Remarks on $I=0$ $J^{PC}=0^{++}$ states: σ/ϵ and $f_0(975)$

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With t-channel ρ and $f_2(1275)$ exchange and the s-channel resonance $f_0(975)$, we reproduce the $\pi\pi$ S-wave scattering phase shift up to 1.1 GeV quite well. A broad pole $\sigma(400)$ is produced by the t-channel ρ exchange. The $f_0(975)$ has a large decay width due to its broad third-sheet pole and a narrow peak width due to its narrow second-sheet pole.

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An understanding of low-energy S-wave $\pi\pi$ scattering is important in many contexts. Pion exchange gives the largest range force in strong interaction physics, and 2π exchange the next largest. It is relevant, for example, to the old σ model [1,2], and secondly, in the recently extended Nambu–Jona-Lasinio model [3,4] and in nucleon-nucleon physics [5,6]. It is also important to chiral perturbation theory and the σ commutator [7]. In another context, it is relevant to quark model calculations for hadron spectra [8,9]. The lightest glueball is expected to have quantum numbers $J^{PC}=0^{++}$. On the experimental side, a large phase shift for $\pi\pi$ S-wave scattering below 900 MeV [10] has been attributed to the σ/ϵ resonance. New data on $p\bar{p}\to 3\pi^0$ [11] reveal two resonances in the 1300–1500 MeV range together with a large enhancement around 700 MeV for the $\pi\pi$ S wave.

As to the $f_0(975)$, it used to be considered as a well-defined narrow resonance [12]. But in a recent paper reanalyzing the S-wave $\pi\pi$ scattering phase shifts and some $\pi\pi$ production processes, we concluded that the $f_0(975)$ is not a simple narrow resonance; instead it has a large decay width corresponding to its third-sheet pole ($797-185i$ MeV), but it shows up as a narrow structure in $\pi\pi$ and $K\bar{K}$ invariant-mass spectra due to its second-sheet pole ($988-23i$ MeV) [13].

In that work, we fitted $\pi\pi$ elastic scattering data empirically. With the coupled-channel resonance $f_0(975)$ alone, we could not reproduce the large $\pi\pi$ S-wave phase shift at low energies. A slowly increasing background contribution was needed. In our parametrization using the Dalitz-Tuan form [13], this background contribution was provided by a term with a broad pole at $408-342i$ MeV. It has been pointed out to us by Speth that the physical origin of this background is dominantly the t-channel ρ exchange [14,15]. We concur with this. In Ref. [14], a Lagrangian field theory model was used with the [1,1] Padé approximant; the $\pi\pi$ phase-shift data were successfully reproduced with a broad pole ($460-338i$) in the S-wave amplitude. In Ref. [15], the contribution of the t-channel ρ exchange was calculated by solving an integral Lippmann-Schwinger equation, but without investigating the s-channel pole position. Neither calculation gave a simple formula for the $\pi\pi$ S-wave amplitude and both have some free parameters. An essential point of the present paper is to give simple K-matrix formulas without free parameters and incorporating data from the left-hand cut. The latter constraint eliminates ambiguities in the choice of parametrization for the background term and significantly stabilizes the determination of pole parameters.

The starting point is the Born term of the $\pi\pi$ scattering amplitude by ρ exchange:

$$T^{\text{Born}}(I=0)=2G\left[\frac{s-u}{m_\rho^2-t}+\frac{s-t}{m_\rho^2-u}\right],$$

$$T^{\text{Born}}(I=1)=G\left[\frac{2(t-u)}{m_\rho^2-s}+\frac{s-u}{m_\rho^2-t}-\frac{s-t}{m_\rho^2-u}\right], \quad (1)$$

$$T^{\text{Born}}(I=2)=-G\left[\frac{s-u}{m_\rho^2-t}+\frac{s-t}{m_\rho^2-u}\right],$$

where s,t,u are the usual Mandelstam variables and $G=g^2_{\rho\pi\pi}/32\pi$. The forces are attractive in isospin $I=0,1$ and repulsive in isospin $I=2$. G and m_ρ^2 are fixed by the width and the mass of the ρ resonance in isospin 1. Our normalization is such that the unitarity relation for partial-wave amplitude reads

$$\text{Im}T_l(s)=\rho_1(s)|T_l(s)|^2 \quad (2)$$

with $\rho_1(s)=(1-4m_\pi^2/s)^{1/2}$. The partial-wave amplitudes are obtained from the full amplitude by the standard projection formula [16]:

$$T_l(s)=\frac{1}{s-4m_\pi^2}\int_{4m_\pi^2-s}^{0}dt P_l\left[1+\frac{2t}{s-4m_\pi^2}\right]T(s,t,u), \quad (3)$$

where $P_l(x)$ is the Legendre function.

