PHYSICAL REVIEW D VOLUME 50, NUMBER 1 1 JULY 1994

Mass equations of Higgs and weak gauge bosons in the fermion condensate scheme with n generations of fermions

Bang-Rong Zhou
*Chinese Center for Advanced Science and Technology (World Laboratory), P.O. Box 8730, Beijing 100080, China and Department of Physics, Graduate School, Academia Sinica, Beijing 100039, China**

(Received 30 November 1993)

The mass equations of the Higgs boson ϕ_s^0 and the gauge bosons W and Z are expounded in a dynamical symmetry-breaking model of Nambu–Jona-Lasinio type with three quark-lepton generations and a heavier degenerate fourth U-fermion generation. The equation of $m_{\phi_s^0}$ with an explicit momentum cutoff dependence and derived approximate formula of $m_{\phi_s^0}$ are numerically solved and a more stringent mass constraint $m_U+m_t\le m_{\phi_s^0}\le 2m_U$ is proven. The results show that the determination of the Higgs boson mass will be the most important experimental test of the model with heavy U fermions. The mass of the W boson is argued to be independent of the momentum cutoff and almost independent of the masses of the heavy t quarks and U fermions. Its equation could take the form in the standard model and is dominated by light fermions. The mass of the Z boson, in addition to having the same feature as m_W, will be affected by the weak isospin breaking caused by the mass difference m_t-m_b. Such an effect makes not only the mass equation of the boson but also the β function responsible for the running of the gauge coupling constants g_1 and g_2 in the mass equation deviate from the respective standard forms. It is also argued that if the electroweak gauge bosons were replaced by composites of fermions then the possibility of the existence of heavy U fermions would be removed.

PACS number(s): 12.60.Rc, 11.15.Bt, 11.15.Ex, 14.80.Bn

I. INTRODUCTION

The top-quark condensate scheme [1,2] of electroweak symmetry breaking may be generalized so as to include n generations of fermions. Different limiting cases of such a generalization were discussed by some authors [2–5] and the general realization of the minimal version has been recently proven [6–8].

In this version, the basic relation to define the vacuum expectation value v responsible for spontaneous symmetry breaking becomes [8]

$$\frac{G_F}{\sqrt{2}}=\frac{1}{8\bar{f}^2(0)}=\frac{1}{2v^2}\ , \tag{1}$$

where G_F is the Fermi constant and

$$\bar{f}^2(0)=\frac{1}{32\pi^2}\sum_{\alpha=1}^{n}d_{Q_\alpha}(R)m^2_{U_\alpha}\left[(1+\gamma_{U_\alpha})\left[\ln\frac{\Lambda^2}{m^2_{U_\alpha}}-\frac{1}{2}\right]+\frac{\gamma^2_{U_\alpha}}{1-\gamma_{U_\alpha}}\ln\gamma_{U_\alpha}\right]\ ,$$
$$\gamma_{U_\alpha}=\frac{m^2_{D_\alpha}}{m^2_{U_\alpha}}\ , \tag{2}$$

with the fermions

$$Q_{\alpha L}=\begin{bmatrix}U_\alpha\\ D_\alpha\end{bmatrix}_L\ ,\quad Q_{\alpha R}=U_{\alpha R},D_{\alpha R}\ ,$$
$$\alpha=1,\ldots,n\quad\text{(generation number)} \tag{3}$$

being assigned in anomaly-free $SU_L(2)\times U_Y(1)$ representations and in the representations R of the color gauge group G_c with dimensions $d_{Q_\alpha}(R)$. These fermions Q_α have the dynamical masses m_{Q_α} which are assumed to be much less than the momentum cutoff Λ so that $m^2_{Q_\alpha}\ll\Lambda^2$. It has been shown [7] that, from the requirement to saturate relation (1) with Λ and m_{Q_α}, it is possible that, when one or two generations of the heavier fermions than the top quarks are added in the model, the acceptable momentum cutoff Λ could be lowered down to the region of 10^6–5×10^3 GeV and this will greatly alleviate the fine-tuning problem of the coupling constants. Certainly, the number and masses of these heavy fer-

*Mailing address.

mions depend on the selection of the momentum cutoff Λ; hence, the model must be Λ dependent. On the other hand, we have derived the mass equations of the Higgs boson ϕ_s^0 and the W and Z bosons in the model with n generations of fermions [8]. Therefore, several interesting questions deserve to be raised. How and to what extent do the masses $m_{\phi_s^0}$, m_W, and m_Z depend on the momentum cutoff Λ and the heavy fermion masses? Is there any relation independent of Eq. (1) here which could be drawn out to give a further limit to the values of Λ or the heavy fermion masses? In addition, among the n generations of fermions, the light fermions will be bound to appear in the mass equations; then what kind of role will they play in determining the masses of those bosons, especially ones of the W and Z bosons? In this paper we will answer the above questions by analyzing the equations of $m_{\phi_s^0}$, m_W, and m_Z. For the sake of definiteness, in any concrete discussions, we will illustrate with a model which contains three generations of ordinary quarks and leptons and one extra heavy degenerate fourth generation of quark-lepton-like fermions. The fourth generation will be generally called U fermions except when it is necessary for them to be distinguished into the quarklike doublet (U,D) and the leptonlike one (N,E), and their degenerate masses will be denoted by m_U.

In Sec. II we will discuss the mass equation of the Higgs boson ϕ_s^0 and an approximate mass formula derived from the equation, then obtain their numerical solutions and a more stringent constraint on $m_{\phi_s^0}$. In Sec. III the equation of m_W will be analyzed so as to examine the dependence of m_W upon the momentum cutoff Λ and the masses of both heavy and light fermions. In Sec. IV a similar analysis to one conducted in Sec. III will be made for the mass equation of the Z boson but emphasis will be put on the effect of the weak isospin breaking induced by the large mass difference $m_t - m_b$. Throughout Secs. II–IV, we will also indicate the different form and/or different results between the mass equations for the models of Nambu–Jona-Lasinio type [9] with gauge and composite electroweak bosons. Finally, in Sec. V we will come to our conclusions.

II. MASS OF HIGGS BOSON

In the fermion condensate scheme with n generations of fermions, the mass $m_{\phi_s^0}$ of the composite Higgs boson ϕ_s^0 is determined by the equation [8]

$$m^2_{\phi_s^0} = \sum_Q 4m_Q^4 K_Q(p^2) \Big/ \sum_Q m_Q^2 K_Q(p^2)\big|_{p^2=m^2_{\phi_s^0}} , \qquad (4)$$

where

$$K_Q(p^2) = -\frac{d_Q(R)}{8\pi^2}\int_0^1 dx \left[\ln\frac{\Lambda^2}{m_Q^2 - p^2 x(1-x)} - 1\right] \qquad (5)$$

and the Q in the sums runs over all of the (light and heavy) fermions. It is seen from Eqs. (4) and (5) that $m_{\phi_s^0}$ depends on Λ both explicitly through $\ln\Lambda^2$ and implicitly through the masses of the heavier fermions than the top quarks in the sums.

However, $m_{\phi_s^0}$ is not sensitive to its explicit Λ dependence. Considering the fact that $m_{\phi_s^0}$ must be smaller than and near the double mass of the heaviest fermions [8], we can make the approximation

$$\int_0^1 dx \ln\frac{\Lambda^2}{m_Q^2 - m^2_{\phi_s^0} x(1-x)} \simeq \ln\frac{\Lambda^2}{m^2_{\phi_s^0}} . \qquad (6)$$

Obviously, the higher is Λ, the better is the approximation (6). Substituting Eq. (6) into (4) we will obtain the mass formula of ϕ_s^0,

$$m^2_{\phi_s^0} = \sum_Q 4m_Q^4 d_Q(R) \Big/ \sum_Q m_Q^2 d_Q(R) , \qquad (7)$$

in which the explicit dependence on Λ has disappeared. Equation (7) is a good approximation of the mass equation (4). In fact, it is just the tree formula of $m_{\phi_s^0}$ in the low-energy effective Lagrangian approach of the model with n generations of fermions. Formula (7) first emerged from a model of Nambu–Jona-Lasinio type with both composite Higgs and composite gauge bosons [3] and its specific expression also appeared in Ref. [4]. Actually, it results from only the assumption that the Higgs boson is viewed as composites of many generations of fermions. As will be shown in Secs. III and IV, in the formula of Ref. [3] no heavier fermions than the top quarks are allowed to exist due to the supposition of composite gauge bosons. However, such heavy fermions could be included in Eq. (7) when the electroweak bosons are fundamental gauge ones.

