Can a logarithmically running coupling mimic a string tension?

Michael Grady
Department of Physics, SUNY Fredonia, Fredonia, New York 14063 *
and Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400 005, India
(Received 20 April 1994)

A Coulomb potential using a running coupling slightly modified from the perturbative form produces an interquark potential that appears nearly linear over a large distance range, and fits well to lattice gauge theory data. This calls into question the accuracy of string tension measurements which are based on the assumption of a constant coefficient for the Coulomb term. It also opens up the possibility of obtaining an effectively confining potential from gluon exchange alone.

PACS number(s): 12.38.Gc, 11.10.Hi, 11.15.Ha, 12.39.Pn

It is surprising that the interquark potential for pure-gauge SU(2) and SU(3) lattice gauge theories fits as well as it does to a simple linear+Coulomb law. Even the extremely high statistics SU(2) results of the UKQCD Collaboration at $\beta = 4/g^2 = 2.85$, which include distances to $R/a = 24$ and with a relatively small physical lattice spacing a ($a^{-1} \simeq 6.56$ GeV), probing well both the short and long distance potential, require no additional terms to fit the data [1]. What is surprising about this is that one of the most definite predictions of perturbation theory, backed up by high-energy scattering experiments, is that the effective coupling is a running coupling, so one would expect the coefficient of the Coulomb term, which can be taken to be a renormalized coupling, to depend upon distance. At weak couplings corresponding to short distances this should match the logarithmic dependence given by renormalization-group improved low-order perturbation theory. If lattice gauge theory is to be successfully matched onto perturbation theory, then at least the short distance part of the potential should be allowed to run. This has been tried and gives indications of a reasonable match to perturbation theory [2]. For longer distances (say $R/a \gtrsim 6$ on the above lattice) the coupling is generally assumed to stop running, to allow an accurate determination of the string tension. However, the fits which show a running coupling at shorter distances show no indication that the running is slowing down. In addition, the values of running coupling obtained from these analyses are considerably larger than the fixed value obtained in the linear + Coulomb fit. The stopping of the running coupling has been justified by the strong-coupling string model of Lüscher which predicts the coefficient of $1/R$ in the potential to be the constant value of $\pi/12$ [3]. The problem with this is that the couplings for which this string picture become valid are probably much stronger than those of the simulations being discussed here [4]. There is very little independent evidence for the stopping of the running of the renormalized coupling.

In the following I investigate the effects of relaxing the assumption that the coupling stops running at long distances. It is shown that even the one-loop perturbative potential roughly mimics the linear+Coulomb form over the large distance range included in the aforementioned simulation. That is, the rising running coupling can temporarily counteract the falling Coulomb law to produce an approximately linear potential over a certain range. In the end the one-loop perturbative potential is ruined by the Landau pole singularity. A phenomenological potential consistent with perturbative functional forms is introduced which contains an additional piece which can kill the Landau pole. This form is found to fit the data every bit as well as the Coulomb+linear form. The potential eventually falls off the linear trend, but not until $R/a \simeq 50$, well beyond the range of the simulation or planned simulations for some time to come. Thus it will be shown that a logarithmically running coupling is capable of producing a phenomenologically confining potential. The potential is not absolutely confining; however, it should be remembered that this is not necessary in the real world with light quarks, since beyond a certain separation a meson pair will form, breaking the "string." Thus the pure-gauge simulation is only relevant to real world physics up to a distance of order 1 fm.

In what follows, the focus is first shifted to the interquark force, as opposed to the potential, because it is more easily compared to the perturbative result [5,6]. For lattice SU(2) the magnitude of the force may be written

$$F(R) = (1 + 3a^2/4R^2)3\alpha(R)/4R^2. \quad (1)$$

The prefactor in parentheses is from an approximation to the lattice Coulomb propagator [valid for $(R/a) \geq 2$] which differs slightly from that of the continuum for small R/a [4]. It actually has very little effect on the fits. The potential is taken to be the integral of the force, which may become complicated for complicated $\alpha(R)$. For this reason it is much easier to work with the force. The main disadvantage to working with the force is that the Monte Carlo data give more directly the potential itself. Data for the force can be obtained from the potential data by taking finite differences. This introduces some horizontal (i.e., ΔR) uncertainty into the data, since one is not sure where within the R interval to plot the force value. Usually the midpoint is chosen, but because of the inverse relationship here the geometric mean is used,

*Present address.

 ©1994 The American Physical Society

which gives exact results for a $1/R$ potential.

