Performance of Newtonian filters in detecting gravitational waves from coalescing binaries

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As coalescing binary systems are one of the most promising sources of gravitational waves, it becomes necessary to devise efficient detection strategies. The detection strategy should be efficient enough so as not to miss any detectable signal and at the same time minimize the false alarm probability. The technique of matched filtering used in the detection of gravitational waves from coalescing binaries relies on the construction of accurate templates. Until recently filters modeled on the quadrupole or the Newtonian approximation were deemed sufficient. Such filters or templates have, in addition to the amplitude, three parameters which are the chirp mass, the time of arrival, and the initial phase. Recently it was shown that post-Newtonian effects contribute to a secular growth in the phase difference between the actual signal and its corresponding Newtonian template. This affects the very foundation of the technique of matched filtering, which relies on the correlation of the signal with the filter and hence is extremely sensitive to errors in phase. In this paper we investigate the possibility of compensating for the phase difference caused by the post-Newtonian terms by allowing for a shift in the Newtonian filter parameters. The analysis is carried out for cases where one of the components is a black hole and the other a neutron star or a small black hole. The alternative strategy would be to increase the number of parameters of the lattice of filters which might prove to be prohibitive in terms of computing power. We find that Newtonian filters perform reasonably for the purpose of detecting the presence of the signal for both the initial and the advanced LIGO detectors. As such a strategy may be used for a preliminary analysis a lower threshold can be used.

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I. INTRODUCTION

Coalescing binaries are the most promising sources of gravitational waves [1] for laser interferometric gravitational wave detectors. The basic reason for the importance of these types of sources is their broadband nature which makes them ideally suited for their detection by the interferometers. The binary systems which are of relevance here are those consisting of compact objects, i.e., black holes and neutron stars. It has been estimated that three such coalescences occur per year out to a distance of 200 Mpc [2,3]. A lot of attention has recently been focused on the issues of detecting the presence of the signal and the extraction of astrophysical information from the estimated parameters of the signal.

There are plans to construct such laser interferometers around the globe and by the end of this century the American Laser Interferometric Gravitational Wave Observatory (LIGO) [4] and French-Italian VIRGO [5] will be in operation. The emphasis in their construction is on the reduction of noise which may be thermal, seismic, quantum, or photon shot noise. In laser interferometric detectors the lower cutoff is decided by the seismic noise which is very dominant at low fequencies. It is expected that the LIGO will be able to go down to 40 Hz in its initial stage and to 10 Hz in its final stage. This means that that we can start observing the binary when its orbital frequency is 20 Hz in the case of the initial detectors and 5 Hz in the case of the advanced ones. This leads to sufficiently large integration times which enhances the signal to noise ratio. It was suggested by Thorne [1] that matched filtering would be an ideal filtering technique for this purpose. Matched filtering is a standard technique used in signal analysis when the waveform is known. It determines for us an optimal linear filter which can decide on the presence or absence of the signal waveform in a given data train [6–10]. This requires accurate modeling of the waveforms, which is possible for the coalescing binary systems. They are clean systems and their inspiral waveform depends on a few parameters such as the individual masses and spins. Tidal interactions do not matter until the very end of the inspiral [11,12]. A lot of research activity has gone in the direction of obtaining accurate templates under the various approximation schemes such as the quadrupole and post-Newtonian [13–16]. Recently it has been shown that post-Newtonian (PN) corrections and spin-orbit (SO) and spin-spin (SS) couplings produce in the waveform an accumulating phase error as compared to the Newtonian expression [14]. Therefore, a template constructed from the Newtonian waveform would go out of phase with the signal and the so-called "matched filtering" technique for detection would woefully fail. In this paper we show that as long as we are only *searching* for signals a Newtonian filter would perform remarkably well even though the signal contains PN corrections. The key idea here is that we allow the parameters of the Newtonian filter to vary and adjust so as to produce the maximum possible correlation with the signal. We have found that this flexibility allows for fairly high values of the correlation. In many cases of interest the correlation obtained is 70% of its maximum possible value which would have been obtained had the template been perfectly matched to the signal. On the other hand, a template with the same parameters as those of the signal produces correlations of about 10–20%. We have carried out the analysis for the two noise curves assuming a LIGO-type detector. The two noise curves are the power spectral densities of the noise for the LIGO in its initial and advanced stages, as given in [17]. In the case of the initial LIGO detector the analysis is also carried out for the case of white noise for the sake of comparision. Also a correspondence between the parameters of the filter and the signal could be set up; it might be possible to estimate the parameters of the signal from those of the filter. In other words the filter parameters may be "renormalized."

The paper is divided as follows. In Sec. II we elaborate on the chirp waveform and the conventional detection strategy. We discuss the technique of matched filtering and define a quantity which shall be a measure of how well a Newtonian waveform can match with a post-Newtonian signal. We also make some comments about the signal power spectrum. In Sec. III we discuss the numerical results of the simulations carried out. And finally in Sec. IV we summarize our results and indicate future directions.