Let us consider the model with three quark-lepton generations and a degenerate heavy U-fermion generation. In view of the fact that each term in the numerator and denominator of Eq. (4) is proportional to m_Q^4 and m_Q^2, respectively, it is certainly a good approximation to suppose the masses of all the lighter fermions than the U fermions and the top quarks to be zero and to keep only the masses m_U and m_t in Eq. (4). Then it was deduced from Eq. (4) [7] that $m_{\phi_s^0}$ must obey the general constraint

$$2m_t \le m_{\phi_s^0} \le 2m_U . \qquad (8)$$

However, when Eq. (4) contains only the two different fermion masses m_U and m_t, we may actually have further constraint on $m_{\phi_s^0}$:

$$m_U + m_t \le m_{\phi_s^0} \le 2m_U ; \qquad (9)$$

i.e., the lower bound of $m_{\phi_s^0}$ may be raised up to the arithmetic mean value of $2m_U$ and $2m_t$. In order to prove Eq. (9), let us rewrite Eq. (4) in the form that

$$m^2_{\phi^0_s}=4[2m_U^4K_U(p^2)+m_t^4K_t(p^2)]/[2m_U^2K_U(p^2)+m_t^2K_t(p^2)]|_{p=m^2_{\phi^0_s}}=(m_U+m_t)^2$$
$$+(m_U-m_t)[(3m_U+m_t)2m_U^2K_U(p^2)-(m_U+3m_t)m_t^2K_t(p^2)]/[2m_U^2K_U(p^2)+m_t^2K_t(p^2)]|_{p^2=m^2_{\phi^0_s}} . \tag{10}$$

Since $m_U > m_t$, the condition of $m^2_{\phi^0_s} \geq (m_U+m_t)^2$ will be that

$$\frac{|K_t(m^2_{\phi^0_s})|}{|K_U(m^2_{\phi^0_s})|} \leq 2\frac{m_U^2}{m_t^2}\frac{3m_U+m_t}{m_U+3m_t} . \tag{11}$$

Noting that the expression (5) for $K_Q(p^2)$ and the result that $0\leq x(1-x)\leq \frac{1}{4}$ for $0\leq x\leq 1$, we may obtain the following sequence of inequalities:

$$\frac{|K_t(m^2_{\phi^0_s})|}{|K_U(m^2_{\phi^0_s})|} < \frac{|K_t(m^2_{\phi^0_s})|}{|K_U(0)|} = \frac{d_t(R)}{d_U(R)}\left[1+\frac{\ln\lambda-\int_0^1 dx\,\ln[1-x(1-x)/z]}{\ln\frac{\Lambda^2}{m_U^2}-1}\right]$$
$$<2\frac{d_t(R)}{d_U(R)}\leq 2<2\frac{m_U^2}{m_t^2}\cdot\frac{3m_U+m_t}{m_U+3m_t} \quad \text{if } m_U>m_t\,,\quad d_U(R)\geq d_t(R)\,, \tag{12}$$

where we have used the facts that both the quantities

$$\lambda=\frac{m_U^2}{m_t^2} \quad\text{and}\quad z=\frac{m_t^2}{m^2_{\phi^0_s}} \tag{13}$$

have the order of magnitude of unity and the assumption that $\Lambda^2 \gg m_U^2$ as well. This shows that condition (11) is surely satisfied and the mass constraint (9) is valid indeed. The mass constraint (9) could also be proven from the mass formula (7) by the same method.

In order to obtain the numerical values of $m_{\phi^0_s}$ in the model, we may integrate out x in the integral of $K_Q(m^2_{\phi^0_s})$ and reduce Eq. (4) to the form

$$\frac{\ln\frac{\Lambda^2}{m_t^2}+1+\sqrt{1-4z}\,\ln\left[\frac{1-\sqrt{1-4z}}{2z}-1\right]}{\ln\frac{\Lambda^2}{m_U^2}+1-\sqrt{4\alpha z-1}\arctan\frac{\sqrt{4\lambda z-1}}{2\lambda z-1}} = \frac{2d_U(R)\lambda(4\lambda z-1)}{3(1-4z)} . \tag{14}$$

We will take $m_t=160$ GeV, $d_U(R)=3+1$ and the corresponding values of Λ and m_U from Table I in Ref. [7], but the values of Λ will be limited to the acceptable region 10^6–5×10^3 GeV. In Table I we list the numerical results of $m_{\phi^0_s}$ obtained by Eqs. (14) and (7), respectively.

It is seen from Table I that $m_{\phi^0_s}$ varies as Λ and m_U rapidly. When Λ is high, for instance, $\Lambda>10^6$ GeV, the values of $m_{\phi^0_s}$ given by the exact equation (14) and those by the approximate formula (7) are almost identical. Only at the lower values of Λ do some small discrepancies between both results appear and gradually enlarge as Λ descends. They indicate the approximate precision of formula (7). In the last line of Table I we also list the values of m_U+m_t. By comparing them with the results of $m_{\phi^0_s}$, we may not only verify the correctness of the mass constraint (9) but also find that the bigger m_U is, the smaller is the ratio $(2m_U-m_{\phi^0_s})/(m_U-m_t)$. This fact clearly displays the tendency that $m_{\phi^0_s}$ more and more approaches $2m_U$ as m_U increases. The full results above show that the determination of the Higgs boson mass will be the most important experimental test of the model which contains the heavy U fermions.

TABLE I. The numerical results of the Higgs boson mass $m_{\phi^0_s}$ for acceptable momentum cutoff Λ and corresponding heavy fermion mass m_U. $m_t=160$ GeV and $d_U(R)=4$ are taken.

Λ	(GeV)	10^6	10^5	10^4	5×10^3
m_U	(GeV)	163	205	294	353
$m_{\phi^0_s}$ by Eq. (14)	(GeV)	324.42	395.23	569.0	688.66
$m_{\phi^0_s}$ by Eq. (7)	(GeV)	324.42	394.82	566.94	685.64
m_U+m_t	(GeV)	323	365	454	513

III. MASS EQUATION OF W BOSON

The mass equation of the W boson has been obtained [8] by setting the inverse propagator of the W-boson including the insertion of the composite charged Goldstone bosons in the vacuum polarizations to be equal to zero and has the form

$$m_W^2=\bar{f}^2(p^2)\bar{g}_2^2(p^2)|_{p^2=m_W^2} , \tag{15}$$

where

$$\bar{f}^2(p^2)=\sum_{\alpha=1}^{n}\frac{d_{Q_\alpha}(R)}{16\pi^2}\int_0^1 dx[m_{U_\alpha}^2(1-x)+m_{D_\alpha}^2 x]\left[\ln\frac{\Lambda^2}{M_{U_\alpha D_\alpha}^2(p^2)}-1\right], \tag{16}$$

$$\frac{1}{\bar{g}_2^2(p^2)}=\frac{1}{g_2^2}+\sum_{\alpha=1}^{n}\frac{d_{Q_\alpha}(R)}{8\pi^2}\int_0^1 dx\, x(1-x)\left[\ln\frac{\Lambda^2}{M_{U_\alpha D_\alpha}^2(p^2)}-1\right], \tag{17}$$

$$M_{U_\alpha D_\alpha}^2(p^2)=m_{U_\alpha}^2(1-x)+m_{D_\alpha}^2 x-p^2x(1-x)\,. \tag{18}$$