A simple K-matrix unitarization of Eqs. (1)–(3) gives

$$T_s^{I=0}(s)=\frac{K_S(s)}{1-i\rho_1(s)K_S(s)}, \quad (4)$$

$$T_P^{I=1}(s)=\frac{K_P(s)}{1-i\rho_1(s)K_P(s)}, \quad (5)$$

where $K_S(s)$ and $K_P(s)$ are partial-wave Born amplitudes for $I=0$ S wave and $I=1$ P wave, respectively:

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$$K_P(s)=\frac{2G(s-4m_\pi^2)}{3(m_\rho^2-s)}+\frac{2G(2s+m_\rho^2-4m_\pi^2)}{s-4m_\pi^2}\times\left[\left[1+\frac{2m_\rho^2}{s-4m_\pi^2}\right]\ln\left[1+\frac{s-4m_\pi^2}{m_\rho^2}\right]-2\right],$$

$$K_S(s)=4G\left[\frac{2s+m_\rho^2-4m_\pi^2}{s-4m_\pi^2}\ln\left[1+\frac{s-4m_\pi^2}{m_\rho^2}\right]-1\right]. \qquad (6)$$

The relation between the amplitude $T_l(s)$ and the phase-shift parameters δ_l and η_l is

$$T_l(s)=\frac{\eta_l(s)e^{2i\delta_l(s)}-1}{2i\rho_1(s)}. \qquad (7)$$

In the $I=1$ P-wave channel, near the ρ pole at $s\approx m_\rho^2$, we have

$$T_P^{I=1}(s)\approx\frac{2G(s-4m_\pi^2)/3}{m_\rho^2-s-i\rho_1(s)2G(s-4m_\pi^2)/3}. \qquad (8)$$

By identification with a Breit-Wigner formula, we obtain

$$G\approx\frac{3m_\rho^2\Gamma_\rho}{2(m_\rho^2-4m_\pi^2)^{3/2}}\approx 0.364, \qquad (9)$$

where the ρ mass $m_\rho=768.1$ MeV and width $\Gamma_\rho=151.5$ MeV from [12] are used.

From Eqs. (4), (6), (7), and (9), we get the $I=0$ $\pi\pi$ S-wave phase shift $\delta_0(s)$ shown by the dashed line in Fig. 1. The result sits between the previous two calculations [14,15]. A broad pole ($370-356i$ MeV) is found in the amplitude. The remarkably similar results from these three different methods show that the t-channel ρ exchange is responsible for the large background term in the $\pi\pi$ S-wave phase shift and produces a broad pole in the amplitude at a mass of about 400 MeV with a width about 700 MeV; we denote it as $\sigma(400)$.

We combine the t-channel ρ exchange background term with the s-channel $f_0(975)$ resonance term using the Dalitz-Tuan representation [13,17]. The basic ansatz of the Dalitz-Tuan representation is that the phases caused by different sources add. In this representation the position of a pole produced by one source does not depend on whether or not other sources exist, so the location of the $\sigma(400)$ pole remains unchanged. The result is

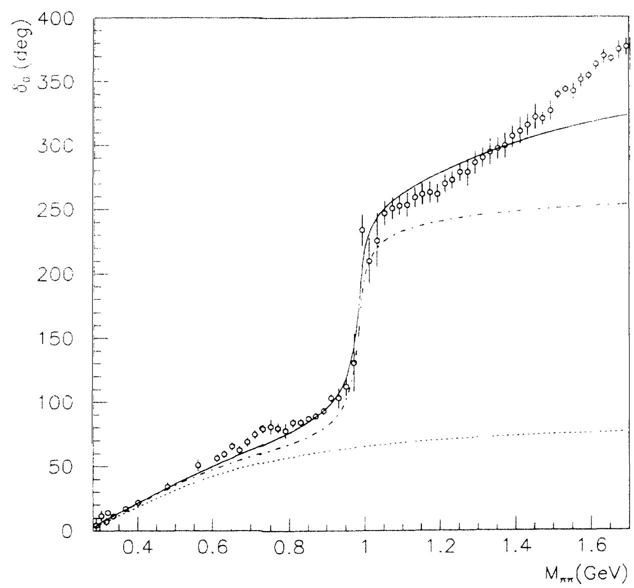

FIG. 1. The isoscalar S-wave phase shift δ_0 for $\pi\pi$ scattering [18–20]. The dashed line includes only the contribution from the t-channel ρ exchange. The dot-dashed line includes the contribution of the s-channel resonance $f_0(975)$ in addition. The solid line further includes the contribution from the t-channel $f_2(1275)$ exchange.

$$T_S^{I=0}(s)=\frac{K_S(s)}{1-i\rho_1(s)K_S(s)}+\frac{g_1}{M_R^2-s-i[\rho_1(s)g_1+\rho_2(s)g_2]}\times\frac{1+i\rho_1(s)K_S(s)}{1-i\rho_1(s)K_S(s)}, \qquad (10)$$

where $\rho_2(s)=(1-4m_K^2/s)^{1/2}$ is the phase-space factor for the $K\bar{K}$ channel, $M_R=0.9535$ GeV, $g_1=0.1108$ GeV^2, and $g_2=0.4229$ GeV^2 [13]. The resulting phase shift is shown by the dot-dashed line in Fig. 1. It already fits the data quite closely.

