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Strong rescattering in $K \to 3\pi$ decays and low-energy meson dynamics

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We present a consistent analysis of final state interactions in $K \to 3\pi$ decays in the framework of chiral perturbation theory. The result is that the kinematical dependence of the rescattering phases cannot be neglected. The possibility of extracting the phase shifts from future K_S-K_L interference experiments is also analyzed.

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I. INTRODUCTION

Unitarity requires that final state strong interactions should be active in kaon nonleptonic decays. As is well known, they contribute imaginary parts (or phase factors) to the decay amplitudes that, otherwise, could be chosen as purely real. Clearly, such effects directly reflect the properties of low-energy meson interactions. Therefore, they could significantly test theoretical approaches to meson dynamics, in particular the framework of chiral perturbation theory (ChPT) and the related amplitude expansions in meson momenta, which incorporate general features of long-distance QCD [1,2].

Final state interactions in $K \to 2\pi$ decays have been extensively discussed, e.g., in Refs. [3] and [4]. In this channel, the π-π phase shifts at $\sqrt{s} = m_K$ are found to have a sizable effect on the amplitudes for the various decay modes. In $K \to 3\pi$ decays, final state strong interactions operate at substantially lower energy due to the limited phase space, and the corresponding phases are expected to be small. In current fits to Dalitz plot distributions and partial rates [5,6], to extract the values of $K \to 3\pi$ amplitudes the assumption of negligible phases had to be adopted in order to limit the number of free parameters. Indeed, this assumption is consistent with the experimental information available at present.

Nevertheless, attempts to extract $K \to 3\pi$ strong phases from improved, future experiments should still be pursued, in view of the theoretical interest that such information would have. For example, in the approximation of considering only two-body strong interactions, thus neglecting the irreducible 3π rescattering diagrams which should be suppressed by phase space, $K \to 3\pi$ final state interactions can provide a complementary way to study π-π phase shifts near threshold. Indeed, this is the region where these phase shifts can be most reliably predicted in the framework of ChPT. Therefore, such an analysis should add a significant test of this theoretical approach, to be combined with, e.g., those from K_{l4} decays [7], also relevant to the π-π low-energy region.

As another important point of interest, we should recall that the knowledge of rescattering is crucial in order to estimate direct CP-violating asymmetries in $K \to 3\pi$ [8–11]. Manifestations of such asymmetries would allow one to determine the existence of direct CP violation in a channel alternative to $K \to 2\pi$, and to improve our knowledge of this phenomenon, which is predicted by the standard model but is not clearly established yet.

With these motivations, in what follows we shall discuss in detail the possibility, suggested in [12] and [13], that a convenient access to final state interactions in $K \to 3\pi$ could be provided by K_L-K_S interference in vacuum as a function of time, to be studied at "interferometry" machines such as the CERN Low Energy Antiproton Ring (LEAR) [14] and the ϕ factory DAΦNE [15,16]. The typical interference term has the form

$$\mathrm{Re}\left[\langle 3\pi|K_S\rangle^* \langle 3\pi|K_L\rangle \exp\left(i\Delta m t\right)\right] \exp\left(-\frac{\Gamma_S+\Gamma_L}{2}t\right), \tag{1}$$

where $\Delta m = m_L - m_S$ is the K_L-K_S mass difference. Studies of the time dependence of Eq. (1) should lead to a determination of both the real part and the (expected small) imaginary part of the amplitudes originating from final state interactions. The advantage of this kind of measurement should be that, in Eq. (1), the strong phases appear linearly, whereas they appear quadratically in Dalitz plot distributions and partial rates.

To obtain an order of magnitude estimate of the effect of the interference term, limited to the case of the ϕ factory, momentum-independent strong phases, with the values predicted by ChPT at the center of the Dalitz plot, were assumed in Ref. [12]. Actually, in the case of $K \to 3\pi$, it is not quite appropriate to use the notion of constant phase shifts because (i) there are two independent $I = 1$ final states which can be connected by strong interactions, so that one should introduce a 2×2 mixing (or rescattering) matrix, and (ii) in general the rescattering matrix elements are functions of pion momenta.

Phenomenologically, $K \to 3\pi$ transition amplitudes for the various modes are expanded in powers of the kinematical variables with constant coefficients, so that the momentum dependence of rescattering should be taken into account for a consistent expansion. This is particularly desirable also in connection with momentum expansions predicted by ChPT. Previous theoretical estimates of momentum-dependent $K \to 3\pi$ strong rescattering were performed in the nonrelativistic approximation in Ref. [17], and in leading order ChPT for charged kaons in Refs. [8] and [9]. The complete calculation of all $K \to 2\pi$ and $K \to 3\pi$ amplitudes to one loop in ChPT was performed in Ref. [6]. In particular, as far as the effects of interest here are concerned, in Ref. [6] are reported the numerical results (but not the explicit expressions) of the imaginary parts of the $K \to 3\pi$ amplitudes, expanded up to quadratic terms. In fact, the explicit kinematical dependence is needed in order to unambiguously reconstruct the rescattering matrix, which will be central to our treatment.

