

ARTICLES

**Testing scalar-tensor gravity with gravitational-wave observations
of inspiralling compact binaries**

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Observations of gravitational waves from inspiralling compact binaries using laser-interferometric detectors can provide accurate measures of parameters of the source. They can also constrain alternative gravitation theories. We analyze inspiralling compact binaries in the context of the scalar-tensor theory of Jordan, Fierz, Brans, and Dicke, focusing on the effect on the inspiral of energy lost to dipole gravitational radiation, whose source is the gravitational self-binding energy of the inspiralling bodies. Using a matched-filter analysis we obtain a bound on the coupling constant ω_{BD} of Brans-Dicke theory. For a neutron-star-black-hole binary, we find that the bound could exceed the current bound of $\omega_{\text{BD}} > 500$ from solar-system experiments, for sufficiently low-mass systems. For a $0.7M_{\odot}$ neutron star and a $3M_{\odot}$ black hole we find that a bound $\omega_{\text{BD}} \approx 2000$ is achievable. The bound decreases with increasing black-hole mass. For binaries consisting of two neutron stars, the bound is less than 500 unless the stars' masses differ by more than about $0.5M_{\odot}$. For two black holes, the behavior of the inspiralling binary is observationally indistinguishable from its behavior in general relativity. These bounds assume reasonable neutron-star equations of state and a detector signal-to-noise ratio of 10.

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I. INTRODUCTION AND SUMMARY

The regular detection of gravitational radiation from astrophysical sources by large-scale laser-interferometer systems as the U.S. Laser Interferometric Gravitational Wave Observatory (LIGO) or the French-Italian VIRGO projects will usher in a new era of gravitational-wave astronomy [1]. One of the most promising sources for detection by laser-interferometric systems is the inspiralling compact binary, a binary system of neutron stars or black holes whose orbit is decaying toward a final coalescence under the dissipative influence of gravitational radiation reaction. For much of the late-time evolution of such systems, the gravitational wave form signal is accurately calculable [2], given by a "chirp" signal, increasing in amplitude, and sweeping in frequency through the detectors' typical sensitive bandwidth between 10 and 1000 Hz. Astrophysical estimates of the rate of such inspiral events are promising: for the advanced version of LIGO, capable of detecting the inspiral wave form to distances of hundreds of Mpc, the estimated rate is 3 per year, and could be as large as 100 per year [3].

In addition to simple detection of the waves, it will be possible to determine important parameters of the inspiralling systems, such as the masses and spins of the bodies [4]. This is made possible by the technique of matched filtering of theoretical wave form templates, which depend on the system parameters, against the broadband detector output [5]. The method exploits the fact that, depending on the source, between 500 and 16 000 cycles of the waves may be observable in the sensitive band-

width, and so the matching of a template to the signal will be extremely sensitive to the evolution of the gravitational-wave frequency with time. That evolution depends, of course, on gravitational radiation reaction, which depends on the parameters of the system. Very accurate determinations of the masses of the components should be possible, while less accurate estimates of spins and other parameters may be feasible [6,7].

It is said that the first detection of gravitational radiation will also constitute a verification of general relativity, since that is the basic theory used in all calculations of gravitational radiation from such systems. It would be useful, however, to quantify that statement, by assessing how accurately such observations could actually constrain or bound alternative theories of gravity. This is not a straightforward question to answer with generality. Whereas in the slow-motion, weak-field, nonradiative limit appropriate to solar-system dynamics, most alternative metric theories of gravity can be encompassed by one simple framework, known as the parametrized post-Newtonian (PPN) formalism (see [8] for review and references), no correspondingly simple framework exists for describing radiative systems, or systems containing strong-internal-field, compact objects, such as neutron stars or black holes. On the other hand, for one simple, but popular class of alternatives, the scalar-tensor theory of Fierz, Jordan, Brans, and Dicke [9,10], and some of its generalizations, the full details have been worked out. In this paper, we shall explore the extent to which observations of gravitational waves from inspiralling compact binaries could usefully constrain scalar-tensor gravity.

For simplicity, we focus on the version of scalar-tensor gravity known for short as the Brans-Dicke (BD) theory. That theory augments general relativity (GR) by the addition of a scalar gravitational field that couples universally to matter (hence the theory, like GR, is a metric theory, satisfying all fundamental equivalence principle tests [11]) and determines the gravitational coupling strength G via $G \propto \phi^{-1}$. The relative importance of the scalar field is parametrized by a coupling constant ω_{BD} (in generalized scalar-tensor theories, ω_{BD} can itself be a function of the scalar field). Roughly speaking, in the limit of large ω_{BD} , the relative difference between effects in GR and effects in BD is $O(1/\omega_{\text{BD}})$. As $\omega_{\text{BD}} \rightarrow \infty$, BD tends smoothly toward GR. The best current empirical bound is $\omega_{\text{BD}} > 500$, from solar-system measurements of the Shapiro time delay and the deflection of radio waves by the Sun [12].

For systems involving gravitational radiation and compact objects, BD introduces three effects [13–15]

(i) Modifications to the effective masses of the bodies. These modifications depend on the internal structure of the bodies, as parametrized by “sensitivities” s_A , which roughly measure the gravitational binding energy per unit mass. These effects violate the strong equivalence principle [16], in that the motion of such bodies now depends on their internal structure (apart from tidal interactions). For neutron stars, $s \approx 0.1 - 0.2$, and for black holes $s \equiv 0.5$.

(ii) Modifications of quadrupole gravitational radiation. BD predicts monopole as well as quadrupole gravitational radiation, whose combined effect is to modify the effective GR quadrupole formula for two-body energy loss,

$$\frac{dE}{dt} = -\frac{8}{15} \frac{\mu^2 m^2}{r^4} (12v^2 - 11\dot{r}^2), \quad (1.1)$$

by corrections of $O(1/\omega_{\text{BD}})$. Here, μ and m are the reduced and total mass, respectively; r , v , and \dot{r} are the orbital separation, velocity, and radial velocity, and the units are such that $G = c = 1$.

(iii) Dipole gravitational radiation. The center of gravitational binding energy need not be coincident with the fixed center of inertial mass, if the two bodies are different, and in BD, the resulting varying dipole moment is a source of scalar radiation. Because it is a dipole rather than a quadrupole effect, the dipole contribution to the energy loss has two fewer time derivatives, and thus is $O(v^{-2})$ larger than the quadrupole contribution. It also depends on the difference in sensitivities, $\mathcal{S} \equiv s_1 - s_2$, between the two bodies. Specifically,

$$\left(\frac{dE}{dt}\right)_{\text{dipole}} = -\frac{2}{3} \frac{\mu^2 m^2}{r^4} \left(\frac{\mathcal{S}^2}{\omega_{\text{BD}}}\right). \quad (1.2)$$

We work here to first order in $1/\omega_{\text{BD}}$.