II. SEARCHING FOR THE SIGNAL

A. Newtonian waveform and conventional stategy

The waveform of the signal from the coalescing binary system, henceforth called the "chirp," has been modeled under various approximations. In the quadrupole approximation the chirp has three parameters other than the amplitude. These are the initial phase ϕ_0, the time of arrival t_a (i.e., the time at which the instantaneous frequency of the gravitational wave equals the lower cutoff of the detector), and the coalescence time ξ which form a convenient set of parameters for our purpose [18,19]. The Newtonian waveform $h(t;\xi,\phi_0,t_a)$ is given by

$$h(t;\xi,\phi_0,t_a) = \mathcal{N} a(t)^{-1/4} \times \cos\left[\frac{16\pi}{5} f_a \xi [1 - a(t)^{5/8}] + \phi_0\right], \quad (2.1)$$

where

$$\xi = 3.003 \left(\frac{\mathcal{M}}{M_\odot}\right)^{-5/3} \left(\frac{f_a}{100 \text{ Hz}}\right)^{-8/3} \text{sec} \quad (2.2)$$

and

$$a(t) = 1 - \frac{t - t_a}{\xi}. \quad (2.3)$$

Here the lower cutoff frequency is denoted by f_a, and $\mathcal{M} = M^{2/5}\mu^{3/5}$ is called the chirp mass where M is the total mass and μ the reduced mass of the binary system. $M_\odot$ is the solar mass and it is a convenient unit for our purpose.

Given the form of the signal and the statistical description of the noise one has to design an adequate set of filters to detect the signal. The noise is assumed to be stationary and is further specified by its power spectral density $S_h(f)$ which is defined by the relation

$$\overline{\tilde{n}(f)\tilde{n}^*(f')} = S_h(f)\delta(f - f'), \quad (2.4)$$

where $\tilde{n}(f)$ is the Fourier transform of a particular realization of noise and the overbar indicates an ensemble average. The $S_h(f)$ defined above is the two-sided power spectral density. We are primarily in search of a filter with an impulse response $q(t)$ which correlates best with the signal, i.e., when the correlation as defined below takes its maximum value for a particular value of the time shift Δt:

$$C(\Delta t) = \int_{-\infty}^{+\infty} h(t) q(t + \Delta t) dt. \quad (2.5)$$

This implies that the Fourier transform of the matched filter $q(t)$ to detect the signal $h(t)$ is given by the relation

$$\tilde{q}(f) = \frac{\tilde{h}(f)}{S_h(f)} e^{2\pi i f \Delta t}. \quad (2.6)$$

For the numerical computations that follow we use the fast Fourier transform algorithm as given in *Numerical Recipes* [20]. The definition of the Fourier transform is the same as given there: i.e.,

$$\tilde{n}(f) = \int_{-\infty}^{+\infty} n(t) e^{2\pi i f t} dt. \quad (2.7)$$

The impulse response of the filter $q(t)$ depends on the parameters ξ, t_a, ϕ_0. It also depends the time shift Δt. It now becomes important to judiciously space out the filters in the parameter space keeping in mind the constraints of computing power. Such an analysis has been carried out in great detail for both white and colored noise by Sathyaprakash and Dhurandhar (see [18,19]). We discuss briefly their major results.

(1) It was found that ξ the coalescence time is a convenient parameter to use since the filters are equally spaced in this parameter, where the spacing is decided by a fixed drop in the correlation.

(2) For the phase ϕ_0 we require just two filters for every value of the ξ parameter, one with $\phi_0 = 0$ and the other with $\phi_0 = \pi/2$. Because of their orthogonality, the correlation is maximised over the phase by simply taking the square root of the sum of squares of the individual correlations: i.e.,

$$C(\Delta t) = \sqrt{C_0^2(\Delta t) + C_{\pi/2}^2(\Delta t)}\,, \quad (2.8)$$

where C_0 and $C_{\pi/2}$ are the correlations corresponding to filters with phases $\phi_0 = 0$ and $\phi_0 = \pi/2$, respectively.

We assume $t_a = 0$ in the design of the filter and therefore the value of Δt for which the maximum of the correlation occurs is equal to the time of arrival of the signal.

Such a procedure of maximizing the correlation over the phase and the time is carried out for each value of ξ. The final maximization of the correlation is then carried over the ξ parameter. The parameters for which the correlation is maximum are then presumed to be the most likely values of the parameters of the gravitational wave signal.

