CP-conserving and -violating contributions to $K_L \to \pi^+\pi^-\nu\bar{\nu}$

C. Q. Geng
Department of Physics, National Tsing Hua University, Hsinchu, Taiwan, Republic of China

I. J. Hsu and Y. C. Lin
Department of Physics, National Central University, Chung-Li, Taiwan, Republic of China
(Received 8 June 1994)

We study both CP-conserving and -violating contributions to the decay $K_L \to \pi^+\pi^-\nu\bar{\nu}$. We find that the decay branching ratio is dominated by the CP-conserving part. In the standard CKM model, we estimate that for $m_t \sim 174$ GeV, the branching ratio due to the CP-conserving (-violating) contributions can be as large as 4.4×10^{-13} (1.0×10^{-14}).

PACS number(s): 13.20.Eb, 11.30.Er, 12.15.Ji, 12.39.Fe

I. INTRODUCTION

With the prospect of a new generation of ongoing kaon experiments a number of rare kaon decays [1] have been suggested to test the Cabibbo-Kobayashi-Maskawa (CKM) [2] paradigm: Quarks of different flavor are mixed in the charged weak currents by means of an unitary matrix V. However it is sometimes a hard task to extract the short-distance contribution, which depends on the CKM matrix, because of large theoretical uncertainties in the long-distance contribution to the decays [3]. To avoid this difficulty, much of recent theoretical as well as experimental attention has been on searching for the two modes $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. It is believed that these two decays are free of long-distance and other theoretical uncertainties [4,5].

It has been shown that the decay branching ratio of $K^+ \to \pi^+\nu\bar{\nu}$ is at the level of 10^{-10} [6,7] arising predominantly from the short-distance loop contributions containing virtual charm and top quarks. This decay is a CP-conserving process and probably the cleanest one, in the sense of theoretical uncertainties, to study the absolute value of the CKM element V_{td}. The current experimental limit is $B(K^+ \to \pi^+\nu\bar{\nu})_{\text{expt}} \le 5 \times 10^{-10}$ [8] given by the ongoing E787 experiment at BNL. It is expected that the experiment will reach the standard-model predicted level in a few years. On the other hand, the decay $K_L \to \pi^0\nu\bar{\nu}$ depending on the imaginary part of V_{td} is a CP-violating process [9] and offers clear information about the origin of CP violation. In the standard model, it is dominated by the Z-penguin and W-box loop diagrams with virtual top quark. But there has been no dedicated experimental search for this decay yet. Although there are several interesting proposals to study this mode at the next round KEK and Fermilab experiments [10], the experimental sensitivities can only be around 10^{-9}, whereas the decay branching ratio in the CKM model is at the level of 10^{-11} [6]. From an experimental point of view very challenging efforts are necessary to perform the experiments. This is because all the final state particles are neutral and the only detectable particles are 2γ's from π^0.

In this paper, we examine the decay $K_L \to \pi^+\pi^-\nu\bar{\nu}$. Like the decays of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, we expect that this mode is also a clean one due to the absence of photon intermediate states.[1] Moreover, in contrast with $K_L \to \pi^0\nu\bar{\nu}$, it contains two charge particles π^+ and π^- in the final states and it could be relatively easy to do an experiment [12]. Therefore, it should be interesting to give a theoretical analysis on this decay to see whether it could be tested experimentally in future kaon facilities.

The paper is organized as follows. In Sec. II, we study the decay rate of $K_L \to \pi^+\pi^-\nu\bar{\nu}$ from the short and long distance contributions. We present our numerical results in Sec. III. The conclusions are given in Sec. IV.

II. DECAY RATES

We start by writing the decay as

$$K_L(p_K) \to \pi^+(p_+)\pi^-(p_-)\nu(k_+)\bar{\nu}(k_-) \tag{1}$$

where p_K, p_+, p_-, k_+, and k_- are the four-momenta of K_L, π^+, π^-, ν, and $\bar{\nu}$, respectively. Similar to the decays of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, the short distance contributions, arising from the box and penguin loop diagrams with virtual charm and top quarks, dominate the decay branching ratio of $K_L \to \pi^+\pi^-\nu\bar{\nu}$. The effective interaction relevant for the process is given by [13]

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi \sin^2\theta_W} \sum_{i=c,t} V_{is}^* V_{id} \eta_i C_\nu(x_i) \bar{s}\gamma_\mu(1-\gamma_5) \times d\bar{\nu}_l \gamma^\mu (1-\gamma_5)\nu_l \tag{2}$$