The most important consequence of dipole gravitational radiation is that it modifies the evolution of the orbital radius and thence the gravitational-wave frequency f , because $\dot{f}/f = -(3/2)\dot{r}/r = -(3/2)\dot{E}/|E|$. In the matched-filtering method, any difference between the frequency evolution of the theoretical template and that of the actual signal will ultimately cause the two to go out

of phase, and the signal-to-noise ratio will drop. A rough measure of the accuracy of the template, then, can be obtained by determining how much of a change in the template is sufficient to cause a change of $\approx \pi$ radians in the total accumulated gravitational-wave phase over the cycles in the detector’s sensitive bandwidth. The accumulated phase is

$$\Phi_{\text{GW}} = \int_{t_{\text{in}}}^{t_{\text{out}}} 2\pi f dt = \int_{f_{\text{in}}}^{f_{\text{out}}} 2\pi (f/\dot{f}) df, \quad (1.3)$$

where the subscripts in and out denote the values when the signal enters and leaves, respectively, the detector’s bandwidth. By demanding that the change in Φ_{GW} caused by the dipole term be smaller than π , one obtains the bound

$$\frac{\mathcal{S}^2}{\omega_{\text{BD}}} < \frac{5376\pi}{25} (\pi \mathcal{M} f_{\text{in}})^{7/3} \eta^{-2/5}, \quad (1.4)$$

where $\eta = \mu/m$, and $\mathcal{M} = \eta^{3/5} m$ is the “chirp mass,” the mass that determines the lowest-order quadrupole effects. For LIGO and/or VIRGO systems, f_{in} is typically chosen to be 30 Hz. A more accurate estimate obtained using the formalism of matched filtering (including post-Newtonian effects — see below) weakens this bound by about a factor of 1.3, assuming a signal-to-noise ratio (S/N) of 10. The resulting bound can be fit by the analytic formula

$$\frac{\mathcal{S}^2}{\omega_{\text{BD}}} < 1.46 \times 10^{-5} \left(\frac{\mathcal{M}}{M_{\odot}}\right)^{7/3} \eta^{-2/5} \left(\frac{10}{S/N}\right). \quad (1.5)$$

Whether this bound provides a useful constraint on the theory depends on the system in question.

(i) *Neutron star and black hole.* Since $s_{\text{BH}} = 0.5$, and $s_{\text{NS}} \leq 0.2$, $\mathcal{S} \geq 0.3$. The resulting bound on ω_{BD} is given, from Eq. (1.5), by

$$\omega_{\text{BD}} > 6156 \eta^{2/5} \left(\frac{M_{\odot}}{\mathcal{M}}\right)^{7/3} \left(\frac{S/N}{10}\right) \left(\frac{\mathcal{S}}{0.3}\right)^2. \quad (1.6)$$

The resulting bounds on ω_{BD} are plotted against the black-hole mass, for various neutron-star masses, in Fig. 1.

(ii) *Two neutron stars.* For neutron stars, s_{NS} varies weakly with mass (see for example Table 3 of [15]), so that typically \mathcal{S} is smaller than 0.05, and for neutron stars each around $1.4M_{\odot}$, is very small indeed. Thus, the small value of \mathcal{M} in Eq. (1.5) is compensated by the smallness of \mathcal{S} , and the resulting bound on ω_{BD} is generally weaker than solar-system results unless the difference in mass between the two neutron stars exceeds about $0.5M_{\odot}$. For the extreme case of $0.7M_{\odot}$ and $1.4M_{\odot}$ neutron stars, the bound could be as large as 1100. The inferred bounds are sensitive to the assumed equation of state for neutron-star matter. For a particular assumption about the dependence of \mathcal{S} on mass, Fig. 2 shows the bounds that could be achieved, assuming a signal-to-noise ratio of 10.

(iii) *Two black holes.* Because $s_{\text{BH}} \equiv 0.5$, $\mathcal{S} \equiv 0$, and there is no dipole radiation at all (see [15] for discus-

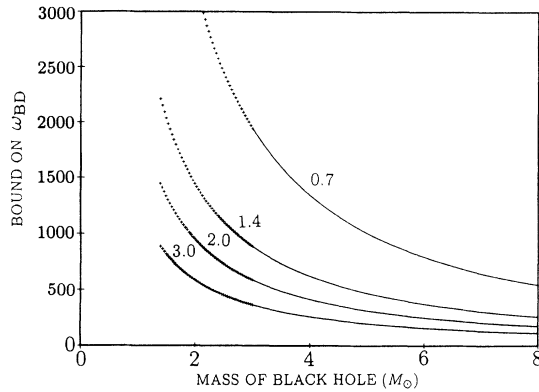


FIG. 1. Bounds on ω_{BD} from inspiralling neutron-star-black-hole binaries, plotted against black-hole mass, for various neutron-star masses. Hatched portion indicates black holes with mass less than $3.0M_{\odot}$, where identification as a black hole may be ambiguous. Curves assume $S = 0.3$ and a signal-to-noise ratio of 10.

sion). In fact the evolution of the system and the resulting gravitational radiation are identical to the general relativistic results, except that the effective gravitational mass of each black hole is given by Hawking's "tensor mass" m_T [17], related to the inertial mass by $m_T = (3 + 2\omega_{\text{BD}})/(4 + 2\omega_{\text{BD}})m$. Since the effective gravitational masses are the only parameters determined from the gravitational-wave signal, no test of BD is possible from inspiralling two-black-hole systems.

In order to place a bound on ω_{BD} , we must be able to decide among cases (i), (ii), and (iii). This requires that both the chirp mass \mathcal{M} and the reduced mass parameter η have been measured with sufficient accuracy that the mass of one of the bodies is known to be greater

than the maximum mass for a neutron star, and thus is a black hole, and that the mass of the other is known to be less than the maximum mass, and is thus likely (though not certain) to be a neutron star, or that the two masses, if both less than the maximum neutron-star mass, are sufficiently different to provide an interesting bound. We have extended the matched-filtering analysis to include post-Newtonian effects in the evolution of the gravitational-wave frequency, effects that depend explicitly on the reduced mass parameter η , and found that the accuracy in determining \mathcal{M} and η simultaneously with the dipole radiation effect is sufficient for this purpose. The dipole radiation effect varies as $v^{-2} \approx r/m$ relative to quadrupole radiation, while post-Newtonian correction terms vary as m/r ; hence the two effects are relatively uncorrelated in the matched filtering (correlation coefficients of order 0.90).

The main result is that for a neutron star of mass typical of those of well-measured pulsars ($1.3\text{--}1.5M_{\odot}$), and a relatively light black hole ($3M_{\odot}$), a bound on the Brans-Dicke parameter around two times the current solar-system bound could be obtained. For a low-mass system ($0.7M_{\odot}$ neutron star, $3M_{\odot}$ black hole), a bound around 2000 could be obtained. For double neutron-star systems of sufficiently different mass, interesting bounds could also result.