B. Post-Newtonian signal

The post-Newtonian corrections lead to corrections to the phase and the amplitude of the Newtonian signal and also lead to additive terms which are qualitatively different from the quadrupole term. In the case of a general binary system it is tedious and difficult to get the various corrections to the evolution of the orbits of the binary. If one of the bodies is large compared to the other as in the case of a black hole neutron star binary system, one can apply the Regge-Wheeler perturbation formalism [21] to get the evolution of the orbit. This provides us with the evolution of the orbital frequency as a function of time. This has been worked out [14] and is given by

$$\frac{\dot{f}}{f^2} = \frac{96\pi}{5}\frac{\mu}{M}\frac{x^{2.5}}{F(x)}, \tag{2.9}$$

where

$$F(x) = \frac{1 - \frac{3}{2}x - \frac{81}{8}x^2 - \frac{675}{16}x^3}{1 - \frac{1247}{336}x + 4\pi x^{1.5} - 4.9x^2 - 38x^{2.5} + 135x^3}. \tag{2.10}$$

Here $\dot{f}$ represents the first time derivative of frequency and $x = (\pi M f)^{2/3}$ the PN expansion parameter. The Newtonian waveform is obtained from the above equation by setting $F(x) = 1$. The phase is obtained by integrating Eq. (2.9). For the amplitude we use the Newtonian dependence on the frequency, i.e., $A(f(t) \approx \text{const}\times f^{2/3}$. This waveform shall be called "restricted post-Newtonian" henceforth. Although this is not exact, we do not expect the errors in the amplitude to affect the correlation significantly. We assume the initial phase and the arrival time of the signal to be 0. As the matched filtering process can also be viewed as a correlation between the incoming signal and the filter it is evident that any secular growth of the phase difference will reduce the correlation drastically. Thus to have a matched filter one must add one or more parameters. This would increase the number of filters enormously with corresponding increase in computational time. It is worthwhile to explore whether we can substantially increase the correlation by allowing for a shift in the parameters of the Newtonian filter, i.e., whether the signal is able to achieve better correlation with a Newtonian filter whose parameters are different from those of the signal. Obtaining large correlations depends on the function space spanned by the signal and filter waveforms and to what extent they overlap. We obtain reasonably large correlations. Here effects due to SO and SS coupling are not taken into account. The addition of such terms will not alter the thrust of the argument in that some other Newtonian filter would perform best.

C. Filtering the post-Newtonian signal

A detailed account of the formalism and notation used here and the theory of hypothesis testing using maximum likelihood methods as applied to detection of gravitational waves from coalescing binaries is given in [17,22]. We define a scalar product and its corresponding norm in the function space between two functions $s(t)$ and $q(t)$ for future use:

$$\langle s(t), h(t)\rangle = \int_{-\infty}^{\infty} \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_h(f)} df \tag{2.11}$$

and

$$\|s\| = \langle s(t), s(t)\rangle^{1/2}. \tag{2.12}$$

If an exact matched filter were present, then the signal to noise ratio (SNR) ρ would be simply equal to $\|s\|$ where s is the signal. Note that our definition of the SNR is different by a factor of 2 from the one given in [17] as they work with the one-sided power spectral density. The quantity we are interested in computing is

$$\eta = \frac{\langle s, h\rangle}{\|s\|\|h\|}, \tag{2.13}$$

where $h(t)$ is the chirp corresponding to the filter. We shall term η as the normalized correlation. Henceforth when we use the word "correlation" we shall mean the quantity η unless specified otherwise. As mentioned above the initial phase and the time of arrival of the signal are taken to be 0. The aim is to maximize η over the range of parameters of the filter. The quantity η takes the value between 0 and 1 and tells us how well a Newtonian filter can substitute for a post-Newtonian one. Geometrically one can visualize η as the cosine of the angle between the signal vector and the chirp vector.

In Fig. 1 we show the impulse response of a filter. As the noise is very high at lower frequencies the amplitude of the impulse response is very small at earlier times and becomes appreciable only at the end. Because of the same reason, the increase of amplitude with time is different from that of the Newtonian chirp. Figure 2 justifies the high correlations obtained. It shows how well the filter matches the restricted post-Newtonian signal. It is to be noted that the filter matches the signal very well during the late stages where the amplitude is largest.

Figure 3 shows the power spectrum of the signal which is the square of the magnitude of the Fourier transform of the signal divided by the power spectral density of noise as a function of frequency:

$$\Omega(f) = \frac{|\tilde{s}(f)|^2}{S_h(f)}. \tag{2.14}$$

This quantity peaks near 200 Hz and it is in this fre-

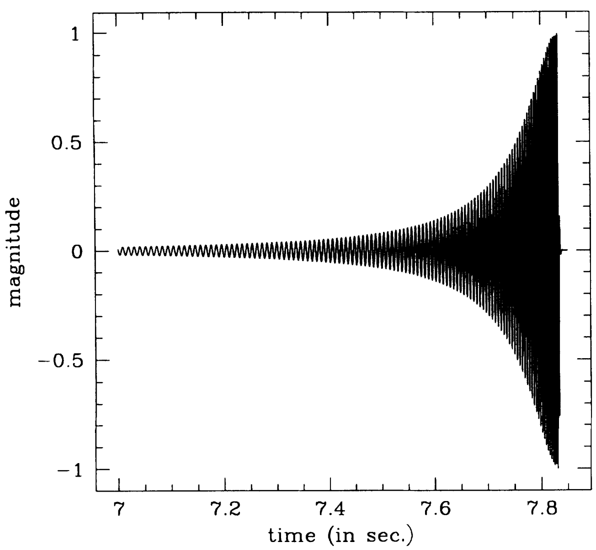

FIG. 1. The impulse response of a typical Newtonian filter is shown. The value of the coalescence time for the Newtonian signal corresponding to the filter is 9.1703 sec. The other two parameters, the arrival time and initial phase, are set to zero.

quency range therefore that the filter must match the signal very well, i.e., it should try to keep in phase with the signal to yield a high correlation. This is borne out in Fig. 2. The stationary phase approximation used in the Fourier transform of the chirp waveform [18] predicts the power spectrum to be a smooth power law. This, however, is not true, and numerical results and further investigations into the stationary phase approximation bear this out. The stationary phase analysis leads to a Fresnel integral and the smooth power law falloff ($f^{-7/3}$) in the power spectrum is obtained if the limits of integration extend from $-\infty$ to ∞. However, since the chirp waveform is taken to be of finite duration, we actually get an incomplete Fresnel integral. This leads to oscillations in the power spectrum of the signal. The oscillations are pronounced at the two ends of the bandwidth of the power spectrum of the signal as the limits of the integration are curtailed from the ideal $-\infty$ to ∞. As the noise is very high at low frequencies the amplitude of the oscillations in $\Omega(f)$ is less at low frequencies. It is easy to explain these oscillations using the Cornu's spiral [23]. The Cornu's spiral does not get wound up before the limits of the integration are reached. The thickness of the line indicates the presence of a substructure in the power spectrum.

III. NUMERICAL RESULTS

A. Correlations and shifts in parameters

The signal waveform was obtained by numerically integrating Eq. (2.9). We get time as a function of frequency which we then invert to get frequency as a function of time. This is now used to generate the phase and the amplitude of the signal. We take the initial phase and the time of arrival of the signal to be zero. We now present the results of the numerical simulations. We have considered black hole masses in the range $(5\text{–}10)M_\odot$. The masses taken for the smaller mass are $0.5M_\odot$, $1.0M_\odot$, and $1.4M_\odot$. The analysis has been carried out for the LIGO