[1] We note that the decay of $K_L \to \pi^+\pi^-\nu\bar{\nu}$ is different from that of $K_L \to \pi^+\pi^-e^+e^-$ in which it is dominated by the long distance due to the photon intermediate states [11].

where $\eta_c \simeq 0.71$ and $\eta_t \simeq 1$ are the QCD correction factors [13], $x_i = m_i^2/M_W^2$, and

$$C_\nu(x_i) = \frac{x_i}{4}\left[\frac{3(x_i-2)}{(x_i-1)^2}\ln x_i + \frac{x_i+2}{x_i-1}\right] . \quad (3)$$

To obtain the matrix element, we follow the analysis of K_{l4} decays by Pais and Treiman [15]. We define the following combinations of four-momenta:

$$P = p_+ + p_-, \quad Q = p_+ - p_- ,$$
$$L = k_+ + k_-, \quad N = k_+ - k_- . \quad (4)$$

Similar to K_{l4} decays [15], the decay $K_L \to \pi^+\pi^-\nu\bar{\nu}$ can be kinematically parametrized by five variables: $s_\pi = P^2$, the invariant mass of the $\pi^+\pi^-$ pair; $s_\nu = L^2$, the invariant mass of the $\nu\bar{\nu}$ pair; θ_π, the angle between $\mathbf{p}_+$ and $\mathbf{L}$ as measured in the $\pi^+\pi^-$ c.m. frame; θ_ν, the angle between $\mathbf{k}_+$ and $\mathbf{P}$ as measured in the $\nu\bar{\nu}$ c.m. frame; and ϕ, the angle between the normals to the $\pi^+\pi^-$ and $\nu\bar{\nu}$ planes. The ranges of the variables are [16]

$$4M_\pi^2 \le s_\pi \le M_K^2 ,$$
$$0 \le s_\nu \le (M_K - \sqrt{s_\pi})^2 ,$$
$$0 \le \theta_\pi, \quad \theta_\nu \le \pi ,$$
$$0 \le \phi \le 2\pi , \quad (5)$$

respectively. For the hadronic matrix element, we use the standard parametrization

$$\langle\pi^+\pi^-|\bar{s}\gamma_\mu(1-\gamma_5)d|K^0\rangle$$
$$= \frac{i}{M_K}\left[FP_\mu + GQ_\mu + i\frac{H}{M_K^2}\epsilon_{\mu\nu\rho\sigma}L^\nu P^\rho Q^\sigma\right] , \quad (6)$$

where the form factors F, G, and H can be related by isospin to the corresponding form factors in the matrix element of $\langle\pi^+\pi^-|\bar{s}\gamma_\mu(1-\gamma_5)u|K^+\rangle$ in K_{l4} decay. These form factors have been evaluated in chiral perturbation theory (ChPT) at order p^4 [17,18]. It is found that

$$F = G = \frac{M_K}{f_\pi} ,$$
$$H = \frac{M_K^3}{2\pi^2 f_\pi^3} \quad (7)$$

with $f_\pi = 130$ MeV. From Eqs. (2) and (6), we obtain the amplitude of the decay $K^0 \to \pi^+\pi^-\nu\bar{\nu}$ for each neutrino flavor as

$$A(K^0 \to \pi^+\pi^-\nu\bar{\nu}) = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi\sin^2\theta_W}\sum_{i=c,t} V_{is}^* V_{id}\eta_i C_\nu(x_i)\frac{i}{M_K}\left[FP_\mu + GQ_\mu + i\frac{H}{M_K^2}\epsilon_{\mu\nu\rho\sigma}L^\nu P^\rho Q^\sigma\right]\bar{\nu}_l\gamma^\mu(1-\gamma_5)\nu_l . \quad (8)$$

With the CPT theorem with $K_L \simeq K_2 + \epsilon K_1 \simeq (K^0 - \bar{K}^0)/\sqrt{2}i$, we find

$$A(K_L \to \pi^+\pi^-\nu\bar{\nu}) = \frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi\sin^2\theta_W}\frac{\sqrt{2}\lambda}{M_K}\Big\{iGQ_\mu[-A^2\lambda^4\eta C_\nu(x_t)] + \Big(FP_\mu + i\frac{H}{M_K^2}\epsilon_{\mu\nu\rho\sigma}L^\nu P^\rho Q^\sigma\Big)[\eta_c C_\nu(x_c)$$
$$+A^2\lambda^4(1-\rho)C_\nu(x_t)]\Big\}\bar{\nu}_l\gamma^\nu(1-\gamma_5)\nu_l \quad (9)$$

where $\lambda = 0.22$ is the Cabibbo angle, A, ρ, and η are the parameters in the Wolfenstein parametrization [19] of the CKM matrix, and we have ignored the contribution from K_1 part because of the smallness of the ϵ parameter. In Eq. (9), the terms proportional to F and H, which represent $I = 0$ s wave and $I = 1$ p wave for the $\pi^+\pi^-$ system, are CP conserving and that to G, $I = 1$ p wave, is CP violating.