The remainder of this paper provides details. In Sec. II, we summarize the relevant BD equations for gravitational radiation and orbital motion in systems containing compact objects. Section III derives bounds on the dipole radiation effect using both the crude estimate of accumulated phase, and a full matched-filtering analysis. In Sec. IV we discuss the possibility of observing BD effects in the gravitational-wave amplitude, including testing for the existence of a third, scalar polarization mode in the gravitational wave form. Section V discusses the results.

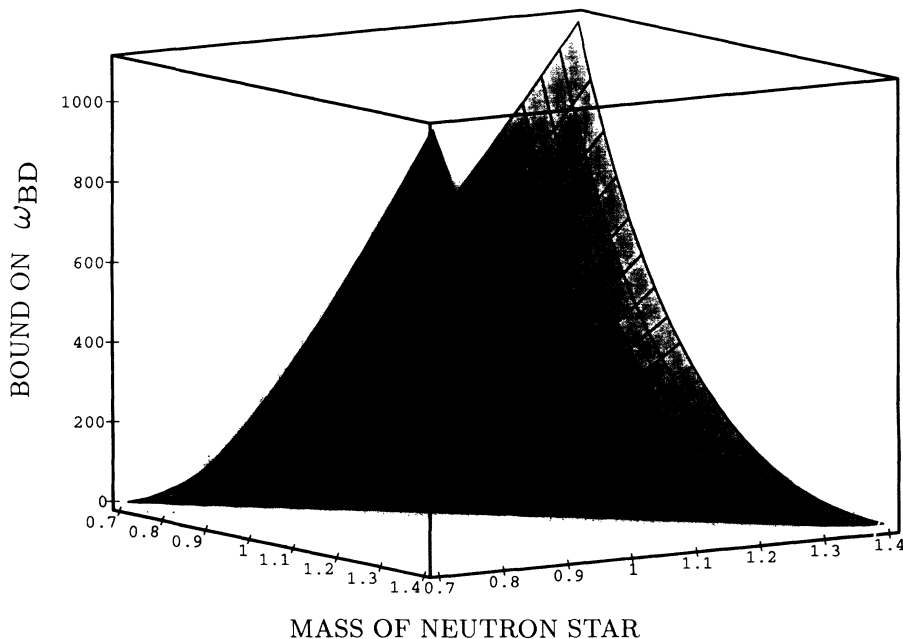


FIG. 2. Bounds on ω_{BD} from inspiralling double neutron-star binaries, plotted against mass of neutron stars. For equal masses, dipole radiation is suppressed, and no bound on ω_{BD} results. Curves assume a linear dependence of sensitivity on mass, with $S = 0.2\delta m/M_{\odot}$, and a signal-to-noise ratio of 10.

II. COMPACT OBJECTS AND GRAVITATIONAL RADIATION IN SCALAR-TENSOR GRAVITY

The equations of motion and of gravitational radiation for systems containing compact objects in Brans-Dicke theory were first derived by Eardley [13] and extended by Will and Zaglauer [14,15]. Here we present the basic background and formulas needed for our purpose, taken from [15].

We work to lowest order in an expansion in powers of $v^2 \approx m/r$, corresponding to Newtonian order for orbital motion, and to what in GR would be called quadrupole order for gravitational radiation. Yet the equations include contributions due to the self-gravitational binding energy of the compact objects, contributions that are formally of post-Newtonian order, but that could be sizable because of the relativistic nature of neutron stars or black holes. These effects are determined by the "sensitivity" of the inertial mass of each body A to changes in the local value of the effective gravitational constant G (caused by changes in the scalar field). It is defined by

$$s_A \equiv -\partial(\ln m_A)/\partial(\ln G). \quad (2.1)$$

In the weak-field limit, it is straightforward to show that s_A is the negative of the usual gravitational self-energy, given by $s_A = (1/2) \int_A \rho \rho' |\mathbf{x} - \mathbf{x}'|^{-1} d^3x d^3x'$, where ρ is the mass density. For neutron stars, s depends on the equation of state; representative values are given in Table 3 of [15]. For instance, for a $1.4M_\odot$ neutron star, $s = 0.143$ and 0.125 for equations of state O and M , respectively. On the other hand, s can vary slowly with mass: for equation of state O , for instance, $s = 0.1432$ for $m = 1.40M_\odot$ and $s = 0.1440$ for $m = 1.44M_\odot$. For black holes, a dimensional argument shows that $m \propto G^{-1/2}$, so $s_{\text{BH}} \equiv 0.5$.

A. Two-body orbits

The equation of motion and Kepler's third law for two-body orbits with orbital frequency ω (orbital period P) take the form

$$d^2\mathbf{x}/dt^2 = -\mathcal{G}m\mathbf{x}/r^3, \quad \omega = 2\pi/P = (\mathcal{G}m/r^3)^{1/2}, \quad (2.2)$$

where

$$\mathcal{G} = 1 - \xi(s_1 + s_2 - 2s_1s_2), \quad (2.3a)$$

$$\xi = (2 + \omega_{\text{BD}})^{-1}. \quad (2.3b)$$

For a circular orbit, the energy and orbital velocity are, respectively, given by

$$E = -(1/2)\mathcal{G}\mu m/r, \quad v^2 = \mathcal{G}m/r. \quad (2.4)$$

B. Gravitational radiation energy loss

The rate of energy loss for a quasicircular two-body orbit (that is, circular, apart from an adiabatic inspiral) is given by

$$\frac{dE}{dt} = -\frac{8}{15} \frac{\mu^2 m^2}{r^4} \left(12\kappa v^2 + \frac{5}{8}\kappa_D \mathcal{S}^2 \right), \quad (2.5)$$

where the first term is the combined quadrupole-mono-pole contribution, and the second term is the dipole contribution, and where

$$\kappa = \mathcal{G}^2 \left(1 - \frac{1}{2}\xi + \frac{1}{12}\xi\Gamma^2 \right), \quad (2.6a)$$

$$\kappa_D = 2\mathcal{G}^2\xi, \quad (2.6b)$$

$$\mathcal{S} = s_1 - s_2, \quad (2.6c)$$

$$\Gamma = 1 - 2(m_1s_2 + m_2s_1)/m. \quad (2.6d)$$

The gravitational waves are dominantly at the frequency $f = \omega/\pi$, corresponding to twice the orbital frequency.