Thus, we shall first review the general symmetry and unitarity constraints on $K \to 3\pi$ amplitudes, and then we will construct a convenient, model-independent parametrization of the strong rescattering matrix which is unique for all decay modes. We shall finally write the analytical expression of the rescattering matrix in the framework of leading order ChPT. The results can be applied to make more reliable predictions for the $K \to 3\pi$ time correlations of Eq. (1). In addition, our analysis will allow us to clarify some delicate questions regarding direct CP violation in $K^\pm \to (3\pi)^\pm$.

Specifically, the plan of the paper is as follows: in Sec. II we set the formalism to expand $K \to 3\pi$ amplitudes; in Sec. III we define the rescattering matrix and evaluate it in lowest order ChPT; in Sec. IV we discuss some consequences for the Dalitz plot analysis and CP violation; in Sec. V we present the expectations for the interference term; finally, Sec. VI contains some concluding remarks.

II. $K \to 3\pi$ FORMALISM

In the limit of CP conservation, there are five different channels for $K \to 3\pi$ decays:

$$\begin{array}{ll} K^\pm \to \pi^\pm\pi^\pm\pi^\mp & (I=1,2), \\ K^\pm \to \pi^\pm\pi^0\pi^0 & (I=1,2), \\ K_L \to \pi^\pm\pi^\mp\pi^0 & (I=1), \\ K_L \to \pi^0\pi^0\pi^0 & (I=1), \\ K_S \to \pi^\pm\pi^\mp\pi^0 & (I=2). \end{array} \tag{2}$$

Here in parentheses we indicate the isospin values relevant to the final (3π) states, assuming only $\Delta I = 1/2, 3/2$ transitions. In principle, the K_S decay to the $I = 0$ state is not forbidden, but due to Bose symmetry it is strongly suppressed by a high angular momentum barrier [18] and we neglect it. The first four modes are dominated by $\Delta I = 1/2$ transitions, while the last one is a pure $\Delta I = 3/2$ transition and only recently has become accessible through time-dependent interference experiments [19].

For $K(p) \to \pi_1(p_1)\pi_2(p_2)\pi_3(p_3)$ decays we introduce the familiar kinematical invariants

$$s_i = (p_K - p_i)^2 \qquad \text{and} \qquad s_0 = \tfrac{1}{3}\sum_i s_i = \tfrac{1}{3}m_K^2 + m_\pi^2, \tag{3}$$

where the index $i = 3$ refers to the "odd" charge pion. Neglecting isospin-breaking effects and following, e.g., Refs. [18,20], we can decompose the decay amplitudes in the general form

$$\begin{aligned} A_{++-} &= 2A_c(s_1,s_2,s_3) + B_c(s_1,s_2,s_3) + B_2(s_1,s_2,s_3), \\ A_{+00} &= A_c(s_1,s_2,s_3) - B_c(s_1,s_2,s_3) + B_2(s_1,s_2,s_3), \\ A^L_{+-0} &= A_n(s_1,s_2,s_3) - B_n(s_1,s_2,s_3), \\ A^L_{000} &= 3A_n(s_1,s_2,s_3), \\ A^S_{+-0} &= \widetilde{B}_2(s_1,s_2,s_3). \end{aligned} \tag{4}$$

Here, due to Bose symmetry and the assumed CP conservation, all amplitudes A_j and B_j ($j = c, n, 2$) are symmetric under exchange $(1 \leftrightarrow 2)$. Furthermore, the amplitudes A_j are completely symmetric for any permutation of the indices 1, 2, and 3. Conversely, the amplitudes B_j do not have this symmetry, and under permutations of indices only obey the relation

$$B_j(s_1,s_2,s_3) + B_j(s_3,s_2,s_1) + B_j(s_1,s_3,s_2) = 0. \tag{5}$$

Finally, the amplitude $\widetilde{B}_2$ is antisymmetric for the exchange $(1 \leftrightarrow 2)$. It is not independent from the other ones, and can be expressed in terms of B_2 as

$$\widetilde{B}_2(s_1,s_2,s_3) = \tfrac{2}{3}\left[B_2(s_3,s_2,s_1) - B_2(s_1,s_3,s_2)\right]. \tag{6}$$

Concerning isotopic spin, the amplitudes A_j and (B_c, B_n) correspond to $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions to the $I = 1$ final three-pion state, while B_2 is associated with the $\Delta I = 3/2$ transition to $I = 2$.

From the decomposition above we note that there are two amplitudes leading to $I = 1$ final states, which differ by the pion exchange symmetry properties, namely, the A's are fully symmetric whereas the B's have mixed symmetry. Accordingly, it is convenient to introduce the two matrices

$$T_c = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}, \qquad T_n = \begin{pmatrix} 1 & -1 \\ 3 & 0 \end{pmatrix}, \tag{7}$$

which in the $I = 1$ sector transform the symmetric and nonsymmetric amplitudes into the physical ones for charged and neutral kaons, respectively. Thus,

$$\begin{pmatrix} A^{(1)}_{++-} \\ A^{(1)}_{+00} \end{pmatrix} = T_c \begin{pmatrix} A_c(s_i) \\ B_c(s_i) \end{pmatrix},$$

$$\begin{pmatrix} A^{(1)}_{++-} \\ A^{(1)}_{+00} \end{pmatrix} \begin{pmatrix} A^{L}_{+-0} \\ A^{L}_{000} \end{pmatrix} = T_n \begin{pmatrix} A_n(s_i) \\ B_n(s_i) \end{pmatrix}. \quad (8)$$

