New approach for measuring $|V_{ub}|$ at future B factories

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(Received 6 June 1994)

It is suggested that the measurements of hadronic invariant mass (m_X) distributions in the inclusive $B \to X_{c(u)} l\nu$ decays can be useful in extracting the CKM matrix element $|V_{ub}|$. We investigated hadronic invariant mass distributions within the various theoretical models of HQET, FAC and the chiral Lagrangian as well as the ACCMM model. It is also emphasized that the m_X distributions even at the region $m_X > m_D$ in the inclusive $b \to u$ are effective in selecting the events, experimentally viable at the future asymmetric B factories, with better theoretical understanding.

PACS number(s): 12.15.Hh, 12.39.Fe, 12.39.Hg, 13.20.He

I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ub} is important to the standard model description of CP violation. If it were zero, there would be no CP violation from the CKM matrix element (i.e., in the standard model), and we have to seek for other sources of CP violation in $K_L \to \pi\pi$. Observations of semileptonic $b \to u$ transitions by the CLEO [1] and ARGUS [2] collaborations imply that V_{ub} is indeed nonzero, and it is important to extract the modulus $|V_{ub}|$ from semileptonic decays of B mesons as accurately as possible. Presently, the charged lepton energy is measured, and the $b \to u$ events are selected from the high end of the charged lepton energy spectrum. This method is applied to both inclusive and exclusive semileptonic B decays.

However, this cut on E_l is not very effective, since only below 10% of $b \to u$ events survive this cut. As first discussed in [3], the measurements of hadronic invariant mass (m_X) distributions in $B \to X_{c,u} l\nu$ (inclusive decays) can be useful to extract a CKM matrix element V_{ub}. For $b \to cl\nu$, one necessarily has $m_X \geq m_D = 1.86$ GeV. So, if we impose a condition $m_X < m_D$, the resulting events come from $b \to ul\nu$. According to the work of [3], $\sim 90\%$ of the $b \to u$ events survive this cut. This is in sharp contrast the usual cut on E_l. Thus one could get an order of magnitude improvement in selecting data.

Inclusive m_X distributions for $b \to u$ transition can be obtained using the Altarelli-Cabibbo-Corbo-Maiani-Martinelli (ACCMM) model [4], or the work by Bigi, Shifman, Uraltsev, and Vainshtein. (BSUV) [5]. In this work, we follow the result of Ref. [3] based on the ACCMM model, with a remark that theoretical uncertainties for the inclusive $b \to u$ decays are less compared to an exclusive mode.

Resonance contributions to the m_X distributions in $B \to X_{c,u} l\nu$ can be estimated invoking various models. Once the decay rate for $B \to Rl\nu$ (where R is a resonance) is known, the corresponding m_X distribution can be written as

$$\frac{d\Gamma}{dm_X} \approx \frac{2m_X\Gamma(B \to Rl\nu)}{\pi} \frac{m_R\Gamma_R}{(m_X^2 - m_R^2)^2 + m_R^2\Gamma_R^2}, \tag{1}$$

in the narrow width approximation. Here, m_R and Γ_R are the mass and the width of the resonance R. In the limit of $\Gamma_R \to 0$, we get

$$\frac{d\Gamma}{dm_X} = \Gamma(B \to R)\ \delta(m_X - m_R), \tag{2}$$

so that the correct decay rate for $B \to Rl\nu$ comes out upon integrating Eq. (2) over dm_X.

In Sec. II, we discuss the result for $B \to D, D^*, (D^{**})$ in the heavy quark effective theory. In Sec. III, $B \to (\pi, \rho) l\nu$ are considered in two different types of approaches: a nonrelativistic quark model and the chiral Lagrangian with heavy mesons as well as light vector mesons. In Sec. IV, the m_X distributions for $B \to X_{c(u)} l\nu$ are shown, and it is emphasized that the m_X distributions even at the region $m_X > m_D$ in the inclusive $b \to u$ are effective in selecting almost 100% of the events experimentally viable at the future asymmetric B factories. Since one can calculate the inclusive decay more reliably, one can achieve a better determination of V_{ub} both statistically and systematically.

