

# Measure on a subspace of FRW solutions and “the flatness problem” of standard cosmology

H. T. Cho \*

*Tamkang University, Department of Physics, Tamsui, Taipei, Taiwan, Republic of China*

R. Kantowski †

*University of Oklahoma, Department of Physics and Astronomy, Norman, Oklahoma 73019*

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We use the metric on the space of gravity fields given by DeWitt to construct a unique kinematic measure on the space of FRW simple fluids and show that when the mass parameter  $\Omega$  is used as a coordinate this measure is singular at  $\Omega = 1$ . This singularity, combined with the time evolution of  $\Omega$ , distorts distributions of  $\Omega$  values to be concentrated in the neighborhood of 1 at early times. It is a distorted distribution of  $\Omega$  values that sometimes misleads the casual observer to conclude that  $\Omega$  must be exactly equal to 1.

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## I. INTRODUCTION

For decades now we have believed that general relativity determines the dynamics of our Universe and in fact one of the Friedmann-Robertson-Walker (FRW) models closely approximates what we can observe. At the current epoch the dynamics of such a model is dominated by inertia and the matter density (pressure now being insignificant and the cosmological constant  $\Lambda = 0$ ). Our task has been to find two observed numbers, e.g.,  $H_0$  the Hubble parameter and  $\Omega_0$  the mass density parameter [see (3) and (6) for definitions], and hence to tie down completely the global structure of the Universe, as well as where we are in its time development. The accepted value of the Hubble parameter is somewhere between 40 and 90 km/s Mpc, depending on how it is estimated [1]. Advocates for one extreme value or the other are not supported by some fundamental principle which makes their value more appealing. The same is not true for the other parameter  $\Omega_0$ . Its accepted<sup>1</sup> value from observation is between 0.01 and 0.2 (luminous-dynamical mass) [4,5] with the frequently advocated value being 1. When  $\Omega = 1$ , the Universe is on the verge of being closed even though the spatial sections are flat. If currently  $\Omega \approx 1$ , then at earlier times (as argued below)  $\Omega \rightarrow 1$  and as can be seen in (3), the spatial curvature of the Universe ( $k/R^2$ ) had negligible effect on its early dynamics. This is referred to as the “flatness problem” of standard cosmology. The advent of inflation has added fervor to the

debate because, in addition to solving some long-standing problems of cosmology (in particular the horizon problem), it would guarantee the almost sanctified value of  $\Omega = 1$ . Sometimes when listening to advocates for inflation the audience is misled to think that the “flatness problem” implies that  $\Omega$  is exactly equal to 1 in the early Universe and hence inflation must be correct. The failure to now observe the value  $\Omega_0 = 1$ , becomes the devotee’s “ $\Omega$  problem” or equivalently an additional missing-mass problem.

The argument goes something like the following [6–8]: If at present, the value  $\Omega = \Omega_0 = 1 - \delta_0$ , then at earlier times  $\Omega = \Omega_0(1+z)/(1+\Omega_0 z) \approx 1 - \delta_0/z$ . This value gets closer and closer to 1 as you choose earlier and earlier times, e.g., when the effects of pressure on the expansion of the Universe are no longer negligible  $z_R \approx 10^4$  and  $\delta_R \approx \delta_0 \times 10^{-4}$ . Before this period, when radiation is dominating the expansion,  $\Omega$  is approaching 1 even faster,  $\Omega = \Omega_R(1+z)^2/(1+2\Omega_R z + \Omega_R z^2) \approx 1 - \delta_R/z^2$ . At the time of nucleosynthesis, where  $z \approx 10^{10} = 10^6$  relative to  $z_R$ , we are sure of our  $\approx 1$  MeV physics and we have  $\Omega = 1 - \delta_0 \times 10^{-16}$ . If the Universe would have undershot 1 by some reasonable value such as  $10^{-5}$  at this early epoch then there would not be much around now, including us; and if the Universe had overshot 1 by such a reasonable value then it would have collapsed long ago. The misleading conclusion drawn from such or similar arguments is that  $\Omega$  must exactly equal one, after all, “how could it be so close and not be 1?” This conclusion is based on an unstated assumption that at some early epoch our value of  $\Omega$  should have been chosen from some possible set of values (by either a classical or quantum mechanical process) of which  $\Omega = 1$  was no more likely than any other value (see [9] for a discussion of initial data). By introducing a measure on a subspace of FRW solutions we expose  $\Omega$  as the problem, i.e., that probability distributions will be skewed towards  $\Omega = 1$ , and that if a “better” coordinate is used the flatness problem clearly does not imply  $\Omega = 1$ . In Sec. II we introduce a “better”