Defining the dimensionless Dalitz plot variables

$$Y = \frac{s_3 - s_0}{m_\pi^2} \quad \text{and} \quad X = \frac{s_1 - s_2}{m_\pi^2}, \quad (9)$$

and taking into account the symmetry properties of A's and B's, we can expand the five independent amplitudes in Eq. (5) in powers of X and Y up to quadratic terms:

$$\begin{aligned} A_j &= a_j + c_j(Y^2 + X^2/3), \\ B_j &= b_jY + d_j(Y^2 - X^2/3). \end{aligned} \quad (10)$$

Substituting Eq. (10) in Eq. (5), we obtain

$$\begin{aligned} A_{++-} &= 2a_c + (b_c + b_2)Y + 2c_c(Y^2 + X^2/3) \\ &\quad +(d_c + d_2)(Y^2 - X^2/3), \\ A_{+00} &= a_c - (b_c - b_2)Y + c_c(Y^2 + X^2/3) \\ &\quad -(d_c - d_2)(Y^2 - X^2/3), \\ A^L_{+-0} &= a_n - b_nY + c_n(Y^2 + X^2/3) - d_n(Y^2 - X^2/3), \\ A^L_{000} &= 3a_n + 3c_n(Y^2 + X^2/3), \\ A^S_{+-0} &= \tfrac{2}{3}b_2X - \tfrac{4}{3}d_2XY. \end{aligned} \quad (11)$$

This decomposition can be easily related to the one introduced in Refs. [5,6].

III. 3π FINAL STATE INTERACTION

Since strong interactions are expected to mix the two $I = 1$ final states, we must introduce a strong interaction rescattering matrix which mixes the corresponding decay amplitudes. Projecting the final state $(3\pi)_{I=1}$ by means of the matrices T_c^{-1} and T_n^{-1} in the symmetric-nonsymmetric basis, we can define the scattering matrix R, common to charged and neutral channels, as

$$\begin{pmatrix} A^{(1)}_{++-} \\ A^{(1)}_{+00} \end{pmatrix}_R = T_cR \begin{pmatrix} A_c \\ B_c \end{pmatrix} = T_cRT_c^{-1} \begin{pmatrix} A^{(1)}_{++-} \\ A^{(1)}_{+00} \end{pmatrix}, \quad (12)$$

$$\begin{pmatrix} A^{L}_{+-0} \\ A^{L}_{000} \end{pmatrix}_R = T_nR \begin{pmatrix} A_n \\ B_n \end{pmatrix} = T_nRT_n^{-1} \begin{pmatrix} A^{L}_{+-0} \\ A^{L}_{000} \end{pmatrix}. \quad (13)$$

Here the subscript R means that in the decay amplitude rescattering has been included. The matrix R defined above has diagonal elements which preserve the symmetry properties under pion exchanges, as well as off-diagonal elements which connect symmetric amplitudes to nonsymmetric ones and vice versa.

The unitarity conditions are obtained by imposing conservation of probability: namely,

$$\int d\Phi \left[|(A^{(1)}_{++-})_R|^2 + |(A^{(1)}_{+00})_R|^2 \right] = \int d\Phi \left[|A^{(1)}_{++-}|^2 + |A^{(1)}_{+00}|^2 \right], \quad (14)$$

$$\int d\Phi \left[|(A^{L}_{+-0})_R|^2 + |(A^{L}_{000})_R|^2 \right] = \int d\Phi \left[|A^{L}_{+-0}|^2 + |A^{L}_{000}|^2 \right], \quad (15)$$

where $d\Phi$ represents the phase space element.

We can now perform the calculation of R using ChPT. At the lowest order p^2, there are no quadratic terms and the coefficients a_j,b_j in (10) are real if CP is conserved. At order p^4 loops and counterterms will appear, generating real parts with higher powers of X and Y and also imaginary parts proportional to the $O(p^2)$ constants a_j and b_j. These imaginary parts define the rescattering matrix relevant to the constant and linear terms. In principle, imaginary parts of quadratic terms can occur similarly, but since these appear at the higher order p^6 we neglect them, as is also justified by the smallness of the experimental values of quadratic slopes. In this approximation, we replace $A_{c,n}$ and $B_{c,n}$ in (12) and (13) by $a_{c,n}$ and $b_{c,n}Y$, respectively. The unitarity conditions resulting from (14) and (15) are equivalent, and take the form

$$\int d\Phi [|R_{11}|^2 + \tfrac{2}{5}|R_{21}|^2] = \int d\Phi, \quad (16)$$

$$\int d\Phi [|R_{22}|^2 + \tfrac{5}{2}|R_{12}|^2]Y^2 = \int d\Phi \, Y^2, \quad (17)$$

$$\int d\Phi \, [5R_{11}R^*_{12} + 2R_{21}R^*_{22}]\, Y = 0. \quad (18)$$

As anticipated, at $O(p^2)$ there are only tree diagrams that can be easily computed with the leading order chiral weak Lagrangian [21], and there is no final state interaction so that $R = I$ (trivial case). At order p^4 loop diagrams (Fig. 1) generate imaginary parts, corresponding to on-shell propagators in internal lines, so that we can write