To write the partial decay rate for (1), it is convenient to introduce the following combination of kinematic and form factors:

$$F_1 = -iFX[\eta_c C_\nu(x_c) + A^2\lambda^4(1-\rho)C_\nu(x_t)] - \sigma_\pi(P\cdot L)\cos\theta_\pi G[A^2\lambda^4\eta C_\nu(x_t)] ,$$
$$F_2 = -\sigma_\pi(s_\pi s_\nu)^{1/2}G[A^2\lambda^4\eta C_\nu(x_t)] , \quad (10)$$
$$F_3 = -\sigma_\pi X(s_\pi s_\nu)^{1/2}\frac{iH}{M_K^2}[\eta_c C_\nu(x_c) + A^2\lambda^4(1-\rho)C_\nu(x_t)] ,$$

where

$$\sigma_\pi = \left(1 - \frac{4M_\pi^2}{s_\pi}\right)^{1/2}, \quad X = [(P\cdot L)^2 - s_\pi s_\nu]^{1/2}, \quad P\cdot L = \frac{1}{2}(M_K^2 - s_\pi - s_\nu) . \quad (11)$$

The differential decay rate is

$$d^5\Gamma = \frac{G_F^2}{2^{12}\pi^6 M_K^5}\left(\frac{\alpha\sqrt{2}\lambda}{4\pi\sin^2\theta_W}\right)^2 X\sigma_\pi I(s_\pi, s_\nu, \theta_\pi, \theta_\nu, \phi) ds_\pi ds_\nu d\cos\theta_\pi d\cos\theta_\nu d\phi \ . \tag{12}$$

The dependence of I on θ_ν and ϕ is given by

$$\begin{aligned} I = {} & I_1 + I_2\cos 2\theta_\nu + I_3\sin^2\theta_\nu\cos 2\phi + I_4\sin 2\theta_\nu\cos\phi + I_5\sin\theta_\nu\cos\phi \\ & + I_6\cos\theta_\nu + I_7\sin\theta_\nu\sin\phi + I_8\sin 2\theta_\nu\sin\phi + I_9\sin^2\theta_\nu\sin 2\phi \ , \end{aligned} \tag{13}$$

where $I_1, \ldots, I_9$ depend on s_π, s_ν, and θ_π. By integrating over the angles θ_ν and ϕ we obtain

$$\begin{aligned} I(s_\pi, s_\nu, \theta_\pi) &= 4\pi\left[I_1 - \frac{1}{3}I_2\right] \\ &= \frac{4\pi}{3}[|F_1|^2 + (|F_2|^2 + |F_3|^2)\sin^2\theta_\pi] \ , \end{aligned} \tag{14}$$

where we have used the formulas for the form factors I_1 and I_2 given by

$$\begin{aligned} I_1 &= \frac{1}{4}\left[|F_1|^2 + \frac{3}{2}(|F_2|^2 + |F_3|^2)\sin^2\theta_\pi\right] \ , \\ I_2 &= -\frac{1}{4}\left[|F_1|^2 - \frac{1}{2}(|F_2|^2 + |F_3|^2)\sin^2\theta_\pi\right] \ . \end{aligned} \tag{15}$$