C. Gravitational wave forms

In the far zone, the spatial components of the radiative metric perturbation $\bar{h}^{\mu\nu} \equiv \eta^{\mu\nu} - \sqrt{(-g)}g^{\mu\nu}$ are given by

$$\bar{h}^{ij} = \theta^{ij} - \frac{1}{2}\theta\delta^{ij} - (\varphi/\phi_0)\delta^{ij}, \quad (2.7)$$

where Greek and Roman indices denote spacetime and spatial components, respectively, where φ is the perturbation of the scalar field ϕ about its asymptotic, cosmological value ϕ_0 and where, to leading order in $v^2 \approx m/r$,

$$\begin{aligned} \theta^{ij} &= 2 \left(1 - \frac{1}{2}\xi \right) R^{-1} (d^2/dt^2) \sum_A m_A x_A^i x_A^j \\ &= (4\mu/R) \left(1 - \frac{1}{2}\xi \right) (v^i v^j - \mathcal{G}m x^i x^j / r^3), \end{aligned} \quad (2.8a)$$

$$\begin{aligned} \varphi/\phi_0 &= \xi(\mu/R) \left\{ \Gamma[(\hat{\mathbf{N}} \cdot \mathbf{v})^2 - \mathcal{G}m(\hat{\mathbf{N}} \cdot \mathbf{x})^2 / r^3] \right. \\ &\quad \left. - (\mathcal{G}\Gamma + 2\Lambda)m/r - 2\mathcal{S}(\hat{\mathbf{N}} \cdot \mathbf{v}) \right\}, \end{aligned} \quad (2.8b)$$

where R and $\hat{\mathbf{N}}$ are the distance and direction unit vector, respectively, of the observer, and $\Lambda = 1 - s_1 - s_2 + O(\xi)$.

The components of the Riemann tensor R^{0i0j} measured by a detector can be shown to be given by $R^{0i0j} = -\frac{1}{2}(d^2/dt^2)h^{ij}$, where h^{ij} is the effective gravitational wave form, given by

$$h^{ij} = \theta_{\text{TT}}^{ij} - \frac{1}{2}(\varphi/\phi_0)(\delta^{ij} - \hat{\mathbf{N}}^i \hat{\mathbf{N}}^j), \quad (2.9)$$

where TT denotes the transverse-traceless projection. Note that the full gravitational wave form is transverse but not traceless because of the presence of the scalar contribution.

For quasicircular orbits, the wave form becomes

$$h^{ij} = \frac{2\mu}{R} \left[Q_{\text{TT}}^{ij} + S(\delta^{ij} - \hat{N}^i \hat{N}^j) \right], \quad (2.10a)$$

$$Q^{ij} = 2 \left(1 - \frac{1}{2}\xi \right) \frac{\mathcal{G}m}{r} (\hat{\lambda}^i \hat{\lambda}^j - \hat{n}^i \hat{n}^j), \quad (2.10b)$$

$$S = -\frac{1}{4}\xi \left\{ \frac{\Gamma \mathcal{G}m}{r} [(\hat{N} \cdot \hat{\lambda})^2 - (\hat{N} \cdot \hat{n})^2] - (\mathcal{G}\Gamma + 2\Lambda) \frac{m}{r} - 2\mathcal{S} \left(\frac{\mathcal{G}m}{r} \right)^{1/2} \hat{N} \cdot \hat{\lambda} \right\}, \quad (2.10c)$$

where $\hat{n} \equiv \mathbf{x}/r$, and $\hat{\lambda} \equiv \mathbf{v}/v$.

III. TESTING SCALAR-TENSOR GRAVITY USING MATCHED FILTERING OF GRAVITATIONAL WAVE FORMS

A. Phase-shift estimate

Because broadband detectors such as the free-mass laser-interferometric systems detect the gravitational wave form $h^{ij}(t)$ superimposed on the noise, and because hundreds to tens of thousands of cycles may be observed in the bandwidth, the observations are especially sensitive to the evolution of the frequency and phase of the wave. By combining Eqs. (2.2), (2.4), and (2.5), one can show that the frequency of the wave form evolves according to

$$\dot{f} = \frac{96}{5} \eta \frac{\mathcal{G}^{1/2}}{\pi m^2} \left(\frac{m}{r} \right)^{11/2} \left(\kappa + \frac{5}{96} \frac{\kappa_D}{\mathcal{G}} \frac{r}{m} \mathcal{S}^2 \right). \quad (3.1)$$

We define the Brans-Dicke chirp mass \mathcal{M} and the dipole parameter b according to

$$\begin{aligned} \mathcal{M} &\equiv (\kappa^{3/5} \mathcal{G}^{4/5} \eta^{3/5} m), \\ b &\equiv (5/96) (\kappa^{-3/5} \mathcal{G}^{-6/5}) \kappa_D \mathcal{S}^2. \end{aligned} \quad (3.2)$$

Defining $u \equiv \pi \mathcal{M} f$, we put Eq. (3.1) into the form

$$\dot{u} = \mathcal{M}^{-1} (96/5) u^{11/3} (1 + b \eta^{2/5} u^{-2/3}). \quad (3.3)$$

Integrating, we get

$$u^{-8/3} [1 - (4/5) b \eta^{2/5} u^{-2/3}] = (256/5) (t_c - t) / \mathcal{M}, \quad (3.4)$$

where t_c is the time at which $u \rightarrow \infty$. We have expanded the expression (3.4) to first order in $b \eta^{2/5} u^{-2/3}$, using the fact that

$$\begin{aligned} b \eta^{2/5} u^{-2/3} &\leq 5 \times 10^{-3} \left(\frac{500}{\omega_{\text{BD}}} \right) \left(\frac{\mathcal{S}}{0.5} \right)^2 \\ &\times \left(\frac{M_\odot}{\mathcal{M}} \right)^{2/3} \left(\frac{30 \text{ Hz}}{f} \right)^{2/3}. \end{aligned} \quad (3.5)$$

From Eq. (1.3), the number of cycles observed in a given bandwidth can be written $\Phi_{\text{GW}} = (2/\mathcal{M}) \int_{u_{\text{in}}}^{u_{\text{out}}} (u/\dot{u}) du$, giving

$$\Phi_{\text{GW}} = \frac{1}{16} (u_{\text{in}}^{-5/3} - u_{\text{out}}^{-5/3}) - \frac{5}{112} b \eta^{2/5} (u_{\text{in}}^{-7/3} - u_{\text{out}}^{-7/3}). \quad (3.6)$$

Demanding that the phase contribution of the dipole term be no more than π , we obtain $b < (112\pi/5) \eta^{-2/5} u_{\text{in}}^{7/3}$, where we assume that $f_{\text{out}} \gg f_{\text{in}}$ (1000 Hz vs 10 Hz). To lowest order in $1/\omega_{\text{BD}}$, $\kappa = \mathcal{G} = 1$, $\kappa_D = 2/\omega_{\text{BD}}$, and thus $b \approx (5/48) \mathcal{S}^2 / \omega_{\text{BD}}$, resulting in the bound given in Eq. (1.4).

