to the DEP in the forbidden region and C to the DEP in the allowed region. The solution along C is obtained by analytically continuing the DEP solution by setting $t = -i\tau$ (> 0), which describes the classical motion after tunneling. The lowest WKB order wave function is described by this classical solution. Now along this path, we solve Eq. (2.23) for $K^i_{\mathbf{a}}$ and $Q^i_{\mathbf{a}}$ with the initial conditions at $t \to -\infty$ as

$$\begin{aligned} \sqrt{\frac{\hbar}{2\omega_{\mathbf{a}}}} Q^i_{\mathbf{a}} &= u^i_{\mathbf{a}}, \\ \sqrt{\frac{\hbar}{2\omega_{\mathbf{a}}}} K^i_{\mathbf{a}} &= u^{i*}_{\mathbf{a}}, \end{aligned} \tag{2.28}$$

where $u^i_{\mathbf{a}}$ and $u^{i*}_{\mathbf{a}}$ are the positive and negative frequency functions, respectively, in the false vacuum. In this way, we obtain the first WKB order wave function before and after the tunneling.

To close this section, perhaps it is worthwhile to mention that the time t (or τ) discussed here is *not* the external time in the original Schrödinger equation but is a parameter that naturally arises from the characterization of the lowest WKB configuration (or the internal time).

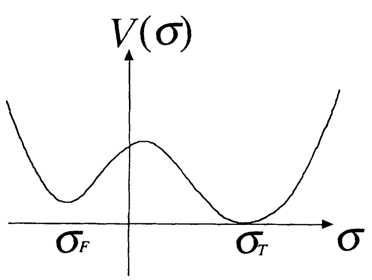

FIG. 2. The potential form of the tunneling field, where σ_F and σ_T represent the values of σ in the false and true vacua, respectively.

III. INCORPORATION OF GRAVITATIONAL EFFECT

We consider the system that consists of two scalar fields: i.e., the tunneling field σ and another field ϕ, which represents the fluctuating degrees of freedom, both coupled to gravity. We consider the situation in which the potential $U(\sigma)$ is in the form shown in Fig. 2 and σ is initially at the false vacuum, $\sigma = \sigma_F$. The interaction between the two fields is assumed to be described by the σ-dependent mass term of ϕ; $m^2(\sigma)\phi^2/2$. As in paper II we ignore the fluctuations of σ and the metric $g_{\mu\nu}$ for simplicity. We begin with the Lagrangian of the form

$$\mathcal{L} = \mathcal{L}_{\text{grav}} + \mathcal{L}_\sigma + \mathcal{L}_\phi, \tag{3.1}$$

where

$$\begin{aligned} \mathcal{L}_{\text{grav}} &= \alpha\sqrt{\gamma}\frac{1}{16\pi G}R, \\ \mathcal{L}_\sigma &= -\alpha\sqrt{\gamma}\left[\frac{1}{2}g^{\mu\nu}\nabla_\mu\sigma\nabla_\nu\sigma + U(\sigma)\right], \\ \mathcal{L}_\phi &= -\alpha\sqrt{\gamma}\left[\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \left[m^2(\sigma) + \xi R\right]\phi^2\right], \end{aligned} \tag{3.2}$$

and G and R are the gravitational constant and the scalar curvature, respectively. Here and in what follows we use the notation of the $3+1$ decomposition of the spacetime metric:

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta_s\beta^s & \beta_n \\ \beta_m & \gamma_{nm} \end{pmatrix}. \tag{3.3}$$

The Hamiltonian is given by

$$H = H_{\text{grav}} + H_\sigma + H_\phi, \tag{3.4}$$

where H_{grav} and H_σ are the Hamiltonians of the gravitational and tunneling fields, respectively, and H_ϕ is that of the fluctuating field:

$$\begin{aligned} H_\phi = \int d^3x \Big[&\frac{1}{2}\frac{\alpha}{\sqrt{\gamma}}p^2 + p\beta^m\nabla_m\phi \\ &+ \frac{\alpha\sqrt{\gamma}}{2}\left\{\gamma^{km}\nabla_k\phi\nabla_m\phi + \left[m^2(\sigma) + \xi R\right]\phi^2\right\}\Big], \end{aligned} \tag{3.5}$$

with p being the momentum conjugate to ϕ.

We should note that when gravity comes into play, there exists no external time, and the Schrödinger equation becomes the Wheeler-DeWitt equation

$$H\Psi = 0, \tag{3.6}$$

which is essentially the Hamiltonian constraint for the total system and the superspace metric has an indefinite signature, presumably with only one timelike component. However, discussion of the nature of the Wheeler-DeWitt equation is beyond the scope of this paper. Since we ignore fluctuations in $g_{\mu\nu}$ as well as in σ, we simply assume that the lowest WKB state is described by a classical solution of the $(\sigma, g_{\mu\nu})$ system and ignore problems associated with the Wheeler-DeWitt equation. Then, as we shall see below, there arises no conceptual problem with the construction of the wave functional for ϕ.

A. Instanton with gravity

Let us first construct a nontrivial solution of the Euclidean Einstein-scalar field equations, an instanton (or bounce) with gravity, to obtain the lowest WKB order picture. In the absence of gravity, it has been shown that the classical solution with the minimum action is O(4) symmetric [9]. Although it is not proved when gravity is present, it seems reasonable that it is also the case in the presence of gravity. Hence we assume so. The O(4)-symmetric instanton with gravity was investigated by Coleman and De Luccia [10]. Here we shall not repeat the details but discuss only those features of the instanton that will be necessary for our purpose.

The O(4)-symmetric instanton takes the form

$$\begin{aligned} ds_E^2 &= n^2(\eta)d\eta^2 + a^2(\eta)\left(dr^2 + \sin^2 r d\Omega_{(2)}^2\right), \\ \sigma &= \sigma(\eta), \\ \phi &= 0. \end{aligned} \tag{3.7}$$

Since we ignore the fluctuations in σ and $g_{\mu\nu}$, we denote the instanton configuration simply by $\sigma(\eta)$ and $g_{\mu\nu}(\eta)$.

Then the Euclidean action becomes

$$S_E = 2\pi^2 \int d\eta \left[\frac{1}{n}\left(-\frac{3}{8\pi G}a\dot{a}^2 + \frac{1}{2}a^3\dot{\sigma}^2\right) + n\left(-\frac{3a}{8\pi G} + a^3 U(\sigma)\right)\right], \tag{3.8}$$

where an overdot means the derivative with respect to η. We see that n plays the role of a Lagrange multiplier; a consequence of the time reparametrization invariance of the system. The variation of S_E with respect to σ gives

$$\ddot{\sigma} + \left(3\frac{\dot{a}}{a} - \frac{\dot{n}}{n}\right)\dot{\sigma} = \frac{dU}{d\sigma}, \tag{3.9}$$

and that with respect to n gives

$$\dot{a}^2 - \frac{4\pi G}{3}a^2\dot{\sigma}^2 = n^2\left(1 - \frac{8\pi G}{3}a^2 U(\sigma)\right), \tag{3.10}$$

which is nothing but the Hamiltonian constraint. Because of this constraint, the variation with respect to a does not give an independent equation.