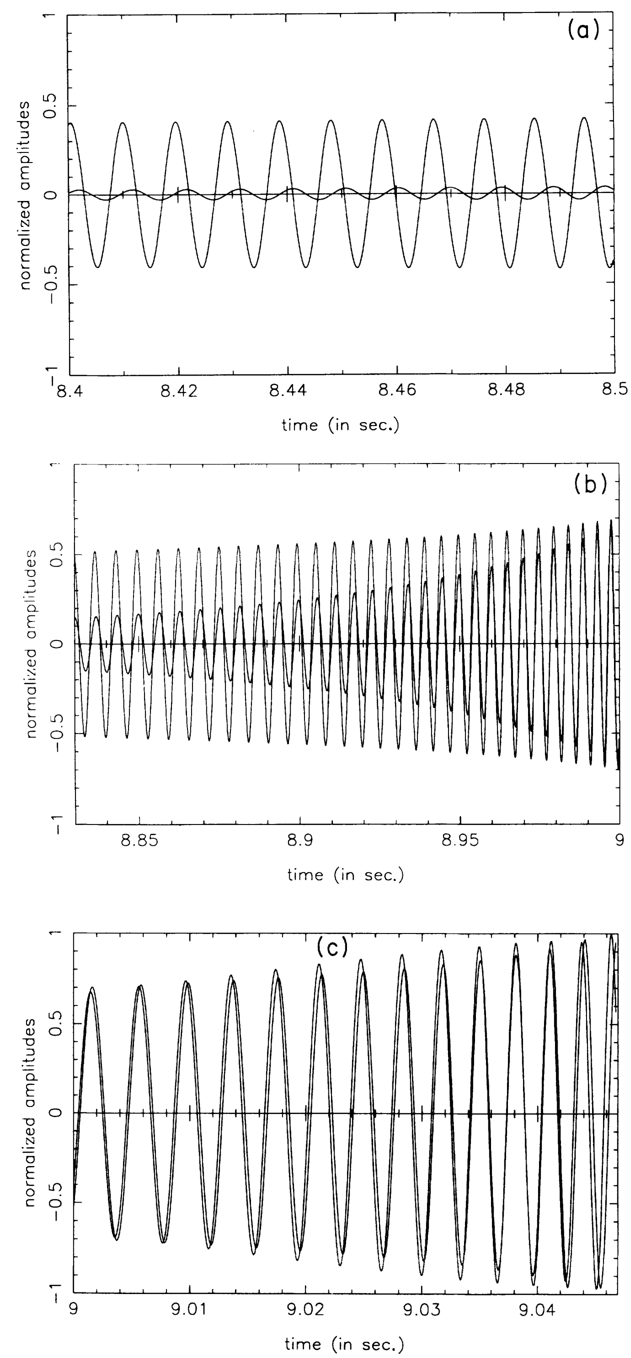

FIG. 2. The figure illustrates how well the post-Newtonian signal and the optimally matched Newtonian filter correlate. The curve whose magnitude is smaller represents the filter. The best matching occurs where the signal strength is high, i.e., near coalescence. The three figures correspond to the frequency ranges 103.4–109.56 Hz, 149.24–168.90 Hz, and 234.856–400.0 Hz, respectively.

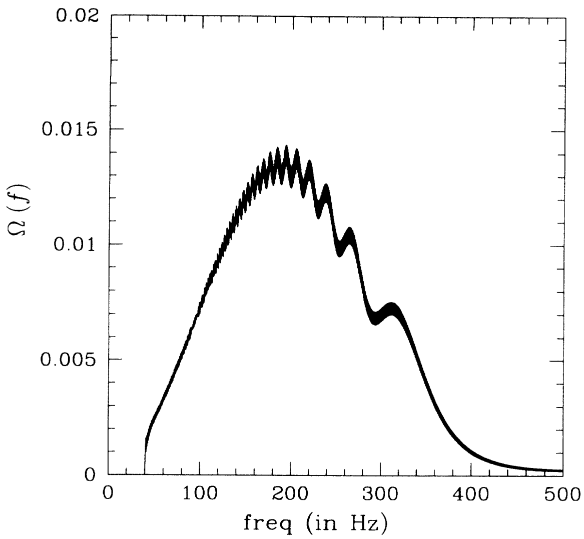

FIG. 3. The figure shows $\Omega(f)$ in the frequency range 40–400 Hz. The thickness of the line is due to the substructure present in the Fourier transform of the signal.

detector both in the initial and the advanced stages. We retain ξ as a parameter for the restricted post-Newtonian waveform also defined by Eq. (2.2) though this quantity does not represent the coalescence time of the signal anymore. In general the amount of time the signal lasts is less for the post-Newtonian signal as compared with the Newtonian one which follows from the fact that the quantity $F(x)$ in Eq. (2.10) is less than 1 in the frequency range considered.

In Table I we list the normalized correlations obtained for the LIGO detector in the initial stage. The mass of the larger component of the binary (M_1) increases from left to right along each row. The mass of the other component (M_2) increases from top to bottom in each column. The values of the masses are listed accordingly in the table. The correlations show a very regular behavior in the table. There are two factors controlling the drop of the correlation: (1) increase of the magnitude of phase corrections with increase of total mass of the binary system and (2) decrease of the integration time due to the increase of total mass of the system.

These two factors work against each other in producing the total amount of phase error between the Newtonian filter and the restricted post-Newtonian signal. Thus when we increase M_1, the increase in the magnitude of the phase corrections dominates over the loss in integration time and we get lower correlations when we go from left to right. Exactly the opposite happens when we increase M_2; i.e., the effect of the decrease of integration time dominates and the correlations increase. In Table II we list the correlations for the advanced LIGO. The same pattern is observed in this table too. We observe that the correlations for the larger frequency range are smaller as may be expected since the filter is more likely to go out of phase in a broader bandwidth. However, it should be emphasized that these correlations are *normalized.* The correlation would be unity if the filter were exactly matched to the signal. In absolute terms, if we consider a signal with given parameters having the same amplitude, then the correlation for the advanced LIGO will be much larger than the initial LIGO since the noise is less; first, we get a larger integration time, and second, the power spectral density is an order of magnitude less in the common bandwidth. We find that for the parameters considered, the absolute values of the correlations are larger by a factor of 20 for the advanced LIGO.

We next take up the issue of the shift in the parameters of the filters which produce maximum correlations. In Table III we list the shift in the parameters ξ and t_a for the case of the initial LIGO detector. The phase parameter ϕ_0 is an extremely sensitive parameter and its shifts are not regular. The value of $\Delta\xi$ is always negative. This is because as mentioned earlier a post-Newtonian signal will last for a smaller length of time as compared to a Newtonian signal with the same values of M_1 and M_2. Also it can be seen from Eq. (2.9) and the definition of ξ that the first derivative of the frequency $\dot{f}$ is approximately proportional to $\xi^{5/3}$. Therefore in order to obtain a higher value of $\dot{f}$ the value of ξ is reduced. Here again the integration time and magnitude of the phase corrections compete against each other in determining how $\Delta\xi$ varies with an increase in either of the masses. The value of $\Delta\xi$ decreases with an increase in M_1 and increases with an increase in M_2. Also Δt_a is always negative. This parameter tries to compensate for the reduction in the coalescence time by pushing the filter forward in time. As the table shows there is apparently a very strong covariance between these two parameters. The value of Δt_a also decreases with an increase in M_1 and increases with an increase in M_2. Typically for $M_1 = 5M_\odot$, and $M_2 = 1.4M_\odot$ we get shifts of $\Delta\xi = -1.32$ sec and $\Delta t_a = -1.218$ sec. This should be compared with the coalescence time of the waveform which is about 9 sec.