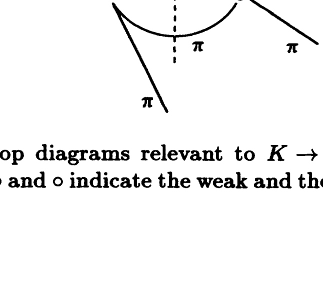

FIG. 1. Loop diagrams relevant to $K \to 3\pi$ rescattering. The symbols • and ∘ indicate the weak and the strong vertices, respectively.

$$R = I + i \begin{pmatrix} \alpha(s_i) & \beta'(s_i) \\ \alpha'(s_i) & \beta(s_i) \end{pmatrix} . \tag{19}$$

Using the strong $O(p^2)$ chiral Lagrangian

$$\mathcal{L}_S^{(2)} = \frac{F_\pi^2}{4} \mathrm{tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger + M(\Sigma + \Sigma^\dagger)\right], \tag{20}$$

where $\Sigma = \exp(i\phi/\sqrt{2}F_\pi)$, ϕ is the octect matrix of the pseudoscalar fields, F_π is the pion decay constant ($F_\pi \simeq$ 93 MeV), and $M = \mathrm{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$, one finds

$$\alpha(s_i) = \frac{1}{32\pi F_\pi^2} \sum_{i=1}^{3} \tfrac{1}{3} v_i (2s_i + m_\pi^2), \tag{21}$$

$$\alpha'(s_i) = \frac{1}{32\pi F_\pi^2} \sum_{i=1}^{2} \tfrac{5}{3} \left[v_i(s_i - m_\pi^2) - v_3(s_3 - m_\pi^2)\right], \tag{22}$$

$$\beta(s_i) = \frac{1}{32\pi F_\pi^2} \left[\frac{1}{3} \sum_{i=1}^{3} v_i (s_i - 4m_\pi^2) + \sum_{i=1}^{2} m_\pi^2 \frac{v_3(s_3 - s_0) - v_i(s_i - s_0)}{s_3 - s_0} \right], \tag{23}$$

$$\beta'(s_i) = \frac{1}{32\pi F_\pi^2} \sum_{i=1}^{3} \tfrac{2}{3} v_i \frac{(s_i - m_\pi^2)(s_0 - s_i)}{s_3 - s_0}, \tag{24}$$

where v_i are the "velocities:" $v_i = (1 - 4m_\pi^2/s_i)^{1/2}$. At this order, only the unitarity condition (18) is nontrivial and implies

$$\int d\Phi \left[2\alpha'(s_i) - 5\beta'(s_i)\right] Y = 0. \tag{25}$$

This condition is exactly verified by the functions in Eqs. (22) and (24), as expected since ChPT is an effective field theory where unitarity is perturbatively satisfied. Final state interactions operate in a quite similar way in $K \to 3\pi$ and in $\eta \to 3\pi$ decays. Therefore, analogous results were obtained in Ref. [22], where the ChPT leading order amplitudes for $\eta \to 3\pi$ were unitarized and the partial decay rates were determined. In fact, our purpose here somewhat differs from [22], as our main interest is the possibility of measuring $K \to 3\pi$ strong phases from interference experiments, not available to η decay, while the calculation in ChPT is simply a means to assess the size of the rescattering effects (and the corresponding required sensitivity) on the basis of a well-founded theoretical model.[1] In this regard, we should remark that the rescattering matrix R could have been directly evaluated by just integrating the π-π scattering amplitude over the phase space of intermediate particles. Actually, once the matrix R has been defined, one could improve the lowest order Eqs. (21)–(24) by including all higher orders in strong interactions, or even by replacing them by any other available phenomenological information on π-π scattering. Analogously, for the weak amplitudes a_c, b_c and a_n, b_n one could use either the ChPT predictions or the available experimental determinations.

For the decay into the $I = 2$ final states there is only one amplitude, with definite symmetry under pion exchange which must be preserved by strong interactions. Thus, we can write

$$\begin{aligned} (B_2)_R &= b_2 Y[1 + i\delta(s_i)], \\ (\widetilde{B}_2)_R &= \tfrac{2}{3} b_2 X[1 + i\widetilde{\delta}(s_i)], \end{aligned} \tag{26}$$

where the two functions δ and $\widetilde{\delta}$ are again not independent because $(B_2)_R$ and $(\widetilde{B}_2)_R$ must satisfy Eq. (6). At the lowest nontrivial order in ChPT one finds

$$\delta(s_i) = \frac{1}{32\pi F_\pi^2} \left[\frac{1}{3} \sum_{i=1}^{3} v_i (s_i - 4m_\pi^2) + \frac{1}{3} \sum_{i=1}^{2} \frac{v_i(s_i - s_0)(2s_i - 5m_\pi^2) - v_3(s_3 - s_0)(2s_3 - 5m_\pi^2)}{s_3 - s_0} \right]. \tag{27}$$

Regarding three-body rescattering, which will appear at two loops, we would expect its contribution to the functions α, α', β, β', and δ to be rather small, as being suppressed by phase space, assuming that the three-body coupling is not anomalously large. This is indeed the case for the leading order Lagrangian (20). Concerning higher orders in ChPT, $O(p^4)$ contributions to R might be relevant, similar to the case of π-π phase shifts where the scattering lengths turn out to be affected at the 20%–30% level [4]. We can take these figures as an indication for the accuracy of the subsequent applications of Eqs. (21)–(27).

