Hairy black holes in string theory

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Solutions of bosonic string theory are constructed which correspond to four-dimensional black holes with axionic quantum hair. The basic building blocks are the renormalization group flows of the CP^1 model with a θ term and the SU(1,1)/U(1) WZW coset conformal field theory. However the solutions are also found to have negative energy excitations, and are accordingly expected to decay to the vacuum.

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The classical end point of gravitational collapse is expected to be a simple object: a stationary black hole characterized by its mass, charge, and angular momentum. This expectation is strengthened by the famous "no-hair" theorems [1]. These theorems assume a specific field content, but they generally suggest that the stationary, stable black hole configurations are labeled only by conserved quantities associated with local gauge symmetries.

Quantum mechanics leads to a further "balding" of black holes. For example, Hawking radiation will cause a charged black hole to radiate away mass until it reaches the extremal (mass = charge) limit. Thus one less quantum number is required to characterize the stable end point.

Several years ago it was suggested that quantum mechanics might lead to hair growth as well as hair loss [2]. In theories with axion strings, an arbitrary phase can be associated with the process of lassoing a black hole with an axion string. This phase arises from the axion-string interaction Lagrangian

$$S_I = T \int_\Sigma B \ . \tag{1}$$

The integral extends over the string world sheet Σ, T is the string tension, and B is the axion two-form potential. If B is given by the closed but not exact two-form obeying $\int_{S^2} B = \theta L^2$ (where θ is dimensionless and L is the appropriate distance scale) for any two-sphere surrounding the horizon, it follows that a string which lassos the black holes picks up a phase $\theta T L^2/\hbar$. θ is classically unobservable, but may be measured quantum mechanically in Aharonov-Bohm-type interference experiments. It is thus a new quantum number, or "quantum hair" associated with a quantum black hole. Generalizations of this idea involving discrete Z_N gauge symmetries were discussed in [3].

Quantum hair has no perturbative effect on the Hawking radiation rate, so large hairy black holes will evaporate as usual. However, for small black holes, nonperturbative effects could be important and one might suspect the existence of hairy extremal black holes [2]. These objects would be stabilized against Hawking radiation by quantum hair, just as charged extremal black holes are stabilized by their classical electromagnetic hair.

Evidence for such extremal objects was found (in the context of discrete gauge hair) in the elegant analysis of Coleman, Preskill, and Wilczek [4]. A Euclidean instanton which can be described as a virtual string lassoing the black holes was shown to slow down the Hawking radiation rate, suggesting that it might actually turn off at a critical value of the mass. However, unlike electromagnetic charge, axion (or discrete) charge is periodic and cannot be made arbitrarily large. Hairy extremal black holes are therefore typically Planckian objects, and their existence cannot ordinarily be determined from low-energy semiclassical gravity. A complete quantum theory of gravity, including all higher dimension operators, is required.

In this article we will use bosonic string theory to demonstrate the existence of hairy extremal black holes. In string theory, nonzero θ leads to a deformation of the classical solutions (which vanishes exponentially at large distances) and some exact classical solutions will be found as conformal field theories. The general solution will be qualitatively described in terms of two-dimensional renormalization group flows. We will find a tachyonic excitation in the spectrum, so these objects are unstable–this is in addition to the usual tachyonic instability of string theory. We briefly discuss the possibility of generalization to the superstring.

Classical solutions of bosonic string theory are provided by conformally invariant two-dimensional σ models The σ model corresponding to a four-dimensional hairy black hole with the line element

$$ds^2 = -N^2(\rho)dt^2 + d\rho^2 + R^2(\rho)d^2\Omega \ , \tag{2}$$

axion hair

$$B = \frac{\theta\epsilon\alpha'}{2} \ , \tag{3}$$

and dilaton field $\Phi(\rho)$ may be written[1]

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[1]Note that, following convention, the θ term in this action differs from (1) by a factor of $\hbar$.

$$S_\sigma = \frac{1}{\pi\alpha'}\int d^2\sigma\left(-N^2(\rho)\partial_+ t\partial_- t + \partial_+\rho\partial_-\rho + \alpha'\mathcal{R}_{+-}\Phi(\rho) + R^2(\rho)G_{\mu\nu}\partial_+ X^\mu\partial_- X^\nu + \frac{\alpha'\theta}{2}\epsilon_{\mu\nu}\partial_+ X^\mu\partial_- X^\nu\right) , \quad (4)$$

where $G_{\mu\nu}(\epsilon_{\mu\nu})$, $\mu,\nu = 1,2$ is the unit metric (volume form) on the two-sphere, $\alpha' = \hbar/2\pi T$ and $\int d^2\sigma\sqrt{g}\mathcal{R} = 4\int d^2\sigma\mathcal{R}_{+-} = 8\pi$ on S^2. We wish to find $c = 4$ conformal field theories of the form (4). The additional $c = 22$ conformal field theory (CFT) required for $c = 26$ will be suppressed.