Combining Eqs. (10)–(14), we get the differential decay rate of $K_L \to \pi^+\pi^-\nu\bar{\nu}$ for three generations of neutrinos as

$$\frac{d^3\Gamma}{ds_\pi ds_\nu d\cos\theta_\pi} = \left(\frac{d^3\Gamma}{ds_\pi ds_\nu d\cos\theta_\pi}\right)_{\rm CPC} + \left(\frac{d^3\Gamma}{ds_\pi ds_\nu d\cos\theta_\pi}\right)_{\rm CPV} \tag{16}$$

with

$$\begin{aligned} \left(\frac{d^3\Gamma}{ds_\pi ds_\nu d\cos\theta_\pi}\right)_{\rm CPC} = {} & \frac{G_F^2}{2^{10}\pi^5 M_K^5}\left(\frac{\alpha\sqrt{2}\lambda}{4\pi\sin^2\theta_W}\right)^2 X^3\sigma_\pi\left(F^2 + \sigma_\pi^2 s_\pi s_\nu\frac{H^2}{M_K^4}\sin^2\theta_\pi\right) \\ & \times[\eta_c C_\nu(x_c) + A^2\lambda^4(1-\rho)C_\nu(x_t)]^2 \end{aligned} \tag{17}$$

and

$$\left(\frac{d^3\Gamma}{ds_\pi ds_\nu d\cos\theta_\pi}\right)_{\rm CPV} = \frac{G_F^2}{2^{10}\pi^5 M_K^5}\left(\frac{\alpha\sqrt{2}\lambda}{4\pi\sin^2\theta_W}\right)^2 X\sigma_\pi^3(X^2\cos^2\theta_\pi + s_\pi s_\nu)G^2[A^2\lambda^4\eta C_\nu(x_t)]^2 \ , \tag{18}$$

corresponding to the CP conserving and violating contributions, respectively. As comparisons, we give the branching ratios for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ from the short distance contributions:

$$B(K^+ \to \pi^+\nu\bar{\nu}) = \frac{3}{2}\left(\frac{\alpha}{2\pi\sin^2\theta_W}\right)^2 B(K^+ \to \pi^0 e^+\nu)\left[(\eta_c C_\nu(x_c) + A^2\lambda^4(1-\rho)C_\nu(x_t))^2 + (A^2\lambda^4\eta C_\nu(x_t))^2\right] \ ,$$

$$B(K_L \to \pi^0\nu\bar{\nu}) = \frac{3}{2}\left(\frac{\alpha}{2\pi\sin^2\theta_W}\right)^2 B(K^+ \to \pi^0 e^+\nu)\frac{\tau(K_L)}{\tau(K^+)}(A^2\lambda^4\eta C_\nu(x_t))^2 \ , \tag{19}$$

where $B(K^+ \to \pi^0 e^+\nu) = 0.048$. It is interesting to note that the $K_L \to \pi^+\pi^-\nu\bar{\nu}$ decay rate of the CP violating part in Eq. (18) has a CKM dependence similar to $K_L \to \pi^0\nu\bar{\nu}$ in Eq. (19) while that of the CP conserving part in Eq. (17) is somewhat different from the CP conserving decay of $K^+ \to \pi^+\nu\bar{\nu}$.

The long distance contribution can be calculated in the framework of chiral perturbation theory. There are three kinds of terms which contribute to the process of interest, $L_{(2)}^{\Delta S=1}$ [5], reducible anomaly ($L_{\rm RA}$), and direct anomaly ($L_{\rm DA}$) [21]. $L_{(2)}^{\Delta S=1}$ is the weak chiral Lagrangian of $O(p^2)$:

$$L_{(2)}^{\Delta S=1} = \frac{G_8 f_\pi^4}{4} \mathrm{Tr}\lambda_6 D_\mu U^\dagger D^\mu U \ , \tag{20}$$

where

$$U = \exp\left(\frac{i\sqrt{2}}{f_\pi}\phi^a\lambda^a\right) \tag{21}$$

is the nonlinear realization of the octet meson fields and

$$D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu \tag{22}$$

is the covariant derivative with

$$\begin{aligned} l_\mu &= \frac{g}{\cos\theta_W} Z_\mu\left(Q - \frac{\xi}{6} - \sin^2\theta_W Q\right) \ , \\ r_\mu &= \frac{g}{\cos\theta_W} Z_\mu(-\sin^2\theta_W Q) \ . \end{aligned} \tag{23}$$