The squared classical $SU_L(2)$ gauge coupling constant g_2^2 in Eq. (17) may be replaced by some boundary value of the running coupling $\bar{g}_2^2(p^2)$, e.g., at $p^2=0$ through the relation

$$\frac{1}{g_2^2}=\frac{1}{\bar{g}_2^2(0)}-\sum_{\alpha=1}^{n}\frac{d_{Q_\alpha}(R)}{8\pi^2}\int_0^1 dx\, x(1-x)\left[\ln\frac{\Lambda^2}{M_{U_\alpha D_\alpha}^2(0)}-1\right]; \tag{19}$$

thus $1/\bar{g}_2^2(m_W^2)$ may be reexpressed by

$$\frac{1}{\bar{g}_2^2(m_W^2)}=\frac{1}{\bar{g}_2^2(0)}+\sum_{\alpha=1}^{n}\frac{d_{Q_\alpha}(R)}{8\pi^2}\int_0^1 dx\, x(1-x)\ln\frac{1-(1-\gamma_{U_\alpha})x}{1-(1-\gamma_{U_\alpha}+a_{U_\alpha})x+a_{U_\alpha}x^2} \tag{20}$$

with the denotations

$$a_{U_\alpha}=\frac{m_W^2}{m_{U_\alpha}^2}\quad\text{and}\quad\gamma_{U_\alpha}=\frac{m_{D_\alpha}^2}{m_{U_\alpha}^2}\,. \tag{21}$$

It is seen from Eq. (20) that $1/\bar{g}_2^2(m_W^2)$ does not depend explicitly upon the momentum cutoff Λ which has been eliminated by the subtraction procedure. After integrating out x, the general expressions for $\bar{f}^2(m_W^2)$ and $1/\bar{g}_2^2(m_W^2)$ become

$$\begin{aligned}\bar{f}^2(m_W^2)=\sum_{\alpha=1}^{n}\frac{d_{Q_\alpha}(R)}{32\pi^2}m_{U_\alpha}^2\Bigg[&(1+\gamma_{U_\alpha})\left[\ln\frac{\Lambda^2}{m_{U_\alpha}^2}+1\right]+\left[\frac{1-\gamma_{U_\alpha}}{2a_{U_\alpha}^2}\Delta_\alpha-\gamma_{U_\alpha}\right]\ln\gamma_{U_\alpha}-\frac{(1-\gamma_{U_\alpha})^2}{a_{U_\alpha}}\\&\mp\frac{1}{a_{U_\alpha}}\left[1+\gamma_{U_\alpha}-\frac{(1-\gamma_{U_\alpha})^2}{a_{U_\alpha}}\right]\sqrt{\pm\Delta_\alpha}\\&\times\begin{cases}\arctan\sqrt{B_\alpha^2-1} & \text{if } B_\alpha^2>1\,,\\ \ln\dfrac{|\varepsilon(B_\alpha)+\sqrt{1-B_\alpha^2}\,|}{|B_\alpha|} & \text{if } B_\alpha^2<1\,,\end{cases}\Bigg],\end{aligned} \tag{22}$$

$$\begin{aligned}\frac{1}{\bar{g}_2^2(m_W^2)}=\frac{1}{\bar{g}_2^2(0)}+\sum_{\alpha=1}^{n}\frac{d_{Q_\alpha}(R)}{48\pi^2}\Bigg[&\frac{5}{6}+\frac{2\gamma_{U_\alpha}}{(1-\gamma_{U_\alpha})^2}+\frac{2(1+\gamma_{U_\alpha})}{a_{U_\alpha}}-\frac{2(1-\gamma_{U_\alpha})^2}{a_{U_\alpha}^2}\\&+\left[\frac{(3-\gamma_{U_\alpha})\gamma_{U_\alpha}^2}{(1-\gamma_{U_\alpha})^3}+\frac{3(1-\gamma_{U_\alpha})^2}{2a_{U_\alpha}^2}-\frac{(1-\gamma_{U_\alpha})^3}{a_{U_\alpha}^3}-\frac{1}{2}\right]\ln\gamma_{U_\alpha}\\&\mp\left[1+\gamma_{U_\alpha}+a_{U_\alpha}-\frac{2(1-\gamma_{U_\alpha})^2}{a_{U_\alpha}}\right]\frac{\sqrt{\pm\Delta_\alpha}}{a_{U_\alpha}^2}\times\begin{cases}\arctan\sqrt{B_\alpha^2-1} & \text{if } B_\alpha^2>1\\ \ln\dfrac{|\varepsilon(B_\alpha)+\sqrt{1-B_\alpha^2}\,|}{|B_\alpha|} & \text{if } B_\alpha^2<1\end{cases}\Bigg],\end{aligned} \tag{23}$$

where

$$\Delta_\alpha=4a_{U_\alpha}-(1-\gamma_{U_\alpha}+a_{U_\alpha})^2\,,\quad B_\alpha=\frac{2\gamma_{U_\alpha}^{1/2}}{1+\gamma_{U_\alpha}-a_{U_\alpha}}\,,\quad\text{and}\quad\varepsilon(B_\alpha)=\frac{B_\alpha}{|B_\alpha|}\,. \tag{24}$$

It seems by expressions (22) and (23) that m_W^2 will depend upon Λ not only implicitly through the heavy fermion mass $m_U(\Lambda)$ but also explicitly through $\ln(\Lambda^2/m_{U_\alpha}^2)$ in $\bar{f}^2(m_W^2)$. However, we will show that m_W contains no explicit dependence upon Λ completely and its dependence upon the heavy fermion masses only appears in nonleading contributions to $1/\bar{g}_2^2(m_W^2)$ as well.

Let us still take the model with three quark-lepton generations and the fourth U-fermion generation. Since each term in $\bar{f}^2(m_W^2)$ is proportional to the corresponding squared fermion mass, we can neglect all fermion masses except m_U and m_t in Eq. (22) and obtain that

$$\bar{f}^2(m_W^2)=\frac{1}{32\pi^2}\left[2d_U(R)m_U^2\left[\ln\frac{\Lambda^2}{m_U^2}-1+U(a_U)\right]+d_t(R)m_t^2\left[\ln\frac{\Lambda^2}{m_t^2}-\frac{1}{2}+T(a_t)\right]\right], \tag{25}$$

where

$$U(a_U)=2-\frac{1}{a_U}\sqrt{a_U(4-a_U)}\arctan\frac{\sqrt{a_U(4-a_U)}}{2-a_U}\;,\quad a_U=\frac{m_W^2}{m_U^2}\;, \tag{26}$$

$$T(a_t)=\frac{3}{2}-\frac{1}{a_t}-\frac{(1-a_t)^2}{a_t^2}\ln(1-a_t)\;,\quad a_t=\frac{m_W^2}{m_t^2}\;. \tag{27}$$

For the allowed values of m_U and m_t [7] and the experimental value of m_W [10] we give the estimations of the orders of magnitude of $U(a_U)$ and $T(a_t)$ in Table II.

The results in Table II indicate that, in the whole region $10^6-5\times10^3$ GeV of the acceptable momentum cutoff Λ, we always have

$$U(a_U)\Big/\left[\ln\frac{\Lambda^2}{m_U^2}-1\right]\ll 1 \quad\text{and}\quad T(a_t)\Big/\left[\ln\frac{\Lambda^2}{m_t^2}-\frac{1}{2}\right]\ll 1\;. \tag{28}$$

Therefore, it is a very good approximation to neglect $U(a_U)$ and $T(a_t)$ from formula (25) and to write $\bar{f}^2(m_W^2)$ as

$$\bar{f}^2(m_W^2)\simeq\frac{1}{32\pi^2}\left[2d_U(R)m_U^2\left[\ln\frac{\Lambda^2}{m_U^2}-1\right]+d_t(R)m_t^2\left[\ln\frac{\Lambda^2}{m_t^2}-\frac{1}{2}\right]\right]=\bar{f}^2(0)\;; \tag{29}$$

i.e., $\bar{f}^2(m_W^2)$ can be safely replaced by $\bar{f}^2(0)$. Then by means of the basic relation (1), the mass equation (15) may be expressed through the classical vacuum expectation value v as

$$m_W^2=\frac{v^2}{4}\bar{g}_2^2(m_W^2) \tag{30}$$

which exactly has the form consistent with the standard electroweak model. Therefore, the squared W boson mass, as $\bar{g}_2^2(m_W^2)$ indicated above, does not explicitly rely on the momentum cutoff Λ as well.

