Covariant and heavy quark symmetric quark models

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There exist relativistic quark models (potential or MIT bag) which satisfy the heavy quark symmetry (HQS) relations among meson decay constants and form factors. A covariant construction of the momentum eigenstates, developed here, can correct for spurious center-of-mass motion contributions. The proton form factor and $M1$ transitions in quarkonia are calculated. An explicit expression for the Isgur-Wise function is found and model-determined deviations from HQS are studied. All results depend on the model parameters only. No additional *ad hoc* assumptions are needed.

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I. INTRODUCTION

A simple, but covariant quark model [1–4], used previously to calculate meson form factors [5], possess also the heavy quark symmetry (HQS) [6–14]. Actually this might be true for a whole class of quark models. This class contains models in which quarks are confined by a central potential. Their wave functions must be Lorentz boosted [2–5]. It is hoped that such models might serve as a useful semiempirical tool. They can be used to roughly estimate physical quantities and effects and to illustrate HQS relations.

Once the model confinement parameters [15–17], the quark masses, and the interaction hypersurface [3,5] are selected, everything else follows from our formalism. No additional assumptions, such as, for example, about the Q^2 dependence of form factors [18], are needed. HQS is intimately connected with the Lorentz-covariant character of the model.

Model hadron states, used previously [1–3], were not momentum eigenstates [19–24]. This can be remedied by a projection [19–25] of model states into momentum eigenstates. A Lorentz-covariant projection [25] is developed here. It is shown that this removal of the spurious center-of-mass motion improves the model description of proton electromagnetic form factors. Such corrections are not important if the hadron contains heavy quarks c or b. In that case they are smaller than 5%.

The model calculations give some corrections to the extreme HQS. Some of those, for example, concerning meson decay constants f_D and f_{D_*} agree with QCD sum rule results [26]. Model predictions for meson form factors in the heavy quark limit (HQL) follow exactly the HQS requirements. One can extract a model prediction for the Isgur-Wise function ξ [7].

II. RELATIVISTIC MODEL

Any static model in which quarks are confined by a central force can be relativized [1–5]. Earlier the MIT bag model was employed [3–5]. Here a harmonic oscillator confining potential [16,17] will be used.

In any of them one can envisage a hadron as located around y. The quark q_i coordinate is

$$x_i = y + z_i. \tag{2.1}$$

The confining "ball" of mass M can be boosted, acquiring the four-momentum P. Individual quark wave functions ψ_n depend on z and P:

$$\psi_n(z^P) = S(P)\eta_n(z_\perp^P)\exp(-iz_\parallel^P\epsilon_n). \tag{2.2}$$

Here

$$\begin{aligned} S(P) &= (\not P\gamma_0 + M)/[2M(E+M)]^{1/2}, \\ E &= (\mathbf{P}^2 + M^2)^{1/2}, \\ z_\perp(P)_\mu &= z_\mu - \beta^\mu(\beta\cdot z), \\ z_\parallel(P) &= \beta_\mu z^\mu, \\ \beta_\mu &= P_\mu/M, \end{aligned} \tag{2.3}$$

and ϵ_n is model energy. For $\beta_\mu = 0$ the Dirac spinor η has a generic form

$$\eta_n(\mathbf{r}) = \begin{pmatrix} U_n(|\mathbf{r}|)\chi \\ i\boldsymbol{\sigma}\cdot\frac{\mathbf{r}}{|\mathbf{r}|}V_n(|\mathbf{r}|)\chi \end{pmatrix}. \tag{2.4}$$

Here χ is the Pauli spinor.

One can introduce the quark field operator

$$\Psi(z^P) = \sum_n [a_n\psi_n(z_P) + b_n^*\overline{\psi_n}(z_P)] \tag{2.5}$$

and define model states for meson "m": for example,

$$|m, M, P, s, y\rangle = \sum_{r,r',f,f'} C^s_{rr'ff'} a^{*rf}_{nP} b^{*r'f'}_{nP}|0\rangle e^{-iPy}. \tag{2.6}$$

Here m is the flavor (B, D, etc.), M is the meson mass, and s is the spin.

Using the configuration space operators [27] (2.5), one can obtain a model wave function whose generic form is

$$\langle 0|\Psi(z_1^P)\Psi(z_2^P)\Psi(z_3^P)|b,M,P,s,y\rangle$$
$$= N_P e^{-iPy} F_b(\psi_{f_1}(z_1^P)\psi_{f_2}(z_2^P)\psi_{f_3}(z_3^P))$$
$$= h_b^s(P,z_1,z_2,z_3)e^{-iPy}. \quad (2.7)$$

Here N_P is the norm and F_b symbolizes the symmetrized combination of quark flavors.

A quark line in the configuration space, in the nonrelativistic limit, corresponds to the normalization integral

$$\int \psi^* \psi \, d^3z = 1. \quad (2.8)$$

This can be generalized as

$$Z = J(z)\bar{\psi}(z^{P_f}) \not{L} \psi(z^{P_i}). \quad (2.9)$$

Here

$$J(z) = \int d^4z \, \delta(Lz). \quad (2.10)$$

Among all possible hypersurfaces

$$L \cdot z = 0 \quad (2.11)$$

only the one defined by

$$L_\mu = (\beta_\mu^i + \beta_\mu^f)/[(\beta_i + \beta_f)^2]^{1/2} \quad (2.12)$$

leads to the proton electromagnetic form factors f_i which satisfy the conserved current constraint $f_3(Q^2) \equiv 0$ (5.2). A model defined on a hyperplane is connected [2,3] with the quasipotential approximation [28] of the Bethe-Salpeter equation.

The vertex spatial dependence follows from (2.9) by replacing

$$\not{L} \to \Gamma_\mu,$$
$$\Gamma_\mu = \gamma_\mu, \gamma_\mu\gamma_5, \text{etc.} \quad (2.13)$$

Figure 1 shows the vertex for the semileptonic $B \to D$ transitions. For mesons m_f, m_i (2.6) a current matrix element is

$$V(\Gamma^\mu) = \int d^4y \langle y, s_f, P_f, M_f, m_f | J(z_1)\bar{\Psi}(z_1^{P_2})\Gamma^\mu \Psi(z_1^{P_1})$$
$$\times J(z_2)\bar{\Psi}(z_2^{P_4}) \not{L} \Psi(z_2^{P_3}) | m_i, M_i, P_i, s_i, y\rangle. \quad (2.14)$$

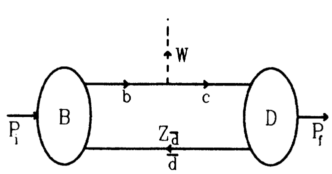

FIG. 1. Vertex for the semileptonic $B \to D$ transitions. Quark lines $(b, c, \bar{d})$, momenta $P_{i,f}$, and the overlap integral Z are indicated.