B. Matched-filter analysis

To obtain a more accurate estimate of the bound that can be placed on the dipole parameter b , we carry out a full matched-filter analysis, following the method described by Chernoff and Finn [6] and Cutler and Flanagan [7]. To the accuracy needed, we approximate the observed gravitational wave form, Eq. (2.10), in a given detector by $h(t) \approx \mathcal{R}\{h^0(t)e^{i\Phi(t)}\}$, where $h^0(t)$ is the slowly varying Newtonian-order contribution to the wave form amplitude, dependent upon the distance to the source, its location on the sky, the orientation of the detector, and on the source parameters \mathcal{M} , η , and r ; $\Phi(t)$ is the gravitational-wave phase, dominantly at twice the orbital phase, and \mathcal{R} denotes the real part. The phase includes the dipole and higher order post-Newtonian corrections, in principle. Calculating the Fourier transform of $h(t)$ in the stationary phase approximation, we obtain

$$\tilde{h}(f) = \begin{cases} \mathcal{A} f^{-7/6} e^{i\Psi}, & 0 < f < f_{\text{max}}, \\ 0, & f > f_{\text{max}}, \end{cases} \quad (3.7)$$

where $\mathcal{A} \propto R^{-1} \mathcal{M}^{5/6} \times$ [function of angles and detector orientation], and $f_{\text{max}} = O(m^{-1})$ corresponds to the frequency when the inspiral turns into a plunge toward coalescence, and

$$\begin{aligned} \Psi(f) &= 2\pi f t_c - \Phi_c - \pi/4 \\ &+ \frac{3}{128} u^{-5/3} \left(1 - \frac{4}{7} b \eta^{2/5} u^{-2/3} \right), \end{aligned} \quad (3.8)$$

where Φ_c formally is the gravitational-wave phase at time t_c .

With a given noise spectrum $S_n(f)$, one defines the inner product of signals h_1 and h_2 , by

$$(h_1|h_2) \equiv 2 \int_0^\infty \frac{\tilde{h}_1^* \tilde{h}_2 + \tilde{h}_2^* \tilde{h}_1}{S_n(f)} df. \quad (3.9)$$

The signal-to-noise ratio for a given signal h is given by

$$\rho[h] \equiv S/N[h] = (h|h)^{1/2}. \quad (3.10)$$

If the signal depends on a set of parameters θ^a which are to be estimated by the matched filter, then the rms error

in θ^a in the large S/N limit is given by

$$\Delta\theta^a \equiv \sqrt{\langle(\Delta\theta^a)^2\rangle} = \sqrt{\Sigma^{aa}}, \quad (3.11)$$

where Σ^{aa} is the corresponding component of the inverse of the ‘‘covariance matrix’’ or ‘‘Fisher information matrix’’ Γ_{ab} defined by

$$\Gamma_{ab} \equiv \left(\frac{\partial h}{\partial\theta^a} \middle| \frac{\partial h}{\partial\theta^b} \right). \quad (3.12)$$

The correlation coefficient between two parameters θ^a and θ^b is

$$c^{ab} = \Sigma^{ab}/\sqrt{\Sigma^{aa}\Sigma^{bb}}. \quad (3.13)$$

For a noise spectrum, we adopt the analytic fit to the LIGO ‘‘advanced detector’’ noise spectral density:

$$S_n(f) = \begin{cases} \infty, & f < 10 \text{ Hz}, \\ S_0\{(f_0/f)^4 + 2[1 + (f/f_0)^2]\}/5, & f > 10 \text{ Hz}, \end{cases} \quad (3.14)$$

where $S_0 = 3 \times 10^{-48} \text{ Hz}^{-1}$ and $f_0 = 70 \text{ Hz}$. The cutoff at 10 Hz corresponds to seismic noise, while the f^{-4} and f^2 dependences correspond to thermal and photon-shot noise, respectively [18].

We shall adopt the following five parameters to be estimated: $\ln \mathcal{A}$, Φ_c , $f_0 t_c$, $\ln \mathcal{M}$, and \tilde{b} , where $\tilde{b} = b\eta^{2/5}$. The corresponding partial derivatives of $\tilde{h}(f)$ are

$$\frac{\partial \tilde{h}(f)}{\partial \ln \mathcal{A}} = \tilde{h}(f), \quad (3.15a)$$

$$\frac{\partial \tilde{h}(f)}{\partial f_0 t_c} = 2\pi i (f/f_0) \tilde{h}(f), \quad (3.15b)$$

$$\frac{\partial \tilde{h}(f)}{\partial \Phi_c} = -i \tilde{h}(f), \quad (3.15c)$$

$$\frac{\partial \tilde{h}(f)}{\partial \ln \mathcal{M}} = -\frac{5}{128} i u^{-5/3} \tilde{h}(f) \left(1 - \frac{4}{5} \tilde{b} u^{-2/3} \right), \quad (3.15d)$$

$$\frac{\partial \tilde{h}(f)}{\partial \tilde{b}} = -\frac{3}{224} i u^{-7/3} \tilde{h}(f). \quad (3.15e)$$

The signal-to-noise ratio is given by

$$\rho^2 = 20 |\mathcal{A}|^2 f_0^{-4/3} I(7)/S_0, \quad (3.16)$$

where we define the integrals

$$I(q) \equiv \int_{1/7}^{\infty} x^{-q/3} (x^{-4} + 2 + 2x^2)^{-1} dx. \quad (3.17)$$

We also define the coefficients $B_q \equiv I(q)/I(7)$. Since our *a priori* expectation is the validity of GR, we are looking for a bound on $|\tilde{b}|$. Equivalently, we wish to determine the error in estimating \tilde{b} about the nominal value $\tilde{b} = 0$; thus we set $\tilde{b} = 0$ in Eq. (3.15d). Then the elements of the covariance matrix turn out to be proportional to $\rho^2 u_0^{-n/3} B_q$ for various integers n and q , where $u_0 = \pi \mathcal{M} f_0$. Inverting the matrix to obtain Σ^{ab} , we obtain from the elements $\Sigma^{\mathcal{M}\mathcal{M}}$, Σ^{bb} , and $\Sigma^{\mathcal{M}b}$, $\Delta(\ln \mathcal{M}) = 57.6 u_0^{5/3}/\rho$, $\Delta(\tilde{b}) = 53.2 u_0^{7/3}/\rho$, and $c^{\mathcal{M}\tilde{b}} = -0.98$.