II. THE $B \to X_c l\nu$ DECAY IN THE HEAVY QUARK EFFECTIVE THEORY

Let us first consider $B \to X_c l\nu$, which is dominated by resonance contributions with $X_c = D, D^*, D^{**}$. Theoretical predictions based on the heavy quark effective field theory (HQET) [6] depend on one hadronic form factor $h_{A_1}(w)$, where $w = v \cdot v'$ is the inner product of the four velocities of the initial and final heavy quarks. In order to calculate a decay rate, one has to know the shape of the form factor over the whole kinematic range of w. However, this form factor is not calculable from first principles, except for the normalization $h_{A_1}(1) = 0.99 \pm 0.04$,

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which is one of the predictions of HQET [6]. Thus one necessarily resorts to some model for the shape of the form factor. If one adopts the results of the QCD sum rule, one may approximate the form factor as

$$h_{A_1}(w) \approx 0.99 \left(\frac{2}{w+1}\right)^{2\varrho^2_{A_1}}, \tag{3}$$

with $\varrho^2_{A_1} \approx 0.8$. Since this result is based on the QCD sum rule, there exist some systematic uncertainties associated with the sum rule. This systematic uncertainty may be taken into account by allowing $\varrho^2_{A_1}$ to vary between 0.5 and 1.1, where the latter is on the border of the limit given by Voloshin's sum rule [7]. For this range of $\varrho^2_{A_1}$, one can predict the decay rates for $B \to D^{(*)}l\nu$ [6]:

$$\Gamma(B \to Dl\nu) = (0.86 - 1.35) \times 10^{13}|V_{bc}|^2/\text{sec}, \tag{4}$$

$$\Gamma(B \to D^*l\nu) = (2.59 - 3.48) \times 10^{13}|V_{bc}|^2/\text{sec}, \tag{5}$$

where the smaller decay rates correspond to the larger $\varrho^2_{A_1}$. For $B \to D^{**}l\nu$, we use the observation by the CLEO Collaboration [8]:

$$B(B \to D^{**}l\nu) \simeq 0.5 \times B(B \to Dl\nu). \tag{6}$$

Our approach concerning the measurement of $|V_{ub}|$ from the m_X distributions can be regarded independently of the uncertainties in Eqs. (4)–(6), because the m_X distributions from the three resonances D, D^*, andD^{**} are essentially δ functions, Eq. (2), and we just require to exclude small regions around $m_X = m_D, m_{D^*}, m_{D^{**}}$. For more details, see Sec. IV and Fig. 1.

III. THE $B \to X_u l\nu$ DECAY

Unlike the $b \to c$ transition, the $b \to u$ transition is largely nonresonant and multiple jetlike final states dominate [9]. The whole inclusive decay can be theoretically well understood in most of the kinematical region. The electron energy spectrum or the hadronic mass distribution for the inclusive semileptonic decay can be calculated rather reliably. In contrast, for the exclusive decays for $b \to u$ such as $B \to (\pi, \rho) + l\nu$, the model dependence becomes more pronounced, especially for the shape of the form factors. Here, we consider two classes of models: the factorization ansatz (FAC) model (a nonrelativistic quark model) and the chiral Lagrangian with heavy mesons. The results are compared with the m_X distribution obtained by the ACCMM model in Sec. IV.

A. The FAC model

The FAC model is based on the nonrelativistic quark model assuming the form factors are factorized as [10]

$$f_i^{\text{FAC}}(q^2) = f_i^{\text{FQM}}(q^2) \times F(q^2), \tag{7}$$

where $f_i^{\text{FQM}}(q^2)$ are the free quark model form factors arising from the overlap of the spin wave functions, and $F(q^2)$ comes from the overlap of the spatial wave functions. For $B \to D^{(*)}$ transitions, one can impose the normalization condition $F(q^2_{\text{max}}) = 1$ by heavy quark flavor symmetry. For $B \to \pi$ (or ρ) transitions, this normalization is not valid and it may be reasonable to assume that the form factor suppression for $B \to \pi$ begins at the $B \to \rho$ threshold. With these assumptions, one gets

$$\Gamma(B \to Dl\nu) = (0.71 - 0.85) \times 10^{13}|V_{bc}|^2/\text{sec}, \tag{8}$$