We also investigated the contribution from t-channel $f_0(975)$ and $f_2(1275)$ exchange. We have found that the t-channel f_0 contribution is negligible, consistent with the result of Ref. [15], but the t-channel f_2 contribution is not negligible.

The Born amplitude for the $I=0$ S-wave from the t-channel f_2 exchange is

$$K_{f2}(s)=2G_{f2}\left\{-\frac{11}{3}s-\frac{2}{3}m_{f2}^2+4m_\pi^2+\frac{(2s+m_{f2}^2-4m_\pi^2)^2-(m_{f2}^2-4m_\pi^2)^2/3}{s-4m_\pi^2}\ln\left[1+\frac{s-4m_\pi^2}{m_{f2}^2}\right]\right\}, \qquad (11)$$

with $G_{f2}\approx 1.9$ GeV^{-2} determined from the width of $f_2(1275)$.

Incorporating the t-channel f_2 contribution into the background term by the Dalitz-Tuan method, i.e., replacing $K_S(s)$ in Eq. (10) by $K_S'(s)$,

$$K_S'(s)=\frac{K_S(s)+K_{f2}(s)}{1-\rho_1^2(s)K_S(s)K_{f_2}(s)}, \qquad (12)$$

we obtain the result shown in Fig. 1 by the solid line. In this fit, the scattering length is $a_0 = 0.24 m_\pi^{-1}$ where the t-channel ρ exchange contributes $0.19 m_\pi^{-1}$, the s-channel $f_0(975)$ contributes $0.05 m_\pi^{-1}$, and the t-channel f_2 contribution is negligible. Though the t-channel $f_0(975)$ has a negligible contribution for the overall fit, it is significant near threshold and contributes an additional amount $(+0.08 m_\pi^{-1})$ to the scattering length. Our simple parameter-free calculation reproduces the experimental phase-shift data remarkably well up to the energy of 1.1 GeV. For higher energies, the situation is complicated due to the opening of the four-pion channel and the emergence of s-channel resonances such as $f_0(1365)$ and $f_0(1520)$ [11]. We have shown in [13] that the $\pi\pi$ phase shifts can be fitted accurately up to 1700 MeV by including a further pole in this mass range, and the poles below 1 GeV change very little.

We conclude that the isoscalar $\pi\pi$ S-wave amplitude has three poles below 1.0 GeV: i.e., a broad pole of width about 700 MeV for $\sigma(400)$, a third-sheet pole of width about 400 MeV, and a second-sheet pole of width about 46 MeV for $f_0(975)$. The $\sigma(400)$ pole is produced by the t-channel ρ exchange and gives the dominant contribution for the large $\pi\pi$ S-wave phase shifts below 900 MeV. The narrow second-sheet pole of $f_0(975)$ is responsible for the sharp increase of the phase shift around 980 MeV. The broad third-sheet pole of $f_0(975)$ gives a modest additional contribution which indicates a large decay width for $f_0(975)$. A delicate interplay of these three poles causes the broad peak around 800 MeV and the narrow dip structure around 980 MeV in the $\pi\pi$ S-wave elastic-scattering cross section. The coupled-channel resonance $f_0(975)$ appears here as an s-channel resonance. Dynamically it requires a large $K\bar{K}$ component, but may have an additional $q\bar{q}$ contribution [3,4,14,15,21].

Finally we give some comments about a recent analysis by Morgan and Pennington [22,23]. They got almost the same narrow second-sheet pole ($988-24i$ MeV) as ours ($988-23i$ MeV) for $f_0(975)$, and a very different third-sheet pole ($978-28i$ MeV) compared with our ($797-185i$ MeV). Their data sets are limited to the energy range 0.87–1.1 GeV. We got our broad third-sheet pole by fitting the data sets for the energy range 0.3–1.7 GeV. They claim [23] that the decay $J/\psi \to \phi(MM)$ demands the $f_0(975)$ to be narrow and that our $f_0(975)$ solution cannot fit the J/ψ decay data. Figure 2 shows that our solution, in fact, gives a fit strictly comparable in quality with theirs. The details for the fit are as follows. The amplitudes for J/ψ decay are given by [23]