Let us present the solution of the above equations in the thin-wall case. We also assume that the true vacuum energy density $U(\sigma_T)$ is non-negative. We choose the gauge $n(\eta) = a(\eta)$ and indicate the wall position by $\eta = \eta_w$. The result is

$$\begin{aligned} a(\eta) &= \begin{cases} \dfrac{1}{H_F \cosh\eta} & (\eta < \eta_w), \\ \dfrac{1}{H_F\left[\cosh(\eta-\eta_w)\cosh\eta_w + \sinh(\eta-\eta_w)\sqrt{\cosh^2\eta_w - (H_T/H_F)^2}\right]} & (\eta > \eta_w), \end{cases} \\ \sigma(\eta) &= \begin{cases} \sigma_F & (\eta < \eta_w), \\ \sigma_T & (\eta > \eta_w), \end{cases} \end{aligned} \tag{3.11}$$

where $\eta_w > 0$ always and

$$H_F^2 = \frac{8\pi G}{3}U(\sigma_F), \quad H_T^2 = \frac{8\pi G}{3}U(\sigma_T). \tag{3.12}$$

A schematic picture of the instanton is shown in Fig. 3. It has the topology of S^4. In the thin-wall approximation, the metric and tunneling field configurations are identical to those of the false vacuum at $\eta < \eta_w$, hence, at $\eta < 0$. Here, we restrict our attention to the case this holds, but it should be mentioned that this is not true in general once the thin-wall approximation breaks down.

As was pointed out in [11], the coordinate η cannot play the role of the "time" parameter τ, which distinguishes each spatial configuration of the instanton, the sequence of which connects the false vacuum and the critical bubble configuration corresponding to the turning point. A relevant choice of it is obtained by the coordinate transformation

$$\begin{cases} \cosh\eta = \frac{1}{\sqrt{\sin^2\tau + R^2\cos^2\tau}}, \\ \sin r = \frac{R}{\sqrt{\sin^2\tau + R^2\cos^2\tau}} \end{cases} (-\infty < \eta < \infty,\ 0 \le r \le \pi), \quad \Leftrightarrow \begin{cases} \sin\tau = -\frac{\cos r}{\sqrt{\cosh^2\eta - \sin^2 r}}, \\ R = \frac{\sin r}{\cosh\eta} \end{cases} (-\pi \le \tau \le \pi,\ 0 \le R \le 1). \tag{3.13}$$

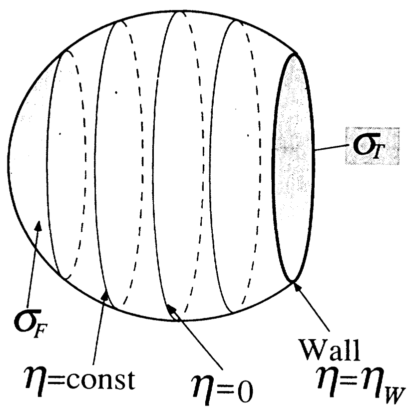

FIG. 3. A schematic picture of the Coleman DeLuccia instanton solution in the thin-wall limit. Two dimensions are suppressed.

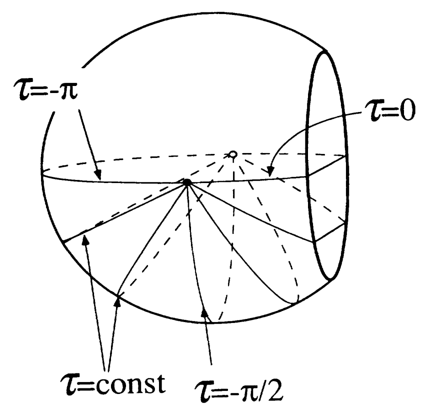

FIG. 4. Foliation of the Coleman DeLuccia instanton solution by τ = const hypersurfaces.

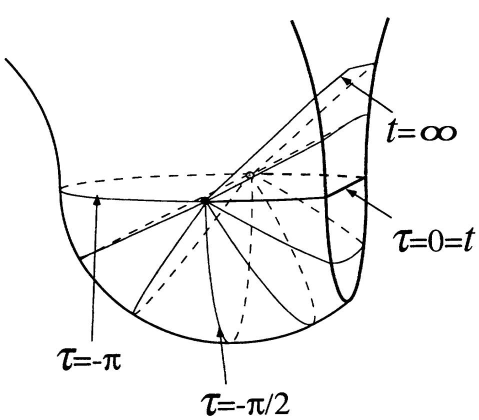

FIG. 5. The analytic continuation to the Lorentzian region of the Coleman DeLuccia instanton solution. The lower and upper halves are Euclidean and Lorentzian regions, respectively.

Then we have

$$ds_E^2 = \cosh^2\eta\, a^2(\eta)\left\{(1-R^2)d\tau^2 + \frac{dR^2}{1-R^2} + R^2 d\Omega_{(2)}^2\right\}. \qquad (3.14)$$

How these coordinates span the Euclidean spacetime is schematically shown in Fig. 4.

There are two main differences from the original coordinates. First, these coordinates reduce to the static de Sitter coordinates when $a(\eta) = 1/H_F \cosh\eta$. Therefore at $\tau < -\pi/2$, the spatial metric and the tunneling field configuration on each τ =const surface are identical to those at the false vacuum origin, and it is also possible to extend this solution beyond $\tau = -\pi$ to $\tau = -\infty$. Second, the configuration on the $\tau = 0$ surface corresponds to the turning point, where the solution can be analytically continued to the Lorentzian region by $t = -i\tau$ (> 0). How these coordinates are analytically continued to the Lorentzian region is shown in Fig. 5. Using these coordinates, a full description of the false vacuum decay at the lowest WKB order is obtained and our formalism of the multidimensional wave function can be applied to investigate the next WKB order effects.

B. Quantum state of the fluctuating field

Next we turn to the issue of the quantum fluctuations during and after this quantum tunneling. As mentioned previously, we neglect the fluctuations of the metric and the tunneling field to avoid difficulties. For convenience, we use the symbol X to represent the tunneling degree of freedom in the superspace, i.e., the spatial configurations of the metric and the tunneling field, $g_{\mu\nu} = g_{\mu\nu}(X)$ and $\sigma = \sigma(X)$. We assume X is suitably normalized so that the reduced Hamiltonian takes the form

$$H_{\rm red} = -\frac{\hbar^2}{2}\frac{\partial^2}{\partial X^2} + U(X) + H_\phi(X,\phi). \qquad (3.15)$$

We then investigate the quantum state described by $H_\phi(X,\phi)$ with X now representing the DEP parametrized by τ, $X = X(\tau)$.

To apply our formalism developed in the preceding subsection to the present case, we replace the suffices $i, j, \ldots$ with the spatial coordinates $\boldsymbol{x}, \boldsymbol{y}, \ldots$ and $\mathbf{a}, \mathbf{b}, \ldots$ with certain eigenvalue indices $\mathbf{k}, \mathbf{p}, \ldots$ for a complete set of mode functions, say $w_{\mathbf{k}}$:

$$\phi^i \to \phi(\boldsymbol{x}), \qquad \sqrt{\frac{\hbar}{2\omega_{\mathbf{a}}}}K^i_{\mathbf{a}}(\tau) \to w_{\mathbf{k}}(\boldsymbol{x},\tau),$$

$$G^{ij}(\tau) \to \frac{\alpha(\boldsymbol{x},\tau)}{\sqrt{\gamma(\boldsymbol{x},\tau)}}\delta^3(\boldsymbol{x}-\boldsymbol{y}). \qquad (3.16)$$

Here one comment is in order. If we wish to deal with the spacetime metric with nonvanishing shift vector β^i, we would need to generalize the formalism in Sec. II to allow the superspace metric to have $G_{0i} \neq 0$. However, since there is no shift vector in our metric, Eq. (3.14), this generalization is unnecessary at the moment.