$$\Gamma(B \to D^*l\nu) = (2.17 - 2.44) \times 10^{13}|V_{bc}|^2/\text{sec}, \tag{9}$$

$$\Gamma(B^0 \to \pi^+l\nu) = (0.24 - 0.86) \times 10^{13}|V_{ub}|^2/\text{sec}, \tag{10}$$

$$\Gamma(B^0 \to \rho^+l\nu) = (0.77 - 2.10) \times 10^{13}|V_{ub}|^2/\text{sec}, \tag{11}$$

for certain ranges of pole masses (see Ref. [10] for more details.) Note that the FAC model predictions for $B \to D^*$ are consistent with (although they are systematically lower than) those by the HQET discussed in the previous section. Hence, the FAC model for the $B \to D^{(*)}$ transitions is rather reliable. For $B \to \pi$ (or ρ) transitions, the predictions are very sensitive to the shape of the form factors because of the large phase space available. Therefore, we simply regard the above numbers for $B \to \pi$ (or ρ) transitions as exemplary values in a nonrelativistic quark model, without giving much meaning to the specific values.

B. The chiral Lagrangian with heavy mesons

Recently, the chiral Lagrangian with heavy mesons and baryons has been developed [11]–[13]. This Lagrangian was originally invented in order to describe interactions among heavy mesons and light mesons such as π and K in the soft pion limit. Then heavy baryons [14] as well as ρ [15],[16] have been incorporated in the leading order in $1/m_Q$ and chiral expansions. The weak current can be represented in terms of physical fields such as heavy hadrons and light mesons, allowing us to calculate the matrix element of the weak current between hadrons and thus the semileptonic decays of heavy hadrons.

However, this apporach has its own limitations. First of all, the hadronic form factors given by this chiral Lagrangian are valid only in very limited regions of the whole kinematic region. Therefore one often assumes certain shapes of form factors and normalizes them at a point to a value given by the chiral Lagrangian with heavy hadrons. Furthermore, if one considers the next-to-leading-order corrections in $1/m_Q$ and chiral expansions, there come in a lot of unknown parameters and one essentially loses predictability. Although the reparametrization invariance of the heavy quark field leads to some constraints to the parameters in the next-to-leading-order terms, it still leaves many other parameters unconstrained. Therefore results based on the chiral Lagrangian with heavy baryons should be understood, keeping in mind the uncertainties just mentioned above.

One of the extensive studies of semileptonic decays of heavy mesons in the framework of the chiral Lagrangian with heavy hadrons is the work by Casalbuoni and his collaborators [15]. Their results are

$$\Gamma(B^0 \to \pi^- l\nu) = 38.8 \left(\frac{f_B \text{ (MeV)}}{200}\right)^2 \times 10^{13} |V_{ub}|^2/\text{sec}, \quad (12)$$

$$\Gamma(B^0 \to \rho^- l\nu) = 22.7 \left(\frac{f_B \text{ (MeV)}}{200}\right)^2 \times 10^{13} |V_{ub}|^2/\text{sec}. \quad (13)$$

At this point, a remark on the B-meson decay constant f_B in Eqs. (12) and (13) is in order. In the lowest order in the $1/m_Q$ expansion,

$$\frac{f_B}{f_D} = \left(\frac{M_D}{M_B}\right)^{1/2}. \quad (14)$$

On the other hand, the lattice QCD and the QCD sum rule [17] suggest that

$$f_B \approx f_D \approx 200 \text{ MeV}, \quad (15)$$

which violates the scaling relation, Eq. (14). Thus the results in Ref. [15] are expressed as above, although it is not systematic in $1/m_Q$ expansion to use Eq. (15).

We note that the results of Ref. [15] are subtantially larger than those based on the FAC model. Especially, the relative ratios between $B \to \pi$ and $B \to \rho$ are opposite in the two models, and may be checked in the near future. For the isospin-related decay $B^- \to \rho^0 l^- \bar{\nu}_l$, the predicted decay rate is half of Eq. (13), with the corresponding branching ratio

$$B(B^- \to \rho^0 l^- \bar{\nu}_l) = 0.44 \times 10^{-3} \left(\frac{f_B \text{ (MeV)}}{200}\right)^2 \times \left|\frac{V_{ub}}{0.0045}\right|^2, \quad (16)$$

assuming $\tau_B = 1.29$ psec. The data from the ARGUS and CLEO Collaborations seem contradictory with each other:

$$B(B^- \to \rho^0 l^- \bar{\nu}_l) = (1.13 \pm 0.36 \pm 0.26) \times 10^{-3} \text{ (ARGUS)}, \quad (17)$$
$$< 0.3 \times 10^{-3} \text{ (CLEO)}. \quad (18)$$