The building blocks in our constructions are σ models on S^2, or CP^1 models, with θ terms, corresponding to the last two terms in (4) with constant R. These have been the object of extensive investigations. The renormalization group flows in the θ, R planes are depicted in Fig. 1. The renormalization group flows have a fixed point at $\theta = \pi, R = R_c$ corresponding to a $c = 1$ CFT, a free scalar at its self-dual SU(2)-invariant radius. This was first conjectured by Haldane [5], who argued that the critical behavior of the spin-s antiferromagnetic chain was governed by the CP^1 σ model, at $\theta = 0$ for integer spin and at $\theta = \pi$ for half-integer. The critical behavior for $s = \frac{1}{2}$ was already known, by bosonization and Bethe ansatz, to be given by the scalar at the self-dual radius. Subsequent work ([6] and references therein; see also [7] for a review) confirmed this conjecture and filled out the phase diagram. The CP^1 σ model with θ term has been applied previously to string theory black holes by Kogan [8], with a different interpretation of the renormalization group flow.

A $c = 4$ CFT can be obtained (following previous constructions [9,10]) by simply taking the tensor product with an SU(1,1)/U(1) level $k = 8, c = 3$ coset model. By representing this coset model as a σ model, it was identified [11] as a two-dimensional black hole. The full σ model action with four-dimensional target space is, to leading order in α',

$$S_\sigma = \frac{1}{\pi\alpha'}\int d^2\sigma\Big[-\tanh^2(\rho/\sqrt{6\alpha'})\partial_+ t\partial_- t + \partial_+\rho\partial_-\rho + \alpha'\mathcal{R}_{+-}(-\ln\cosh(\rho/\sqrt{6\alpha'}) + \Phi_h)$$
$$+\left(R_c^2 G_{\mu\nu} + \frac{\pi\alpha'}{2}\epsilon_{\mu\nu}\right)\partial_+ X^\mu\partial_- X^\nu\Big] , \quad (5)$$

where Φ_h is an arbitrary constant. Equation (5) corresponds to a special type of four-dimensional hairy black hole. There is no asymptotically flat region; the two-spheres have radius R_c for all ρ and θ is restricted to equal π.

The one free parameter is the constant Φ_h, the value of the dilaton at the horizon $\rho = 0$, which is related to the Arnowitt-Deser-Misner (ADM) mass of the black hole [11]. At the quantum level, Hawking radiation presumably drives the mass to zero. In this limit the horizon moves off to infinity and the action is simply (after a coordinate transformation)

$$S_\sigma = \frac{1}{\pi\alpha'}\int d^2\sigma\left[-\partial_+ t\partial_- t + \partial_+\rho\partial_-\rho - \sqrt{\frac{\alpha'}{6}}\mathcal{R}_{+-}\rho + \left(R_c^2 G_{\mu\nu} + \frac{\pi\alpha'}{2}\epsilon_{\mu\nu}\right)\partial_+ X^\mu\partial_- X^\nu\right] . \quad (6)$$

This represents an extremal black hole with quantum hair.

It is of interest to find hairy extremal black holes which are asymptotically flat and thus might exist in our Universe. These can be described starting from the renormalization group flows in Fig. 1. Consider a point $\theta = \pi$, with R just above the fixed point R_c. For small $R - R_c$, the corresponding σ model is nearly conformally invariant and has a small β function:

$$\mu\partial_\mu \ln R(\mu) = \gamma(R - R_c)^2 , \quad (7)$$

where $\gamma > 0$. The approach to the fixed point is along the perturbation $j_z^a j_{\bar z}^a$ and so is marginal as indicted by the quadratic β function [6]. A conformal field theory to order $R - R_c$ can be constructed by dressing the action with the ρ field

$$S \sim \frac{1}{\pi\alpha'}\int d^2\sigma\Big[-\partial_+ t\partial_- t + \partial_+\rho\partial_-\rho - \sqrt{\frac{\alpha'}{6}}\mathcal{R}_{+-}\{\rho + O(\rho^{-2})\}$$
$$+\left(\{R_c^2 - 2R_c/\gamma\rho + O(\rho^{-2})\}G_{\mu\nu} + \frac{\pi\alpha'}{2}\epsilon_{\mu\nu}\right)\partial_+ X^\mu\partial_- X^\nu\Big] \quad (8)$$

near $\rho = -\infty$. Presumably it is possible to correct the action (8) to obtain a CFT in a power series in $R - R_c = -1/\gamma\rho$ in a neighborhood of the fixed point. As $\rho \to -\infty$, R approaches R_c, and the geometry approaches the previously discussed extremal black hole. As ρ increases, the radius R increases, and the black hole "throat" begins to open up. When $R - R_c$ is of order one the radius rapidly increases and one enters the mouth region. At this point the expansion parameter used in constructing CFT's by dressing renormalization group flows breaks down, and we have no quantitative tools to analyze the theory. However, if one assumes that R passes through the mouth region to larger values, the theory can be analyzed in an expansion in $1/R$ on the other side of