For the case of the advanced LIGO detector (see Table IV) the magnitude of the shifts is much more, but the

TABLE I. This table displays the correlations for the initial LIGO detector for a wide range of masses in units of solar masses. The black hole mass varies from $5M_\odot$ to $10M_\odot$ and the other mass takes the values $0.5M_\odot$, $1.0M_\odot$ and $1.4M_\odot$.

	$5.0M_\odot$	$6.0M_\odot$	$7.0M_\odot$	$8.0M_\odot$	$9.0M_\odot$	$10.0M_\odot$
$0.5M_\odot$	0.6403	0.6204	0.5979	0.5830	0.5762	0.5608
$1.0M_\odot$	0.7182	0.7045	0.6999	0.6838	0.6711	0.6411
$1.4M_\odot$	0.7474	0.7434	0.7309	0.7269	0.7190	0.6988

TABLE II. This table displays the correlations for the advanced LIGO detector. The value of masses is the same as in the previous table.

	$5.0M_{\odot}$	$6.0M_{\odot}$	$7.0M_{\odot}$	$8.0M_{\odot}$	$9.0M_{\odot}$	$10.0M_{\odot}$
$0.5M_{\odot}$	0.4420	0.4230	0.4078	0.3963	0.3866	0.3781
$1.0M_{\odot}$	0.5118	0.4956	0.4812	0.4702	0.4607	0.4526
$1.4M_{\odot}$	0.5420	0.5276	0.5157	0.5055	0.4978	0.4891

TABLE III. Shown below are the shifts $\Delta\xi$ and Δt_a in sec ($\Delta\xi$ above and Δt_a below) in the case of the initial LIGO detector.

	$5.0M_{\odot}$	$6.0M_{\odot}$	$7.0M_{\odot}$	$8.0M_{\odot}$	$9.0M_{\odot}$	$10.0M_{\odot}$
	−3.300	−3.800	−4.200	−4.650	−5.100	−5.4
$0.5M_{\odot}$	−3.160	−3.540	−3.835	−4.180	−4.541	−4.753
	−1.660	−1.875	−1.980	−2.310	−2.460	−2.56
$1.0M_{\odot}$	−1.552	−1.708	−1.760	−2.04	−2.144	−2.191
	−1.320	−1.475	−1.560	−1.600	−1.640	−1.755
$1.4M_{\odot}$	−1.218	−1.331	−1.379	−1.385	−1.396	−1.478

TABLE IV. Shown below are the shifts $\Delta\xi$ and Δt_a in sec ($\Delta\xi$ above and Δt_a below) in the case of the advanced LIGO detector.

	$5.0M_{\odot}$	$6.0M_{\odot}$	$7.0M_{\odot}$	$8.0M_{\odot}$	$9.0M_{\odot}$	$10.0M_{\odot}$
	−3.000	−7.700	−11.495	−15.90	−19.20	−22.80
$0.5M_{\odot}$	−12.09	−15.64	−18.44	−21.94	−24.42	−27.263
	−2.750	−4.750	−7.750	−9.250	−11.50	−13.00
$1.0M_{\odot}$	−7.421	−8.774	−11.22	−12.24	−14.05	−15.15
	2.700	−4.440	−5.500	−7.050	−8.85	−9.450
$1.4M_{\odot}$	−6.091	−7.328	−7.971	−9.156	−10.63	−10.94

TABLE V. This table displays the correlations for the case of band-limited white noise. The bandwidth ranges from 40 to 400 Hz. The masses are given in units of solar masses. The value of the masses is same as in the previous tables.

	$5.0M_{\odot}$	$6.0M_{\odot}$	$7.0M_{\odot}$	$8.0M_{\odot}$	$9.0M_{\odot}$	$10.0M_{\odot}$
$0.5M_{\odot}$	0.6363	0.6182	0.6043	0.5933	0.5838	0.5765
$1.0M_{\odot}$	0.6977	0.6850	0.6753	0.6659	0.6576	0.6517
$1.4M_{\odot}$	0.7238	0.7135	0.7054	0.6969	0.6912	0.6874

TABLE VI. Shown below are the shifts $\Delta\xi$ and Δt_a in sec ($\Delta\xi$ above and Δt_a below) in the case of band-limited white noise.