The $v^2/4$ in Eq. (30), or say $\bar{f}^2(0)\simeq\bar{f}^2(m_W^2)$ by Eqs. (1) and (29), is mainly contributed by the heavy U fermions and t quarks and the contributions from the lighter fermions are always negligible. Physically this means that only the condensates of these heavy fermions dominate the spontaneous symmetry breaking. Conversely, to the sum in $1/\bar{g}_2^2(m_W^2)$ in Eq. (30) we will show that the contributions from the heavy fermions could be omitted but the light fermions, e.g., ordinary light quarks and leptons, will give the leading contributions. The reason is that the sum in $1/\bar{g}_2^2(m_W^2)$ merely comes from pure fermion loops without the insertions of the composite Goldstone modes in the vacuum polarizations [8]. Since the light fermions including massless neutrinos should not be ignored in $1/\bar{g}_2^2(m_W^2)$, the fermion condensate scheme in which only the heavy U fermions and/or the (t,b) doublets are included cannot be regarded as a theoretically complete and consistent one. In order to make a comparison between the contributions to $1/\bar{g}_2^2(m_W^2)$ from the light and the heavy fermions, we notice that

$$\begin{aligned}&m_U,m_t>m_W,\\&m_W^2\gg m_Q^2\quad\text{for } Q=u,d,s,c,b,e,\mu,\tau\;,\\&m_Q=0\quad\text{for } Q=\nu_e,\nu_\mu,\nu_\tau\;.\end{aligned} \tag{31}$$

Then it may be obtained from Eq. (23) that

TABLE II. Estimations of order of magnitude of $U(a_U)$ and $T(a_t)$ for an acceptable momentum cutoff Λ. Taking $m_t=160$ GeV, m_U to be the values satisfying Eq. (1), and $m_W=80.6$ GeV by experiments.

Λ (GeV)	10^6	10^5	10^4	5×10^3
m_U (GeV)	163	205	294	353
$U(a_U)/[\ln(\Lambda^2/m_U^2)-1]$	2.5×10^{-3}	2.3×10^{-3}	2.1×10^{-3}	2.0×10^{-3}
$T(a_t)/[\ln(\Lambda^2/m_t^2)-1/2]$	5.3×10^{-3}	7.3×10^{-3}	1.2×10^{-2}	1.4×10^{-2}

$$\frac{1}{\bar{g}_2^2(m_W^2)}=\frac{1}{\bar{g}_2^2(0)}+\frac{1}{48\pi^2}\left[d_U(R)P(a_U)+d_t(R)N(a_t)+\sum_{Q=u,c}^{e,\mu,\tau}d_Q(R)\left[\frac{5}{6}-\ln\frac{m_W^2}{m_Q^2}\right]+\sum_{U_\alpha=u,c}d_{U_\alpha}(R)H(\gamma_{U_\alpha})\right], \tag{32}$$

where

$$P(a_U)=\frac{5}{3}+\frac{4}{a_U}-\frac{2+a_U}{a_U^2}\sqrt{a_U(4-a_U)}\arctan\frac{\sqrt{a_U(4-a_U)}}{2-a_U}, \tag{33}$$

$$N(a_t)=\frac{5}{6}-\frac{2(1-a_t)}{a_t^2}-\frac{(1-a_t)^2(a_t+2)}{a_t^3}\ln(1-a_t), \tag{34}$$

$$H(\gamma_{U_\alpha})=\frac{2\gamma_{U_\alpha}}{(1-\gamma_{U_\alpha})^2}+\frac{(3-\gamma_{U_\alpha})\gamma_{U_\alpha}^2}{(1-\gamma_{U_\alpha})^3}\ln\gamma_{U_\alpha}. \tag{35}$$

If we take $m_U=163$ GeV ($\Lambda=10^6$ GeV), $m_t=160$ GeV, $m_W=80.6$ GeV, and

$$m_u=4.2\times10^{-3}\text{ GeV},\quad m_c=1.2\text{ GeV},\quad m_e=0.5\times10^{-3}\text{ GeV},\quad m_\mu=0.106\text{ GeV},\quad m_\tau=1.781\text{ GeV}, \tag{36}$$

then the ratios between the different terms in the square brackets of Eq. (32) can be calculated and turn out to be that

$$d_U(R)P(a_U):d_t(R)N(a_t):\sum_{Q=u,c}^{e,\mu,\tau}\frac{5}{6}d_Q(R):-\sum_{Q=u,c}^{e,\mu,\tau}d_Q(R)\ln\frac{m_W^2}{m_Q^2}:\sum_{U_\alpha=u,c}d_{U_\alpha}(R)H(\gamma_{U_\alpha})=0.20:0.41:7.50:-129.24:4.71. \tag{37}$$

They show that the leading terms are those proportional to $\ln(m_W^2/m_Q^2)$ ($Q=u,c,e,\mu,\tau$) and come from only the light fermions. On the other hand, the heavy fermion terms $d_U(R)P(a_U)$ and $d_t(R)N(a_t)$ are almost negligible. We note that $P(a_U)$ and $N(a_t)$ may have the expressions

$$P(a_U)=\frac{m_W^2}{m_U^2}g\left[\frac{m_W^2}{m_U^2}\right]\quad\text{with } g(0)=\text{const}, \tag{38}$$

$$N(a_t)=\frac{m_W^2}{m_t^2}f\left[\frac{m_W^2}{m_t^2}\right]\quad\text{with } f(0)=\text{const}; \tag{39}$$

hence, the strong suppression of the heavy fermion terms is only due to the general principle of decoupling.

After appropriately redefining the boundary value $\bar{g}_2^2(\mu^2)$ at some low energy scale μ by absorbing the constant terms and the small terms in Eq. (32) we will find that the running coupling $\bar{g}_2^2(m_W^2)$ is consistent with the contributions of two generation light quark doublets and three generation lepton doublets to the usual β functions. The conventional renormalization-group evolution of g_2 at the scale $p^2\sim m_W^2$ possesses the form [11]

$$\frac{1}{\bar{g}_2^2(m_W^2)}=\frac{1}{\bar{g}_2^2(\mu^2)}+\frac{1}{16\pi^2}\frac{1}{3}\left[22-\sum_{Q=u,c}^{e,\mu,\tau}d_Q(R)-\frac{1}{2}\right]\ln\frac{m_W^2}{\mu^2}. \tag{40}$$

The coefficient before $\ln m_W^2/16\pi^2$ in Eq. (32) is obviously the same as the fermion loop contributions to the β function in Eq. (40). Therefore, when determining the $\bar{g}_2^2(m_W^2)$ in the mass equation (30) of the W boson we can use the conventional renormalization-group evolution of g_2.