Now as the state before tunneling, the most natural false vacuum state is the so-called Euclidean vacuum, which is de Sitter invariant and exhibits the same short-distance behavior of the field as the Minkowski vacuum [12]. Although the latter property is essential to single out the Euclidean vacuum, it can be made explicit only when we deal with a specific theory. Hence we focus on the de Sitter invariance of the vacuum in this subsection. Since the natural mode functions associated with the form of the metric (3.14) [with $a(\eta) = (H_F \cosh\eta)^{-1}$] does not respect the de Sitter invariance, the relation between these mode functions and those for the Euclidean vacuum is nontrivial. Specifically, they are related by a Bogoliubov transformation. This implies that the Euclidean vacuum is described as some kind of an excited state relative to the ground state constructed with respect to the static time coordinate (we call the latter the static vacuum for convenience). This is the reason why it was necessary to extend our formalism to the case of excited states at false vacuum.

To prepare the Euclidean vacuum and to obtain the Schrödinger wave functional relevant for the false vacuum decay, in what follows, we first consider a general static spacetime and overview the relation between the usual Heisenberg representation of a vacuum state and the corresponding Schrödinger wave functional. Then we construct the Euclidean vacuum over the static vacuum and translate the result to the Schrödinger picture. Once this is done, it is then straightforward to obtain the tunneling wave functional according to the prescription given in the preceding section.

Let $u_{\mathbf{k}}(x,t)$ be a set of mode functions (not necessarily the positive frequency functions with respect to the static time coordinate) and $A_{\mathbf{k}}$ be the corresponding annihilation operator; $A_{\mathbf{k}}|O\rangle = 0$, where $|O\rangle$ is the "vacuum" in the Heisenberg picture. Then we have

$$\hat{\phi}(\boldsymbol{x},t) = \sum_{\mathbf{k}}[u_{\mathbf{k}}(\boldsymbol{x},t)A_{\mathbf{k}} + u^*_{\mathbf{k}}(\boldsymbol{x},t)A^\dagger_{\mathbf{k}}],$$
$$\hat{p}(\boldsymbol{x},t) = \frac{\sqrt{\gamma(\boldsymbol{x})}}{\alpha(\boldsymbol{x})}\sum_{\mathbf{k}}[\dot{u}_{\mathbf{k}}(\boldsymbol{x},t)A_{\mathbf{k}} + \dot{u}^*_{\mathbf{k}}(\boldsymbol{x},t)A^\dagger_{\mathbf{k}}]. \quad (3.17)$$

We assume the mode functions $u_{\mathbf{k}}(\boldsymbol{x},t)$ are orthonormalized with respect to the Klein-Gordon inner product:

$$(u_{\mathbf{k}}, u_{\mathbf{p}}) := -i\int d^3x \frac{\sqrt{\gamma}}{\alpha}\left(u_{\mathbf{k}}\dot{u}^*_{\mathbf{p}} - \dot{u}_{\mathbf{k}}u^*_{\mathbf{p}}\right) = \hbar\delta_{\mathbf{kp}}. \quad (3.18)$$

To consider the Schrödinger representation, we introduce time-dependent annihilation and creation operators $a_{\mathbf{k}}(t)$ and $a^\dagger_{\mathbf{k}}(t)$, respectively, as

$$a_{\mathbf{k}}(t) = e^{-i\hat{H}t}A_{\mathbf{k}}e^{i\hat{H}t}, \quad a^\dagger_{\mathbf{k}}(t) = e^{-i\hat{H}t}A^\dagger_{\mathbf{k}}e^{i\hat{H}t}, \quad (3.19)$$

where $\hat{H}$ is the Hamiltonian operator. The Schrödinger representations of the field operators $\hat{\phi}_S(\boldsymbol{x})$ and $\hat{p}_S(\boldsymbol{x})$ are given by

$$\begin{aligned}\hat{\phi}_S(\boldsymbol{x}) &= e^{-i\hat{H}t}\hat{\phi}(\boldsymbol{x},t)e^{i\hat{H}t} \\ &= \sum_{\mathbf{k}}[u_{\mathbf{k}}(\boldsymbol{x},t)a_{\mathbf{k}}(t) + u^*_{\mathbf{k}}(\boldsymbol{x},t)a^\dagger_{\mathbf{k}}(t)], \\ \hat{p}_S(\boldsymbol{x}) &= e^{-i\hat{H}t}\hat{p}(\boldsymbol{x},t)e^{i\hat{H}t} \\ &= \frac{\sqrt{\gamma(\boldsymbol{x})}}{\alpha(\boldsymbol{x})}\sum_{\mathbf{k}}[\dot{u}_{\mathbf{k}}(\boldsymbol{x},t)a_{\mathbf{k}}(t) + \dot{u}^*_{\mathbf{k}}(\boldsymbol{x},t)a^\dagger_{\mathbf{k}}(t)].\end{aligned} \quad (3.20)$$

Using these operators, the Schrödinger representation of the vacuum, i.e., $|O(t)\rangle_S = e^{-i\hat{H}t}|O\rangle$, is determined by the condition

$$a_{\mathbf{k}}(t)|O(t)\rangle_S = 0. \quad (3.21)$$

On the other hand, using the orthonormality of the mode functions, $a_{\mathbf{k}}(t)$ and $a^\dagger_{\mathbf{k}}(t)$ are expressed as

$$\begin{aligned}\hbar a_{\mathbf{k}}(t) &= i\int d^3x\left(u^*_{\mathbf{k}}(\boldsymbol{x},t)\hat{p}_S(\boldsymbol{x}) - \frac{\sqrt{\gamma}}{\alpha}\dot{u}^*_{\mathbf{k}}(\boldsymbol{x},t)\hat{\phi}_S(\boldsymbol{x})\right), \\ \hbar a^\dagger_{\mathbf{k}}(t) &= i\int d^3x\left(-u_{\mathbf{k}}(\boldsymbol{x},t)\hat{p}_S(\boldsymbol{x}) + \frac{\sqrt{\gamma}}{\alpha}\dot{u}_{\mathbf{k}}(\boldsymbol{x},t)\hat{\phi}_S(\boldsymbol{x})\right).\end{aligned} \quad (3.22)$$

Then, going over to the coordinate representation by the replacements

$$\hat{p}_S(\boldsymbol{x}) \to -i\hbar\frac{\delta}{\delta\phi(\boldsymbol{x})}, \quad \hat{\phi}_S(\boldsymbol{x}) \to \phi(\boldsymbol{x}), \quad (3.23)$$

we find from Eq. (3.21) that

$$|O(t)\rangle_S = \mathcal{N}\exp\left(-\frac{1}{2\hbar}\int\int d^3x d^3y\ \Omega(\boldsymbol{x},\boldsymbol{y};t) \times\phi(\boldsymbol{x})\phi(\boldsymbol{y})\right), \quad (3.24)$$

where $\mathcal{N}$ is a normalization constant and

$$\Omega(\boldsymbol{x},\boldsymbol{y};t) = \frac{1}{i}\frac{\sqrt{\gamma(\boldsymbol{x})}}{\alpha(\boldsymbol{x})}\sum_{\mathbf{k}}\dot{u}^*_{\mathbf{k}}(\boldsymbol{x},t)u^{*-1}_{\mathbf{k}}(\boldsymbol{y},t), \quad (3.25)$$

where $u^{-1}_{\mathbf{k}}$ is defined as

$$\sum_{\mathbf{k}}u_{\mathbf{k}}(\boldsymbol{x},t)u^{-1}_{\mathbf{k}}(\boldsymbol{y},t) = \delta^3(\boldsymbol{x}-\boldsymbol{y}). \quad (3.26)$$