\*Electronic address:

bsp15%twntku10.bitnet@pucc.princeton.edu

†Electronic address: kantowski@phyast.nhn.uoknor.edu

<sup>1</sup>The dynamical values obtained by [2,3] are much larger. We chose to quote and use values obtained from more established methods simply to make our point about the problems encountered when  $\Omega$  is used as a coordinate.

coordinate called  $C$  and in Sec. III we introduce the essentially unique measure (the kinematic measure) on the space of solutions and express it in both the "good" coordinate  $C$  and the not so good coordinate  $\Omega$ . In Sec. IV we make the point about  $\Omega$  being a "bad" coordinate by following a hypothetical distribution to larger and larger redshifts. We also conjecture the relationship of the kinematic measure proposed here to the dynamical measure proposed by Henneaux [10] and Gibbons *et al.* [11].

## II. A COORDINATE FOR SIMPLE PERFECT FLUID FRW SOLUTIONS

The Robertson-Walker metrics can be found in every book on cosmology, e.g., see [12]:

$$ds^2 = c^2 dt^2 - R(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 (\sin^2 \theta d\phi^2 + d\theta^2) \right], \quad (1)$$

where  $k = -1, 0, 1$  and  $R(t)$  is arbitrary. For simple perfect fluid solutions [ $p = (\gamma - 1)\rho c^2$ ] of the standard theory,  $R(t)$  is determined by the Einstein equations which reduce to

$$\frac{8\pi G}{3c^2} \rho R^{3\gamma} = \text{const} \equiv C^{3\gamma-2}, \quad (2)$$

and

$$\frac{H^2}{c^2} = \frac{8\pi G}{3c^2} \rho - \frac{k}{R^2} = \frac{1}{R^2} \left[ \left( \frac{C}{R} \right)^{3\gamma-2} - k \right], \quad (3)$$

where  $H \equiv \dot{R}/R$ . The constant in (2) has been written in terms of another constant  $C$  whose units are the same as those of  $R$ . The one parameter family of solutions  $R(t, C)$  is given by integrating (3):

$$\int dR \left[ \left( \frac{C}{R} \right)^{3\gamma-2} - k \right]^{-1/2} = c \int dt. \quad (4)$$

For the spatially flat  $k = 0$  case,  $C$  can be scaled to any desired value by scaling the  $r$  coordinate and hence only one such solution exists. The same is not true for the spatially curved  $k = \pm 1$  solutions;  $C$  remains as the single parameter ( $0 < C < \infty$ ) distinguishing between possible models. For the closed FRW models  $C$  is clearly the maximum value of  $R$ . The current value of  $C$  ( $\gamma = 1$  for pressure = 0) corresponding to the above observed range of small  $\Omega_0$  values is  $C_0 = (0.01 - 0.3)c/H_0$ . Once  $C$  is fixed another parameter (e.g.,  $t_0$  or  $H_0$ ) must be given to fix our epoch. Giving the Hubble parameter  $H_0 = H(t_0 = t_{\text{now}})$  is equivalent to giving the current critical mass density  $\rho_c$  of the Universe:

$$\rho_c = \frac{3H_0^2}{8\pi G}. \quad (5)$$

The mass-density parameter  $\Omega_0$  is normally used as a label for solutions rather than the  $C$  introduced above.

It is defined in terms of the current mass density  $\rho_0$  and its critical value:

$$\Omega_0 \equiv \frac{\rho_0}{\rho_c}. \quad (6)$$

In what follows we use  $C$  and  $\Omega_0$  as two different parametrizations of the above set of gravity fields.