Figure 1 shows a fit to the UKQCD data from Ref. [1] for $R/a \geq 4$ (the $R/a = 2$ point was also excluded from the fits in Ref. [1]). The short-dashed line is a one-parameter fit to the one-loop renormalization-group improved force with

$$\alpha(R) = [1 + 8\pi b_0 \ln(R_0/R)]^{-1}, \qquad (2)$$

where $b_0 = 11/24\pi^2$. The fit gives R_0/a=12.28. Considering that it includes only effects from (summed) lowest-order perturbation theory, it falls suprisingly close to the data even in the high-R region. If b_0 is allowed to be a free parameter, only a marginally better fit is produced with b_0 increasing 5% over its perturbative value. Such a fit can be thought of as a determination of the scaling parameter b_0 directly from the data. Thus the hypothesis that it is the running coupling which is responsible for the form of the force even at intermediate and large distances appears reasonably consistent with asymptotic scaling. Note that the force function produced by the running coupling is relatively flat in the region $13 \leq R/a \leq 22$, varying only 10% from a constant. Thus the force in this region is much closer to a constant force, which would mimic a confining string tension, than it is to a $1/R^2$ Coulomb force. The Coulomb decrease is nearly matched by the increase in effective coupling.

Although this function roughly fits the data it does not fit these high statistics data well enough in detail, giving a $\chi^2/N_{\rm DF} = 4.4$. Clearly the fit is spoiled by the appearance of the Landau pole at $R/a \simeq 28$. This is widely believed to be an unphysical result due to the partial summation of the series. The fit to the two-loop renormalization-group improved force, using the running coupling

$$\alpha(R) = (4\pi b_0\{\ln[(R_0/R)^2] + (b_1/b_0^2)\ln\ln[(R_0/R)^2]\})^{-1}$$

with $b_1/b_0^2 = 102/121$ gives the large-dashed line, which is clearly worse than the one-loop fit. This one-parameter fit gives $R_0/a = 32.5$. If one allows the constants to deviate from their perturbative values, not much improvement results. The worse fit to the two-loop force can be attributed to the even earlier appearance of the Landau pole, around $R/a = 24$. There is still a relatively constant region from about $10 \leq R/a \leq 20$, but the value is about 50% too low. One can understand why the two-loop result might be worse at long distances by considering the renormalization-group β function. If $\alpha(R)$ is to remain finite, the true β function, which diverges from the axis like g^3 at small g, must eventually turn back toward the axis. The two-loop β function adds a g^5 term of the same sign as the g^3 term, which causes the β function to diverge even faster. This will cause it to disagree even more with the assumed behavior of the true β function at large g, even though it is more accurate than the one-loop result at small g.

If the Landau pole could somehow be removed, then it would not take a running coupling much different from the one-loop form to fit the data, because the one-loop force already gives a qualitatively successful fit. The approach to be taken is that of a phenomenological extension of the one-loop force, consistent with functional forms of (summed) perturbation theory. It is reasonable to assume that $\alpha(R)$ is given by a power series in $\ln(R_0/R)$, but with coefficients that differ from the simple geometric series of the one-loop bubble diagrams. A reasonable generalization for the geometric series summation is that of the Padé approximant:

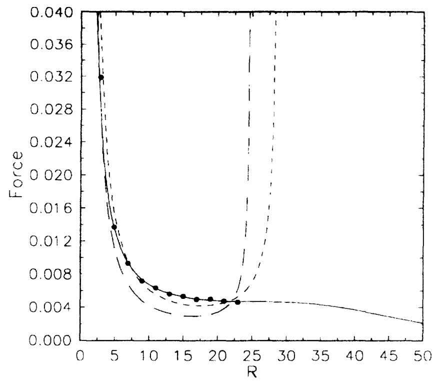

FIG. 1. SU(2) force data from Ref. [1] with fits to one-loop (short dash), two-loop (long dash), and modified Coulomb (solid line) force. The force and R are in lattice units. Error bars range from about 1/2 the size of plotted points up to the size of the points.

$$\alpha(R) = \frac{\sum_{j=0}^{n} a_j[\ln(R_0/R)]^j}{\sum_{k=0}^{m} c_k[\ln(R_0/R)]^k}.$$

Excellent agreement with the data is found with a one-term extension to the one-loop form, namely, $n = 0$, $m = 2$. The solid line in Fig. 1 is a fit to the force associated with the phenomenological running coupling:

$$\alpha(R) = \{1 + 8\pi b_0 c \ln(R_0/R) + d[\ln(R_0/R)]^2\}^{-1},$$

where the three-parameter fit gives $c = 1.1163$, $d = 0.4856$, and $R_0 = 10.953$. The fit is extrapolated beyond the data to show a large region of approximately constant force extending at least to $R/a \simeq 35$. This is, for all practical purposes, indistinguishable from the force due to a string tension. The force does eventually fall off around $R/a = 50$, but this region is well beyond the reach of today's simulations. It is remarkable that a two-term logarithmic form could so effectively destroy the $1/R^2$ Coulomb dependence over such a large distance range. The fit has a $\chi^2/N_{\rm DF} = 0.7$, compared with 0.9 reported for the Coulomb + linear fit in Ref. [1]. Note that the point at $R/a = 2.83$ is not included in the fits. Vertically it is quite far off the fit, but a small horizontal correction would easily place it on. Note also the close agreement with the two-loop result at small R. This was not in any way built in, but came out as a result of the fit. The agreement with the two-loop result is quite close in the range $2.5 \leq R/a \leq 4$. The phenomenological force then begins to disagree with the two-loop force again for smaller R. Since there are no data in this region, it is not an issue for the present fit; but for work which includes shorter distances, it would probably be best to match the new $\alpha(R)$ onto the two-loop result in this region of approximate agreement and to use the two-loop $\alpha(R)$ for smaller distances than this, because it is almost certainly valid here. This would also cure the new $\alpha(R)$ of its major flaw, namely, that it does not obey the two-loop perturbative renormalization-group equation. Of course, this is what was given up to get rid of the Landau pole. However, the above noted agreement with the two-loop result for $2.5 \leq R/a \leq 4$ means that numerically it does agree with perturbative scaling here.

To compare with the the linear+Coulomb fit as well as to check the form of the potential resulting from the new phenomenological running coupling, a numerical integration of the new force was performed. A one-parameter fit was then made to the potential data to determine the constant of integration. This is shown in Fig. 2 along with the potential data and the Coulomb+linear fit of Ref. [1]. Neither fit used the point at $R/a = 2$. The fits are seen to be nearly identical from $R/a = 4$ to $R/a \simeq 40$. Thus the modified running coupling is capable of producing a phenomenologically confining potential which is only distinguishable from a linear string tension potential at very large distances. In this way the potential being discussed here also differs from the Richardson potential [7], since that potential is also ultimately linear at long distances.

The two interpretations also give a suprisingly close physical scale for the lattice spacing. Interpreting the linear term as a string tension and using a physical value of $\sqrt{\sigma} = 0.44$ GeV gives $a^{-1} = 6.56$ GeV [1]. This can be used to determine $\Lambda_L = 9.80$ MeV and $\Lambda_R = 20.78\Lambda_L = 204$ MeV. Using again the two-loop form from which Λ_R is defined ($\Lambda_R = 1/R_0$ for the two-loop force) [6], and fitting this form to the new force (with parameters fixed to values previously given) in the short distance region, $2.5 \leq R/a \leq 4.0$, gives $R_0/a = 30.57$ in the two-loop force. Thus $a^{-1} = 30.57\Lambda_R = 6.22$ GeV, using the same physical value of Λ_R as above. Thus the two interpretations nearly agree on the physical scale of the lattice.

The same approach was also successfully tried for the SU(3) potential at $\beta = 6/g^2 = 6.2$ and 6.4 using the data of Ref. [8]. The SU(3) data are not as good statistically as the SU(2) data, however, nor do they go to as far a distance, so it is not as stringent a test as the fits given above.

What is one to make of the fact that the interquark potential can be fitted solely to a Coulomb force with a logarithmically running coupling? First, a true string tension term has not been ruled out by any means. One can have a string tension term in addition to the modified Coulomb potential, with the Coulomb potential running more slowly. Hovever, a fit cannot easily tell the difference between these. At the very least what is being shown here is that, unless it can be proven that the coupling stops running at some accessible distance, then the current quoted values for the string tension should be taken as upper limits, with one-sided error bars of the order of 100%. It is therefore very important to determine

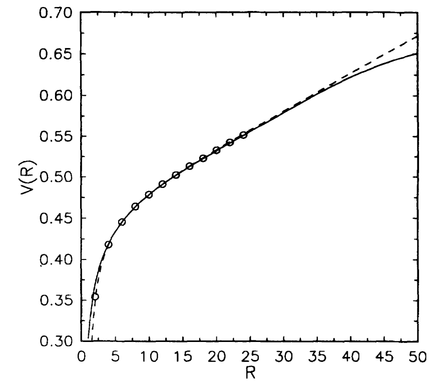

FIG. 2. SU(2) potential data from Ref. [1]. Dashed line is Coulomb+linear fit; solid line is fit to modified Coulomb potential. Error bars range from 1/10 to 1/3 the size of plotted points.

the true behavior of the running coupling in the intermediate and long distance region. It should also be pointed out that the modification introduced by the log-squared term *reduces* the force from the one-loop perturbative value. The problem with the perturbative force is that it is too strong, not too weak, at long distances.