IV. CONSEQUENCES FOR DALITZ PLOT ANALYSIS AND *CP* VIOLATION

As expected from the smallness of the available phase space, the functions α, α', β, β', and δ are smaller than unity over the whole Dalitz plot. Indeed, by expanding in powers of X and Y up to quadratic terms, we obtain

$$\begin{array}{lll} \alpha(X,Y) \cong \alpha_0 + \alpha_1(Y^2 + X^2/3) & [\alpha_0 \simeq 0.13, & \alpha_1 \simeq -2.9 \times 10^{-3}] \\ \alpha'(X,Y) \cong \alpha'_0 Y + \alpha'_1(Y^2 - X^2/3) & [\alpha'_0 \simeq -0.12, & \alpha'_1 \simeq 3.4 \times 10^{-3}] \\ \beta(X,Y) \cong \beta_0 + \beta_1(Y^2 - X^2/3)/Y & [\beta_0 \simeq 0.047, & \beta_1 \simeq 4.7 \times 10^{-3}] \\ \beta'(X,Y) \cong \beta'_0(Y^2 + X^2/3)/Y & [\beta'_0 = \alpha'_0/5] & \\ \delta(X,Y) \cong \delta_0 + \delta_1(Y^2 - X^2/3)/Y & [\delta_0 = -\beta_0, & \delta_1 \simeq -0.020] \\ \widetilde{\delta}(X,Y) \cong \delta_0 - 2\delta_1 Y. & & \end{array} \tag{28}$$

[1] If measured, $K \to 3\pi$ rescattering phases would determine the $\eta \to 3\pi$ ones, and in this sense our discussion should be relevant to the η decay also.

Using Eqs. (12), (13), (26), and (28) we can expand both real and imaginary parts of all $K\to 3\pi$ amplitudes up to linear terms.

As a first application of the formalism we can discuss the role of rescattering in CP-odd charge asymmetries in $K^{\pm}\to 3\pi$. Contrary to $K\to 2\pi$, where direct CP violation is suppressed by the smallness of the $\Delta I=3/2$ amplitude, in $K\to 3\pi$ an observable effect can potentially arise also from the interference of the two $\Delta I=1/2$ amplitudes. For a nonvanishing effect it is crucial that the relevant amplitudes have different electroweak phases (which can be the case only at order p^4 in the framework of ChPT) as well as different rescattering phases.

Let us consider, for example, the amplitudes for $K^+\to(\pi^+\pi^+\pi^-)_{I=1}$. From the preceding relations we easily obtain

$$\mathrm{Re}(A^{(1)}_{++-})=2a_c+b_cY, \tag{29}$$

$$\begin{aligned}\mathrm{Im}(A^{(1)}_{++-})&=2a_c\alpha_0+a_c\alpha_0'Y+b_c\beta_0Y\\&=2a_c\alpha_0+b_cY\left(\beta_0+\frac{a_c}{b_c}\alpha_0'\right).\end{aligned}\tag{30}$$

In Eq. (30) the contribution of α_0' is multiplied by the sizable factor $|a_c/b_c|\simeq 3.5$–4.0, and dominates over the one of β_0 by almost one order of magnitude. Nevertheless, such a large imaginary contribution to the term linear in Y does not help in generating the large CP-violating interference between the two $I=1$ amplitudes suggested in Ref. [23]. Indeed, of the two Y-dependent terms in Eq. (30), the one proportional to a_c has the same weak phase as the constant term and consequently, as already noticed in Refs. [9,10], the CP-violating interference between the two amplitudes to $I=1$ states must be proportional to the small difference $(\alpha_0-\beta_0)$. This example shows that the full kinematical dependence of the rescattering functions is relevant in constructing the imaginary parts of the amplitudes.

In principle, the rescattering phases should be included in the analysis of CP-conserving Dalitz plot parameters. Their contribution could affect the determination of the linear and the quadratic slopes. However, since in this case the imaginary parts appear quadratically, their effect is of order p^8 in ChPT and thus for completeness also the other contributions of the same order should be included. As a curiosity, we estimate the contribution of $\mathrm{Im}A^L_{000}$ to the quadratic slope h, in the Dalitz plot of $K_L\to 3\pi^0$, defined by

$$\begin{aligned}|A^L_{000}|^2&=(\mathrm{Re}A^L_{000})^2+(\mathrm{Im}A^L_{000})^2\\&\propto 1+h(Y^2+X^2/3)+\cdots.\end{aligned}\tag{31}$$

Using Eqs. (13) and (28), we find

$$\mathrm{Im}A^L_{000}=3a_n\alpha_0+3(b_n\beta_0'+a_n\alpha_1)(Y^2+X^2/3), \tag{32}$$

which gives the following contribution to h:[2]

$$h^{(\mathrm{Im})}=2\alpha_0\left(\alpha_1+\frac{b_n}{a_n}\beta_0'\right)\simeq +1.4\times 10^{-3}. \tag{33}$$

This number turns out to be of the same order of the experimental value of h [24],

$$h=-(3.3\pm 1.1)\times 10^{-3}, \tag{34}$$

but is substantially smaller than the p^6 contribution theoretically estimated in Ref. [25].