III. MOMENTUM EIGENSTATES

The factor $\exp(-iPy)$ (2.6) describes the motion of the center of force (c.f.). The center of mass (c.m.) of centrally confined quarks oscillates about the c.f. As is well known [1–6,25] the spurious center-of-mass-motion (c.m.m.) persist even in the static ($\mathbf{P} = 0$) case. Thus the boosted centrally confined model (BCCM) states (2.6) are not the momentum eigenstates. This can be remedied by decomposing a BCCM state into momentum eigenstates $|l, s\rangle$ as follows [19–24]:

$$|h,M,P,s,y\rangle = 2M \int d^4l \, \delta(l^2 - M^2)\theta(\omega)e^{-ily}\phi_P(l)|l,s\rangle$$
$$= 2M \int \frac{d^3l}{2\omega} e^{-ily}\phi_P(l)|l,s\rangle. \quad (3.1)$$

Here h denotes a hadron. The momentum eigenstates normalization is

$$\langle l's'|ls\rangle = \delta_{ss'}\delta(\mathbf{l}' - \mathbf{l}),$$
$$l = (\omega, \mathbf{l}). \quad (3.2)$$

For a BCCM state one has

$$\langle h,M,P,s,0|0,s,P,M,h\rangle = 1$$
$$= \int d^3l \frac{M^2}{\omega^2}|\phi_P(l)|^2. \quad (3.3)$$

This provides a normalization of the components ϕ of the momentum eigenstates.

The momentum eigenstates in (3.1) are not the exact physical hadron states but the model hadron states, i.e., some kind of "mock" hadron states [29].

In the occupation number space one finds, for a baryon b, for example,

$$\langle y=0,s,P,M,b|b,MP,s,y=\zeta_\perp(P)\rangle$$
$$= M^2 \int \frac{d^3l}{\omega^2}|\phi_P(\mathbf{l},\omega)|^2 e^{-il\zeta_\perp(P)}. \quad (3.4)$$

In coordinate space this becomes

$$\langle y=0,s,P,M,b|b,M,P,s,y=\zeta_\perp(P)\rangle = \mathcal{M}(P,\zeta_\perp(P))$$
$$= J(z_1)J(z_2)J(z_3)h_b^{*s}(P,z_1,z_2,z_3)\not{L}_1\not{L}_2\not{L}_3 \quad (3.5)$$
$$\times h_b^s(P,z_1-\zeta_\perp(P),z_2-\zeta_\perp(P),z_3-\zeta_\perp(P))$$

For the proton, with all light quark masses equal ($m_u = m_d$), one finds

$$\mathcal{M}(P,\zeta_\perp(P)) = \left(J(z)\bar{\psi}(z_\perp(P))S^{-1}\left(-\frac{\mathbf{P}}{E}\right)\right.$$
$$\left.\times \not{L} S\left(-\frac{\mathbf{P}}{E}\right)\psi([z(P)-\zeta(P)]_\perp)\right)^3. \quad (3.6)$$

Integrating (3.4) and (3.6) over ζ, one finds

$$J(\zeta)\mathcal{M}(P,\zeta_\perp(P))e^{il\zeta_\perp(P)} = M^2 J(\zeta)\int \frac{d^3k}{\omega_k^2}|\phi_P(k,\omega_k)|^2 \times e^{i(l-k)\zeta_\perp(P)}. \tag{3.7}$$

The end result is the Lorentz-covariant expression for the components of the momentum eigenstates:

$$\frac{M}{\omega_l}|\phi_P(l,\omega_l)|^2 = \frac{l\cdot P}{(2\pi)^3M^2}\int d^4\zeta\,\delta(L\cdot\zeta) \times\mathcal{M}\left(P,\frac{\mathbf{P}\cdot\zeta}{E},\zeta\right)e^{il\zeta_\perp(P)}. \tag{3.8}$$

Some explicit expressions for ϕ's are listed in the Appendix.

IV. CONFINEMENT

The Dirac equation for quarks can be solved for the potential

$$V(r,P) = \frac{1}{2}\left(1+\frac{P}{M}\right)[V_0 - \tfrac{1}{2}Kz_\perp(P)^2], \tag{4.1}$$

which in the hadron rest frame has the harmonic oscillator (HO) shape [16]

$$V(r) = \tfrac{1}{2}(1+\gamma^0)(V_0+\tfrac{1}{2}Kr^2). \tag{4.2}$$

Here V_0 and K are model parameters. The rest frame solution has a general form (2.4), with

$$U_a = \exp(-r^2/2R_{0a}^2),$$
$$V_a = r\beta_a U_a/R_{0a}, \tag{4.3}$$
$$N_a = [R_{0a}^3\pi^{3/2}(1+\tfrac{3}{2}\beta_a^2)]^{-1/2}.$$

The index a denotes the quark's flavor. The quantities R_{0a} and β_a depend on the constituent mass m_a and the energy E_a:

$$E_a = m_a + V_0 + 3[K/2(m_a+E_a)]^{1/2},$$
$$R_{0a}^4 = 2/K(m_a+E_a), \tag{4.4}$$
$$\beta_a = R_{0a}^{-1}(m_a+E_a)^{-1}.$$

An approximate solution [6] for the linear potential $V(r) = \frac{1}{2}(1+\gamma^0)(V_0+\lambda r)$ would also have the form (4.3), with an accuracy of $\sim 6\%$. All general HQS features, discussed below would thus apply for that potential also.

In the heavy quark limit (HQL), where $m_a\to\infty$ and $E_a\to m_a$, one has

$$\frac{\beta_a}{R_{0a}} \to \frac{1}{2}\sqrt{\frac{K}{m_a}} \to 0. \tag{4.5}$$

Thus only the "large" component U survives in (4.3).

One can also show that in the MIT bag model [15] a "small" component V vanishes in the HQL. In the numerical evaluation the MIT bag model parameters employed previously by Ref. [5] will be used.

The HO model parameters are

$$V_0 = -0.35 \text{ GeV},\qquad K = 0.035 \text{ GeV}^3. \tag{4.6}$$

The constituent quark masses and related quantities β, E, and R_0 are listed in Table I.

Table II shows model hadron masses calculated using either model states (2.6) or model-dependent momentum eigenstates (3.2). The relevant formula for the valence quark contribution to the hadron mass $\tilde{M}_Q$ is

$$\tilde{M}_Q^h = \langle h,M,0,s,0|\int T^{00}d^3x|h,M,0,s,0\rangle = \langle h,M,0,s,0|P^0|h,M,0,s,0\rangle. \tag{4.7}$$

Here T^{00} is the momentum-energy tensor. One must add magnetic $\Delta\tilde{M}_M$ and electric $\Delta\tilde{M}_E$ effective one gluon exchange contributions [15,16] which for the HO potential model can be calculated explicitly. Finally one has a BCCM-based hadron mass without c.m.m. corrections:

$$\tilde{M} = \tilde{M}_Q + \Delta\tilde{M}_M + \Delta\tilde{M}_E. \tag{4.8}$$

Using momentum eigenstates, one obtains the following identities for a meson m or a baryon b:

$$\tilde{M}^m = \int\frac{d^3k}{4\omega^2}|\varphi^m(k)|^2\sqrt{M^{m2}+\mathbf{k}^2}, \tag{4.9}$$

$$\tilde{M}^b = \int d^3k\frac{M^{b^2}}{\omega^2}|\phi^b(k)|^2\sqrt{M^{b^2}+\mathbf{k}^2}. \tag{4.10}$$

Here $\tilde{M}$'s and ϕ's are determined by parameters from Table I. The c.m.m. corrected masses $M^{m,b}$ can be found numerically. Inspection of Table II reveals that c.m.m. corrections improve the agreement with the experimental values [6]. The mass of the pion is quite wrong, as in all valence quark models which do not account for the Goldstone-boson nature of the pion. Other theoretical masses are correct within 10% or better. c.m.m. corrections increase the mass difference in a SU(6) multiplet, (p, Δ, etc.), bringing theory closer to experiment. Corrections decrease with the increase of the heavy quark mass. Thus, for example, $(\tilde{M}_B - M_B)/M_B \cong 1.6\%$.