Substituting $f_0 = 70 \text{ Hz}$, and working to first order in $1/\omega_{\text{BD}}$, so that $\mathcal{M} = \eta^{3/5} m$ and $\tilde{b} = (10/96)\eta^{2/5} S^2/\omega_{\text{BD}}$ [see Eq. (3.2)], we obtain

$$\Delta(\ln \mathcal{M}) = 6.56 \times 10^{-5} \left(\frac{\mathcal{M}}{M_\odot} \right)^{5/3} \left(\frac{10}{S/N} \right), \quad (3.18a)$$

$$\Delta \left(\frac{S^2}{\omega_{\text{BD}}} \right) = 6.14 \times 10^{-6} \eta^{-2/5} \left(\frac{\mathcal{M}}{M_\odot} \right)^{7/3} \left(\frac{10}{S/N} \right). \quad (3.18b)$$

C. Post-Newtonian effects

The interpretation of gravitational-wave observations as testing scalar-tensor gravity relies upon determining the masses of the two bodies with sufficient accuracy (i) that one can decide with confidence whether each body is more or less massive than the accepted neutron-star maximum mass (modulo the many uncertainties in that number) in the neutron-star-black-hole case, or (ii) that one can establish that the mass difference exceeds some critical value, in the double neutron-star case. The determination of the individual masses makes use of post-Newtonian corrections to the orbital phase, which depend explicitly on the reduced mass parameter η . It is then necessary to check whether such determinations can be made *simultaneously* with the bound on the parameter \tilde{b} . To this end, we extend the matched-filter analysis to include post-Newtonian corrections. Those corrections have not been fully calculated to date in the context of BD, but one expects them to be the same as those of GR, within corrections of $O(1/\omega_{\text{BD}})$. To incorporate them into our analysis in a first approximation, then, it suffices to add the appropriate GR post-Newtonian terms of [7], Eq. (3.13), to the Fourier transform phase, Eq. (3.8), to obtain

$$\Psi(f) = 2\pi f t_c - \Phi_c - \pi/4 + \frac{3}{128} u^{-5/3} \left[1 - \frac{4}{7} b \eta^{2/5} u^{-2/3} + \frac{20}{9} \left(\frac{743}{336} + \frac{11}{4} \eta \right) \eta^{-2/5} u^{2/3} - 16\pi \eta^{-3/5} u \right], \quad (3.19)$$

where the final term is the ‘‘tail’’ effect. Adding the parameter $\ln \eta$ to the set to be estimated, we find the partial derivatives

$$\frac{\partial \tilde{h}(f)}{\partial \ln \mathcal{M}} = -\frac{5i}{128} u^{-5/3} \tilde{h}(f) \left[1 - \frac{4}{5} \tilde{b} u^{-2/3} + \frac{4}{3} \left(\frac{743}{336} + \frac{11}{4} \eta \right) \eta^{-2/5} u^{2/3} - \frac{32}{5} \pi \eta^{-3/5} u \right], \quad (3.20a)$$

$$\frac{\partial \tilde{h}(f)}{\partial \ln \eta} = \frac{i}{96} u^{-5/3} \tilde{h}(f) \left[\left(-\frac{743}{168} + \frac{33}{4} \eta \right) \eta^{-2/5} u^{2/3} + \frac{108}{5} \pi \eta^{-3/5} u \right]. \quad (3.20b)$$

TABLE I. The rms errors for signal parameters, the corresponding bound on ω_{BD} , and the correlation coefficients $c_{\mathcal{M}\eta}$, $c_{\mathcal{M}\tilde{b}}$, and $c_{\tilde{b}\eta}$. General relativistic post-Newtonian effects are included; the noise spectrum is that of the advanced LIGO system, given by Eq. (3.14), and a signal-to-noise ratio of 10 is assumed. Masses are in units of M_{\odot} , Δt_c is in msec. For neutron-star-black-hole systems, $\mathcal{S} = 0.3$ is assumed; while for double neutron-star systems, $\mathcal{S} = 0.2\delta m/M_{\odot}$ is assumed.

m_1	m_2	$\Delta\phi_c$	Δt_c	$\Delta\mathcal{M}/\mathcal{M}$	$\Delta\eta/\eta$	ω_{BD}	$c_{\mathcal{M}\eta}$	$c_{\mathcal{M}\tilde{b}}$	$c_{\tilde{b}\eta}$
Neutron-star-black-hole systems									
1.0	2.0	2.21	0.83	0.028%	2.23%	2140	-0.950	-0.991	-0.909
1.0	5.0	2.21	0.83	0.056%	2.20%	682	-0.949	0.991	0.908
1.4	5.0	2.19	0.82	0.079%	2.73%	470	-0.953	-0.991	-0.911
1.4	10.0	2.19	0.82	0.131%	2.46%	194	-0.952	0.991	0.909
Double neutron-star systems									
0.7	1.4	2.24	0.83	0.015%	1.84%	1092	-0.945	0.992	-0.905
1.4	1.4	2.19	0.82	0.029%	2.34%	0	-0.953	-0.991	-0.911

The other partial derivatives in Eqs. (3.15) are unchanged. Setting $\tilde{b} = 0$, we then calculate and invert the covariance matrix and evaluate the errors in the five relevant parameters (the parameter $\ln \mathcal{A}$ decouples from the rest, and is not important for our purposes), along with the correlation coefficients between \mathcal{M} , η , and \tilde{b} . Notice that the fact that \tilde{b} involves η implicitly does not affect the outcome, because we are considering the results centered around $\tilde{b} = 0$. For various double neutron-star and neutron-star-black-hole systems, the results are shown in Table I. Note that \tilde{b} is less strongly correlated with η than is \mathcal{M} , a result to be expected because of the very different dependences of the dipole and post-Newtonian effects on m/r or on u . Nevertheless, a result of the correlation is to weaken the bound on $\mathcal{S}^2/\omega_{\text{BD}}$ in Eq. (3.18b) by a factor of about 2.3. Equation (1.5) provides an analytic fit to the result.

Notice from Table I that the accuracy in determining η is a few percent, consistent with the results of [7]. Together with the high accuracy in determining \mathcal{M} , this will lead to accurate values for the two masses, except for the degenerate region around equal masses, $\eta = 0.25$. However, that region is excluded from our considerations because of the weakness of the bound in the nearly equal mass, double neutron-star case, and because of the impossibility of determining unambiguously that one of two objects of nearly equal mass is a black hole while the other is not, in the mixed case. Figure 3 shows the regions in $\mathcal{M} - \eta$ space corresponding to the three types of system.

D. Dependence on the nature of the system

The nature of the system being observed is important to the interpretation of the bound (1.5) as a test of gravitation theory.