	$5.0M_\odot$	$6.0M_\odot$	$7.0M_\odot$	$8.0M_\odot$	$9.0M_\odot$	$10.0M_\odot$
	−0.0385	−0.135	−0.210	−0.288	−0.370	−0.43
$0.5M_\odot$	−0.062	−0.069	−0.068	−0.073	−0.082	−0.082
	−0.0735	−0.125	−0.165	−0.205	−0.240	−0.280
$1.0M_\odot$	−0.051	−0.055	−0.055	−0.057	−0.058	−0.063
	−0.084	−0.12	−0.15	−0.184	−0.215	−0.235
$1.4M_\odot$	−0.047	−0.049	−0.050	−0.055	−0.059	−0.057

time for which the signal spends in the frequency range 10–400 Hz is about a factor of 50 more than that for the initial LIGO for similar masses. Here the same pattern is observed in the variation of $\Delta\xi$ and Δt_a as in the initial LIGO. The typical values of the shifts observed are $\Delta\xi = -2.7$ sec and $\Delta t_a = -6.0912$ sec for $M_1 = 5M_\odot$ and $M_2 = 1.4M_\odot$.

Simulations were also done for the band-limited white noise with the power spectral density having a constant value between 40 and 400 Hz. The results were compared with those of the initial stage of LIGO. The effect of colored noise of the type considered here is to narrow band the signal. Thus the Newtonian filter has to match the signal over a smaller range of frequencies. However, if the narrow banding occurs at higher frequencies, for the chirp, the magnitude of the phase corrections is more. Thus in addition to changing the first derivative of the frequency through the parameter ξ the values of the higher derivatives of the frequencies would also have to be changed to get a good match. Had the narrow banding been at lower frequencies where the time derivatives of the frequency are relatively less, the correlation would have been much higher as the shifts in the Newtonian parameters would have been sufficient for the purpose. In Table V we show the correlations obtained for band-limited white noise and Table VI shows the corresponding parameter shifts. We observe that in the case of the initial LIGO the correlations obtained are less than those for the white noise case for higher values of the total mass and vice versa. This can also be seen as an effect of narrow banding. The values of the shifts in the parameters are also much smaller in the case of white noise as most of the contribution to the correlation comes from the lower frequencies where a small shift in ξ is sufficient for the filter to match well with the signal.

B. Effect of discreteness of the bank of filters

Until now we have considered our filter bank to have an infinite number of filters; i.e., we have allowed for a continuous variation of ξ. However, one is limited by the computing power available and one must confine oneself to a finite number of filters. Thus in general the signal will be unable to achieve its maximum correlation. Our aim is to estimate the drop in the correlation for a given computing speed. The maximum drop in the correlation because of the finiteness of the filter bank will have to be kept small. We consider a discrete set of Newtonian filters corresponding to distinct values of the ξ parameter. The filter spacing in the ξ parameter is taken to be constant ($\delta\xi_c$) across the entire range of values ξ can take (see [18]).

We first consider the initial LIGO and assume a 1 Gflop machine on which we intend to do an on-line search. The maximum time the signal lasts is found to be 25 sec for the mass range considered. However, the data train needs to be padded with zeros to 4 times the original length which is optimal for computational purposes (see [8]). This will increase the length of the data train to 100 sec. We allow for an overlap of 25 sec between consecutive data trains. Thus we have 75 sec in which to calculate n_f correlations where n_f is the number of filters. We sample the waveform at 1000 Hz. Thus we get approximately 2^{17} data points per data train. We have to perform one fast Fourier transform (FFT) operation per filter. The Fourier transforms will have already been calculated once and for all for the filters in the bank and one inverse Fourier transform will have to be performed to obtain the correlation as a function of the time lag Δt. The computation time will be mostly taken up by the FFT's as each FFT involves $3N\log_2(N)$ operations where N is the number of points in the data train. In this particular case therefore each Fourier transform will require about 6.4 million floating point operations (MFO's). This has to be compared with the number of floating point operations which can be carried out over the period of 75 sec which is 75×10^3 MFO's. Thus the number of filters that can be accommodated is about 11 700 filters. We require two filters for the phase for each value of ξ. As the maximum value of the coalescence time ξ is 25 sec for the range of masses considered, we get a filter spacing in the ξ parameter of around 4.3 msec. In the case of the advanced LIGO this number is about 172 msec.