TABLE I. HO model parameters.

Flavor	m (GeV)	E (GeV)	β	R_0 (GeV^{-1})
u,d	0.315	0.426	0.455	2.96
s	0.525	0.557	0.343	2.70
c	1.850	1.710	0.140	2.00
b	5.450	5.221	0.062	1.52

TABLE II. Hadron masses in HO model. All masses are in GeV.

Hadron	$\tilde{M}$	M	$M_{\rm expt}$	$M/\tilde{M}$	$\lvert \frac{M-M_{\rm expt}}{M_{\rm expt}} \rvert \%$
p	1.191	0.928	0.938	0.78	1.1
Δ	1.365	1.138	1.236	0.83	8.0
π	0.679	0.329	0.139	0.48	
ρ	0.910	0.677	0.770	0.74	12.1
K	0.817	0.528	0.498	0.65	6.0
K^*	1.019	0.798	0.892	0.78	10.5
η_c	3.381	3.267	2.979	0.97	9.7
Φ	3.433	3.322	3.097	0.97	7.3
D^+	1.906	1.752	1.869	0.92	6.3
D^{+*}	2.005	1.858	2.010	0.93	7.6
D_s	2.138	1.994	1.969	0.93	1.2
D_s^*	2.229	2.091	2.110	0.94	0.9
B^+	5.207	5.125	5.279	0.98	2.9
B^{+*}	5.249	5.168	5.325	0.98	2.9
B_s	5.580	5.501	5.384	0.99	2.1
B_s^*	5.620	5.541	5.431	0.99	2.0

V. PROTON FORM FACTORS

Calculation of the proton form factors is a useful test of any quark model. All calculational details have been discussed and described in Refs. [3] and [5]. It remains to be shown that the inclusion of c.m.m. corrections improves upon earlier results.

These corrections are included by the equality

$$\int d^4y \prod_{i=1}^{3} J(z_i)\langle M, P_f, y| \sum_{i,j,k,\mathrm{perm}} V^\mu(z_i)C(z_j)C(z_k)e^{-iQx_i}|M, P_i, y\rangle$$
$$= (2\pi)^4\delta(P_f - P_i - Q)J(z)\int \frac{d^3l\, d^3l'}{\omega\omega'} M^2 \phi^*_{P_f}(\boldsymbol{l}', \omega')\phi_{P_i}(\boldsymbol{l}, \omega)\langle \boldsymbol{l}'|V^\mu(z)e^{-iQz}|\boldsymbol{l}\rangle. \qquad (5.1)$$

Here,

$$V^\mu(z_i) = \bar{\Psi}(z_i)\gamma^\mu\Psi(z_i),$$
$$C(z_k) = \bar{\Psi}(z_k)\not{L}\Psi(z_k), \qquad (5.2)$$
$$\langle l'|V^\mu(0)|l\rangle = \bar{u}(l')[f_1(s^2)\gamma^\mu + f_2(s^2)i\sigma^{\mu\nu}s_\nu + f_3(s^2)s^\mu]u(l),$$
$$s = l' - l, \quad f_3(s^2) \equiv 0.$$

The left-hand side (LHS) of (5.1) is the expression used earlier [5] to calculate electromagnetic form factors. Here it is written in the occupation number space.

In general one cannot invert the expression (5.1). However, at the momentum transfer $Q^2 = O$ one can determine [23] the Sach's form factor $G_M(0)$.

The LHS of (5.1) can be written as

$$(5.1)(\mathrm{LHS}) = (2\pi)^4\delta(P_f - P_i - Q) \times\chi^\dagger\left(W^0 + \frac{i}{2M}\boldsymbol{\sigma}\times\mathbf{Q}W^2\right)\chi. \qquad (5.3)$$

Here

$$W^0 = I_0 Z^2,$$
$$W^2 = I_2 Z^2,$$
$$Z = \frac{M_f}{E_f}4\pi\int dr\, r^2 j_0(\rho)[U^2 + V^2],$$
$$I_0 = 4\pi\frac{M_f}{E_f}\int dr\, r^2 j_0(\tilde{\rho})[U^2 + V^2],$$
$$I_2 = 4\pi\frac{M_f}{E_f}\int dr\, r^2\left(j_0(\tilde{\rho})U^2 - [\tfrac{1}{3}j_0(\tilde{\rho}) - \tfrac{2}{3}j_2(\tilde{\rho})]V^2 + \frac{2E_f}{|\mathbf{P}_f|}j_1(\tilde{\rho})UV\right),$$
$$\rho = \frac{|\mathbf{P}_f|}{E_f}2\epsilon|\mathbf{r}|; \quad \tilde{\rho} = \frac{|\mathbf{P}_f|}{E_f}2(M - \epsilon)|\mathbf{r}|.$$

The quantities W^α were identified [3,5] as Sachs form factors:

$$W^0 \sim G_E, \qquad W^2 \sim G_M. \qquad (5.4)$$

However, (5.4) was obtained using BCCM states, which are not momentum eigenstates. More accurate approach is based on the equality

$$(5.1)(\mathrm{RHS}) = (2\pi)^4\delta(P_f - P_i - Q)D^\mu,$$

$$D^\mu = 2\pi \int l^2 dl \sin\theta\, d\theta \frac{M^3}{(\omega\omega')^{3/2}} \phi^*_{P_f}(\boldsymbol{l},\omega')\phi_{P_i}(\boldsymbol{l},\omega)\frac{1}{\sqrt{4M^2(\omega+M)(\omega'+M)}}$$

$$\times \frac{1}{(1-\frac{q^2}{4M^2})}\left[G_E(q^2)\left(\delta^\mu - \frac{\eta^\mu}{2M}\right) + G_M(q^2)\left(\frac{\eta^\mu}{2M} - \frac{q^2}{4M^2}\delta^\mu\right)\right]\chi^\dagger\Gamma(\mu)\chi, \tag{5.5}$$

$$\chi^\dagger\Gamma(\mu)\chi = \chi^\dagger[\delta_{\mu 0} + \delta_{\mu 3} + \delta_{\mu 1} i\sigma_2 - \delta_{\mu 2} i\sigma_1]\chi.$$