## III. THE INVARIANT MEASURE ON THE SPACE OF FRW SOLUTIONS

To statistically weigh a set of possible fields  $\{\phi^i\}$ , two structures must be given: (i) a measure (e.g., a volume element) on the space of fields and (ii) a scalar function normalized with the given measure. For many fields [including the metric fields  $g_{\alpha\beta}(x)$  of gravity] the only known measure is proportional to the volume element of some field metric  $G_{ij}(\phi)$  on the space of fields:

$$ds^2 = G_{ij} d\phi_{\perp}^i d\phi_{\perp}^j = G_{ij}^{\perp} d\phi^i d\phi^j. \quad (7)$$

The parallel projection  $d\phi_{\parallel}^i = P_{\parallel j}^i d\phi^j$  selects the gauge-dependent part of the difference of two neighboring fields and the perpendicular projection  $d\phi_{\perp}^i = (\delta_j^i - P_{\parallel j}^i) d\phi^j$  selects the part orthogonal to all possible gauge transformations,

$$G_{ij} P_{\parallel k}^i (\delta_l^j - P_{\parallel l}^j) = 0, \quad (8)$$

giving

$$G_{ij}^{\perp} \equiv G_{ij} - G_{kl} P_{\parallel i}^k P_{\parallel j}^l. \quad (9)$$

The distance between two gauge equivalent fields, computed using (9), clearly vanishes. Other measures can be defined if the set of fields is restricted by some dynamical theory; e.g., a phase space volume can be defined when the dynamics is canonically described. For the non-dynamically restricted metric fields a unique field metric exists and is commonly used when performing a path integral quantization of gravity [13]. It was first given by DeWitt [14] but its absolutely essential role was made clear when Vilkovisky developed the current effective action theory [15,16]. We fix the differential manifold and write the field in a given coordinate patch as

$$\phi^i = g_{\alpha\beta}(x) \quad (i = \{\alpha, \beta, x\}). \quad (10)$$

The field-space metric of DeWitt [14] to be used in (7) to give the distance between two neighboring metrics is

$$G_{ij} = \sqrt{|\det g|}^{\frac{1}{4}} [g^{\alpha\mu} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\mu} - a g^{\alpha\beta} g^{\mu\nu}]_x \times \delta^4(x - y), \quad (11)$$

where  $a$  is an arbitrary unitless constant ( $\neq 1/2$ ). This metric is commonly used in path integral versions of quantum gravity; however, it is a purely classical structure and it is only in that context that we use it here.

For metric fields the gauge group is the set of active coordinate transformations (i.e., the diffeomorphism group) and the difference between two neighboring fields is de-

composed into a part attributable to an active coordinate change and a part which is not, i.e., a part perpendicular to all possible coordinate changes (see Appendix):

$$\delta\phi^i = \delta g_{\alpha\beta}(x) = \delta g_{\parallel\alpha\beta}(x) + \delta g_{\perp\alpha\beta}(x) . \quad (12)$$

Here  $\delta g_{\parallel\alpha\beta}(x) = \nabla_\alpha \delta\xi_\beta + \nabla_\beta \delta\xi_\alpha$  is generated by some small coordinate shift  $x^\alpha \rightarrow x^\alpha + \delta\xi^\alpha(x)$ . The metric as given by (11) is unique (up to the parameter  $a$ ) provided that  $G_{ij}$  is assumed to be local [i.e.,  $\propto \delta^4(x-y)$ ], assumed not to depend on the metric's curvature (i.e., not to depend on derivatives of  $g_{\alpha\beta}$ ), and assumed to be invariant under gauge transformations. In Eqs. (22) and (24) we will see that the value of the arbitrary parameter  $a$  does not affect a normalized probability distribution on the FRW subspace studied here. Equation (7), evaluated using (11), should be thought of as giving the intrinsic (i.e., coordinate-independent) geometrical distance between two metrics  $g_{\alpha\beta}(x)$  and  $g_{\alpha\beta}(x) + \delta g_{\alpha\beta}(x)$

$$ds_C^2 = C^2 \left\{ \frac{d^2\chi}{\chi^{2-3\gamma} - k} - \chi^2 \left[ \frac{dr^2}{1 - kr^2} + r^2(\sin^2\theta d\phi^2 + d\theta^2) \right] \right\} = C^2 ds_{C=1}^2 . \quad (15)$$