V. MEASUREMENTS OF THE RESCATTERING MATRIX IN INTERFEROMETRY MACHINES

As pointed out in Sec. I, measurements of K_L-K_S interference as a function of time should represent a convenient means to determine the (3π) rescattering matrix elements, because this observable depends linearly on $\mathrm{Im}[(A^S_{+-0})^*A^L_{+-0}]$. To this purpose, "interferometry machines" such as DAΦNE and LEAR should have the advantage that interference naturally occurs in vacuum there. It is possible to measure K_L-K_S interference terms also in fixed-target experiments where statistics can be higher; however, in this case an accurate knowledge of the regeneration amplitude is required.

Recent LEAR data [19] give a preliminary indication of the term proportional to $\cos(\Delta mt)$ in Eq. (1) and suggest the possibility of measuring in the near future also the $\sin(\Delta mt)$ component. Consequently, it is worthwhile to improve the order of magnitude estimates of Ref. [12] and, using the ChPT results of Secs. III and IV, to derive predictions based on that definite theoretical model.

Choosing $|\overline{K^0}\rangle=CP|K^0\rangle$, the CP-even and CP-odd eigenstates are $|K_{1,2}\rangle=(|K^0\rangle\pm|\overline{K}^0\rangle)/\sqrt{2}$ and, with the Wu-Yang phase convention, the mass eigenstates are (assuming CPT invariance)

$$|K_{S,L}\rangle=p|K^0\rangle\pm q|\overline{K^0}\rangle\equiv\frac{|K_{1,2}\rangle+\varepsilon|K_{2,1}\rangle}{\sqrt{1+|\varepsilon|^2}}. \tag{35}$$

The proper time evolution of initial K^0 or $\overline{K^0}$ states is

$$\begin{aligned}|K^0(t)\rangle=\frac{\sqrt{1+|\varepsilon|^2}}{\sqrt{2}\,(1+\varepsilon)}\Bigg[&|K_S\rangle\exp\left(\frac{-\Gamma_St}{2}-im_St\right)\\&+|K_L\rangle\exp\left(\frac{-\Gamma_Lt}{2}-im_Lt\right)\Bigg],\end{aligned}$$

$$\begin{aligned}|\overline{K^0}(t)\rangle=\frac{\sqrt{1+|\varepsilon|^2}}{\sqrt{2}\,(1-\varepsilon)}\Bigg[&|K_S\rangle\exp\left(\frac{-\Gamma_St}{2}-im_St\right)\\&-|K_L\rangle\exp\left(\frac{-\Gamma_Lt}{2}-im_Lt\right)\Bigg].\end{aligned}\tag{36}$$

At LEAR, tagged K^0 and $\overline{K^0}$ are produced, and the simplest means to observe interference is represented by the asymmetry

[2]This numerical result is obtained with the value of b_n/a_n resulting from the fit of Ref. [6].

$$A_f^{+-0}(t) = \frac{\int d\Phi\; f(X,Y)\left[|A(K^0\to\pi^+\pi^-\pi^0)|^2 - |A(\overline{K^0}\to\pi^+\pi^-\pi^0)|^2\right]}{\int d\Phi\left[|A(K^0\to\pi^+\pi^-\pi^0)|^2 + |A(\overline{K^0}\to\pi^+\pi^-\pi^0)|^2\right]}, \tag{37}$$

where $f(X,Y)$ is an odd-X function chosen in order to disentangle the different kinematical dependences. Up to first order in ε, the decay amplitude squared as a function of time is given by

$$\begin{aligned}&|A(K^0(\overline{K^0})\to\pi^+\pi^-\pi^0)|^2\\ &\simeq \tfrac{1}{2}(1\mp 2\mathrm{Re}\,\varepsilon)\{\exp(-\Gamma_S t)|A_S|^2+\exp(-\Gamma_L t)|A_L|^2\\ &\quad\pm 2\exp(-\Gamma t)[\mathrm{Re}\,(A_LA_S^*)\cos(\Delta m t)\\ &\quad+\mathrm{Im}\,(A_LA_S^*)\sin(\Delta m t)]\},\end{aligned} \tag{38}$$

where $\Delta m = m_L - m_S$, $\Gamma = (\Gamma_L+\Gamma_S)/2$, and $A_{S,L}\equiv A^{S,L}_{+-0}$. Then Eq. (37) can be rewritten as

$$\begin{aligned}A_f^{+-0}(t) = {}&\frac{2e^{-\Gamma t}\int d\Phi\; f(X,Y)\,\mathrm{Re}A_L\,\mathrm{Re}A_S}{\int d\Phi\,[e^{-\Gamma_S t}|A_S|^2+e^{-\Gamma_L t}|A_L|^2]}\\ &\times\left[\cos(\Delta m t)+\delta_f\sin(\Delta m t)+O(\delta_f^2)\right],\end{aligned} \tag{39}$$

where

$$\delta_f = \frac{\int d\Phi\; f(X,Y)\,[\mathrm{Im}A_L\,\mathrm{Re}A_S - \mathrm{Im}A_S\,\mathrm{Re}A_L]}{\int d\Phi\; f(X,Y)\,\mathrm{Re}A_L\,\mathrm{Re}A_S}. \tag{40}$$