The overall normalization G_8 is determined by the amplitude of $K \to \pi\pi$ and the numerical value is 9×10^{-6} GeV^{-2}. The matrix Q is the quark charge matrix, $Q = \mathrm{diag}(2/3, -1/3, -1/3)$, which characterizes the EM current coupling of Z. The parameter ξ inside the left-handed current l_μ is the coefficient for the singlet current coupling of Z, and it is of unity in the limit of nonet symmetry. Note that we have different identification for l_μ and r_μ than those in [5]. The reducible anomaly arises from the kind of diagrams starting with a K-π (or K-η) weak transition induced from $L_{(2)}^{\Delta S=1}$, then followed by a π (or η) pole and ended by an anomaly vertex derived from L_{WZW}. The relevant pieces to our calculation in L_{WZW} are given by

$$\begin{aligned} L_{\mathrm{WZW}} = &-\frac{i}{16\pi^2}\mathrm{Tr}\epsilon_{\mu\nu\alpha\beta}L^\mu L^\nu L^\alpha l^\beta \\ &+\frac{i}{16\pi^2}\mathrm{Tr}\epsilon_{\mu\nu\alpha\beta}R^\mu R^\nu R^\alpha r^\beta \end{aligned} \tag{24}$$

where

$$\begin{aligned} L_\mu &= iU^\dagger\partial_\mu U \ , \\ R_\mu &= iU\partial_\mu U^\dagger \ . \end{aligned} \tag{25}$$

The direct anomaly can be understood as the bosonization of the product of the left-handed currents arising from $L_{(2)}^{\Delta S=1}$ and L_{WZW}. It reads

$$\begin{aligned} L_{\mathrm{DA}} = \frac{G_8 f_\pi^2}{32\pi^2}\{&2a_1 i\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\lambda_6 L_\mu \mathrm{Tr}L_\nu L_\alpha L_\beta + a_2\mathrm{Tr}\lambda_6[U^\dagger F_R^{\mu\nu}U, L_\mu L_\nu] \\ &+3a_3\mathrm{Tr}\lambda_6 L_\mu \mathrm{Tr}(F_L^{\mu\nu} + U^\dagger F_R^{\mu\nu}U)L_\nu + a_4\mathrm{Tr}\lambda_6 L_\nu \mathrm{Tr}(F_L^{\mu\nu} - U^\dagger F_R^{\mu\nu}U)L_\nu\} \end{aligned} \tag{26}$$

where $F_{R,L}^{\mu\nu}$ are the field strengths associated with the fields r_μ and l_μ correspondingly. The coefficients a_i are in principle of order one and they can be extracted from the anomalous radiative modes of kaon. In terms of the kinetics variables defined before, the decay amplitude resulting from long distance effect is given by

$$\begin{aligned} A_L(K_L \to \pi^+\pi^-\nu\bar{\nu} = &+\frac{i\sqrt{2}g^2G_8}{32\pi^2 f_\pi M_Z^2\cos^2\theta_W}\bar{\nu}_l\gamma_\mu(1-\gamma_5)\nu_l \\ &\times\left\{\epsilon^{\mu\nu\alpha\beta}L_\nu P_\alpha Q_\beta\left[2(3a_1 - 3a_3 - a_4) + 2\sin^2\theta_W(a_2 + 2a_4) + \xi\frac{M_K^2}{M_K^2 - M_\pi^2}\right]\right. \\ &\left.+8i\pi^2 f_\pi^2(1 - 2\sin^2\theta_W)(P^\mu + L^\mu)\right\} \ , \end{aligned} \tag{27}$$

and the corresponding differential decay rate is then given by

$$\begin{aligned} \left(\frac{d^3\Gamma}{ds_\pi ds_\nu d\cos\theta_\pi}\right)_L = &\frac{g^4 G_8^2\sigma_\pi X^3}{2^{19}\pi^9 f_\pi^2 M_Z^4 M_K^3}\cos^4\theta_W\{64\pi^4 f_\pi^4(1 - 2\sin^2\theta_W)^2 \\ &+\sigma_\pi^2\sin^2\theta_\pi s_\pi s_\nu[2(3a_1 - 3a_3 - a_4) + 2\sin^2\theta_\pi(a_2 + 2a_4) + \xi M_K^2/(M_K^2 - M_\pi^2)]^2\} \ . \end{aligned} \tag{28}$$

III. NUMERICAL RESULTS

The validity of relating m_t to the decay rate depends upon the negligibility of long distance contribution. Therefore, it is important to learn the branching ratio arising from the long distance effect. Because of the absence of m_t in the amplitude, the decay rate of long distance contribution is relatively suppressed by at least two orders. Numerically we find

$$\begin{aligned} B(L_{\mathrm{DA}} + L_{\mathrm{RA}}) &\sim 4.7 \times 10^{-20} \ , \\ B(L_{(2)}^{\Delta S=1}) &= 5.0 \times 10^{-17} \ . \end{aligned} \tag{29}$$

As we shall see below it is safe to ignore the long distance effect and we shall concentrate on the short distance effect only in the following analysis of decay rate.