Now let us specialize the above to the case of de Sitter space and construct the Euclidean vacuum over the static vacuum. Since the Euclidean vacuum is de Sitter invariant, we need a set of mode functions which are defined over a complete Cauchy surface. However, it cannot be covered by one static chart. Just as in the case of describing the Minkowski vacuum in terms of Rindler mode functions in the Minkowski spacetime [13], we therefore need to prepare two static charts. We label the quantities associated with these two charts by the indices (1) and (2). The two regions are causally disconnected. Furthermore, as clear from the metric (3.14), the bubble nucleation takes place only in one of the two regions. For convenience we regard region (1) to be the one in which the bubble nucleation occurs. As the time direction is opposite in the two regions, we fix it by identifying the future direction with the time direction in region (1). Thus a complete Cauchy surface is given by a hypersurface $t = t^{(1)} = t^{(2)}$ and the Hamiltonian operator is positive in region (1) and negative in region (2):

$$\hat{H}_\phi = \hat{H}_\phi^{(1)} - \hat{H}_\phi^{(2)}, \tag{3.27}$$

where both the operators $H_\phi^{(i)}$ $(i = 1, 2)$ are positive and have the same form.

The positive frequency functions in each static chart behave as

$$u_{\mathbf{k}}^{(1)} \propto e^{-i\omega_{\mathbf{k}} t^{(1)}}, \quad u_{\mathbf{k}}^{(2)} \propto e^{i\omega_{\mathbf{k}} t^{(2)}}, \tag{3.28}$$

where $t^{(1)}$ and $t^{(2)}$ are the static time coordinates in regions (1) and (2), respectively. We orthonormalize them by the Klein-Gordon inner product (3.18). Note that the lapse function α is negative in region (2). The static vacuum is expressed as

$$|O\rangle = |O^{(1)}\rangle \otimes |O^{(2)}\rangle; \quad A_{\mathbf{k}}^{(i)}|O^{(i)}\rangle = 0 \quad (i = 1, 2), \tag{3.29}$$

where $A_{\mathbf{k}}^{(i)}$ are the annihilation operators associated with the positive frequency functions $u_{\mathbf{k}}^{(i)}$.

The positive frequency functions for the Euclidean vacuum are expressed in terms of a Bogoliubov transformation from the static vacuum positive frequency functions. Since regions (1) and (2) are completely symmetric, there exist two independent positive frequency functions for the Euclidean vacuum for each $\mathbf{k}$, which we denote by $\bar{u}_{\mathbf{k}}^{(1)}$ and $\bar{u}_{\mathbf{k}}^{(2)}$. Then in matrix notation with indices $\mathbf{k}$ suppressed, the Bogoliubov transformation takes the form

$$\begin{pmatrix} \bar{u}^{(1)} \\ \bar{u}^{(2)} \end{pmatrix} = \alpha \begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix} + \beta \begin{pmatrix} u^{(1)*} \\ u^{(2)*} \end{pmatrix};$$

$$\alpha = \begin{pmatrix} \alpha^{(1)} & \alpha^{(2)} \\ \alpha^{(2)} & \alpha^{(1)} \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta^{(1)} & \beta^{(2)} \\ \beta^{(2)} & \beta^{(1)} \end{pmatrix},$$

$$\alpha\alpha^\dagger - \beta\beta^\dagger = I, \tag{3.30}$$

where I denotes the unit matrix. Further, by a unitary transformation, we can always make the matrix α diagonal. Hence we may assume

$$\alpha_{\mathbf{kk'}}^{(1)} = \alpha_{\mathbf{k}}\delta_{\mathbf{kk'}}, \quad \alpha_{\mathbf{kk'}}^{(2)} = 0. \tag{3.31}$$

Let $|\bar{O}\rangle$ be the Euclidean vacuum. Then it is characterized by

$$\bar{A}_{\mathbf{k}}^{(i)}|\bar{O}\rangle = 0 \quad (i = 1, 2), \tag{3.32}$$

where $\bar{A}_{\mathbf{k}}^{(i)}$ are the annihilation operator associated with the positive frequency functions $\bar{u}_{\mathbf{k}}^{(i)}$. The corresponding Bogoliubov transformation for $\bar{A}_{\mathbf{k}}^{(i)}$ is

$$\begin{pmatrix} \bar{A}^{(1)} \\ \bar{A}^{(2)} \end{pmatrix} = \alpha^* \begin{pmatrix} A^{(1)} \\ A^{(2)} \end{pmatrix} - \beta^* \begin{pmatrix} A^{(1)\dagger} \\ A^{(2)\dagger} \end{pmatrix}. \tag{3.33}$$

Then using the commutation relation between creation and annihilation operators, we find that the vacuum $|\bar{O}\rangle$ is expressed as

$$|\bar{O}\rangle = \mathcal{N} \exp\left(\frac{1}{2} \sum_{i,j,\mathbf{k},\mathbf{k'}} B_{\mathbf{kk'}}^{(i)(j)} A_{\mathbf{k}}^{(i)\dagger} A_{\mathbf{k'}}^{(j)\dagger}\right) \times |O^{(1)}\rangle \otimes |O^{(2)}\rangle, \tag{3.34}$$

where $\mathcal{N}$ is some normalization constant and

$$B_{\mathbf{kk'}}^{(i)(j)} = \left(\alpha^{*-1}\beta^*\right)_{\mathbf{kk'}}^{(i)(j)}. \tag{3.35}$$

The Euclidean vacuum state is invariant under the action of any generator of the de Sitter group. Suppose the indices $\mathbf{k}$ of $u_{\mathbf{k}}^{(i)}$ represent the eigenvalues associated with spherical mode decomposition, (k, ℓ, m), i.e., each of these modes is characterized by an eigenstate of the Hamiltonian $\hat{H}_\phi$, the angular-momentum square $\hat{\mathbf{J}}^2$, and the z component of it $\hat{J}_z$. Since these operators are generators of the de Sitter group, the Euclidean vacuum must be a zero eigenstate of all of them. From the fact that all of these operators have the form, $\hat{Q} = \hat{Q}^{(1)} - \hat{Q}^{(2)}$, we find the matrix B should take the form

$$B = \begin{pmatrix} 0 & b \\ b & 0 \end{pmatrix}; \quad b_{\mathbf{kk'}} = B_{\mathbf{k}}\delta_{\mathbf{kk'}}. \tag{3.36}$$

Then, from Eqs. (3.31), (3.35), and (3.36), the matrix β is found to have the form

$$\beta_{\mathbf{kk'}}^{(1)} = 0, \quad \beta_{\mathbf{kk'}}^{(2)} = \beta_{\mathbf{k}}\delta_{\mathbf{kk'}}, \tag{3.37}$$

and

$$B_{\mathbf{k}} = \frac{\beta_{\mathbf{k}}^*}{\alpha_{\mathbf{k}}^*}. \tag{3.38}$$