A few more remarks should be made upon the original mass equation (15). It is noted that the $1/g_2^2$ term in Eq. (17) comes from the tree contribution to inverse propagator of the W boson and is non-negligible in a gauged $SU_L(2)\times U_Y(1)$ model. However, if the gauge bosons are considered as composites of fermions, as was made in Ref. [3], then the W boson would have no kinetic terms, or equivalently, $g_2\to\infty$; thus, the tree contribution to Eq. (17) would disappear. Considering that $m_W^2/M_{U_\alpha D_\alpha}^2(m_W^2)$ has the order of magnitude of unity, we can take the approximation that

$$\ln\frac{\Lambda^2}{M_{U_\alpha D_\alpha}^2(m_W^2)}=\ln\frac{\Lambda^2}{m_W^2}+\ln\frac{m_W^2}{M_{U_\alpha D_\alpha}^2(m_W^2)}\simeq\ln\frac{\Lambda^2}{m_W^2} \tag{41}$$

and then reduce the mass equation (15) to the mass-sum formula

$$m_W^2\simeq3\sum_Q d_Q(R)m_Q^2\Big/\sum_Q d_Q(R) \tag{42}$$

which coincides with the formula (3.12) in Ref. [3]. In the $\Lambda\rightarrow\infty$ limit Eq. (42) is valid rigorously. Equation (42) is quite similar to the basic relation (1), but there is an important difference between them. Equation (1) allows the existence of the heavier fermions than the top quarks for the limited values of Λ; however, Eq. (42) does not tolerate the similar result. In fact, by inputting into Eq. (42) the experimental value of m_W, it is not difficult to find that when we take $m_t\sim 130-150$ GeV no heavier fermions than the top quarks, even only a single heavier lepton doublet, are allowed to exist, though Eq. (42) could predict the t-quark mass of 131.6 GeV [12] for the model with only the three generations of quarks and leptons. Such a fact maybe represents an essential difference between the gauged and the composite electroweak boson models.

IV. MASS EQUATION OF Z BOSON

The mass equation of the Z boson comes from the condition in which the inverse propagator of the Z boson, including insertions of the composite neutral pseudoscalar Goldstone boson in the vacuum polarizations, is equal to zero and takes the form [8]

$$m_Z^2=f^2(p^2)[g_1^2(p^2)+g_2^2(p^2)]|_{p^2=m_Z^2}\ , \tag{43}$$

where

$$f^2(p^2)=\frac{1}{32\pi^2}\sum_Q d_Q(R)m_Q^2I_1^Q(p^2)+\frac{p^2}{96\pi^2}\sum_Q d_Q(R)Y_{Q_L}\delta_Q I_2^Q(p^2)\ , \tag{44}$$

$$\frac{1}{g_1^2(p^2)}=\frac{1}{g_1^2}+\frac{1}{48\pi^2}\sum_Q d_Q(R)(Y_{Q_L}^2+\tfrac{3}{2}Y_{Q_L}\delta_Q+\tfrac{1}{2})I_2^Q(p^2) \tag{45a}$$

$$=\frac{1}{g_1^2(0)}+\frac{1}{48\pi^2}\sum_Q d_Q(R)(Y_{Q_L}^2+\tfrac{3}{2}Y_{Q_L}\delta_Q+\tfrac{1}{2})J_2^Q(p^2)\ , \tag{45b}$$

$$\frac{1}{g_2^2(p^2)}=\frac{1}{g_2^2}+\frac{1}{96\pi^2}\sum_Q d_Q(R)(Y_{Q_L}\delta_Q+1)I_2^Q(p^2) \tag{46a}$$

$$=\frac{1}{g_2^2(0)}+\frac{1}{96\pi^2}\sum_Q d_Q(R)(Y_{Q_L}\delta_Q+1)J_2^Q(p^2)\ , \tag{46b}$$

$$I_1^Q(p^2)=\int_0^1 dx\left[\ln\frac{\Lambda^2}{M_Q^2(p^2)}-1\right]\ , \tag{47}$$

$$I_2^Q(p^2)=6\int_0^1 dx\,x(1-x)\left[\ln\frac{\Lambda^2}{M_Q^2(p^2)}-1\right]\ , \tag{48a}$$

$$J_2^Q(p^2)=I_2^Q(p^2)-I_2^Q(0)\ , \tag{48b}$$

$$M_Q^2(p^2)=m_Q^2-p^2x(1-x)\ , \tag{49}$$

and Y_{Q_L} is the $U_Y(1)$ charge of the left-handed Q fermions and the sign function

$$\delta_Q=\begin{cases}1 & \text{for } Q=U_\alpha\\ -1 & \text{for } Q=D_\alpha\end{cases}\quad(\alpha=1,\ldots,n)\ . \tag{50}$$

In Eqs. (45b) and (46b) we have replaced the classical gauge couplings $1/g_1^2$ and $1/g_2^2$ by the boundary values $1/g_1^2(0)$ and $1/g_2^2(0)$ of respective running couplings.

The mass equation (43) may be rewritten by means of Eqs. (44)–(46) as

$$m_Z^2=\bar{f}^2(m_Z^2)\Bigg/\left[\frac{1}{g_1^2(m_Z^2)+g_2^2(m_Z^2)}-l(m_Z^2)\right]\ , \tag{51}$$

where

$$\bar{f}^2(m_Z^2)=\frac{1}{32\pi^2}\sum_Q d_Q(R)m_Q^2I_1^Q(m_Z^2) \tag{52}$$

and

$$l(m_Z^2)=\frac{1}{96\pi^2}\sum_Q d_Q(R)Y_{Q_L}\delta_Q I_2^Q(m_Z^2)\ . \tag{53}$$

In the limit of rigorous weak isospin symmetry we will have

$$l(m_Z^2)=0,\qquad \bar{f}^2(m_Z^2)=\bar{f}^2(m_Z^2)\ ,\ \text{ if } m_{U_\alpha}=m_{D_\alpha}\ \text{ for } \alpha=1,\ldots,n\ , \tag{54}$$

where $\bar{f}^2(m_Z^2)$ can be obtained by Eq. (16).

In the same approximation as Eq. (29) (it remains to be valid when m_W^2 is replaced by m_Z^2), we may reduce Eq. (43) to the standard form

$$m_Z^2=\frac{v^2}{4}[g_1^2(m_Z^2)+g_2^2(m_Z^2)]\ . \tag{55}$$

However, spontaneous symmetry breaking will lead to $m_{U_\alpha}\neq m_{D_\alpha}$; hence, Eqs. (54) and (55) are not valid rigorously. Here, among other things, $l(m_Z^2)$ represents an explicit weak isospin-breaking term. Its nonzero value is mainly contributed by the mass difference between the U_α and D_α when either or both of them are heavy fermions (the mass difference between the light U_α and the light D_α is completely negligible for the scale $m_Z \gg m_{U_\alpha}, m_{D_\alpha}$).

For the sake of discussing the weak isospin breaking effects involved in Eq. (51), i.e., in $\bar{f}^2(m_Z^2)$, $l(m_Z^2)$, $g_1^2(m_Z^2)$, and $g_2^2(m_Z^2)$, we integrate out x and give the algebraic expressions for $I_1^Q(m_Z^2)$, $I_2^Q(m_Z^2)$, and $J_2^Q(m_Z^2)$ as

$$I_1^Q(m_Z^2)=\ln\frac{\Lambda^2}{m_Q^2}+1\mp\begin{cases}\sqrt{4/\bar{a}_Q-1}\ \arctan\dfrac{\sqrt{\bar{a}_Q(4-\bar{a}_Q)}}{2-\bar{a}_Q}\ , & \bar{a}_Q<4\ ,\\ \sqrt{1-4/\bar{a}_Q}\ln\left[\dfrac{\bar{a}_Q-\sqrt{\bar{a}_Q(\bar{a}_Q-4)}}{2}-1\right]\ , & \bar{a}_Q>4\ ,\end{cases} \tag{56}$$

$$I_2^Q(m_Z^2)=\ln\frac{\Lambda^2}{m_Z^2}-1+J_2^Q(m_Z^2)\ , \tag{57}$$

$$J_2^Q(m_Z^2)=\frac{5}{3}+\frac{4}{\bar{a}_Q}\mp\frac{2+\bar{a}_Q}{\bar{a}_Q}\begin{cases}\sqrt{4/\bar{a}_Q-1}\ \arctan\dfrac{\sqrt{\bar{a}_Q(4-\bar{a}_Q)}}{2-\bar{a}_Q}\ , & \bar{a}_Q<4\ ,\\ \sqrt{1-4/\bar{a}_Q}\ln\left[\dfrac{\bar{a}_Q-\sqrt{\bar{a}_Q(\bar{a}_Q-4)}}{2}-1\right]\ , & \bar{a}_Q>4\ ,\end{cases} \tag{58}$$