Here

$$\boldsymbol{l}' = \boldsymbol{l} + \mathbf{Q}, \quad \omega'^2 = \boldsymbol{l}'^2 + M^2,$$

$$q = (q^0, \mathbf{Q}), \quad q^0 = \omega' - \omega,$$

$$\delta^0 = a_i a_f + \boldsymbol{l}^2 + \mathbf{Q}\cdot\boldsymbol{l},$$

$$\delta^1 = \delta^2 = (a_i - a_f)|\boldsymbol{l}|\cos\theta + a_i|\mathbf{Q}|,$$

$$\delta^3 = a_f|\boldsymbol{l}|\cos\theta + a_i(|\boldsymbol{l}|\cos\theta + |\mathbf{Q}|),$$

$$\eta^0 = (a_f - a_i)\mathbf{Q}\cdot\boldsymbol{l} - a_i\mathbf{Q}^2, \tag{5.6}$$

$$\eta^1 = \eta^2 = a_i a_f|\mathbf{Q}| + |\mathbf{Q}|(\boldsymbol{l}^2 + \boldsymbol{l}\mathbf{Q}) - \boldsymbol{l}^2|\mathbf{Q}|\sin^2\theta + (\omega' - \omega)(-a_f|\boldsymbol{l}|\cos\theta - a_i|\boldsymbol{l}|\cos\theta - a_i|\mathbf{Q}|),$$

$$\eta^3 = (\omega' - \omega)[a_f|\boldsymbol{l}|\cos\theta - a_i(|\boldsymbol{l}|\cos\theta + |\mathbf{Q}|)],$$

$$a_i = \omega + M, \quad a_f = \omega' + M,$$

$$\mathbf{Q}\cdot\boldsymbol{l} = |\mathbf{Q}||\boldsymbol{l}|\cos\theta.$$

The four-momentum q is an average value of $l' - l$ calculated between two wave packets ϕ_P, which have speeds β_i^μ and β_f^μ, respectively.

For the Sach's form factors one can assume the well known dipole shapes

$$\frac{G_E(q^2)}{G_E(0)} = \frac{G_M(q^2)}{G_M(0)} = \left(1 - \frac{q^2}{\eta^2}\right)^{-1}. \tag{5.7}$$

The magnetic moment $G_M(0) = \mu_P$ can be determined from the equalities (5.4) and (5.6) taken at $Q^2 = O$. One obtains

$$\mathbf{D}|_{\mathbf{Q}=0} = 0,$$

$$\mathbf{Q}\left.\frac{\partial\mathbf{D}}{\partial\mathbf{Q}}\right|_{\mathbf{Q}=0} = \frac{1}{2M}\chi^\dagger i\boldsymbol{\sigma}\times\mathbf{Q}\chi\kappa = \frac{1}{2M}\chi^\dagger i\boldsymbol{\sigma}\times\mathbf{Q}\chi W^2(\mathbf{Q}=0), \tag{5.8}$$

$$\kappa = W^2(\mathbf{Q}=0) = 4\pi\int l^2 dl \frac{M^2}{\omega^2}|\phi_{\mathbf{P}=0}(\boldsymbol{l},\omega)|^2 \times\left[\frac{G_E(0)}{3}\left(\frac{M}{\omega} - 1\right) + \frac{G_M(0)}{3}\left(1 + \frac{M}{\omega} + \frac{M^2}{\omega^2}\right)\right].$$

With $G_E(0) = 1$ one finds

$$G_M(0) = 2.212, \tag{5.9}$$

which is about 20% too small. However, without c.m.m. corrections one would have obtained

$$G_M(0) = 1.738, \tag{5.10}$$

which is much smaller than the experimental value [30] $G_M(0) = 2.793$. The c.m.m. corrections have resulted in 27% improvement of the model value [23].

The equality (5.5) can be used to determine the parameter η. For $Q^2 < 1.17\ \mathrm{GeV}^2$ equality is, within 10% error, satisfied with

$$\eta = 0.70\ \mathrm{GeV}^2 \tag{5.11}$$

which is very close to the experimental value [30] $\eta_{\mathrm{expt}} = 0.71\ \mathrm{GeV}^2$.

The fit (5.11) fails progressively as Q^2 increases above $1.17\ \mathrm{GeV}^2$. Qualitatively this agrees with other model-based calculations; see for example Ref. [31].

An analogous formalism can be used for the nucleon axial vector coupling constant g_A. Without c.m.m. corrections, one finds $g_A = 1.14$. With c.m.m. corrections the theoretical result $g_A = 1.22$ is surprisingly close to the experimental value [30].

A strong point in favor of BCCM with c.m.m. corrections is that corrections are much larger for G_M (27%) than for g_A (7%), just as needed.

VI. $M1$ TRANSITION IN QUARKONIA

The $M1$ transitions

$$V \to P + \gamma,\qquad ({}^3S_1 \to {}^1S_0 + \gamma), \tag{6.1}$$

provide useful informations [32,33] about c.m.m. corrections for systems containing heavy quarks c and b. The decay amplitude is

$$\langle m(P_f)|J^\mu_{\rm em}(0)|v(P_i,\epsilon)\rangle = \frac{1}{(2\pi)^3\sqrt{4E_iE_f}} g\epsilon^{\mu\nu\sigma\rho} \times\epsilon_\nu (P_f - P_i)_\sigma (P_f + P_i)_\rho, \tag{6.2}$$

with the corresponding decay width

$$\Gamma(v \to m\gamma) = \tfrac{4}{3}\alpha\left(\frac{g}{l}\right)^2 \omega_\gamma^3,\qquad \omega_\gamma = \frac{M_i^2 - M_f^2}{2M_i}. \tag{6.3}$$

Here α is the fine structure constant.

In BCCM's the form factor g can be calculated with $[g(s^2)]$ and without $[\tilde g(s^2)]$ c.m.m. corrections. In the first case one starts with

$$(2\pi)^4\delta(P_f + k - P_i)\prod_{i=1}^{2} J(z_i) \sum_{l,n,\text{perm}} \overline{\kappa_m}^{s=0}(P_f, z_1^P, z_2^P, y=0) \times(\gamma^\mu)_l e^{ikz_l} \not{L}_n \kappa_v^{s=1}(P_i, z_1^P, z_2^P, y=0) = (2\pi)^4\delta(P_f + k - P_i)\mathcal{N}^\mu N_{fi}, \tag{6.4}$$

$$N_{fi} = \frac{1}{(2\pi)^3}\sqrt{\frac{M_iM_f}{E_iE_f}}\ .$$

As $\mathcal{N}^\mu$ must have the same form as (6.2) one can identify the form factor $\tilde g(0)$. Here κ_m^s is the meson wave function analogous to (2.7). The calculation was carried out in the generalized Breit frame

$$\frac{E_i}{M_i} = \frac{E_f}{M_f},\quad \frac{\mathbf{P}_i}{M_i} = -\frac{\mathbf{P}_f}{M_f},\quad P_i = P_f + k,$$
$$|\mathbf{k}| = \frac{M_i^2 - M_f^2}{2\sqrt{M_iM_f}},\quad |\mathbf{P}_i| = \sqrt{\frac{M_i}{M_f}}\frac{M_i - M_f}{2}. \tag{6.5}$$