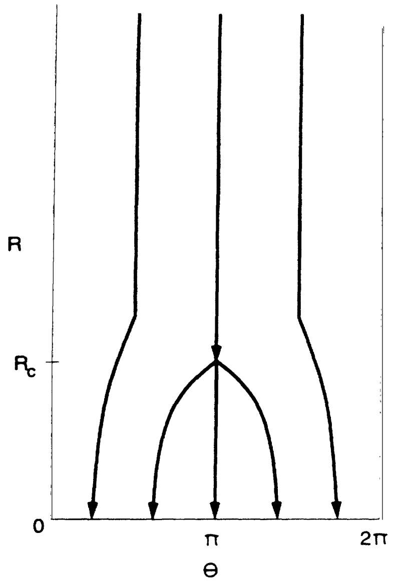

FIG. 1. Shown is a sketch of the renormalization group flows for the CP^1 model. An infrared fixed point lies at $\theta = \pi, R = R_c$.

this region. To leading order in this expansion, conformal invariance and $c = 4$ implies that $R \propto \rho$ and Φ is constant. Thus it is possible to assume that the theory ties on to an asymptotically flat geometry.

Other renormalization group trajectories can similarly be used to construct extremal black holes. For θ near but not equal to π, there is a long throat region produced by R lingering near the fixed point. In this region a construction of the type (8) yields an approximate CFT. However, for $\theta \neq \pi$, R eventually becomes small and the approximations break down. At the $R = 0$ infrared fixed point all excitations are infinitely massive and a spacetime interpretation of the theory is no longer possible [10]. If θ is not near π, a throat region never forms, and one immediately descends into the vicinity of the infrared fixed point.

As indicated in Fig. 1, the perturbation in the θ direction away from $\theta = \pi$ is relevant. It corresponds to the $(j, \tilde{j}) = (\frac{1}{2}, \frac{1}{2})$ primary, of weight $(\frac{1}{4}, \frac{1}{4})$. Expanding around a linear dilaton background, the mass squared [including a term $(\nabla\Phi)^2$ from the linear dilaton] is tachyonic, $m^2 = -17/6\alpha'$. The extremal object is therefore classically unstable, as are the near-extremal objects with long throats. Presumably they will decay by emission of a radial axion gradient to flat R^3 with $\theta = 0$. The would-be hair, like a bad toupee, slips off.

The picture of the $\theta = \pi$ extremal black hole is similar to that found for magnetically charged extremal black holes [12,10] which, for large charge, can be analyzed perturbatively. There is an asymptotically flat region, and a mouth connecting on to a semi-infinite throat region. These also share with the present construction the feature that the time coordinate is a free field and plays only a spectator role in the construction. The solutions with θ neither 0 nor π resemble the $Q = \pm 1$ solutions of [10] in the degeneration to a massive field theory at the origin.

A qualitative, but not quantitative, picture of the structure of hairy black holes can be developed in a minisuperspace-type approximation. The renormalization group flows in Fig. 1 originate entirely from nonperturbative world sheet instantons which wrap around the horizon, since those are the only configurations sensitive to θ. The spacetime effective action which incorporates these instanton effects is nonlocal. However, in an S-wave approximation in which all configurations are required to be spherically symmetric, a world sheet instanton is represented by a point in the two-dimensional ρ, t plane. Summing over spherically symmetric world sheet instantons is then equivalent to summing over ordinary instantons in the two-dimensional effective theory. The effects of such instantons are reproduced by adding to the action the operator whose effects mimic that of the instanton.[2] The result is

$$\begin{aligned} S = \frac{2\pi}{\kappa^2} \int d^2x \sqrt{-g} e^{-2\Phi} \Big[& R^2 \mathcal{R}^{(2)} \\ & + 2(\nabla R)^2 + 2 + 4R^2 (\nabla\Phi)^2 \\ & - 2\nabla^2 R^2 - \frac{\alpha'^2}{4R^2} (\nabla\theta)^2 + C e^{-2R^2/\alpha'} \cos\theta \Big] , \end{aligned} \tag{9}$$

where $x = (\rho, t)$. C is a positive determinant. The first six terms are obtained by spherical reduction of the four-dimensional string action with the ansatz

$$\begin{aligned} ds^2 &= g_{ab}(x) dx^a dx^b + R^2(x) d^2\Omega , \\ B &= \frac{\alpha'}{2} \theta(x) \epsilon . \end{aligned} \tag{10}$$