where we have used the denotation

$$\bar{a}_Q=m_Z^2/m_Q^2\ . \tag{59}$$

For the considered model with the four generations of fermions, similar to $\bar{f}^2(m_W^2)$ in Eq. (25) we can keep in $\bar{f}^2(m_Z^2)$ only the U-fermion and the t-quark terms and then express it by

$$\bar{f}^2(m_Z^2)=\bar{f}^2(0)+\frac{1}{32\pi^2}[2d_U(R)m_U^2J_1^U(m_Z^2)+d_t(R)m_t^2J_1^t(m_Z^2)]\ , \tag{60}$$

where

$$J_1^Q(m_Z^2)=I_1^Q(m_Z^2)-I_1^Q(0)=2-\left[\frac{4}{\bar{a}_Q}-1\right]^{1/2}\arctan\frac{\sqrt{\bar{a}_Q(4-\bar{a}_Q)}}{2-\bar{a}_Q}\ ,\quad Q=U,t\ . \tag{61}$$

It is easy to verify that $J_1^U(m_Z^2)$ and $J_1^t(m_Z^2)$ are so small as to be negligible for the terms containing $\ln\Lambda^2$ in $\bar{f}^2(0)$. In fact, when we take $m_t=160$ GeV, $m_U=163$ GeV ($\Lambda=10^6$ GeV) [7], and the experimental value $m_Z=91.161$ GeV [10], it follows that

$$J_1^U(m_Z^2)=0.054\ \text{ and }\ J_1^t(m_Z^2)=0.056\ . \tag{62}$$

As a result we can make the approximation

$$\bar{f}^2(m_Z^2)\simeq\bar{f}^2(0)=f^2(0)=\frac{1}{32\pi^2}\left[2d_U(R)m_U^2\left[\ln\frac{\Lambda^2}{m_U^2}-1\right]+d_t(R)m_t^2\left[\ln\frac{\Lambda^2}{m_t^2}-1\right]\right]\ . \tag{63}$$

However, $f^2(0)$ does not coincide with $\bar{f}^2(0)$ in Eq. (29) which is used to define the vacuum expectation value v; the difference

$$\bar{f}^2(0)-f^2(0)=\frac{1}{64\pi^2}d_t(R)m_t^2 \tag{64}$$

represents the effect of the weak isospin breaking caused by $m_t\neq m_b$, as indicated in Refs. [2,13]. In fact, if $m_t=m_b$, then it may be seen by comparing Eq. (16) with Eq. (44) that $\bar{f}^2(0)$ and $f^2(0)$ will be identical in the approximation of keeping only the U and t fermions. Here the U fermions do not lead to the weak isospin breaking just owing to the assumption that their masses are degenerate in the bubble approximation.

The isospin breaking term $l(m_Z^2)$ will not only make the mass equation (51) deviate from the standard form, but also bring about an extra change of the β function determining the running gauge couplings $g_1^2(m_Z^2)$ and $g_2^2(m_Z^2)$, since it is also contained in Eqs. (45) and (46). In order to show this effect, we need to deal with the heavy and the light fermions separately once again. Considering the fact that

$$\bar{a}_Q=\frac{m_Z^2}{m_Q^2}\begin{cases}\gg 1 & \text{for } Q=u,d,s,c,b,e,\mu,\tau\ ,\\ <1 & \text{for } Q=U,t\end{cases} \tag{65}$$

we obtain from Eqs. (57) and (58) that

$$I_2^Q(m_Z^2)=\begin{cases}\ln\dfrac{\Lambda^2}{m_Z^2}+\dfrac{2}{3} & \text{for } Q=u,d,s,c,b,e,\mu,\tau,\nu_e,\nu_\mu,\nu_\tau \quad (66a)\\[2ex] \ln\dfrac{\Lambda^2}{m_Q^2}-1+P(\bar a_Q) & \text{for } Q=U,t \quad (66b)\end{cases}$$

and

$$J_2^Q(m_Z^2)=\begin{cases}\dfrac{5}{3}-\ln\dfrac{m_Z^2}{m_Q^2} & \text{for } Q=u,d,s,c,b,e,\mu,\tau\ , \quad (67a)\\[2ex] P(\bar a_Q) & \text{for } Q=U,t\ , \quad (67b)\end{cases}$$

where the function $P(\bar a_Q)$ has the same form as the one given by Eq. (33). Substituting Eqs. (66) and (67) into Eqs. (45b), (46b), and (53) we may obtain that

$$\frac{1}{g_1^2(m_Z^2)}=\frac{1}{g_1^2(0)}+\frac{1}{48\pi^2}\left[\sum_{Q=U,D}^{N,E,t} d_Q(R)(Y_{Q_L}^2+\tfrac{3}{2}Y_{Q_L}\delta_Q+\tfrac{1}{2})P(\bar a_Q) + \sum_{Q=u,d,s,c,b}^{e,\mu,\tau} d_Q(R)(Y_{Q_L}^2+\tfrac{3}{2}Y_{Q_L}\delta_Q+\tfrac{1}{2})\left[\frac{5}{3}-\ln\frac{m_Z^2}{m_Q^2}\right]\right], \quad (68)$$

$$\frac{1}{g_2^2(m_Z^2)}=\frac{1}{g_2^2(0)}+\frac{1}{48\pi^2}\left[\sum_{Q=U,D}^{N,E,t} d_Q(R)\tfrac{1}{2}(Y_{Q_L}\delta_Q+1)P(\bar a_Q)+\sum_{Q=u,d,s,c,b}^{e,\mu,\tau} d_Q(R)\tfrac{1}{2}(Y_{Q_L}\delta_Q+1)\times\left[\frac{5}{3}-\ln\frac{m_Z^2}{m_Q^2}\right]\right], \quad (69)$$

and

$$l(m_Z^2)=\frac{1}{96\pi^2}\left[\ln\frac{m_Z^2}{m_t^2}-\frac{5}{3}+P(\bar a_t)\right], \quad (70)$$

where (U,D) and (N,E), respectively, denote the quark and lepton doublets of the fourth generation of fermions. It is noted that no contribution from the massless neutrinos appears in Eqs. (68) and (69) owing to the fact that

$$Y_{Q_L}^2+\tfrac{3}{2}Y_{Q_L}\delta_Q+\tfrac{1}{2}=Y_{Q_L}\delta_Q+1=0 \quad \text{for } Q=\nu_e,\nu_\mu,\nu_\tau\ , \quad (71)$$

This makes us avoid the unpleasant mass singularities. The weak isospin breaking term $l(m_Z^2)$ now contains only the contribution from the (t,b) doublets with the large mass difference m_t-m_b. The U fermions with the degenerate masses and all the other lighter fermions than the b quarks in the approximation (66a) will not give a nonzero contribution to $l(m_Z^2)$.

Let us compare the running couplings $1/g_1^2(m_Z^2)$ and $1/g_2^2(m_Z^2)$ represented, respectively, by Eqs. (68) and (69) with their conventional renormalization-group evolutions. The main differences between both may be stated as follows.

(1) In Eqs. (68) and (69) there are contributions coming from the fermions with the mass $m_Q>m_Z$. They are represented by

$$P(\bar a_Q)=\frac{m_Z^2}{m_Q^2}g\left[\frac{m_Z^2}{m_Q^2}\right] \quad (Q=U,t) \quad \text{with } g(0)=\text{const}\ . \quad (72)$$

However, as in the case of $1/\bar g_2^2(m_W^2)$, because of the decoupling they only have very small values and may be omitted from these equations.