By expansion of $\mathcal{N}^\mu$ around $Q^2 = O$ one can find, for smaller Q^2,

$$\tilde g(Q^2) = \frac{\tilde g(0)}{1 - Q^2/\Lambda_1 + Q^4/\Lambda_2 + \cdots} \cong \frac{\tilde g(0)}{1 - Q^2/\Lambda_1}. \tag{6.6}$$

The c.m.m. corrections are introduced by using the equality

$$\mathcal{N}^\mu N_{fi} = J(z)\int \frac{d^3l\, d^3l'}{4\omega\omega'}\varphi^*_{P_f}(\boldsymbol{l}', \omega')\varphi_{P_i}(\boldsymbol{l}, \omega)\cdot\langle \boldsymbol{l}'|J^\mu_{\rm em}(z)e^{ikz}|\boldsymbol{l}\rangle, \tag{6.7}$$

$$\mathcal{N}^1 = \int \frac{d^3l}{(4\omega\omega')^{3/2}}\varphi^*_{P_f}(\boldsymbol{l} - \mathbf{k}, \omega')\varphi_{P_i}(\boldsymbol{l}, \omega)\frac{i}{\sqrt{2}} g(q^2)\left[2\omega\left(1 + \frac{M_f}{M_i}\right)|\mathbf{P}_i| - 2(\omega - \omega')|\boldsymbol{l}|\cos\theta\right].$$

Here

$$\omega'^2 = (\boldsymbol{l} - \mathbf{k})^2 + M_f^2,\qquad \cos\theta = \mathbf{P}_i\cdot\boldsymbol{l}/|\mathbf{P}_i||\boldsymbol{l}|,\qquad q = (\omega' - \omega, \mathbf{k}). \tag{6.8}$$

In (6.7) one must introduce the form (6.6) for $g(q^2)$. Then using the equality (6.7), where $\mathcal{N}^\mu$ is determined by the integration over the model wave functions (6.4), one can determine $g(0) \equiv g$.

Model predictions, based on the parameters listed in

TABLE III. The $M1$ transition decay widths without ($\tilde{\Gamma}$) and with (Γ) c.m.m. corrections.

Mode	$\tilde{g}(0)$ (GeV^{-1})	$\tilde{\Gamma}_0$ (10^{-6} GeV)	$g(0)$ (GeV^{-1})	Γ (10^{-6} GeV)	$\frac{g(0)-\tilde{g}(0)}{\tilde{g}(0)}$ (%)
$\Psi \to \eta_c\gamma$	0.367	2.041	0.377	2.148	2.6
$D^{\dagger *} \to D^{\dagger}\gamma$	−0.127	0.393	−0.132	0.429	4.4
$D^{0*} \to D^{0}\gamma$	0.811	16.656	0.847	18.164	4.4
$D_s^{\dagger *} \to D_s^{\dagger}\gamma$	−0.062	0.094	−0.064	0.103	4.2
$B^{\dagger *} \to B^{\dagger}\gamma$	0.639	0.382	0.645	0.389	1.0
$B^{0*} \to B^{0}\gamma$	−0.367	0.126	−0.371	0.128	1.0
$B_s^{0*} \to B_s^{0}\gamma$	0.290	0.084	0.293	0.086	1.0

Table I, are shown in Table III. The decay widths (6.3) are calculated using either $\tilde{g}(0), (\tilde{\Gamma})$ or $g(0), (\Gamma)$. No attempts have been made to select model parameters in order to improve the agreement with the measured value $\Gamma(\Psi \to \eta_c\gamma) = (1.12 \pm 0.35)10^{-6}$ GeV [34]. It is interesting that such, unadjusted, results are in a very reasonable agreement with the unadjusted results of Ref. [33] (their Table II, columns 2,3), which were obtained in a quite different quark model.

The main aim here was to calculate the magnitude of c.m.m. corrections. They turned out to be 4.4% or smaller, decreasing with the increase of the heavy quark mass. With a b quark present c.m.m. corrections are practically negligible. Indeed, when one of the valence quarks is very heavy the c.f. and c.m. almost coincide [6], so that the spurious c.m.m. almost vanishes.

VII. HEAVY QUARK SYMMETRY LIMIT AND MESON DECAY CONSTANTS

The decay constant f_m, for a meson m, can be calculated in the BCCM [22,24]. The Lorentz covariant c.m.m. corrections are introduced through the equality

$$\begin{aligned}&\int d^4y\, J(z)\langle 0|\bar{\Psi}(z^P)\gamma^\mu\gamma_5\Psi(z^P)|m,P,M,s=0,y\rangle e^{iqx}\\&=(2\pi)^4\delta^{(4)}(q-P)Z^\mu(P)\\&=\int d^4y\, J(z)\int\frac{d^3l}{2\omega}\varphi_P(l,\omega)\langle 0|J_5^\mu(z)|l,0,m\rangle e^{i(q-P)y}e^{iqz}.\end{aligned} \quad (7.1)$$

In the RHS of (7.1) is the meson decay constant f_m defined for a momentum eigenstate $|P,0,m\rangle$:

$$\langle 0|J_{\mu 5}(x)|P,0,m\rangle = \frac{i}{\sqrt{2E(2\pi)^3}}P_\mu f_m e^{-iPx}. \quad (7.2)$$

In the HO potential version of the BCCM, integrations in (7.1) can be carried out explicitly. One finds

$$\sqrt{6}\frac{P^\mu}{M}K_{\alpha,\beta} = \frac{(2\pi)|^{3/2}}{2M\sqrt{2E}}\varphi_P(\mathbf{P},E)P^\mu f_m,$$

$$f_m = \sqrt{\frac{12}{M_m(2\pi)^{3/2}R^3_{\alpha,\beta}C^{(1)}_{\alpha\beta}}}K_{\alpha\beta=m}. \quad (7.3)$$

Here

$$K_{\alpha\beta} = 4\pi\int dr\, r^2(U_\alpha U_\beta - V_\alpha V_\beta),$$

$$C^{(1)}_{\alpha\beta} = 1 - 3\left(\frac{c\alpha}{R^2_{0\alpha}} + \frac{c_\beta}{R^2_{0\beta}}\right)R^2_{\alpha\beta} + 15\frac{c_\alpha c_\beta}{R^2_{0\alpha}R^2_{0\beta}}R^4_{\alpha\beta}, \quad (7.4)$$

$$R^2_{\alpha\beta} = \frac{2R^2_{0\alpha}R^2_{0\beta}}{R^2_{0\alpha}+R^2_{0\beta}}, \quad c_\alpha = \frac{\beta_\alpha}{4+6\beta^2_\alpha}.$$

In the HQL (4.5) one has

$$K_{\alpha\beta} \to 4\pi\int dr\, r^2 U_\alpha U_Q = K_{\alpha_{\mathrm{HQL}}},$$

$$R_{\alpha\beta} \to R_{\alpha_{\mathrm{HQL}}}, \quad (7.5)$$

$$f_m \to \frac{1}{\sqrt{M_m}}F_{\alpha_{\mathrm{HQL}}}.$$

Here Q is a heavy quark (c,b) while α denotes a light quark (u,d,s). With (7.5) a meson decay constant has $M_m^{-1/2}$ dependence as required by the heavy quark symmetry (HQS). One obtains, for example,

$$f_{B_{\mathrm{HQL}}} = \sqrt{\frac{M_D}{M_B}}f_{D_{\mathrm{HQL}}} = 0.6 f_{D_{\mathrm{HQL}}}. \quad (7.6)$$

With full expression (7.1), using parameters listed in Table I, one obtains

$$f_D = 130.6\ \mathrm{MeV},\quad f_B = 90.9\ \mathrm{MeV},$$
$$f_B/f_D = 0.696. \quad (7.7)$$

The ratio f_B/f_D (7.7) is in a very good agreement with the result $f_B/f_D \cong 0.69$ obtained by the $1/m_Q$ expansion of the heavy-light currents [14,35]. However, it is about 30% smaller than the results based on QCD sum rules, lattice calculations, and semilocal parton-hadron duality [36].