Let $\Delta\bar{\xi} = \Delta\xi - \Delta\xi_m$ where $\Delta\xi_m$ is the value of $\Delta\xi$ corresponding to the maximum correlation. Figure 4 shows how the correlation for a given signal normalized to its maximum value varies with $\Delta\bar{\xi}$ along a line of curvature, i.e., along the curve parametrized by $\Delta\bar{\xi}$ along which the drop in the correlation is the slowest. In other words the figure shows the correlation maximized over Δt_a and $\Delta\phi_0$ as a function of $\Delta\bar{\xi}$. The curve has been plotted for the initial LIGO and for $M_1 = 5M_\odot$ and $M_2 = 1.4M_\odot$. However, the shape of this curve and the magnitudes in the drop of the correlation are insensitive to the values of M_1 and M_2. We observe that even for shifts of 100

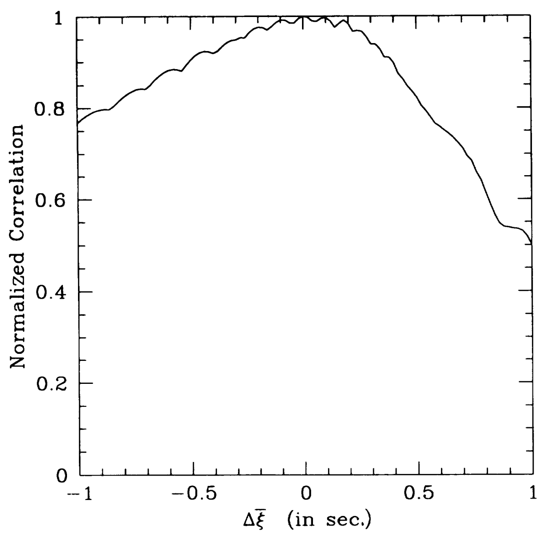

FIG. 4. Variation of the normalized correlation with $\Delta\xi$ along the line of curvature which is the curve along which the matching factor falls least.

msec the correlation does not drop by more than 2%. For the case of the advanced LIGO this drop in the correlation is even lower. Thus the filter spacing which we have calculated is sufficient for our purpose.

IV. CONCLUSION

We have demonstrated here the possibility of using Newtonian filters for detecting the presence of a restricted post-Newtonian signal. Such a strategy would be very useful in providing a preliminary on-line analysis of the data train. The analysis which we have carried out here is valid only for the point mass case where $\mu \ll M$ where μ is the reduced mass and M the total mass. For the initial LIGO the correlation is 0.65 on average and for the advanced LIGO it is around 0.45. These are only the normalized correlations as we have already stressed before. The absolute values of the signal to noise will be much higher (by a factor of about 20) for the advanced LIGO. It must be noted that the drop of the correlation will translate into a loss in the event rate. The distance up to which we can detect the binary will come down by a factor equal to the normalized correlation. This means that for the initial LIGO the distance to which we can detect the binary will be brought down by 35% and for the advanced LIGO it will be brought down by 55% from their respective maximum ranges. In absolute terms the advanced LIGO will still be able to look further than the initial LIGO. This reduction in the distance translates into a reduction in the event rate. The event rate is proportional to the cube of the distance to which we can probe for gravitational wave signals. But it must be emphasized here that the Newtonian filters will be employed mainly in the preliminary stage of the data processing. One may lower the threshold for a coarse search for signals.

The effect of the discreteness of the filter bank in producing a further drop in the correlations was investigated. It was found that for a 1 Gflop machine the drop in correlation due to the discreteness was very small. With better and faster machines we can make the bank of filters still more efficient.

If we consider higher derivatives of frequency f, say, $\ddot{f}$, etc., as parameters [24] we should get a better match, but the computation is very likely to increase. It should be possible to construct filters which not only enable us to save on the computation time but also span the set of signal waveforms adequately. A deeper analysis of the signal waveforms is in order so that efficient techniques can be developed. This work is now in progress.

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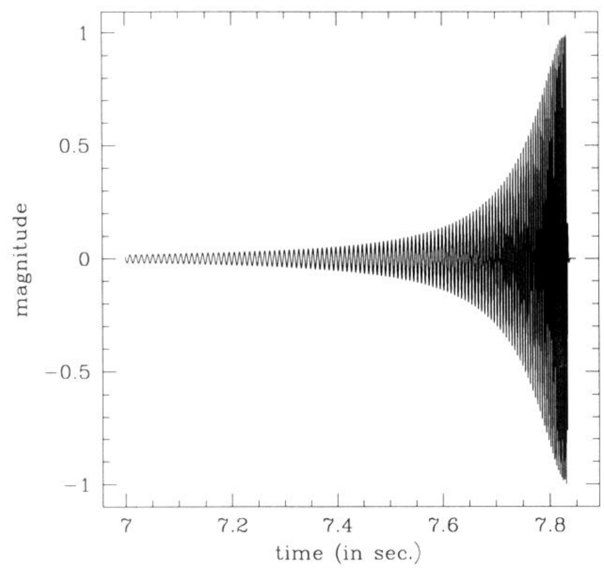

FIG. 1. The impulse response of a typical Newtonian filter is shown. The value of the coalescence time for the Newtonian signal corresponding to the filter is 9.1703 sec. The other two parameters, the arrival time and initial phase, are set to zero.