For two neutron stars, the chirp mass is relatively small ($1.2M_{\odot}$ for two $1.4M_{\odot}$ neutron stars), so the bound on $\mathcal{S}^2/\omega_{\text{BD}}$ can be quite small. The reason is that such systems enter the detectors' bandwidth (say, at 10 Hz) with

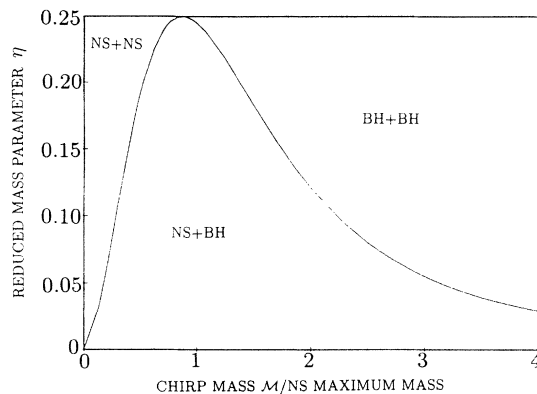


FIG. 3. Chirp-mass/reduced-mass parameter plane, showing location of three different types of compact binaries. Chirp mass is plotted in units of the neutron-star maximum mass.

a large separation relative to the total mass ($r/m \approx 180$ for two $1.4M_{\odot}$ neutron stars). As a result, gravitational radiation damping is weaker, and more cycles occur in the detectors' bandwidth (up to 16 500 cycles for two $1.4M_{\odot}$ neutron stars), hence the matched filter is more sensitive to effects on the template phase evolution. However, the sensitivity difference \mathcal{S} is also small, because of the weak dependence of the sensitivity on the masses of neutron stars, and because the neutron stars are expected to have similar masses, as has been seen in known binary pulsar systems, such as the Hulse-Taylor system PSR 1913+16 [19]. For relatively stiff equations of state, which are required in order to have neutron stars sufficiently massive ($\geq 1.4M_{\odot}$) to agree with masses inferred from known binary pulsars, Table 3 of [15] shows that the sensitivity varies roughly linearly with mass, with $s \sim 0.2m/M_{\odot}$, reaching a maximum near the maximum mass for the given equation of state (around $1.46M_{\odot}$ for the stiff equations of state discussed in [15]). To get a rough idea of the bounds on ω_{BD} that might be possible in double neutron-star systems, we substitute the relation $\mathcal{S} \approx 0.2\beta\delta m/M_{\odot}$ into Eq. (1.5), where $\delta m \equiv m_1 - m_2$, and where β may vary between 0 and 1, and use the fact that $\delta m^2 = (1 - 4\eta)m^2$, to obtain

$$\omega_{\text{BD}} > 2736\beta^2 \left(\frac{M_{\odot}}{m}\right)^{1/3} \left(\frac{1}{\eta} - 4\right) \left(\frac{S/N}{10}\right). \quad (3.21)$$

The resulting bounds for $\beta \approx 1$ are plotted in Fig. 2. For $\beta \approx 1$, the bounds are weaker than solar-system bounds for neutron stars whose masses differ by less than $0.5M_{\odot}$. At the other extreme, the bound could approach 1000 for a $0.7 - 1.4M_{\odot}$ pair.

For black holes in scalar-tensor gravity, $s_{\text{BH}} \equiv 0.5$. This is a consequence of the fact that, in the formation of a black hole, the scalar field is radiated away (see [15,17,20,21] for further discussion). As a result, from Eqs. (2.3) and (2.6), we see that $\mathcal{G} = 1 - \xi/2$, $\kappa = (1 - \xi/2)^3$, $\mathcal{S} = \Gamma = \Lambda = 0$. Substituting these values into Eqs. (2.2), (2.4), (2.5), and (2.8), we see

that the dipole effects vanish, and the equations become equivalent to those of GR if we replace every mass by the “tensor mass” [17], given by

$$m_T \equiv m(1 - \xi/2) = m(3 + 2\omega_{\text{BD}})/(4 + 2\omega_{\text{BD}}). \quad (3.22)$$

If gravitational radiation is the only information about the system, then only the tensor masses are measured in a matched filter. Since the behavior of the system is general relativistic in terms of tensor masses, then no test of BD in double black-hole systems is possible.

For the case of neutron-star-black-hole systems, $s_{\text{BH}} = 0.5$ while $s_{\text{NS}} \approx 0.1 - 0.2$. Although the black hole cannot support scalar field in its vicinity, the companion neutron star can, and consequently dipole gravitational radiation can occur. For stiff equations of state, $s_{\text{NS}} \leq 0.14$, and thus we can approximate $S \geq 0.3$. With these assumptions, Eq. (1.6) leads to the bounds plotted in Fig. 1.

IV. TESTING SCALAR-TENSOR GRAVITY USING WAVE FORM AMPLITUDE AND POLARIZATION

From Sec. II C we see that scalar-tensor gravity affects the amplitude of the gravitational wave form in two important ways: (i) it introduces a third, nontraceless polarization state [Eq. (2.10a)], and (ii) it introduces, through the dipole term, a contribution at the orbital frequency in addition to the quadrupole and/or monopole contributions at twice the orbital frequency [final term in Eq. (2.10c)]. However, these are unlikely to be testable, for the following reason. From Eqs. (3.9) and (3.15), it is simple to see that the cross components between $\ln \mathcal{A}$ and the other four parameters in the covariance matrix vanish, so that Γ_{ab} has the form

$$\Gamma_{ab} = \rho^2 \begin{pmatrix} 1 & 0 \\ 0 & D \end{pmatrix}, \quad (4.1)$$

where D is a 4×4 matrix corresponding to the parameters Φ_c , $f_0 t_c$, $\ln \mathcal{M}$, and \tilde{b} . Thus $\Delta(\ln \mathcal{A}) = (\Sigma^{\mathcal{A}\mathcal{A}})^{1/2} = 1/\rho \approx 0.1$, for a signal-to-noise ratio of 10. However, the scalar-tensor corrections to the amplitude are much smaller than this. The ratio of the monopole part of S [Eq. (2.10c)] to the quadrupole term Q^{ij} [Eq. (2.10b)] is $O(\xi/8) \approx 2 \times 10^{-4} (500/\omega_{\text{BD}})$, while the ratio of the dipole part of S to the quadrupole term is $O[\frac{1}{4}\xi S(r/m)^{1/2}] \approx 10^{-3} (500/\omega_{\text{BD}}) (S/0.3) (r/100m)^{1/2}$. Thus, despite the presence of qualitatively new contributions to the wave form, the lower sensitivity of matched filtering to amplitudes, as compared to phases, makes those contributions difficult to detect or bound.

V. DISCUSSION OF RESULTS

We have shown that interesting, if not spectacular, bounds on the coupling constant ω_{BD} of Brans-Dicke theory could be obtained by matched filtering of a gravitational wave signal from an inspiralling binary of a neutron

star and either a neutron star of very different mass, or a low-mass black hole. The bound that can be obtained decreases rapidly with decreasing neutron-star mass difference and with increasing black-hole mass.