The range of the new coordinate  $\chi$  is  $0 \leq \chi < \chi_{\max}$  where  $\chi_{\max} = \infty$  for  $k = -1$  and  $\chi_{\max} = 1$  for  $k = 1$ . The difference in two neighboring metric fields of fixed  $\gamma$  becomes

$$\delta g_{\alpha\beta}(C; \chi, r, \theta, \phi) = 2C\delta C g_{\alpha\beta}(C=1; \chi, r, \theta, \phi) , \quad (16)$$

written symbolically as

$$\delta\phi_C^i = 2C\delta C\phi_{C=1}^i , \quad (17)$$

and giving from (13) an induced metric

$$G(C)dCdC = 4C^2 G_{ij}^\perp(C)\phi_{C=1}^i\phi_{C=1}^j dCdC . \quad (18)$$

From (11) and (15) it is clear that in four dimensions  $G_{ij}(C) = G_{ij}(C=1)$ , i.e., that the field metric when evaluated at any of the  $\gamma = \text{fixed}$  simple fluids does not depend on  $C$ . In the Appendix we show that the same is true for  $G_{ij}^\perp(C)$  [see (A8)], consequently giving the measure as a simple function of  $C$ :

$$\sqrt{G(C)} = \text{const} \times C . \quad (19)$$

The potentially devastating divergence that occurs (constant  $\rightarrow \infty$ ) for the infinite open models is harmless here because we are keeping the equation of state fixed and a normalization of probability removes the constant. The parameters,  $H_0$  and  $\Omega_0$  rather than  $C$  are ordinarily used to label the FRW solutions. Of these two parameters,  $H_0$  is fixed at its current value and  $\Omega_0$  is used as the free parameter. Eliminating  $R$  between Eqs. (2) and (3) gives

$$C = \frac{c}{H_0} \left( \frac{-k}{1 - \Omega_0} \right)^{\frac{3\gamma}{2(3\gamma-2)}} \Omega_0^{\frac{1}{3\gamma-2}} , \quad (20)$$

which implies

defined on the same manifold. The induced natural (kinematic) measure associated with a set of metric fields is simply proportional to the volume of a neighboring set of fields, i.e.,  $\propto \det|G_{ij}^\perp|$ . The above metric (11) on all metric fields will induce a metric on any subspace of fields; in particular it will induce a metric  $G(C)$  on the  $\gamma = \text{fixed}$  subspaces of  $k = \pm 1$  perfect fluid FRW solutions:

$$ds^2 = G_{ij}^\perp d\phi^i d\phi^j = G(C)dCdC . \quad (13)$$

The  $k = 0$  solution is only a point in the field space. The induced natural measure on the open (closed) simple fluid solutions is  $\propto \sqrt{G(C)} dC$ . To compute it we rewrite (1) replacing  $t$  by a new variable  $\chi \equiv R/C$ :

$$dt = \frac{dR}{HR} = \frac{C d\chi}{c\sqrt{\chi^{2-3\gamma} - k}} . \quad (14)$$

The form of the metric is now

$$dC = -k \frac{c}{H_0} \left( \frac{-k}{1 - \Omega_0} \right)^{\frac{9\gamma-4}{2(3\gamma-2)}} \times \left( \frac{1}{3\gamma-2} + \frac{\Omega_0}{2} \right) \Omega_0^{\frac{3-3\gamma}{3\gamma-2}} d\Omega_0 . \quad (21)$$

The measure as a function of  $\Omega_0$  becomes

$$\begin{aligned} \sqrt{G(C)} dC &= \sqrt{G(\Omega_0)} d\Omega_0 \\ &= \text{const} \times C dC \\ &= \text{const} \times (-k) \left( \frac{c}{H_0} \right)^2 \left( \frac{-k}{1 - \Omega_0} \right)^{\frac{2(3\gamma-1)}{3\gamma-2}} \\ &\quad \times \left( \frac{1}{3\gamma-2} + \frac{\Omega_0}{2} \right) \Omega_0^{\frac{4-3\gamma}{3\gamma-2}} d\Omega_0 , \end{aligned} \quad (22)$$

and clearly diverges on any neighborhood of  $\Omega_0 = 1$  when  $\gamma > 2/3$ . This expression is the distance between two neighboring universes whose coordinates are  $C$  and  $C + dC$ . In the second form the distance is evaluated by comparing the values of  $\Omega$  for these to universes when their Hubble parameters are the same (both =  $H_0$ ).