Note that the two results are incompatible with each other. The ARGUS result [18] is consistent with the prediction by Casalbuoni *et al.*, but is inconsistent with the FAC model prediction. On the other hand, if the result by the CLEO Collaboration [19] is confirmed, the prediction based on the chiral Lagrangian with heavy mesons will be excluded. In this case, there can be many possible reasons for it. First of all, interactions between ρ and heavy mesons may not be well described by the chiral Lagrangian in the lowest order because of the relative heaviness of ρ. This would be contrary to the better known case, the chiral Lagrangian with vector mesons (ρ), where the dynamics of π, ρ are rather well described. Second, and most likely, the usual simple assumption on the shape of the form factor may not be right. In most cases, including Ref. [15], it is assumed that a form factor $f(q^2)$ satisfies a monopole form:

$$f(q^2) = \frac{f(0)}{1 - q^2/M^2_{\text{pole}}}, \quad (19)$$

where M_{pole} is a pole mass. This extrapolation of the form factor through the whole kinematic range does not have justifications from first principles, and is a source of uncertainties in any model.

IV. DISCUSSIONS AND CONCLUSIONS

The resulting m_X distributions for $B \to Rl\nu$ for $R = \pi, \rho, D, D^*, D^{**}$ are shown in Fig. 1, along with the inclusive m_X distribution for the $b \to u$ transition, with $|V_{ub}/V_{cb}| = 1$. The $b \to c$ transition is dominated by the $X_c = D, D^*, D^{**}$, and can be reliably calculated in the HQET as described in the previous section. The regions between the triangles are the range of the predicted rate when the dm_X integration over the δ function

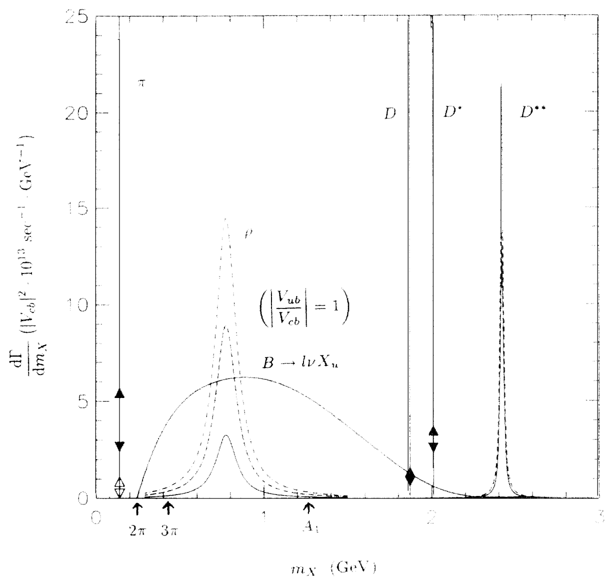

FIG. 1. The m_X distributions in $B \to X_{c,u} l\nu$ with $|V_{ub}/V_{cb}| = 1$. The $b \to c$ transition is dominated by the $X_c = D, D^*, D^{**}$, and can be reliably calculated in the HQET. The regions between the arrows are predicted rates in the units of 10^{13} sec^{-1} when the dm_X integration over the δ function is performed. On the other hand, the $b \to u$ transition is largely nonresonant. The cases with $X_u = \pi, \rho$ are shown explicitly for two different models. For $X_u = \pi$, the region between the white triangles is the predictions by Hagiwara *et al.*, and the region between the black triangles is the predictions by Casalbuoni *et al.* For $X_u = \rho$, the region between the lower two curves is the predictions by Hagiwara *et al.*, whereas the region between the upper two curves are predictions by Casalbuoni *et al.*. The inclusive m_X distribution for $b \to u$ was obtained from the ACCMM model with hadronic mass constraint of $m_X \gtrsim 2m_\pi$.