The BCCM based calculation gives

$$f_{D_s} = 149,2\ \text{MeV},$$
$$f_{D_s}/f_D = 1.14. \tag{7.8}$$

The ratio f_{D_s}/f_D is in reasonable agreement with previous results obtained from lattice QCD or potential models [36]. QCD sum rule analyses gave $f_{D_s}/f_D \cong 1.19$ [26] and $f_{D_s}/f_D \cong 1.1$ [37]. However, absolute values [(7.7),(7.8)] for heavy meson decay constants seem to be smaller than the QCD sum rule or lattice QCD based estimates [14,26,37–40].

The BCCM with c.m.m. corrections predicts

$$f_{K^+} = 171\ \text{MeV},$$

which is in good agreement with the experimental value $f_{K^+} = (160.6 \pm 1.3)$ MeV [34]. The pion decay constant $f_\pi = 271$ MeV is too large [$f_{\pi_{\text{expt}}} = (131.73 \pm 0.15)$ MeV [34]], as is usual in valence quark models.

VIII. MESON DECAY FORM FACTORS AND HQS

The calculation of meson decay form factors has already been described [3,5] so only some examples need to be shown here. The matrix elements for $B \to D(D^*)$ transitions are

$$\langle P_f, s=0, D|\bar{c}\gamma^\mu b|B, s=0, P_i\rangle = \frac{2\pi\delta^{(4)}(P_f+Q-P_i)}{2\sqrt{E_iE_f}}[f_+(Q^2)(P_i+P_f)^\mu + f_-(Q^2)(P_i-P_f)^\mu], \tag{8.1}$$

$$\langle P_f, \epsilon, D^*|\bar{c}\gamma^\mu b|B, s=0, P_i\rangle = \frac{2\pi\delta^{(4)}(P_f+Q-P_i)}{2\sqrt{E_iE_f}} ig(Q^2)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu(P_i+P_f)_\rho(P_i-P_f)_\sigma, \tag{8.2}$$

$$\langle P_f, \epsilon, D^*|\bar{c}\gamma^\mu\gamma_5 b|B, s=0, P_i\rangle = \frac{2\pi\delta^{(4)}(P_f+Q-P_i)}{2\sqrt{E_iE_f}}[f(Q^2)\epsilon^{*\mu} + a_+(Q^2)(\epsilon^*\cdot P_i)(P_i+P_f)^\mu$$
$$+a_-(Q^2)(\epsilon^*\cdot P_i)(P_i-P_f)^\mu]. \tag{8.3}$$

The corresponding BCCM expressions in the generalized Breit frame (6.5) are

$$f_+ = \frac{1}{\sqrt{4M_iM_f}}\left((M_i+M_f)\frac{M_f}{E_f}I^0_{cb} - (M_i-M_f)\frac{M_f}{|\mathbf{P}_f|}I^3_{cb}\right)Z_{\bar{d}},$$

$$f_- = f_+[(M_i+M_f) \leftrightarrow (-)(M_i-M_f)],$$

$$g = \frac{M_f\sqrt{M_fM_i}}{2M_iE_f|\mathbf{P}_f|}V^1_{cb}(\lambda=+1)Z_{\bar{d}},$$

$$f = \sqrt{4M_iM_f}A^1_{cb}(\lambda=+1)Z_{\bar{d}}, \tag{8.4}$$

$$a_+ = \frac{1}{4M_iM_f}\sqrt{\frac{M_f}{M_i}}\left[(M_i-M_f)\left(\frac{M_f}{|\mathbf{P}_f|}\right)^2\left(\frac{M_f}{E_f}A^3_{cb}(\lambda=0) - A^1_{cb}(\lambda=+1)\right)\right.$$
$$\left.+(M_i+M_f)\left(\frac{M_f}{E_f}\right)^2\left(\frac{M_f}{|\mathbf{P}_f|}A^0_{cb}(\lambda=0) - A^1_{cb}(\lambda=+1)\right)\right]Z_{\bar{d}},$$

$$a_- = a_+[(M_i+M_f) \leftrightarrow (-)(M_i-M_f)].$$

Here

$$I^0_{cb} = 4\pi \frac{M_f}{E_f} \int dr\, r^2 j_0(\tilde\rho)[U_cU_b + V_cV_b],$$

$$I^3_{cb} = 4\pi \frac{M_f}{E_f} \int dr\, r^2 j_1(\tilde\rho)[U_cV_b - V_cU_b],$$

$$V^1_{cb}(\lambda = +1) = 4\pi \int dr\, r^2 \left(\frac{|\mathbf{P}_f|}{E_f} \{ j_0(\tilde\rho)U_cU_b - [\tfrac{1}{3}j_0(\tilde\rho) - \tfrac{2}{3}j_2(\tilde\rho)]V_cV_b \} + j_1(\tilde\rho)[U_cV_b + V_cU_b] \right),$$

$$A^1_{cb}(\lambda = +1) = 4\pi \int dr\, r^2 \left(j_0(\tilde\rho)U_cU_b - [\tfrac{1}{3}j_0(\tilde\rho) - \tfrac{2}{3}j_2(\tilde\rho)]V_cV_b + \frac{|\mathbf{P}_f|}{E_f} j_1(\tilde\rho)(U_cV_b + V_cU_b) \right), \tag{8.5}$$

$$A^0_{cb}(\lambda = 0) = -I^3_{cb},$$

$$A^3_{cb}(\lambda = 0) = 4\pi \frac{M_f}{E_f} \int dr\, r^2 \left(j_0(\tilde\rho)(U_cU_b - V_cV_b) + 2[\tfrac{1}{3}j_0(\tilde\rho) - \tfrac{2}{3}j_2(\tilde\rho)]V_cV_b + \frac{|\mathbf{P}_f|}{M_f} j_1(\tilde\rho)U_cV_b \right).$$

c.m.m. corrections have been neglected. For heavy-light quark combination they are always smaller than 5% (see Table III). In (8.5) one has introduced the spherical Bessel functions $j_l(\tilde\rho)$ where

$$\tilde\rho = \frac{M_f}{E_f} B_{cb}|\mathbf{r}|, \tag{8.6}$$

$$B_{cb} = [(M_f + M_i) - (\epsilon_c + \epsilon_b)] \frac{|\mathbf{P}_f|}{M_f}.$$

The symbol λ labels the polarization of the vector meson D^*. The expressions (8.4) contain also the overlap (free line) (2.9) of the light spectator quark:

$$Z_{\bar d} = \frac{M_f}{E_f} 4\pi \int dr\, r^2 j_0(\rho)[U_{\bar d}^2 + V_{\bar d}^2],$$

$$\rho = 2\epsilon_d \frac{|\mathbf{P}_f|}{E_f}|\mathbf{r}|. \tag{8.7}$$

Formulas (8.5) are a version of the more general formulas listed in Appendix [(A1)–(A7)] of Ref. [5]. Such formulas are valid for any BCCM, which includes BBM [5].