#### IV. CONCLUSIONS AND DISCUSSION

We have not proposed any dynamical mechanism to determine the distribution of possible FRW Universes. We only argue that  $\Omega$  is not the best coordinate to use for a label if you wish to consider earlier and earlier times. The only natural measure on the space of FRW polytropic solutions is singular at  $\Omega = 1$  and (as seen below) every neighborhood of 1 shrinks to 1 at early times. If the parameter  $C$  is used, its value is well behaved in the currently observed negligible pressure domain  $C_0 < \infty$  [see Eq. (20) with  $\gamma = 1$ ], and that this value remains con-

stant all the way back to a period when radiation rather than  $p = 0$  models describe the dynamics ( $z_R \approx 10^4$ ). Matching-boundary conditions ( $R$ ,  $\dot{R}$ , and  $\rho$ ) at this redshift where the equation of state changes to  $\gamma = 4/3$  requires a decrease in the value of the constant  $C$ :

$$C_R = \sqrt{R_0 C_0 / (1 + z_R)} = \frac{c}{H_0} (0.001 \rightarrow 0.006). \quad (23)$$

This constant value persists as far back as the equation of state ( $\gamma = 4/3$ ) remains valid, e.g., to the inflation period.

If we assume this observed value exists by choice among some normalized set of possible values, a length scale  $L$  must exist for the distribution function  $P(C^2/L^2)$ ,

$$\int_0^\infty P(C^2/L^2) d(C^2/L^2) = 1, \quad (24)$$

and we can immediately see the true flatness problem: why is  $L \approx C_R \approx 10^{59} \times L_{\text{Planck}}$ ? If this distribution was determined at the time of transition from quantum gravity to classical gravity when the only length around was the Planck length ( $L_{\text{Planck}} = 1.6 \times 10^{-33}$  cm), what inflated it by a factor of  $10^{59}$ ? One of the current forms of inflation is commonly assumed to have done so; however, [17,18] argue that  $\Omega$  could be  $\approx 1$  without inflation. The actual form of  $P(C^2/L^2)$  is of course not known but its origin must be determined by the probability of having sources of gravity which produce a given gravity field, i.e., a given  $C$ . For illustrative purposes we pick a simple normalized example:

$$P(C^2/L^2) = \exp(-C^2/L^2). \quad (25)$$

Using (20) and (21) with  $z = 0$  replaced by  $z_R$  and  $\gamma = 4/3$  along with the redshift dependence of  $\Omega$  computed from (3), (5), and (6), i.e.,

$$\Omega_R = \frac{1}{1 + (1/\Omega - 1)(1 + z)^{3\gamma - 2}}, \quad (26)$$

we can look at the distribution of possible  $\Omega$  values at early times by writing

$$P(\Omega, z) d\Omega = P(C^2/L^2) d(C^2/L^2). \quad (27)$$

In (26)  $z = 0$  is at the end of the radiation phase where the mass parameter is  $\Omega_R$ . What is found (e.g., see Fig. 1) is a distribution rapidly being squeezed (as  $z$  increases) to a peak just less than  $\Omega = 1$ . The narrowing peak follows the implicit solution  $\Omega(z)$  of Eq. (26). It is cut off on the left by the fact that the distribution is normalized [e.g., by the exponential in (25)] and on the right by the singularity in the measure (22). The maximum in the probability curve is going up as  $(1 + z)^2$ , the width is shrinking as  $(1 + z)^{-2}$ , and the difference  $1 - \Omega$  is decreasing as  $(1 + z)^{-2}$ . It is this narrow, extremely high peak being squeezed to  $\Omega = 1$  that frequently misleads a casual observer to think that  $\Omega$  must be "fine-tuned" to 1. In our simple example the probability density actually vanishes at  $\Omega = 1$ .