Specifically, the Euclidean vacuum positive frequency functions are given by

$$\begin{aligned} \bar{u}_{\mathbf{k}}^{(1)} &= \alpha_{\mathbf{k}} u_{\mathbf{k}}^{(1)} + \beta_{\mathbf{k}} u_{\mathbf{k}}^{*(2)}, \\ \bar{u}_{\mathbf{k}}^{(2)} &= \alpha_{\mathbf{k}} u_{\mathbf{k}}^{(2)} + \beta_{\mathbf{k}} u_{\mathbf{k}}^{*(1)}, \end{aligned} \tag{3.39}$$

and the Euclidean vacuum is given by

$$|\bar{O}\rangle = \mathcal{N} \exp\left(\sum_{\mathbf{k}} B_{\mathbf{k}} A_{\mathbf{k}}^{(1)\dagger} A_{\mathbf{k}}^{(2)\dagger}\right) |O^{(1)}\rangle \otimes |O^{(2)}\rangle. \tag{3.40}$$

At this point, we note that $\bar{u}_{\mathbf{k}}^{(1)}$ ($\bar{u}_{\mathbf{k}}^{(2)}$) is proportional to $e^{-i\omega_{\mathbf{k}} t}$ ($e^{i\omega_{\mathbf{k}} t}$) for the time coordinate t $(= t^{(1)} = t^{(2)})$ extended over both regions (1) and (2). This is a result of the fact that, although regions (1) and (2) are causally disconnected in the Lorentzian regime, they are analytically connected through the Euclidean regime as $\tau = it^{(1)}$ and $\tau = \pm\pi + it^{(2)}$. It is known that the positive (negative) frequency functions for the Euclidean vacuum are characterized by the regularity on the upperhalf (lower-half) complex t plane [13]. For later convenience, let us consider the negative frequency functions (i.e., the analytic continuations at $t = 0$ and $t = -i\pi$ through the lower-half plane). Then the negative frequency function in region (1), $\bar{u}_{\mathbf{k}}^{(1)*} \propto e^{i\omega_{\mathbf{k}} t^{(1)}}$, is analytically continued to region (2) as $\bar{u}_{\mathbf{k}}^{(1)*} \propto e^{-\omega_{\mathbf{k}}\pi + i\omega_{\mathbf{k}} t^{(2)}}$. Similarly, the negative frequency function in region (2), $\bar{u}_{\mathbf{k}}^{(2)*} \propto e^{-i\omega_{\mathbf{k}} t^{(2)}}$, is analytically continued to region (1) as $\bar{u}_{\mathbf{k}}^{(2)*} \propto e^{-\omega_{\mathbf{k}}\pi - i\omega_{\mathbf{k}} t^{(1)}}$. Taking the complex conjugates of them we then find

$$\bar{u}_{\mathbf{k}}^{(1)} = \alpha_{\mathbf{k}} \left(u_{\mathbf{k}}^{(1)} + e^{-\omega_{\mathbf{k}}\pi} u_{\mathbf{k}}^{*(2)} \right),$$

$$\bar{u}_{\mathbf{k}}^{(2)} = \alpha_{\mathbf{k}} \left(u_{\mathbf{k}}^{(2)} + e^{-\omega_{\mathbf{k}}\pi} u_{\mathbf{k}}^{*(1)} \right). \tag{3.41}$$

Comparing these with Eq. (3.39) and noting the normalization condition $|\alpha_{\mathbf{k}}|^2 - |\beta_{\mathbf{k}}|^2 = 1$, we obtain

$$\alpha_{\mathbf{k}} = \frac{e^{i\theta_{\mathbf{k}}}}{\sqrt{1 - e^{-2\omega_{\mathbf{k}}\pi}}}, \quad \beta_{\mathbf{k}} = \frac{e^{-\omega_{\mathbf{k}}\pi + i\theta_{\mathbf{k}}}}{\sqrt{1 - e^{-2\omega_{\mathbf{k}}\pi}}}, \tag{3.42}$$

where $\theta_{\mathbf{k}}$ is a constant phase. Also from Eq. (3.38) we find

$$B_{\mathbf{k}} = e^{-\omega_{\mathbf{k}}\pi}. \tag{3.43}$$

Now it is easy to interpret the above result to the language of the Schrödinger wave functional. Applying Eq. (3.24) to the present case we find

$$|\bar{O}(t)\rangle_S = \mathcal{N} \exp\left(-\frac{1}{2\hbar} \int\int d^3x d^3y \, \bar{\Omega}(\boldsymbol{x}, \boldsymbol{y}; t) \times \phi(\boldsymbol{x})\phi(\boldsymbol{y}) \right), \tag{3.44}$$

where

$$\bar{\Omega}(\boldsymbol{x}, \boldsymbol{y}; t) = \frac{\sqrt{\gamma}}{\alpha} \sum_{\mathbf{k}} \left(\dot{\bar{u}}_{\mathbf{k}}^{(1)}(\boldsymbol{x}, t) \bar{u}_{\mathbf{k}}^{(1)-1}(\boldsymbol{y}, t) + \dot{\bar{u}}_{\mathbf{k}}^{(2)}(\boldsymbol{x}, t) \bar{u}_{\mathbf{k}}^{(2)-1}(\boldsymbol{y}, t) \right). \tag{3.45}$$

However, as we have seen, this state is a zero eigenstate of the Hamiltonian. Hence the wave functional (3.44) should be time independent. This can be demonstrated as follows. Operating $e^{-i\hat{H}_\phi t}$ to Eq. (3.40) we obtain

$$|\bar{O}(t)\rangle_S = \exp\left(\sum_{\mathbf{k}} B_{\mathbf{k}} a_{\mathbf{k}}^{(1)\dagger}(t^{(1)}) a_{\mathbf{k}}^{(2)\dagger}(t^{(2)}) \right) \times |O^{(1)}(t^{(1)})\rangle_S \otimes |O^{(2)}(t^{(2)})\rangle_S, \tag{3.46}$$

where $t^{(1)} = t^{(2)} = t$ and $a_{\mathbf{k}}^{(i)\dagger}(t^{(i)})$ are the time-dependent creation operators as defined in Eq. (3.19). Their coordinate representations are given as

$$a_{\mathbf{k}}^{(i)}(t^{(i)}) = \int d^3x \left(\hbar u_{\mathbf{k}}^{*(i)}(\boldsymbol{x}, t^{(i)}) \frac{\delta}{\delta\phi(\boldsymbol{x})} - i\frac{\sqrt{\gamma}}{\alpha} \dot{u}_{\mathbf{k}}^{*(i)}(\boldsymbol{x}, t^{(i)}) \phi(\boldsymbol{x}) \right),$$

$$a_{\mathbf{k}}^{(i)\dagger}(t^{(i)}) = \int d^3x \left(-\hbar u_{\mathbf{k}}^{(i)}(\boldsymbol{x}, t^{(i)}) \frac{\delta}{\delta\phi(\boldsymbol{x})} + i\frac{\sqrt{\gamma}}{\alpha} \dot{u}_{\mathbf{k}}^{(i)}(\boldsymbol{x}, t^{(i)}) \phi(\boldsymbol{x}) \right). \tag{3.47}$$

Since $|O^{(i)}\rangle$ is the true ground state of the Hamiltonian $\hat{H}_\phi^{(i)}$, we have, for $t^{(1)} = t^{(2)} = t$,

$$|O^{(1)}(t)\rangle_S \otimes |O^{(2)}(t)\rangle_S = \left(e^{-iE_0 t} |O^{(1)}\rangle \right) \otimes \left(e^{iE_0 t} |O^{(2)}\rangle \right) = |O^{(1)}\rangle \otimes |O^{(2)}\rangle. \tag{3.48}$$