At the planned DAΦNE machine, a K_S-K_L coherent state will be produced and the interference term of Eq. (1) can be studied by looking at the final state $(l^\mp\pi^\pm\nu,\pi^+\pi^-\pi^0)$ [12]. Following Ref. [26], we define for a generic decay $K_{S,L}K_{L,S}\to f_1(t_1)f_2(t_2)$ an intensity

$$I(f_1,f_2;t) = \frac{1}{2}\int_{|t|}^{\infty} dT|\langle f_1(t_1)f_2(t_2)|i\rangle|^2, \tag{41}$$

where $t=t_1-t_2$ and $T=t_1+t_2$. Choosing $f_1 = l^\mp\pi^\pm\nu$ and $f_2=\pi^+\pi^-\pi^0$, we can define an asymmetry similar to $A_f^{+-0}(t)$: namely,

$$R_f^{\pm}(t) = \frac{\int d\Phi\; f(X,Y)\, I(l^\mp\pi^\pm\nu,\pi^+\pi^-\pi^0;t)}{\int d\Phi\; I(l^\mp\pi^\pm\nu,\pi^+\pi^-\pi^0;t)}. \tag{42}$$

Indeed, for $t>0$ we have

$$\begin{aligned}&I(l^\mp\pi^\pm\nu,\pi^+\pi^-\pi^0;t>0)\\ &=\frac{\Gamma_L(l^\mp\pi^\pm\nu)}{2\Gamma}\Big\{\exp(-\Gamma_S t)|A_L|^2+\exp(-\Gamma_L t)|A_S|^2\\ &\quad\pm 2\exp(-\Gamma t)\,[\mathrm{Re}\;(A_LA_S^*)\cos(\Delta m t)\\ &\quad-\mathrm{Im}\;(A_LA_S^*)\sin(\Delta m t)]\Big\},\end{aligned} \tag{43}$$

and therefore

$$\begin{aligned}R_f^{\pm}(t>0) = \pm&\frac{2e^{-\Gamma t}\int d\Phi\; f(X,Y)\,\mathrm{Re}A_L\,\mathrm{Re}A_S}{\int d\Phi\,[e^{-\Gamma_L t}|A_S|^2+e^{-\Gamma_S t}|A_L|^2]}\\ &\times[\cos(\Delta m t)-\delta_f\sin(\Delta m t)],\end{aligned} \tag{44}$$

where δ_f is the same as defined in Eq. (40) and terms of order δ_f^2 have been neglected. For $t<0$ the analogue of Eq. (44) is

$$\begin{aligned}R_f^{\pm}(t<0) = \pm&\frac{2e^{-\Gamma|t|}\int d\Phi\; f(X,Y)\,\mathrm{Re}A_L\,\mathrm{Re}A_S}{\int d\Phi\,\left[e^{-\Gamma_S|t|}|A_S|^2+e^{-\Gamma_L|t|}|A_L|^2\right]}\\ &\times[\cos(\Delta m|t|)+\delta_f\sin(\Delta m|t|)].\end{aligned} \tag{45}$$

However, due to the exchange $\Gamma_L\leftrightarrow\Gamma_S$, the denominator in Eq. (45) quickly becomes much larger than in Eq. (44), and suppresses the interference effect.

Considering for δ_f the first nonvanishing order in ChPT, which is $O(p^6)$ in the numerator and $O(p^4)$ in the denominator, we obtain

$$\delta_f = \frac{\int d\Phi\; f(X,Y)\left[a_n(\alpha-\alpha'-\widetilde{\delta})X - b_n(\beta-\beta'-\widetilde{\delta})XY\right]}{\int d\Phi\; f(X,Y)\,[a_nX-b_nXY]}. \tag{46}$$

If we use as weight function $f(X,Y)=\mathrm{sgn}(X)$, we obtain numerically the result

$$\delta_X = 0.18\pm0.01\,. \tag{47}$$

Essentially, this turns out to be $\delta_X=\alpha_0-\delta_0$, and is practically independent of the theoretical uncertainties on the small ratio b_n/a_n. For this reason the result (47) is in good agreement with the prediction of Ref. [12].