is performed. On the other hand, the $b \to u$ transition is largely nonresonant. The cases with $X_u = \pi, \rho$ are shown explicitly for two different models discussed in Sec. III. For $X_u = \pi$, the region between the open triangles is the predictions by Hagiwara *et al.* [10], and the region between the closed triangles is the predictions by Casalbuoni *et al.* [15] For $X_u = \rho$, the region between the lower two curves is the predictions by Hagiwara *et al.*, whereas the region between the upper two curves is the predictions by Casalbuoni *et al.* The inclusive m_X distribution for $b \to u$ was obtained from the ACCMM model with hadronic mass constraint of $m_X \gtrsim 2m_\pi$. The excusive decay of $B \to \pi l\nu$ is shown separately.

From Fig. 1, most of the $b \to u$ transition events survive the cut on the hadronic invariant mass, $m_X < m_D$, in contrast to the more conventional cut on the electron energy. In fact, one can relax the condition $m_X < m_D$ because the m_X distribution in $b \to cl\nu$ is completely dominated by contributions by three resonances D, D^*, and D^{**}, which are essentially like δ functions, Eq. (2). In other words, one can use the $b \to u$ events in the region even above $m_X = m_D$, excluding small regions in m_X around $m_X = m_D, m_{D^*}, m_{D^{**}}$. The cut on the m_X is more effective than the cut on the electron energy by a factor of ~ 10, and therefore we have much better statistics. Furthermore, theoretical understanding of exclusive decay modes of $B \to X_u l\nu$ is rather poor, as we discussed in Sec. III. Two different models lead to vastly different predictions for $X_u = \pi$ and ρ. This would induce theoretical uncertainties in the determination of V_{ub} from the measurement of an exclusive semileptonic decay of B mesons. On the other hand, the inclusive decay is better understood, so it would be more reliable to calculate the m_X distribution for inclusive $b \to u$ transitions.

At future B factories with microvertex detectors (symmetrical or asymmetrical), one expects that the efficiency for the event reconstruction will be improved (maybe 30% efficiency). Then, among 10^8 $B\bar{B}$ events, $\sim 10^5$ events without any constraint on m_X may be reconstructed. For more details on the problems of experimental reconstruction and continuum background, see Ref. [3].

Even without a full event reconstruction, one may have good measurements of the missing energy and momentum of the missing neutrino, E_ν and $\vec{p}_\nu$, satisfying the mass-shell condition

$$E_\nu^2 - \mathbf{p}_\nu = m_\nu^2 = 0.$$

In this case, the hadronic mass m_X is not fully constructed, but it is bounded by

$$m_X^2 < m_{X,\max}^2 = m_B^2 + m_{l\nu}^2 - 2\gamma m_B(E_{l\nu} - \beta p_{l\nu}). \tag{20}$$

Here, m_B is the B-meson mass, and $\gamma = (1-\beta^2)^{-1/2} = m_\Upsilon/2m_B$ for the symmetric B factory with $e^+e^- \to \Upsilon(4S) \to B\bar{B}$. Since β is very small, $m_{X,\max}^2$ is close to m_X^2, and we lose very little efficiency. It turns out that $\sim 80\%$ events for $b \to u$ transitions survive the cut on the $m_{X,\max}^2$ [3]:

$$m_{X,\max} < m_D.$$

In any case, studying the hadronic mass distributions in inclusive semileptonic $b \to u$ transitions is experimentally viable. It is also theoretically better described, so theoretical uncertainties in determining $|V_{ub}|$ would be less compared to the $|V_{ub}|$ determined from studies of exclusive decay modes. In summary, we would have better statistics in extracting $|V_{ub}|$ by measuring the m_X distributions in inclusive $b \to u$ semileptonic decays, and have better theoretical handles on the inclusive decays rather than exclusive decays.

ACKNOWLEDGMENTS

We would like to thank A. I. Vainshtein for helpful discussions. This work was supported in part by the Korea Science and Engineering Foundation, in part by Non-Direct-Research-Fund, Korea Research Foundation 1993, in part by the Center for Theoretical Physics, Seoul National University, and in part by the Basic Science Research Institute Program, Ministry of Education, 1994, Project No. BSRI-94-2425.

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