In the HQL (4.6),

$$V_\alpha = 0, \;\; \alpha = b, c,$$

$$U_b = U_c = U_{\rm HQL},$$

$$B_{cb} \to 0, \;\; \tilde\rho \to 0, \;\; j_0(0) = 1,$$

$$I^0_{\rm HQL} = \frac{M_f}{E_f} 4\pi \int dr\, r^2 U^2 = \frac{M_f}{E_f} K_{\rm HQL}, \tag{8.8}$$

$$I^3_{\rm HQL} = 0,$$

$$V^1_{\rm HQL} = \frac{|\mathbf{P}_f|}{M_f} I^0_{\rm HQL}, \;\; A^1_{\rm HQL} = \frac{E_f}{M_f} I^0_{\rm HQL},$$

$$A^0_{\rm HQL} = 0, \;\; A^3_{\rm HQL} = I^0_{\rm HQL}.$$

With

$$|\mathbf{P}_f|^2/M_f^2 = \frac{(M_i - M_f)^2 - Q^2}{4M_iM_f},$$

$$4E_iE_f = (M_i + M_f)^2 \left[1 - \frac{Q^2}{(M_i + M_f)^2} \right], \tag{8.9}$$

one finds

$$f_+ = \frac{2\sqrt{M_i M_f}}{(M_i + M_f)} \left(1 - \frac{Q^2}{(M_i + M_f)^2}\right)^{-1} (Z_{\bar{d}} K_{\rm HQL}),$$

$$g = \frac{1}{(M_i + M_f)} f_+, \tag{8.10}$$

$$a_+ = -g,$$

$$f = 2\sqrt{M_i M_f}(Z_{\bar{d}} K_{\rm HQL}).$$

Very elegant relations among form factors can be found by using the form factors from Ref. [18]: i.e.,

$$F_1 = f_+, \quad V = (M_i + M_f) g,$$

$$A_2 = -(M_i + M_f) a_+, \tag{8.11}$$

$$A_1 = \frac{1}{M_i + M_f} f.$$

As in HQL, $M_{D^*} \equiv M_D = M_f$, one immediately obtains the well-known [14] HQS relations

$$\begin{aligned} F_1(Q^2) &= V(Q^2) \\ &= A_2(Q^2) \\ &= \left(1 - \frac{Q^2}{(M_i + M_f)^2}\right)^{-1} A_1 \\ &= \frac{2\sqrt{M_i M_f}}{(M_i + M_f)} \frac{1}{[1 - \frac{Q^2}{(M_i + M_f)^2}]} Z_{\bar{d}} K_{\rm HQL}. \end{aligned} \tag{8.12}$$

From (8.12) one easily extracts the Isgur-Wise function [7,14] which is actually determined by the overlap $Z_{\bar{d}}$ (8.7). First one must realize that $K_{\rm HQL}$ is actually the HQL of the normalization integral:

$$N = \int dr\, r^2 (U^2 + V^2), \quad N_{\rm HQL} = K_{\rm HQL} = 1. \tag{8.13}$$

Then, when (8.12), (8.13), and the definition [14]

$$\xi(v \cdot v') = \lim_{m_Q \to \infty} R F_1(Q^2),$$

$$v \cdot v' = \frac{M_i^2 + M_f^2 - Q^2}{2 M_i M_f}, \tag{8.14}$$

$$R = \frac{2\sqrt{M_i M_f}}{M_i + M_f},$$

one obtains

$$\xi(v \cdot v') = \frac{4 M_i M_f}{(M_i + M_f)^2} \frac{1}{[1 - \frac{Q^2}{(M_i + M_f)^2}]} Z_{\bar{d}}(Q^2). \tag{8.15}$$

It should be noted that both (8.12) and (8.15) include explicitly the kinematic factor $[1 - Q^2/(M_i + M_f)^2]$. Furthermore, at the maximum momentum transfer $Q^2_{\rm max} = (M_i - M_f)^2$ one finds [14,41]

$$\begin{aligned} Z_{\bar{d}}|_{\mathbf{P}_i = \mathbf{P}_f = 0} &= 1, \\ F_1 = V = A_2 &= A_1^{-1} = R^{-1}. \end{aligned} \tag{8.16}$$

Thus in HQL the BCCM-based relations coincide exactly with QCD-based ones, what is only approximately true for other models [18,29,42].

It might be interesting to compare BCCM prediction for the Q^2 dependence of form factors, including the HQL limit, with other approaches. The results obtained for the HO model are shown in Fig. 2 using the same scale as in corresponding Figs. 1.3 and 5.8 in Ref. [14].

All results presented here can be obtained also in BCCM based on the MIT bag model [15]. Figure 3 shows that both versions of BCCM procedure quite similar results. Model predictions stay close to the HQS limit, which is, up to factor R^{-1}, given by Isgur-Wise function $\xi(v \cdot v')$. In BCCM one always obtains

$$V(Q^2) > A_2(Q^2) \cong \left[1 - \frac{Q^2}{(M_B + M_D)^2}\right]^{-1} A_1 > F_1.$$

This ordering differs from other quark models [14]. It does agree with QCD sum rule results (Fig. 5.8 in Ref. [14]). However, quantitative agreement is not so good. The absolute values of QCD-sum rule form factors are usually larger than the corresponding BCCM values. The gaps separating $V, [1 - \frac{Q^2}{(M_B + M_D)^2}]^{-1} A_1$, A_2, and F_1 curves are also larger. BCCM, as used here, does not take into account the short distance corrections which are responsible [14] for 50% of the enhancement of V relative to F_1 and A_1.

All BCCM-based conclusions seem to be independent of the form of central confinement [3,5,15–17]. However, the precise form of the Q^2 dependence might be influenced by the model details. Thus the selection of the particular version of BCCM could be some kind of fine-tuning.

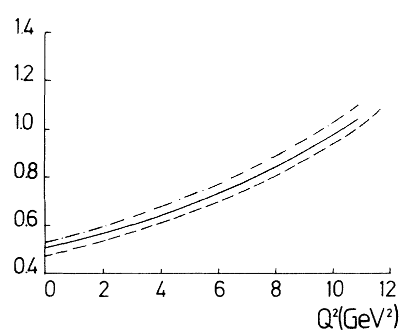

FIG. 2. Predictions for the weak decay form factors in HO based BCCM. Dot-dashed line corresponds to V, solid line to A_2 and $[1 - Q^2/(M_B + M_D)^2]^{-1} A_1$ and the dashed line to F_1. HQS limit coincides with the solid line.