$$\begin{aligned} F_1 &\equiv F(\psi \to \phi\pi^+\pi^-) \\ &= \left[\tfrac{2}{3}\right]^{1/2} [\alpha_1(s)T_{11} + \alpha_2(s)T_{21}] , \\ F_2 &\equiv F(\psi \to \phi K^+K^-) \\ &= \left[\tfrac{1}{2}\right]^{1/2} [\alpha_1(s)T_{12} + \alpha_2(s)T_{22}] , \end{aligned} \tag{13}$$

where T_{ij} are $\pi\pi$-$K\bar{K}$ coupled-channel amplitudes which we take from our Dalitz-Tuan form of [13]; the real coupling functions $\alpha_1(s)$, $\alpha_2(s)$ are parametrized as

$$\alpha_i(s) = \gamma_{i0} + \gamma_{i1} s \; . \tag{14}$$

The relation between the $\pi\pi$ or $K\bar{K}$ invariant mass spectra and F_i is

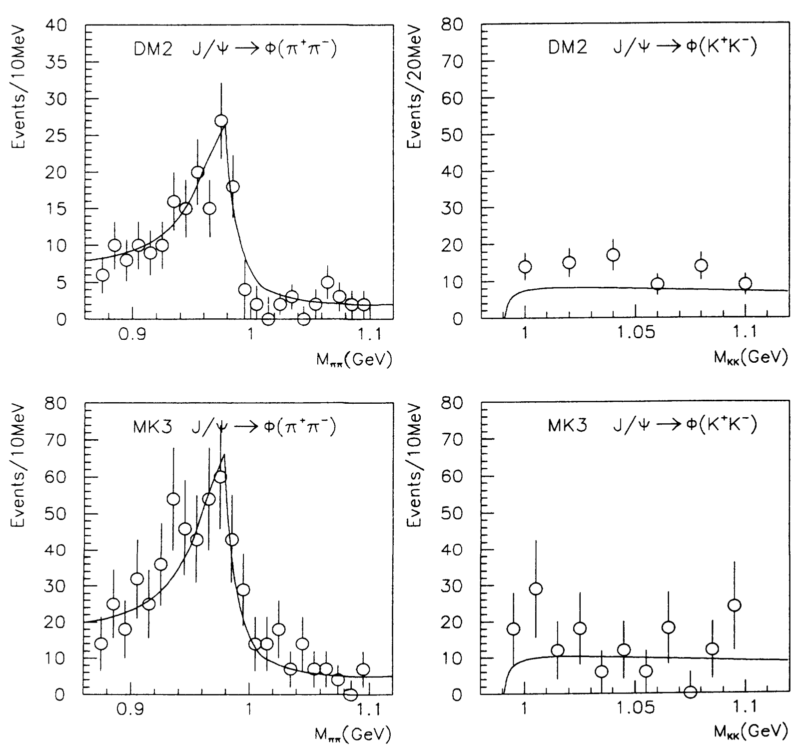

FIG. 2. Fit to DM2 data [24] and Mark III data [25] on $J/\psi \to \phi\pi\pi(K\bar{K})$ production [assuming the final $\pi\pi(K\bar{K})$ system is all $I=0$ S wave].

$$\frac{dN_{J/\psi\to\phi\pi^+\pi^-}}{dM_{\pi\pi}}=N_0|F_1|^2\sqrt{M_{\pi\pi}^2-4m_\pi^2}\sqrt{[m_\psi^2-(M_{\pi\pi}+m_\phi)^2][m_\psi^2-(M_{\pi\pi}-m_\phi)^2]}\ ,$$
$$\frac{dN_{J/\psi\to\phi K^+K^-}}{dM_{KK}}=N_0|F_2|^2\sqrt{M_{KK}^2-4m_K^2}\sqrt{[m_\psi^2-(M_{KK}+m_\phi)^2][m_\psi^2-(M_{KK}-m_\phi)^2]}\ , \quad (15)$$

where N_0 is a constant normalization factor. For our fit in Fig. 2, $\alpha_1(s)=2.7048-2.0508s$, $\alpha_2(s)=0.2601+1.7691s$ with s in GeV^2. $N_0=0.6$ for DM2 data [24] and $N_0=1.5$ for Mark III data [25].

So all available data sets are compatible with our $f_0(975)$ solution. A clear peak structure for the $f_0(975)$ resonance in the $\pi^+\pi^-$ invariant mass spectra of the $J/\psi\to\phi\pi^+\pi^-$ [24,25] as well as $D_s^+\to\pi^+\pi^+\pi^-$ [26] indicates that there is a large $s\bar{s}$ component in the resonance, in these two processes the f_0 is formed mainly through $s\bar{s}$ [24,27] and the background term due to ρ and f_2 exchange is suppressed.

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