Alternatively you could argue that by forcing a uniform

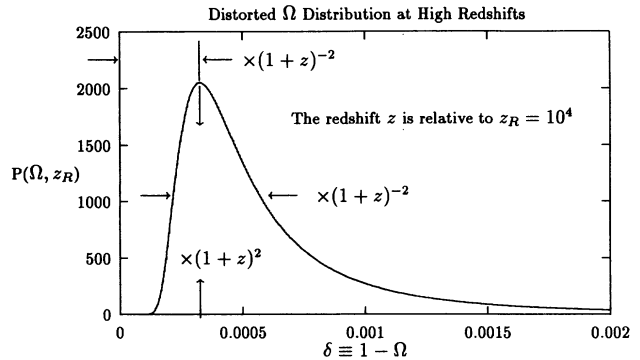


FIG. 1. Plot of  $P(\Omega, z)$  from (27) at redshift =  $z_R$  as a function of  $\delta = 1 - \Omega$ . This probability distribution comes from (25) assuming  $L = C_R$  and is intended for illustrative purposes only. The effects of additional redshifting are indicated by the factors  $\times(1 + z)^{\pm 2}$ .

distribution of  $C^2$  (i.e.,  $L \rightarrow \infty$ ), you force  $\Omega \rightarrow 1$  as the only value allowed for  $\Omega$ . Without a scale for  $C$  the only choices are  $L = 0$  or  $L = \infty$  which correspond to  $\Omega \rightarrow 0$  and  $\Omega \rightarrow 1$ , respectively.

Other measures on the space of FRW solutions have been proposed in conjunction with classical [10,11,17] or quantum [19,20] dynamical theories. The  $\gamma = 2$  case given here can be directly compared with the massless scalar field case of Gibbons *et al.* [11], see Eq. (3.15). Here the gravity-field space is clearly one dimensional ( $C$  is one parameter), but there the Henneaux, Gibbons, Hawking, and Stewart measure is for a two-dimensional initial data space. The extra dimension appearing in the dynamical measure comes from the initial data for the scalar field  $\phi$ . The value of the scalar field does not affect the gravity field (only its rate of change does) and, not surprisingly, their measure is of the form

$$d\mu = \text{const} \times dC^2 \wedge d\phi, \quad (28)$$

when our  $C$  coordinate is used. In the form given by Gibbons *et al.* [11], the measure is of the form of our Eq. (22)  $\wedge d\phi$  (their coordinate  $y \equiv H_0 \sqrt{\Omega_0}$ ). Integrating over the  $\phi$  initial data gives a uniform distribution in  $C^2$ , i.e.,  $L \rightarrow \infty$  and  $\Omega \rightarrow 1$ . The origin of their result is clear. The gravity-field part,  $\sqrt{G(C)} dC$  (the kinematic measure as we call it) is as we say it inevitably must be and the massless scalar field, having no intrinsic scale and having had its initial (dynamical) value uniformly distributed, cannot select any one  $C$  over another, i.e.,  $P(C^2/L^2)$  is constant. Normalization forces this constant to zero and selects the divergent point  $\Omega \rightarrow 1$  as the only possible configuration. For other more complicated cases we expect similar agreement between the unique kinematic measure we propose and dynamical probability distribution coming from the canonical phase-space measure proposed by Henneaux, Gibbons, Hawking, and Stewart. For more complicated cases this agreement is likely to occur only when the parameter  $a = 1$  in (11). This is because the  $a = 1$  metric appears in the kinetic energy term for background field expansions and is hence

built into any dynamical theory containing conventional general relativity (GR).

Our objective here has been limited to evaluating the unique kinematic measure induced on the configuration space of a limited set of gravity fields. We have found that it is not well behaved as  $\Omega \rightarrow 1$ . In addition we hope we have convinced the reader of two things.

(1) That in the absence of knowing the true distribution function of expected values of  $\Omega_0$  or the dynamical mechanism that produces the distribution, one should use the measure given here simply because of its uniqueness. If the probability of producing a given gravity field by the set of all sources were to be known, it would appear as the function  $P(C^2/L^2)$ , normalized with this measure as in (24).