Furthermore, since the Hamiltonian is diagonalized with respect to the static vacuum mode functions, we have

$$a_{\mathbf{k}}^{(1)\dagger}(t) a_{\mathbf{k}}^{(2)\dagger}(t) = \left(e^{-i\omega_{\mathbf{k}} t} A_{\mathbf{k}}^{(1)\dagger} \right) \left(e^{i\omega_{\mathbf{k}} t} A_{\mathbf{k}}^{(2)\dagger} \right) = A_{\mathbf{k}}^{(1)\dagger} A_{\mathbf{k}}^{(2)\dagger}. \tag{3.49}$$

Therefore we find

$$|\bar{O}(t)\rangle_S = \exp\left(\sum_{\mathbf{k}} B_{\mathbf{k}} A_{\mathbf{k}}^{(1)\dagger} A_{\mathbf{k}}^{(2)\dagger} \right) |O^{(1)}\rangle \otimes |O^{(2)}\rangle. \tag{3.50}$$

This form is explicitly time independent.

Now, as we have obtained the asymptotic behavior of the wave functional in the false vacuum, we go back to the problem of constructing the tunneling wave functional. As clear from Eq. (3.50), the Euclidean vacuum can be considered as a superposition of many particle states of the form

$$(A_{\mathbf{k}_1}^{(1)\dagger})^{n_1} \cdots (A_{\mathbf{k}_m}^{(1)\dagger})^{n_m} |O^{(1)}\rangle \otimes (A_{\mathbf{k}_1}^{(2)\dagger})^{n_1} \cdots (A_{\mathbf{k}_m}^{(2)\dagger})^{n_m} |O^{(2)}\rangle. \tag{3.51}$$

As there is no causal connection between regions (1) and (2), the wave functional can be constructed independently in each region. First consider region (1). Applying the results obtained in the preceding section, the wave functional for region (1), which has the asymptotic form in the false vacuum as

$$(A_{\mathbf{k}_1}^{(1)\dagger})^{n_1} \cdots (A_{\mathbf{k}_m}^{(1)\dagger})^{n_m} |O^{(1)}\rangle, \tag{3.52}$$

is given by

$$[A_{\mathbf{k}_1}^{(1)\dagger}(t^{(1)})]^{n_1} \cdots [A_{\mathbf{k}_m}^{(1)\dagger}(t^{(1)})]^{n_m} |O^{(1)}(t^{(1)})\rangle, \tag{3.53}$$

where $|O^{(1)}(t^{(1)})\rangle$ is the *quasi-ground-state* wave functional with the parameter time $t^{(1)}$ along the path shown in Fig. 1. The operators $A_{\mathbf{k}}^{(1)}(t^{(1)})$ and $A_{\mathbf{k}}^{(1)\dagger}(t^{(1)})$ are given by

$$A_{\mathbf{k}}^{(1)}(t^{(1)}) = e^{-i\omega_{\mathbf{k}}t^{(1)}}a_{\mathbf{k}}^{(1)}(t^{(1)}) = e^{-i\omega_{\mathbf{k}}t^{(1)}}\int d^3x\left(\hbar w_{\mathbf{k}}^{(1)}(\boldsymbol{x},t^{(1)})\frac{\delta}{\delta\phi(\boldsymbol{x})} - i\frac{\sqrt{\gamma}}{\alpha}\dot{w}_{\mathbf{k}}^{(1)}(\boldsymbol{x},t^{(1)})\phi(\boldsymbol{x})\right),$$
$$A_{\mathbf{k}}^{(1)\dagger}(t^{(1)}) = e^{i\omega_{\mathbf{k}}t^{(1)}}a_{\mathbf{k}}^{(1)\dagger}(t^{(1)}) = e^{i\omega_{\mathbf{k}}t^{(1)}}\int d^3x\left(-\hbar v_{\mathbf{k}}^{(1)}(\boldsymbol{x},t^{(1)})\frac{\delta}{\delta\phi(\boldsymbol{x})} + i\frac{\sqrt{\gamma}}{\alpha}\dot{v}_{\mathbf{k}}^{(1)}(\boldsymbol{x},t^{(1)})\phi(\boldsymbol{x})\right), \tag{3.54}$$

where $v_{\mathbf{k}}^{(1)}$ and $w_{\mathbf{k}}^{(1)}$ satisfy the field equation

$$\{g^{\mu\nu}\nabla_\mu\nabla_\nu - [m^2(\sigma) + \xi R]\}z_{\mathbf{k}}^{(1)} = 0 \quad (z = v, w), \tag{3.55}$$

along the trajectory on the complex $t^{(1)}$ plane shown in Fig. 1 with the asymptotic initial condition at $t^{(1)} < 0$,

$$v_{\mathbf{k}}^{(1)}(\boldsymbol{x},t^{(1)}) = u_{\mathbf{k}}^{(1)}(\boldsymbol{x},t^{(1)}),$$
$$w_{\mathbf{k}}^{(1)}(\boldsymbol{x},t^{(1)}) = u_{\mathbf{k}}^{(1)*}(\boldsymbol{x},t^{(1)}). \tag{3.56}$$

As for the wave functional for region (2), since the tunneling degree of freedom remains at the false vacuum origin, it is $t^{(2)}$ independent. However, we may express the wave functional in the same manner as that for region (1) by solving the mode functions $v_{\mathbf{k}}^{(2)}$ and $w_{\mathbf{k}}^{(2)}$ along the contour of Fig. 1 on the complex $t^{(2)}$ plane, but with the false vacuum configuration throughout the contour.

Summing up all the terms again, we obtain the wave functional, which describes tunneling from the Euclidean vacuum:

$$\Phi[\phi(\,\cdot\,);t] = \mathcal{N}\exp\left(\sum_{\mathbf{k}} B_{\mathbf{k}}A_{\mathbf{k}}^{(1)\dagger}(t)A_{\mathbf{k}}^{(2)\dagger}\right) \times|O^{(1)}(t)\rangle\otimes|O^{(2)}\rangle. \tag{3.57}$$

Now, as we have discussed when deriving the Bogoliubov coefficients (3.42), when we consider the negative frequency functions, regions (1) and (2) are analytically connected through the lower-half complex t plane while the tunneling field is at the false vacuum origin. Hence we expect the tunneling wave functional to be obtained by finding the mode functions $\bar{w}_{\mathbf{k}}^{(i)}$ for the tunneling background that correspond to the Euclidean vacuum negative frequency functions $\bar{u}_{\mathbf{k}}^{(i)*}$. Namely, first we set the boundary condition

$$\bar{w}_{\mathbf{k}}^{(i)}(\boldsymbol{x},t) = \bar{u}_{\mathbf{k}}^{(i)*}(\boldsymbol{x},t), \tag{3.58}$$

at $t < 0$. Then we find that they solve the field equation (3.55) in the Lorentzian time to $t = t^{(1)} = t^{(2)} = 0$, and further in the Euclidean time with $\tau = it^{(1)}$ and $\tau = -\pi + it^{(2)}$ beyond $\tau = -\pi$ to $-\infty$. Note that from Eq. (3.41), the solutions are automatically consistent with the analytic continuation at both $\tau = 0$ and $-\pi$. Then we solve back to $\tau = 0$ through the nontrivial O(4) bubble background only in the interval $-\pi/2 < \tau \leq 0$ and analytically continue to the Lorentzian time with $t = t^{(2)} = -i(\tau + \pi)$ at $\tau = -\pi$ to the de Sitter background and $t = t^{(1)} = -i\tau$ at $\tau = 0$ to the O(3,1) bubble background.