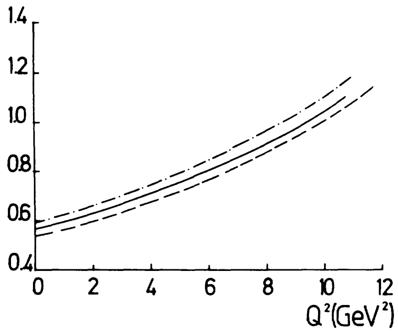

FIG. 3. Predictions for the weak decay form factors in MIT bag based BCCM. Line identification is the same as in Fig. 2.

IX. MAIN CHARACTERISTICS

The main aim of this paper was to demonstrate how one can construct a whole class of quark models which are heavy quark (b, c) symmetric. Such models are also Lorentz covariant, as has been shown in Refs. [3] and [5]. The kinematic factor (8.12), (8.15) that appears in HQS relations is a typical consequence of the Lorentz covariance.

The class of HQS models contains models [15–17] in which each quark is independently centrally confined. As is well known [25], such models experience spurious c.m.m. effects. It is demonstrated here that one can introduce c.m.m. corrections in the manifestly covariant way. They notably improve μ_p, g_A, hadron masses, and other quantities which involve "light" (u, d, s) quarks. For "heavy-light" combinations c.m.m. corrections diminish with the increase of the heavy quark mass (see Tables II and III. In the derivation of HQS relations (8.12) they could have been neglected. However, their presence, as in (7.6), does not spoil HQS character of the model.

A BCCM is based on a static quark model (examples in Refs. [15–17]) with specified boosts (2.3), hyperplane projection (2.12), and overlaps (2.9). After BCCM is formulated all calculations depend only on the parameters of the underlaying static model. Basing BCCM on the MIT bag model one uses the usual bag-model parameters [15]. With a harmonic oscillator potential as a starting point one employs only parameters listed in Table I. All form factors (7.3), (8.4), HQS relations (7.6), (8.12), Isgur-Wise function (8.15), etc. are obtained by a straightforward calculation, without any additional *ad hoc* assumptions. The results in Table II, excellent g_A value and responsible μ_p (5.9), are not due to any "fine-tuning." Playing with parameters one could "improve" some of those outcomes, which would be pointless, as it does not lead to any new physical insights. Of more fundamental importance could be the selection of the type of central confinement. It obviously pays to select the confinement which best mimics the real physics. Some idea about the confinement dependence can be obtained by comparison of Figs. 2 and 3.

As is usual with central valence quark models [15–17], BCCM also fails in the description of pion, by not being able to account for its Goldstone-boson character.

BCCM's can be used to calculate corrections [(7.7), Figs. 2 and 3] to the extreme HQS results. However, only those corrections which depend on the valence quark dynamics are included. Short distance QCD corrections [14] were not incorporated in BCCM. As the model is formulated in the quantum filed formalism (2.5), which can be related to the Furry bound state picture [27,43], some estimates of QCD effects might become feasible.

An important characteristic of the class of BCCM's is that those models describe mesons and baryons within the same formalism. Here BCCM's were mostly applied to mesons, but calculations of the electromagnetic [3–5] (5.4) and of the semileptonic [3] baryon form factors are equally feasible.

APPENDIX

The explicit expressions for the components ϕ of the momentum eigenstates can be found by using (3.8), (4.3), and Table I. For proton (nucleon) one finds

$$\frac{M}{\omega_l}|\phi_P(\boldsymbol{l},\omega_l)|^2 = \frac{\tilde{\omega}_l}{M}\left(\frac{R_0}{\sqrt{3\pi}}\right)^3 e^{\frac{-\tilde{\boldsymbol{l}}^2 R_0^2}{3}} \times\{C_0 + C_1(\tilde{\boldsymbol{l}})^2 + C_2(\tilde{\boldsymbol{l}})^4 + C_3(\tilde{\boldsymbol{l}})^6\}. \tag{A1}$$

Here

$$\tilde{\omega} = \frac{P_\mu l^\mu}{M},\quad \tilde{\boldsymbol{l}} = \boldsymbol{l} + \frac{\mathbf{P}(\mathbf{P}\cdot\boldsymbol{l})}{M(E+M)} - \frac{\mathbf{P}}{M}\omega_l,$$

$$\tilde{\omega} - \tilde{\boldsymbol{l}}^2 = M^2,$$

$$C_0 = 1 - 6c + 20c^2 - \frac{280}{9}c^3,$$

$$C_1 = \frac{4}{3}\left(c - \frac{20}{3}c^2 + \frac{140}{9}c^3\right)R_0^2, \tag{A2}$$

$$C_2 = \frac{16}{27}\left(c^2 - \frac{14}{3}c^3\right)R_0^4,$$

$$C_3 = \frac{64}{729}c^3 R_0^6,$$

$$c = \frac{\beta^2}{4+6\beta^2}.$$

The normalization of the proton component (A1) is

$$\int d^3l \frac{M^2}{\omega_l^2}|\phi_P(\boldsymbol{l},\omega_l)|^2 = 1. \tag{A3}$$

The meson component is

$$\frac{|\varphi_P(\boldsymbol{l},\omega_l)|^2}{2\omega_l} = \frac{2(l\cdot P)}{(2\pi)^{3/2}M} R_{ab}^3 e^{-\tilde{\boldsymbol{l}}^2 R_{ab}^2/2}\cdot [C_{ab}^{(1)} + C_{ab}^{(2)}\tilde{\boldsymbol{l}}^2 + C_{ab}^{(3)}\tilde{\boldsymbol{l}}^4]. \tag{A4}$$

Here, with flavors a, b,

$$\tilde{\omega} = \frac{P_\mu L^\mu}{M}, \quad \tilde{\boldsymbol{l}} = \boldsymbol{l} + \frac{\mathbf{P}(\mathbf{P}\cdot\boldsymbol{l})}{M(E+M)} - \frac{\mathbf{P}}{M}\omega_l,$$

$$C_{ab}^{(1)} = 1 - 3\left(\frac{c_a}{R_{0a}^2} + \frac{c_b}{R_{0b}^2}\right) R_{ab}^2 + 15\frac{c_a c_b}{R_{0a}^2 R_{0b}^2} R_{ab}^4,$$

$$C_{ab}^{(2)} = \left(\frac{c_a}{R_{0a}^2} + \frac{c_b}{R_{0b}^2}\right) R_{ab}^4 - 10\frac{c_a c_b}{R_{0a}^2 R_{0b}^2} R_{ab}^6, \tag{A5}$$

$$C_{ab}^{(3)} = \frac{c_a c_b}{R_{0a}^2 R_{0b}^2} R_{ab}^8,$$

$$R_{ab}^2 = \frac{2R_{0a}^2 R_{0b}^2}{R_{0a}^2 + R_{0b}^2},$$

$$c_a = \frac{\beta_a^2}{4 + 6\beta_a^2}.$$

The normalization is

$$\int d^3 l \frac{1}{4\omega_l^2} |\varphi_P(\boldsymbol{l},\omega_l)|^2 = 1. \tag{A6}$$

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