Going beyond this, it is interesting to entertain the possibility of doing away with the string tension and absolute confinement altogether for the continuum Yang-Mills theory and possibly even for QCD. As mentioned before, it is not necessary for the linear potential to extend beyond a few fm to successfully model meson spectroscopy. The modified Coulomb potential could presumably come from summed higher-order perturbation theory without the need to invoke additional nonperturbative physics. Because of the probable importance of multiple gluon exchange at long distances, due to the high effective coupling, this potential could easily have a substantial Lorentz-scalar piece, which may be necessary to obtain the correct heavy-quark spin-orbit splittings [9]. Of course *lattice* Yang-Mills theories using the Wilson action necessarily confine at strong coupling, and thus must have a real string tension within the region of validity of the strong-coupling expansion. However, it is possible that this region is separated from the weak-coupling region which includes the continuum limit by a phase transition, as occurs in the U(1) lattice gauge theory. The scaling of string tension and pseudo-specific heat are consistent with the existence of a higher-order phase transition around $\beta = 2.5$ for SU(2) and $\beta = 6.7$ for SU(3) [10]. The infinite lattice critical points may be around $\beta = 3.0$ and 7.0, respectively. This transition would correspond to the remnant of the finite-temperature transition which exists on the symmetric lattice [11]. In other words, it is possible that the conventionally interpreted finite-temperature transition is in fact a true four-dimensional bulk deconfining transition which remains at finite β_c as the lattice size becomes infinite [12]. Recently it has been shown that, when an adjoint term is added to the action, the deconfinement transition joins the previously known bulk transition in the fundamental-adjoint plane [13]. If this connection persists as the lattice size becomes infinite, then the transition must be everywhere a bulk transition. It should also be mentioned in this vein that noncompact simulations of SU(2) lattice gauge theory have failed to see definite signs of confinement [14]. Absolute confinement is also questionable [15] for actions which prohibit negative plaquettes [16].

Finally, the possible effects of dynamical quarks should be considered. Even without a fundamental string tension in the pure glue theory, it is very likely that the color force is strong enough to cause chiral symmetry breaking. If the chiral condensate is then polarized by the strong color fields surrounding a quark-antiquark pair, a region of higher than normal vacuum energy surrounding the pair could be formed [10]. This region can form a kind of bag around the meson which contributes a linear term to the interquark force, and may also contribute to the dynamical mass [17]. A diminishing of the vacuum condensate $\langle\bar{\psi}\psi\rangle$ in the neighborhood of a quark source has been observed in a lattice simulation [18], lending support to this hypothesis. Gribov has also presented a scenario somewhat different from this in which light quarks are responsible for confinement [19].

[1] S. P. Booth *et al.*, Phys. Lett. B **275**, 424 (1992).
[2] C. Michael, Phys. Lett. B **283**, 103 (1992); G. S. Bali and K. Schilling, Phys. Rev. D **47**, 661 (1993); S. P. Booth *et al.*, Phys. Lett. B **294**, 385 (1992); M. Lüscher *et al.*, Nucl. Phys. **B389**, 247 (1993).
[3] M. Lüscher, Nucl. Phys. **B180**, 317 (1981).
[4] F. Gutbrod, Z. Phys. C **30**, 585 (1986).
[5] M. Creutz, Phys. Rev. D **23**, 1815 (1981).
[6] A. Billiore, Phys. Lett. **104B**, 472 (1981).
[7] J. L. Richardson, Phys. Lett. **82B**, 272 (1979).
[8] G. S. Bali and K. Schilling, Phys. Rev. D **46**, 2636 (1992).
[9] A. B. Henriques, B. H. Kellet, and R. G. Moorhouse, Phys. Lett. **64B**, 85 (1976).
[10] M. Grady, Z. Phys. C **39**, 125 (1988).
[11] E. Kovacs, Phys. Lett. **118B**, 125 (1982).
[12] A. Patrascioiu, E. Seiler, and I. O. Stamatescu, Nuovo Cimento D **11**, 1165 (1989).
[13] R. V. Gavai, M. Grady, and M. Mathur, "Finite Temperature Phase Transition in SU(2) Lattice Gauge Theory with Extended Action," TIFR Report No. TIFR/TH/94-06 [Nucl. Phys. B (to be published)].
[14] A. Patrascioiu, E. Seiler, and I. O. Stamatescu, Phys. Lett. **107B**, 364 (1981); E. Seiler, I. O. Stamatescu, and D. Zwanziger, Nucl. Phys. **B239**, 177 (1984); K. Cahill, M. Herbert, and S. Prasad, Phys. Lett. B **210**, 198 (1988); K. Cahill, *ibid.* **231**, 294 (1989); **304**, 307 (1993).
[15] M. Grady, Phys. Rev. D **42**, 1223 (1990).
[16] G. Mack and E. Pietarinen, Nucl. Phys. **B205** [FS5], 141 (1982).
[17] M. Grady, Nuovo Cimento A **105**, 1065 (1992).
[18] W. Feilmair, M. Faber, and H. Markum, Phys. Rev. D **39**, 1409 (1989).
[19] V. N. Gribov, Phys. Scr. **T15**, 164 (1987); Phys. Lett. B **194**, 119 (1987).