Then, with mode functions $\bar{w}_{\mathbf{k}}^{(i)}$ obtained thus we can in fact show that the tunneling wave functional satisfies

$$\bar{a}_{\mathbf{k}}^{(i)}(t)\Phi[\phi(\,\cdot\,);t] = 0, \tag{3.59}$$

where

$$\bar{a}_{\mathbf{k}}^{(i)}(t) = \int d^3x\left(\hbar\bar{w}_{\mathbf{k}}^{(i)}(\boldsymbol{x},t)\frac{\delta}{\delta\phi(\boldsymbol{x})} - i\frac{\sqrt{\gamma}}{\alpha}\dot{\bar{w}}_{\mathbf{k}}^{(i)}(\boldsymbol{x},t)\phi(\boldsymbol{x})\right). \tag{3.60}$$

After all, as is clear from the above procedure we do not have to know $\bar{w}_{\mathbf{k}}^{(i)}$ at $\tau \leq -\pi$ but only those in the interval $-\pi \leq \tau \leq 0$. Furthermore $\bar{w}_{\mathbf{k}}^{(i)}$ coincide with $\bar{u}_{\mathbf{k}}^{(i)}$ in the region $\tau \leq -\pi/2$. Once we become aware of these facts, we need not stick to the construction of $\bar{w}_{\mathbf{k}}^{(i)}$ themselves, nor to the coordinates of the metric (3.14). Instead, any complete set of mode functions which are related to $\bar{w}_{\mathbf{k}}^{(i)}$ by a unitary transformation is relevant and any convenient coordinate system can be chosen to solve for them. For this reason, we may drop the superscript (i) for the mode functions and denote them simply by $\bar{w}_{\mathbf{k}}$. Thus the procedure to construct the tunneling wave functional in the case with gravity turns out to be very similar to the case without gravity. In particular, the resulting quantum state after tunneling will be related to the true vacuum state by a Bogoliubov transformation as discussed in papers I and II.

C. Conformal scalar model

Here, as a simple application of our formalism, we consider a conformally coupled scalar field ϕ [i.e., $\xi = 1/6$ in Eq. (3.5)], which is massless except on the bubble wall. In paper II we have investigated a similar model in the absence of the background curvature. We show below that this conformal scalar model gives the quantum state, which is conformally equivalent to the one without gravity in paper II.

A convenient choice of the coordinates for the present case is obtained by the coordinate transformation of the metric (3.7) with $n(\eta) = a(\eta)$ as

$$T_E = -e^{\eta}\cos r,$$
$$\rho = e^{\eta}\sin r, \tag{3.61}$$

or equivalently that of the metric (3.14) as

$$T_E = \frac{\sqrt{1-R^2}\sin\tau(1+\sqrt{1-R^2}\cos\tau)}{\sin^2\tau + R^2\cos^2\tau},$$
$$\rho = \frac{R(1+\sqrt{1-R^2}\cos\tau)}{\sin^2\tau + R^2\cos^2\tau}. \tag{3.62}$$

Then the metric becomes

$$ds_E^2 = \Omega_E^2(\xi_E)\left(dT_E^2 + d\rho^2 + \rho^2 d\Omega_{(2)}^2\right), \qquad (3.63)$$

where

$$\Omega_E^2 = \frac{a^2(\ln \xi_E)}{\xi_E^2}, \quad \xi_E = \sqrt{T_E^2 + \rho^2}. \qquad (3.64)$$

As clear from Eq. (3.62), the analytic continuation to the Lorentzian metric by $\tau = it$ ($t = t^{(1)}$) and $\tau = -\pi + it$ ($t = t^{(2)}$) corresponds to that at $T_E = 0$ by $T_E = iT$. Hence the Lorentzian version of the metric (3.63) is

$$ds^2 = \Omega^2(\xi)\left(-dT^2 + d\rho^2 + \rho^2 d\Omega_{(2)}^2\right), \qquad (3.65)$$

where

$$\Omega^2 = \frac{a^2(\ln \xi)}{\xi^2}, \quad \xi = \sqrt{-T^2 + \rho^2}. \qquad (3.66)$$

At the false vacuum origin, Eq. (3.65) reduces to the de Sitter metric:

$$ds_{\rm deS}^2 = \Omega_{\rm deS}^2(\xi)\left(-dT^2 + d\rho^2 + \rho^2 d\Omega_{(2)}^2\right),$$
$$\Omega_{\rm deS} = \frac{2}{H_F(1 - T^2 + \rho^2)}. \qquad (3.67)$$

As we have seen in the preceding subsection, the procedure to obtain the tunneling wave functional is to solve for the mode functions $\bar{w}_{\mathbf{k}}$ in the interval $-\pi \le \tau \le 0$ of the Euclidean tunneling background and analytically continue them to the Lorentzian background at $\tau = -\pi$ and $\tau = 0$ with the condition that $\bar{w}_{\mathbf{k}}$ are unitarily equivalent to the analytic continuation of the Euclidean vacuum negative frequency functions $\bar{u}_{\mathbf{k}}^*$ in the region $\tau \le -\pi/2$. Hence we may regard Eq. (3.65) to represent the background metric both in the Euclidean and Lorentzian regimes by allowing T to take complex values. We show how the time coordinate T spans the spacetime in Fig. 6. The advantage of using the coordinates (T, ρ) is that the correspondence to the flat spacetime case becomes transparent. The metric (3.65) is conformally equivalent to the flat spacetime metric: $g_{\mu\nu} = \Omega^2 \eta_{\mu\nu}$. Further, the conformal factor Ω is a function of only ξ, i.e., it is O(3,1) [or O(4)] invariant. In particular, Ω may be regarded as a function of the tunneling field σ.

Because of the conformal coupling of the scalar field, the field equation (3.55) for the mode functions $\bar{w}_{\mathbf{k}}$ can be conformally transformed to that on the flat spacetime:

$$[\eta^{\mu\nu}\nabla_\mu\nabla_\nu - m^2(\sigma)\Omega^2(\sigma)]\bar{w}_{f\mathbf{k}} = 0, \qquad (3.68)$$

where we have regarded Ω as a function of σ and

$$\bar{w}_{f\mathbf{k}} = \Omega \bar{w}_{\mathbf{k}}. \qquad (3.69)$$

Thus we can construct the mode functions which satisfy Eq. (3.55), by solving Eq. (3.68) in the flat spacetime.