(2) The leading contributions to Eqs. (68) and (69) come from the light fermions and are proportional to $\ln m_Z^2$. Does the coefficient before $\ln m_Z^2/16\pi^2$ coincide with the fermion loop contributions to the usual β functions responsible for the renormalization-group evolutions of g_1 and g_2? In order to answer this question, first let us consider $1/g_1^2(m_Z^2)$. The coefficient before $\ln m_Z^2/16\pi^2$ in Eq. (68) may be written as

$$\tilde{\beta}_1^f = -\frac{1}{3}\sum_{Q=u,d,s,c,b}^{e,\mu,\tau} d_Q(R)(Y_{Q_L}^2+\tfrac{3}{2}Y_{Q_L}\delta_Q+\tfrac{1}{2})$$
$$= -\frac{1}{3}\left[\sum_{\alpha=1}^{3} d_{Q_\alpha}(R)(Y_{U_{\alpha L}}^2+Y_{D_{\alpha L}}^2+1)\right.$$
$$\left.-d_t(R)(Y_{t_L}^2+\tfrac{3}{2}Y_{t_L}+\tfrac{1}{2})\right]$$
$$=\beta_1^f+\tfrac{1}{6}d_t(R)Y_{t_L}=\beta_1^f+\tfrac{1}{6}d_b(R)Y_{b_L}\ , \tag{73}$$

where the relations [8]

$$Y_{Q_{\alpha R}}=Y_{Q_{\alpha L}}\pm 1 \quad \text{for} \quad \begin{cases} Q=U_\alpha\ ,\\ Q=D_\alpha\ ,\end{cases} \tag{74}$$

have been used and the denotation

$$\beta_1^f=-\frac{1}{6}\left[\sum_{\alpha=1}^{3} d_{Q_\alpha}(R)(Y_{U_{\alpha L}}^2+Y_{U_{\alpha R}}^2+Y_{D_{\alpha L}}^2+Y_{D_{\alpha R}}^2)\right.$$
$$\left.-d_t(R)(Y_{t_L}^2+Y_{t_R}^2)\right] \tag{75}$$

represents the loop contributions of the three generations of quark leptons except the t quarks below the scale m_Z to the usual β_1 function in the renormalization-group evolution:

$$\frac{1}{g_1^2(m_Z^2)}=\frac{1}{g_1^2(\mu^2)}+\frac{\beta_1}{16\pi^2}\ln\frac{m_Z^2}{\mu^2}\ . \tag{76}$$

We note that

$$\Delta\beta_1^f=\tilde{\beta}_1^f-\beta_1^f=\tfrac{1}{6}d_b(R)Y_{b_L}\neq 0 \tag{77}$$

and it effectively makes a $3/5$ reduction in the contribution of the b quarks to the usual β_1^f. This discrepancy between $\tilde{\beta}_1^f$ and β_1^f is due to the mass splitting between the members in the same (t,b) doublet with the feature that $m_t>m_Z>m_b$. As a result, $\tilde{\beta}_1^f$ contains only the contribution from the b quarks but none from the t quarks and the sum $\sum_Q d_Q(R)Y_{Q_L}\delta_Q$ will no longer be equal to zero. It is simply nonzero of this sum that brings about $\tilde{\beta}_1^f\neq\beta_1^f$. In fact, such an effect leading to an extra change of the β function may appear in the whole energy region with $4m_t^2>p^2>4m_b^2$ or generally in the energy regions with $4(m_{Q_\alpha})_{\max}^2>p^2>4(m_{Q_\alpha})_{\min}^2$ $(\alpha=1,\ldots,n)$. Here it is paid special attention only because the mass difference m_t-m_b has to do with the heavy t quarks and it is so large that such an effect seems not to be negligible. On the other hand, when one is determining $g_1^2(p^2)$ for $p^2>4m_t^2$, the corresponding β_1^f sector would have no difference from the one in the conventional renormalization-group evolution of g_1, even though there still exists $m_t\neq m_b$ in this case.

Next let us discuss $1/g_2^2(m_Z^2)$. The coefficient before $\ln m_Z^2/16\pi^2$ in Eq. (69) may be written by

$$\tilde{\beta}_2^f=-\frac{1}{6}\sum_{Q=u,d,s,c,b}^{e,\mu,\tau} d_Q(R)(Y_{Q_L}\delta_Q+1)$$
$$=-\frac{1}{6}\left[\sum_{\alpha=1}^{3}2d_{Q_\alpha}(R)-d_t(R)(Y_{t_L}+1)\right]\ . \tag{78}$$

We note, by comparing Eq. (78) with the corresponding factor in Eq. (40) for $1/\bar{g}_2^2(m_W^2)$, that we have increased the b-quark loop contribution in Eq. (78) and this is necessary for the determination of $1/g_2^2(m_Z^2)$.

Let β_2^f represent the loop contributions of the three generations of quark leptons except the t quarks to the conventional β_2 function in $1/g_2^2(m_Z^2)$; we will have

$$\beta_2^f=-\frac{1}{6}\left[\sum_{\alpha=1}^{3}2d_{Q_\alpha}(R)-d_t(R)\right]\ . \tag{79}$$

We may obtain from Eqs. (78) and (79) that

$$\Delta\beta_2^f=\tilde{\beta}_2^f-\beta_2^f=\tfrac{1}{6}d_b(R)Y_{b_L}\neq 0\ . \tag{80}$$

It effectively makes a 1/3 reduction in the contribution of the b quarks to the usual β_2^f. Such a change of the β_2 function in $1/g_2^2(m_Z^2)$ has the same origin as the change of the β_1 function in $1/g_1^2(m_Z^2)$. In addition, a similar change of β_2 function in $1/g_2^2(p^2)$ should be considered in the whole energy region with $4m_t^2>p^2>4m_b^2$, if one intends to make more precise calculations.

The above discussions indicate that, in the mass equation (43) of the Z boson, in addition to considering the other weak isospin breaking effects induced by $m_t\neq m_b$, we should express the fermion-loop contributions to the β functions in $g_1^2(m_Z^2)$ and $g_2^2(m_Z^2)$ by $\tilde{\beta}_1^f$ in Eq. (73) and $\tilde{\beta}_2^f$ in Eq. (78) instead of the usual β_1^f in Eq. (75) and β_2^f in Eq. (79), respectively. Such replacements should be maintained even after the complete gauge interactions and the Higgs dynamics are taken into account. However, we may point out that it is unnecessary to consider such modification for the usual running electric charge $e^2(m_Z^2)$. In fact, by Eqs. (45), (46), and (67), $e^2(m_Z^2)$ can be expressed by

$$\frac{1}{e^2(m_Z^2)}=\frac{1}{g_1^2(m_Z^2)}+\frac{1}{g_2^2(m_Z^2)}=\frac{1}{e^2(0)}+\frac{1}{12\pi^2}\left[\sum_{Q=U,D}^{E,t} d_Q(R)e_Q^2P(\bar{a}_Q)+\sum_{Q=u,d,s,c,b}^{e,\mu,\tau} d_Q(R)e_Q^2\left[\frac{5}{3}-\ln\frac{m_Z^2}{m_Q^2}\right]\right]\ , \tag{81}$$

where we have used the relation

$$e_Q=\tfrac{1}{2}(Y_{Q_L}+\delta_Q)\ . \tag{82}$$

The coefficient before $\ln m_Z^2/16\pi^2$ in Eq. (81),

$$-\frac{4}{3}\sum_{Q=u,d,s,c,b}^{e,\mu,\tau} d_Q(R)e_Q^2=-\tfrac{4}{3}\mathrm{Tr}\hat{Q}^2\ , \tag{83}$$

is obviously coincident with the fermion loop contributions below the scale m_Z to the β function in conventional renormalization group running of e. No effect of the weak isospin breaking induced by $m_t - m_b$ exists here.