(2) That the assumption  $\Omega_0 \approx 1$  implies  $\Omega = 1$  is based on an unstated assumption that the distribution of possible values of  $\Omega$  is relatively flat at  $\Omega = 1$ . If it were well behaved at 1, finding a value differing from 1 by  $10^{-16}$  or less would be deemed significant. It would imply that some additional mechanism beyond conventional dynamics and probabilities produced the observed early values of  $\Omega \approx 1$ , e.g., inflation might have driven  $\Omega_0$  to this value. However, we know that  $\Omega$  is not a good coordinate to use because a divergence in the measure will amplify the probability distribution as  $\Omega \rightarrow 1$ . Consequently finding an early value near  $\Omega = 1$  might be quite likely even if the probability of finding a value of  $\Omega = 1$  was zero.

Finally, we know the production of a distribution of  $\Omega$ 's is one thing, but observing various values is another. Only those universes or parts of "the Universe" having a limited range of  $H_0$  and  $\Omega_0$  values would likely produce civilizations such as ours asking such questions. This selection effect cannot be denied. However, it may or may not have distorted the original distribution. In any event, this selected distribution is likely to include only universes where  $\gamma \geq 1$  for a significant recent history and for all of these,  $\Omega$  approaches 1 at earlier times.

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#### APPENDIX

What is referred to as the gauge group for metric fields on a fixed differentiable manifold is actually the group of diffeomorphisms of that manifold. All metrics are identified as equivalent that can be actively transformed into one another. For "infinitesimal" transformations these look like  $x^\alpha \rightarrow x^\alpha + \delta\xi^\alpha(x)$  which change the metric by

$$g_{\alpha\beta}(x) \rightarrow g_{\alpha\beta}(x) + \nabla_\alpha \delta\xi_\beta + \nabla_\beta \delta\xi_\alpha, \quad (\text{A1})$$

and which are generically written as

$$\phi^i \rightarrow \phi^i + Q_\sigma^i \delta\xi^\sigma, \quad (\text{A2})$$

where

$$Q_\sigma^i \delta\xi^\sigma = \int d^4y \{g_{\alpha\gamma} \nabla_\beta + g_{\beta\gamma} \nabla_\beta\}_x \delta^4(x-y) \delta\xi^\gamma(y), \quad (\text{A3})$$

i.e., where ( $i = \{\alpha, \beta, x\}$ ) and ( $\sigma = \{\gamma, y\}$ ), repeated discrete indices are summed over, and repeated continuous indices are integrated over. The metric components in the gauge directions are defined by

$$\begin{aligned} N_{\sigma\rho} &= G_{ij} Q_\sigma^i Q_\rho^j \\ &= -\sqrt{-g} \{g_{\sigma\rho} \square + \nabla_\rho \nabla_\sigma - a \nabla_\sigma \nabla_\rho\}_y \delta^4(y-z), \end{aligned} \quad (\text{A4})$$

and are seen to form a local differential operator whose inverse  $N^{\sigma\rho}$  is consequently a nonlocal Green's function,

$$N^{\sigma\lambda} N_{\lambda\rho} = \delta_\rho^\sigma \delta^4(y-z). \quad (\text{A5})$$

The relevant quantity needed for computing  $G_{ij}^\perp$  is the parallel projection operator

$$P_{||j}^i = Q_\sigma^i N^{\sigma\rho} Q_\rho^k G_{kj}, \quad (\text{A6})$$

and is nonlocal because of the  $N^{\sigma\rho}$  term. The perpendicular part of the field metric needed is, consequently,

$$G_{ij}^\perp = G_{ij} - G_{ik} Q_\sigma^k N^{\sigma\rho} Q_\rho^l G_{lj}. \quad (\text{A7})$$

What we wish to show is that  $G_{ij}^\perp$  like  $G_{ij}$  [as we have already pointed out in the paragraph after Eq. (18)] when evaluated at (15) is independent of  $C$ . From (A3) we see  $Q_\sigma^i(C) = C^2 Q_\sigma^i(C=1)$ , and from (A4),  $N_{\sigma\rho}(C) = C^4 N_{\sigma\rho}(C=1)$ . From (A5) we see  $N^{\sigma\rho}(C) = C^{-4} N^{\sigma\rho}(C=1)$ , and consequently from (A7) we have the desired result

$$G_{ij}^\perp(C) = G_{ij}^\perp(C=1). \quad (\text{A8})$$

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