Another class of laser-interferometric detectors that may be relevant to this discussion is the space-based class of systems, such as the Laser Gravitational-Wave Observatory in Space (LAGOS) [22] and SAGITTARIUS [23]. These will be most sensitive to gravitational waves in the frequency range $10^{-4} - 10^{-1}$ Hz, and could detect gravitational waves from the inspiral of a $1M_{\odot}$ star into a black hole in the $10^6 - 10^7 M_{\odot}$ range. Noting that the bound on ω_{BD} scales with the frequency f_0 as $f_0^{-7/3}$, we find from Eq. (1.6) for a $1M_{\odot}$ star and a $10^6 M_{\odot}$ black hole, with $f_0 \approx 10^{-2}$ Hz, that a bound exceeding 50 000 might be possible. This warrants a more detailed matched-filter study, including the use of the appropriate noise curve for space-based detectors analogous to Eq. (3.14), and including the effects of spin of the central hole, which are likely to be important. This study is currently under way.

It is useful to point out that, for generalized scalar-tensor (ST) theories, i.e., those in which the coupling constant becomes effectively a function of the scalar field [24], the foregoing conclusions still apply, with the following principal change: the dipole parameter κ_D , Eq. (2.6b), becomes

$$\kappa_D = \frac{2\mathcal{G}^2}{(2 + \omega_{\text{ST}})} \left(1 + \frac{2\omega'_{\text{ST}}}{(3 + 2\omega_{\text{ST}})^2} \right)^2, \quad (5.1)$$

where $\omega'_{\text{ST}} \equiv d\omega_{\text{ST}}/d\phi$. (There are other changes induced by ω'_{ST} , in κ , Γ and other formulas, but they are unimportant for our purposes.) In the large- ω_{ST} limit, this means replacing ω_{BD} by $\omega_{\text{ST}}/(1 + \omega'_{\text{ST}}/2\omega_{\text{ST}}^2)$ in the bounds of Eqs. (1.4), (1.6), and (3.18b).

Another difference between BD and such generalized scalar-tensor theories is that, while in the former case, large ω_{BD} implies that all physical predictions are close to those of GR [roughly within $O(1/\omega_{\text{BD}})$], in the latter case the statement holds only for weak scalar-field situations ($G\phi \approx 1$), such as in the solar system. However, in the strong-field interiors of neutron stars, where the scalar field may have values very different from its weak-field values, the differences between the scalar-tensor theory and GR may be significant, despite a large exterior value of ω_{ST} [21,25–27]. In evaluating the sensitivities of neutron stars, we used neutron-star models computed using GR, since BD is a small correction for large ω_{BD} throughout the stellar interior [28]. But the strong-field effects in ST theories alter the situation, and quite different values of sensitivities for a given neutron-star mass may result [21,27]. Since our quoted bounds were strongly dependent on the sensitivities, it is impossible to draw strong conclusions about bounds on ST theories from inspiralling binaries without further research.

How do the bounds we have suggested compare with other empirical bounds and with theoretical expectations? Solar-system bounds on scalar-tensor gravity may improve from the current level of $\omega_{\text{BD}} > 500$ during the

coming decade. The Relativity Gyroscope Experiment (Gravity Probe B) aims to measure the geodetic precession of a set of orbiting superconducting gyroscopes at the 10^{-5} level, which would correspond to $\omega_{\text{BD}} > 60\,000$ [29]. Orbiting optical interferometers for astrometry may measure the deflection of light at the 5 microarcsecond level, resulting in bounds on ω_{BD} at the level of 10^5 [30].

In BD, ω_{BD} is an arbitrary constant, so no value has an *a priori* theoretical significance (except $\omega_{\text{BD}} = \infty$). However, in generalized theories, the value of ω_{ST} is coupled to the dynamics of the scalar field (or fields, in multiscalar theories [25]), and its present, average cosmic value may depend on the evolution of the universe. Damour and Nordtvedt [31] have pointed out that, in one class of generalized scalar-tensor theories, in which $\omega(\phi) \equiv -\frac{3}{2} - \frac{1}{2}(\chi \ln \phi)^{-1}$, with $0 \leq \phi \leq 1$, cosmic evolution from strongly non-general-relativistic early universes

tends toward a large- ω “attractor” at the present epoch. For specific models and values of the parameter χ , the present value of ω_{ST} can range from 2×10^4 to 1×10^6 . Unless there are unusual strong-field effects in this theory that would modify our conclusions based on BD, these values are well above the bounds we anticipate from coalescing binaries.

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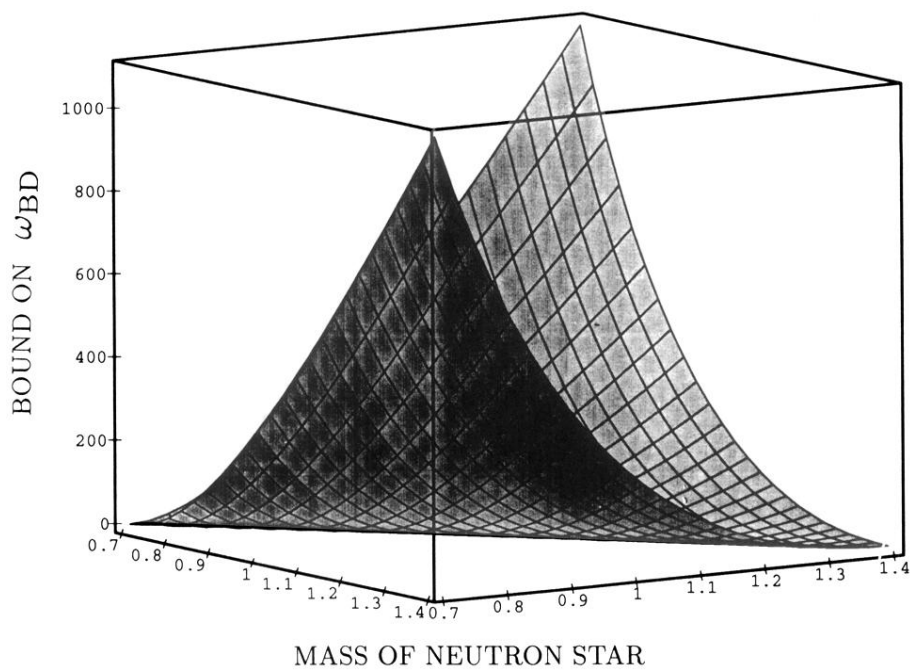


FIG. 2. Bounds on ω_{BD} from inspiralling double neutron-star binaries, plotted against mass of neutron stars. For equal masses, dipole radiation is suppressed, and no bound on ω_{BD} results. Curves assume a linear dependence of sensitivity on mass, with $S = 0.2\delta m/M_{\odot}$, and a signal-to-noise ratio of 10.