We should also mention a distinction between the $SU_L(2)$ running couplings $\bar{g}_2^2(p^2)$ in Eq. (17) and $g_2^2(p^2)$ in Eq. (46a). From Eq. (18) and Eqs. (48a) and (49) it is easy to see that

$$\bar{g}_2^2(p^2)=g_2^2(p^2) \quad \text{only if } m_{U_\alpha}=m_{D_\alpha} \quad \text{or } p^2 \gg m_{U_\alpha}^2, m_{D_\alpha}^2 \quad (\alpha=1,\ldots,n) \tag{84}$$

which means that only if the effect of the weak isospin breaking is absent or it is negligible for very high momentum p^2, could $\bar{g}_2^2(p^2)$ and $g_2^2(p^2)$ just be considered the same one.

Finally we point out a mass sum formula similar to Eq. (42). If all of the electroweak gauge bosons are regarded as composites of the fermions, then they will not have the kinetic terms and the tree contributions $1/g_1^2$ and $1/g_2^2$ in Eqs. (45a) and (45b) could be ignored. Under the approximation

$$\ln\frac{\Lambda^2}{M_Q^2(m_Z^2)}=\ln\frac{\Lambda^2}{m_Z^2}+\ln\frac{m_Z^2}{M_Q^2(m_Z^2)}\simeq\ln\frac{\Lambda^2}{m_Z^2} \tag{85}$$

the mass equation (43) can be reduced to the mass sum formula

$$m_Z^2=3\left[\sum_Q d_Q(R)m_Q^2\right]\left[\sum_Q d_Q(R)4e_Q^2\right]\Big/\left[\sum_Q d_Q(R)(4e_Q^2-1)\right]\left[\sum_Q d_Q(R)\right], \tag{86}$$

where e_Q is defined by Eq. (82). In view of the relation

$$\sum_Q d_Q(R)(4e_Q^2-\tfrac{1}{2})=\frac{1}{2}\sum_{\alpha=1}^{n}\operatorname{tr}\hat{Y}_\alpha^2 , \tag{87}$$

where $\operatorname{tr}\hat{Y}_\alpha^2$ is the trace of the squared $U_Y(1)$ operator over the αth generation of fermions, it is easy to verify that formula (86) could be transformed into the form of Eq. (3.13) in Ref. [3]. Similar to Eq. (42), formula (86) will also remove out the possible existence of the heavier fermions than the top quarks.

V. CONCLUSIONS

We have expounded the mass equations of the Higgs boson, W- and Z-gauge bosons in a model of Nambu–Jona-Lasinio type with the three generations of quark leptons and a heavier degenerate fourth generation of U fermions. In the bubble approximation, it has been shown that the mass $m_{\phi_s^0}$ of the composite Higgs boson ϕ_s^0 is dominated by the heavy fermion masses m_U and m_t and further refined by its explicit momentum cutoff dependence. A new mass restriction $m_U+m_t\le m_{\phi_s^0}\le 2m_U$ has been proven. The results indicate that the mass of the Higgs boson could provide the most important experimental test of such a kind of model with heavy U fermions. Contrary to $m_{\phi_s^0}$, the W-boson mass m_W contains no explicit dependence on Λ and scarcely relies on the heavy fermion masses, since the factor $\bar{f}^2(m_W^2)$ in the mass equation dominated by the heavy fermion masses and explicit Λ dependence can be replaced by the vacuum expectation value v which is simply associated with the Fermi constant G_F, and the other heavy fermion terms appearing in the running coupling $\bar{g}_2^2(m_W^2)$ will be greatly suppressed by the decoupling effect as well. The mass m_Z of the Z boson has the same feature as m_W. Therefore, neither the equation of m_W nor m_Z can be regarded as another independent relation of Eq. (1) between the heavy fermion masses and the momentum cutoff Λ. On the other hand, in the equation of m_W, all of the light fermions with the masses $m_Q<m_W/2$ except the b quarks can never be neglected in $\bar{g}_2^2(m_W^2)$ and their loops exactly give the usual fermion contributions to the β function in the renormalization-group evolution of g_2. Similarly, in the equation of m_Z, we can never ignore the light fermions with the masses $m_Q<m_Z/2$ in the running couplings $g_1^2(m_Z^2)$ and $g_2^2(m_Z^2)$. However in this case there exist the weak isospin breaking effects induced by the large mass difference m_t-m_b (assuming the U fermions to be mass degenerate). They will lead to that not only $f^2(0)\neq\bar{f}^2(0)$ and $l(m_Z^2)\neq 0$ to make the mass equation deviate the standard form but also the change of the β functions; i.e., the fermion contributions to the β functions responsible for $g_1^2(m_Z^2)$ and $g_2^2(m_Z^2)$ in the equation of m_Z will have small differences from the ones to the usual β functions responsible for the renormalization-group evolution of g_1 and g_2. In fact, such deviations of the β functions will appear in the whole energy region with $4m_t^2>p^2>4m_b^2$ including $p^2=m_Z^2$. Only for the running couplings $g_1^2(p^2)$ and $g_2^2(p^2)$ with $p^2>4m_t^2$, will the above effect to alter the β functions disappear. Therefore, it is necessary for accurate calculations to take this effect into account for $4m_t^2>p^2>4m_b^2$ in both $g_1^2(p^2)$ and $g_2^2(p^2)$. Such a consideration will be useful for the precise determination of the masses of the heavy U fermions, the top quarks and the Higgs boson by the calculations of the complete dynamics of these particles where the renormalization group evolutions of g_1 and g_2 in the standard model will be used as a starting point of the discussions.

The research on the mass equations also brings us to the conclusion that an important, probably essential, difference between the models of Nambu–Jona-Lasinio type with gauge and composite electroweak bosons lies in the fact that the former will allow but the latter will remove the possible existence of heavier fermions than the top quarks.

ACKNOWLEDGMENTS

The author is grateful to Professor H. Terazawa for a private communication which brought the author's attention to the model of Nambu–Jona-Lasinio type with composite electroweak bosons proposed in Ref. [3]. This work was partially supported by Grant No. LWTZ-1298 of the Chinese Academy of Sciences and by the National Natural Science Foundation of China.

[1] Y. Nambu, in *New Theories in Physics,* Proceedings of XI International Symposium on Elementary Particle Physics, Kazimierz, Poland, 1988, edited by Z. Ajduk, S. Pokorski, and A. Trautman (World Scientific, Singapore, 1989); V. A. Miransky, M. Tanabashi, and K. Yamawaki, Mod. Phys. Lett. A **4**, 1043 (1989); Phys. Lett. B **221**, 177 (1989); W. J. Marciano, Phys. Rev. Lett. **62**, 2793 (1989).

[2] W. A. Bardeen, C. T. Hill, and M. Lindner, Phys. Rev. D **41**, 1647 (1990).

[3] H. Terazawa, Y. Chikashige, and K. Akama, Phys. Rev. D **15**, 480 (1977).

[4] M. Suzuki, Phys. Rev. D **41**, 3457 (1990).

[5] W. J. Marciano, Phys. Rev. D **41**, 219 (1990); M. Luty, *ibid.* **41**, 2893 (1990); R. F. Lebed and M. Suzuki, *ibid.* **45**, 1744 (1992).

[6] Bang-Rong Zhou, Commun. Theor. Phys. **19**, 337 (1993).

[7] Bang-Rong Zhou, Phys. Rev. D **47**, 2656 (1993).

[8] Bang-Rong Zhou, Phys. Rev. D **47**, 5038 (1993).

[9] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).

[10] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).

[11] H. D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973); D. J. Gross and F. Wilczek, *ibid.* **30**, 1343 (1973); Phys. Rev. D **8**, 3633 (1973).

[12] H. Terazawa, Phys. Rev. D **22**, 2921 (1980); **41**, 3541(E) (1990).

[13] A. Carter and H. Pagels, Phys. Rev. Lett. **43**, 1845 (1979).