On the other hand, choosing $f(X,Y)=\mathrm{sgn}(YX)$ we obtain numerically

$$\delta_{XY} = 0.30\pm0.05. \tag{48}$$

This result is about a factor of 4 larger than the value $\delta_{XY}\simeq0.07$ obtained in Ref. [12], where the momentum dependence of strong phases has been neglected. Indeed, by expanding the rescattering functions, in the present calculation we have

$$\delta_{XY}\simeq\frac{(\beta_0-\delta_0)+\frac{a_n}{b_n}(\alpha_0'-2\delta_1)}{1-\frac{a_n}{b_n}\dfrac{\int d\Phi\;|X|\mathrm{sgn}(Y)}{\int d\Phi\;|XY|}}. \tag{49}$$

Equation (49) shows that also in the case of δ_{XY} the Y-

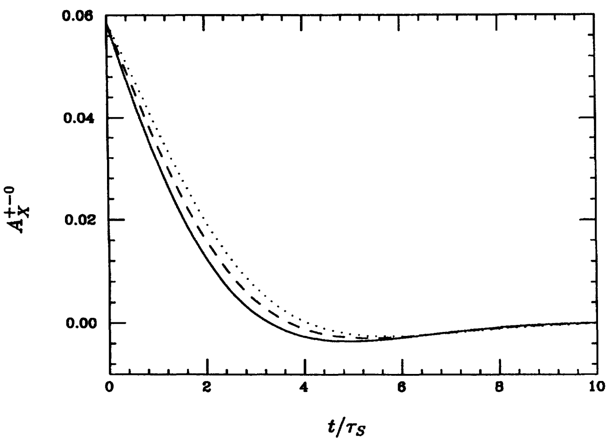

FIG. 2. The asymmetry A_X^{+-0} of Eq. (39) vs t. The full, dashed, and dotted lines correspond to $\delta_X = 0$, 0.2, and 0.4, respectively.

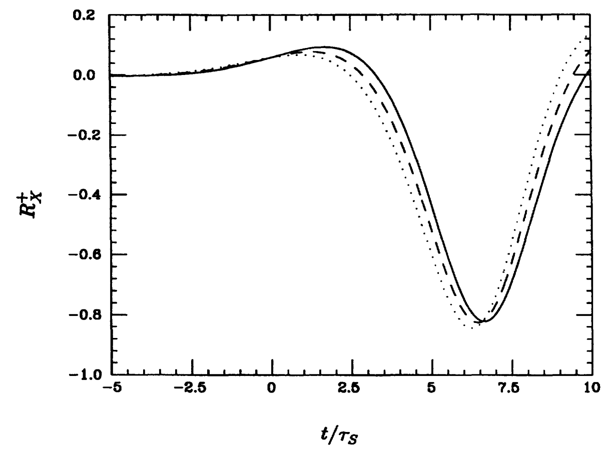

FIG. 3. The asymmetry R_X^+ of Eq. (44) for positive and negative t. The full, dashed, and dotted lines correspond to $\delta_X = 0$, 0.2, and 0.4, respectively.

dependent terms give a sizable contribution, since they are multiplied by the large factor (a_n/b_n). The error in Eq. (48) accounts for the theoretical uncertainty on (a_n/b_n), for which either the experimental value or the $O(p^2)$ ChPT prediction can be used.[3]

VI. CONCLUDING REMARKS

In the previous sections we have introduced a general formalism to consistently account for final state interactions in $K \to 3\pi$ amplitudes, and have used leading order chiral perturbation theory to evaluate the rescattering matrix.

We have considered some potentially observable effects of rescattering on Dalitz plot variables. The results indicate that the off-diagonal elements of the rescattering matrix in the $I = 1$ sector induce sizable imaginary parts in the X- and Y-dependent amplitudes. However, these large imaginary parts are not easily detected from Dalitz plot analyses, in agreement with previous analyses [5,6], and unfortunately cancel in direct CP-violating asymmetries.

Planned experiments at "interferometry machines" can have direct access to the rescattering matrix elements via appropriately defined time-dependent asymmetries, which we have estimated in leading order ChPT. As examples of the typical effects expected in this framework, Fig. 2 shows the asymmetry $A_X^{+-0}(t)$ of Eq. (39) relevant to LEAR. The solid line represents the asymmetry with no rescattering ($\delta_X = 0$), the dashed line corresponds to the leading ChPT estimate of Eq. (47), and finally the dotted line would result by doubling the value of δ_X. To obtain Fig. 2, for the real parts of the amplitudes A^L_{+-0} and A^S_{+-0} we have used the expansion (11) with the values of the parameters obtained in the fit of Ref. [6]. As one can see, the curves in this figure have similar shapes, but possibly could be distinguished in high precision experiments.

In Fig. 3 we report the asymmetry $R_X^+(t)$ of Eq. (44) relevant to DAΦNE, and the three curves refer to the same cases considered in Fig. 2. Here we note that rescattering affects the shape of the curves more significantly, especially for $t > 0$ where the asymmetry can become quite large. However, this occurs for the values of t where the number of events becomes smaller. As an indication, the total expected number of events at $t > 0$, with the planned DAΦNE luminosity 5×10^{32} cm^{-2} sec^{-1}, is of the order of 10^3/yr.

In conclusion, the previous analysis shows the interest of experimental efforts to accurately measure the kind of asymmetries proposed here. The ultimate goal would be the determination of the $K \to 3\pi$ rescattering matrix elements testing ChPT in the strong sector, but in any case even a reasonable upper bound would represent important information in this regard. Furthermore, the direct measurement of the CP-conserving $K_S \to 3\pi$ amplitude is by itself an important achievement, extremely useful in order to test chiral symmetry in nonleptonic weak interactions.

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[3]The unknown quadratic term in A^S_{+-0} can affect to some extent the numerical result for δ_{XY} [6], and its effect can be roughly taken into account by doubling the error in Eq. (48).

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