The last term in (9) reproduces the effects of string instantons. One can think about this in two ways. The first is the usual string σ model point of view, where the string wrapping the black hole is a world-sheet instanton, and the last term in the action represents the contribution of these instantons to the β function. The fact that the dilute instanton approximation suggests a $\theta = \pi$ fixed point of the CP^1 model was noted in [14]. Alternately we can think of them, as in [4], just like ordinary spacetime instantons, involving solitonic strings wrapping around a black hole.[3] This should be valid at large R because, in the spirit of [15], the low-energy effective field theory does not know if the core of the string contains a fundamental string or resembles a smooth soliton.

The equations of motion following from (9) have a solution with constant R and constant $\theta = \pi$. However, it

[2]In the case at hand the single instanton action is infrared divergent. However, as explained in [13], this does not mean that instanton effects cannot be summarized by a local operator. Rather, a single insertion of the appropriate operator must reproduce the infrared divergence.

[3]In [4] the effects of string instantons are, in contrast with the present case, nonperturbative in $\hbar$ because they keep $T = \hbar/2\pi\alpha'$ (rather than α') fixed as $\hbar \to 0$.

is only suggestive since (9) cannot be trusted when R is small and the instantons are not dilute. This approximation also misses the fact that the approach to the fixed point is marginal.

The reduced action (9) can be thought of as describing spherical four-dimensional dilaton gravity coupled to a scalar field $\theta(x)$ with a field-dependent potential that vanishes asymptotically. Nonextremal hairy black hole solutions can be constructed by solving the radial equations with boundary conditions imposed at the horizons. Since the equations degenerate at the horizon, there are (after gauge fixing) only three independent initial data which may be taken to be the horizon values θ_h, Φ_h, and R_h of θ, Φ, and R. R_h directly determines the horizon area, while Φ_h determines the asymptotic value of Φ. If θ were an ordinary massive field there would be two possible asymptotic solutions: one which grows and one which decays exponentially. Unless one takes $\theta_h = 0$, there would be some admixture of the growing solution and the spacetime would not be asymptotically flat. This is in accord with the no-hair theorems for scalar fields. On the other hand, if θ were exactly massless the asymptotic solutions go as a constant plus $1/R$, and there is no danger of destroying asymptotic flatness. In fact, the solution will have $\theta = \theta_h$ everywhere. The action (9) is somewhere between the massive and massless case. Because the mass vanishes asymptotically, there are no growing modes and no fear of destroying asymptotic flatness. On the other hand, since the potential is nonzero θ will not be constant if $\theta_h \neq 0, \pi$. In general the phase measured in string interference will be given by the asymptotic value of $\theta(x)$ which is nontrivially related to θ_h.

It is natural to attempt a similar construction for the superstring. In this case, however, θ can be eliminated by a chiral fermion rotation so there is no analogue of Fig. 1. The analogous effect for black holes with discrete gauge hair was discussed in [16]. However, for a collection of neutral black holes characterized by different values of θ, there is no globally defined chiral rotation which eliminates all the θ's. Thus we do expect quantum hair to arise, although the conformal field theoretic methods described herein are inadequate to describe it.

In closing we wish to discuss the observability of the axion hair in this solution. There is a limit, albeit artificial, in which it can be measured. This is the limit of string tree level, where we are doing conformal field theory in a fixed background. This limit is partly classical and partly quantum mechanical. The background fields do not fluctuate, but the propagation of test strings is quantum mechanical; for example, the vertex operators satisfy wave equations. A spherical world sheet can loop the black hole, so interference effects from θ will appear in tree-level amplitudes. Note also that because the background is classical the field satisfies $\langle \Delta B\rangle^2 = \langle(\Delta B)^2\rangle$. The left-hand side here is a two-instanton order. The right-hand side has both one- and two-instanton contributions, but the first enters at string loop order and so is suppressed in the limit discussed here.

Quantum effects make the axion hair difficult to measure for several reasons. In general θ eigenstates are not energy eigenstates, and so the θ mode of the axion field will rapidly fluctuate (we believe this is equivalent to the arguments of [4]). In the present case this is exacerbated by the presence of the tachyonic mode.

In conclusion, under closer inspection axion hair turns out to be a toupee in bosonic string theory: it does not provide a new quantum label for stable, extremal black holes. It remains a logical and interesting possibility that genuine quantum hair could exist in other contexts, such as superstring theories or discrete gauge symmetries.

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