The remaining task is to impose the correct boundary condition on $\bar{w}_{f\mathbf{k}}$. For this purpose, let us consider the case of a pure de Sitter background. If the conformal vacuum defined by the positive frequency functions $u_{f\mathbf{k}}$ ($\propto e^{-ikT}$) on the flat space agrees with the Euclidean vacuum, the required initial condition for $\bar{w}_{f\mathbf{k}}$ is trivial; $\bar{w}_{f\mathbf{k}} = u_{f\mathbf{k}}^*$. To see this is indeed the case, we examine the symmetric two-point function in the conformal vacuum. It is well known that the positive frequency function $\bar{u}_{\mathbf{k}}$ for this conformal vacuum is given by $\bar{u}_{\mathbf{k}} = \Omega^{-1}\bar{u}_{f\mathbf{k}}$ and as a result the two-point function $G^{(1)}$ is given by

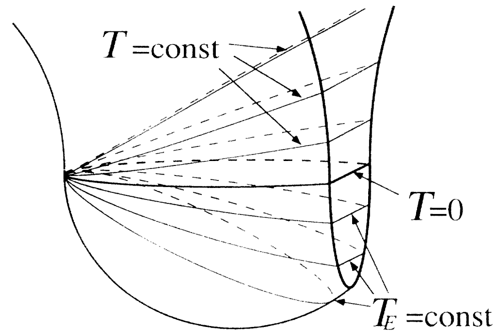

FIG. 6. The same as Fig. 5, but with foliation by the $T =$ const hypersurfaces.

$$G^{(1)}(x, x') = \Omega_{\rm deS}^{-1}(x^2) D^{(1)}(x, x') \Omega_{\rm deS}^{-1}(x'^2), \qquad (3.70)$$

where $\Omega_{\rm deS}$ is given by Eq. (3.67), $x^2 = -(x^0)^2 + \boldsymbol{x}^2 = -T^2 + \rho^2$, and

$$D^{(1)}(x, x') := \frac{1}{2\pi^2}\frac{1}{(x - x')^2} \qquad (3.71)$$

is the symmetric two-point function in the Minkowski vacuum.

If we embed the de Sitter space in five-dimensional Minkowski space,

$$ds^2 = -(d\tilde{x}^0)^2 + (d\tilde{x}^1)^2 + (d\tilde{x}^2)^2 + (d\tilde{x}^3)^2 + (d\tilde{x}^4)^2, \qquad (3.72)$$

we find the coordinates $\tilde{x}^a$ ($a = 0, 1, 2, 3, 4$) on the de Sitter space are expressed in terms of $x^\mu = (T, \boldsymbol{x})$ as

$$(\tilde{x}^\mu, \tilde{x}^4) = \left(\frac{2x^\mu}{H_F(1 + x^2)}, \frac{1 - x^2}{H_F(1 + x^2)}\right). \qquad (3.73)$$

It is then easy to show that

$$G^{(1)}(x, x') = \frac{1}{2\pi^2}\frac{1}{(\tilde{x} - \tilde{x}')^2}. \qquad (3.74)$$

This shows $G^{(1)}$ is de Sitter invariant. Hence, together with the fact that its short-distance behavior is the same as that in the Minkowski vacuum, the present conformal vacuum is found to be the Euclidean vacuum. The relevant mode functions $\bar{w}_{\mathbf{k}}$ for the tunneling wave functional are then given by solving Eq. (3.68) along the contour shown in Fig. 1 on the complex T plane and multiplying the result by the inverse of the conformal factor; $\bar{w}_{\mathbf{k}} = \Omega^{-1}\bar{w}_{f\mathbf{k}}$.

To summarize, in the present case of a conformally coupled scalar field, the effects of gravity to the quantum state after tunneling is to solve the mode functions for the flat background with the mass term m^2 replaced by $m^2\Omega^2$ and multiplying the resultant mode functions by Ω^{-1}, with Ω given in Eq. (3.66). The two-point functions for the quantum state after tunneling are also given by the conformal transformation of those obtained for the flat background $D(x,x')$ as

$$G(x,x') = \Omega^{-1}(x^2)D(x,x')\Omega^{-1}(x'^2). \tag{3.75}$$

However, the evaluation of the energy-momentum tensor seems to require some care. It is known that the regularized vacuum expectation value of the energy-momentum tensor, $\langle T^{\mu\nu}\rangle$, for a conformally coupled field on a conformally flat spacetime consists of term arising from a trivial conformal transformation of $\langle T_f^{\mu\nu}\rangle_{\rm reg}$ in flat space and the terms representing conformal anomalies [14]:

$$\langle T_{\mu\nu}\rangle_{\rm reg} = \Omega^{-2}\left\langle T^f_{\mu\nu}\right\rangle + \text{conformal anomalies}. \tag{3.76}$$

It is also known that if there arises no further divergence apart from the common ones for any spacetime, Eq. (3.76) continues to hold for arbitrary state. However, in the present case, as discussed in paper II, we encounter a new type of divergence, which may be partly due to the δ-function nature of the mass term in our model and is also possibly due to the breakdown of the WKB expansion. Unfortunately at the moment, we are unable to clarify if these new divergences would give rise to terms that are conformally nontrivial. If not, since the finite terms of $\left\langle T^f_{\mu\nu}\right\rangle$ we have found in flat space in paper II will be absent in the case of conformal coupling, except for the term that diverges on the light cone, we would find only the conformal anomaly terms in the present model, provided we impose the regularity on the light cone. Furthermore, if the true vacuum has no vacuum energy, it reduces to the flat space and we would find no finite term at all, which sounds rather paradoxical. In paper II we have argued that the regularity on the light cone is necessary to keep the validity of the WKB expansion. Hence if we allow the presence of the term, which diverges on the light cone, it will be necessary to seriously consider the possible breakdown of the WKB expansion. The resolution of this issue is left for future study.

IV. CONCLUSIONS

We have considered an extension of our previous analysis in paper II of the quantum state after O(4)-symmetric bubble nucleation in flat space to the case with the gravitational effect. In order to do so, we have first extended the formalism developed in paper I to the case of multidimensional tunneling from an excited state at the false vacuum origin. Then using the result of extension, we have developed a method to obtain the tunneling wave functional from the false vacuum to the true vacuum through a nontrivial geometry of the background spacetime described by the O(4)-symmetric bubble with gravity. Provided that the O(4) bubble is described by the thin-wall approximation, we have found the procedure to construct the tunneling wave functional can be formulated in quite a similar manner as in the case of flat spacetime background.

As an explicit demonstration of our formalism, we have considered a simple conformal scalar model, which is massless except on the bubble wall to represent the fluctuations around the O(4) bubble. We have then found the the resulting quantum state is conformally equivalent to that in the absence of gravity, i.e., it is described by a Bogoliubov transformation of the true vacuum state (a squeezed state). However, we have argued that the evaluation of the regularized expectation value of the energy-momentum tensor for this quantum state may be highly nontrivial, apart from the conventional conformal anomalies. We have also pointed out the paradoxical situation that the regularized energy-momentum tensor might vanish due to the conformal coupling nature of our model if the true vacuum has no vacuum energy density.

At the moment, we are unfortunately unable to judge whether these issues are particularities of our oversimplified model or intrinsic difficulties associated with field-theoretical tunneling phenomena. Further research on the present subject is apparently required.

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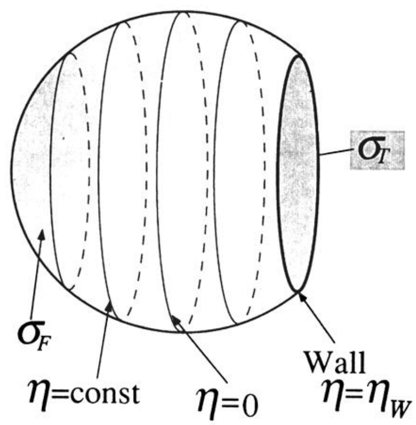

FIG. 3. A schematic picture of the Coleman DeLuccia instanton solution in the thin-wall limit. Two dimensions are suppressed.