To estimate the CP conserving and violating decay rates in (17) and (18), we need to find out the allowed values for the CKM parameters A, ρ, and η, constrained by the experimental measurements such as ϵ, the CP violation parameter in $K \to \pi\pi$; x_d, the B_d^0-$\bar{B}_d^0$ mixing; and the ratios $|V_{cb}/V_{us}^2|$ and $|V_{ub}/V_{cb}|$ of the CKM elements. We use the same fitting procedure and the necessary equations in Refs. [6,20]. In the fits, we take the updated values $|V_{cb}| = 0.041 \pm 0.005$ and $|V_{ub}/V_{cb}| = 0.080 \pm 0.025$ and $f_B = 200 \pm 50$ MeV.

Integrating over all the variables in Eqs. (17) and (18), we can examine the decay rates for both CP conserving and violating parts which depend on the top quark mass and the CKM parameters. In Figs. 1(a) and 1(b), we plot the branching ratios of CP conserving and violating contributions to $K_L \to \pi^+\pi^-\nu\bar{\nu}$ as a function of the top quark mass, showing the lower and higher values allowed at 90% C.L., where

$$B(K_L \to \pi^+\pi^-\nu\bar{\nu}) = \Gamma(K_L \to \pi^+\pi^-\nu\bar{\nu})/\Gamma(K_L \to \text{all}) .$$

We also show the corresponding decay branching ratios of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ in Figs. 1(c) and 1(d) by using Eq. (19), respectively. From the figures, we see that the CP conserving part of the branching ratio is much larger than that of CP violating one. Clearly, measuring the decay rate will not give us information on the CP violation. For $150 \le m_t \le 200$ GeV, we find

$$\begin{aligned} 1.1\times10^{-13} &\le B(K_L \to \pi^+\pi^-\nu\bar{\nu})_{\rm CPC} \le 5.0\times10^{-13} , \\ 5.0\times10^{-16} &\le B(K_L \to \pi^+\pi^-\nu\bar{\nu})_{\rm CPV} \le 1.1\times10^{-14} . \end{aligned} \tag{30}$$

In particular, we obtain that for $m_t \sim 174$ GeV, the branching ratio of $K_L \to \pi^+\pi^-\nu\bar{\nu}$ due to the CP conserving and violating contributions can be as large as 4.4×10^{-13} and 1.0×10^{-14}, respectively. We note that the similar CP violating short distance contribution to the branching ratio of $K_L \to \pi^+\pi^-e^+e^-$ is estimated to be order of 10^{-16} [11].

We now study the differential decay spectrum in terms of s_π (θ_π) by integrating over s_ν and θ_π (s_π and s_ν) in Eqs. (17) and (18) to see whether we would distinguish the CP conserving and violating parts. We define the normalized invariant mass of $\pi^+\pi^-$ as $x = s_\pi/M_K^2$. To illustrate the shapes of the spectra between the CP conserving and violating cases, we choose $m_t \sim 160$ GeV and CKM parameters $A \sim 1$, $\rho \sim -0.2$, and $\eta \sim 0.4$. We plot the differential branching ratios $dB(K_L \to \pi^+\pi^-\nu\bar{\nu}/dx^{1/2}$ vs $x^{1/2}$ and $dB(K_L \to \pi^+\pi^-\nu\bar{\nu}/d\cos\theta_\pi$ vs $\cos\theta_\pi$ in Figs. 2 and 3, respectively. As shown in Fig. 2, the CP conserving and violating spectra of $dB(K_L \to \pi^+\pi^-\nu\bar{\nu})/dx^{1/2}$ have similar shapes and are dominated by small values of s_π. However, in Fig. 3, as expected, $[dB(K_L \to \pi^+\pi^-\nu\bar{\nu}/d\cos\theta_\pi]_{\rm CPV}$ becomes maximum when θ_π is close to 0 or π and minimal when it reaches $\pi/2$, whereas $[dB(K_L \to \pi^+\pi^-\nu\bar{\nu})/d\cos\theta_\pi]_{\rm CPC}$ does the opposite. Unfortunately, the values of the CP violating one around $\theta_\pi = 0, \pi$ may be still too small to be tested.

IV. CONCLUSIONS

We have studied both short and long distance contributions to the decay of $K_L \to \pi^+\pi^-\nu\bar{\nu}$. We have demonstrated that the long distance effect to the decay rate is negligibly small. We have shown that the branching ra-

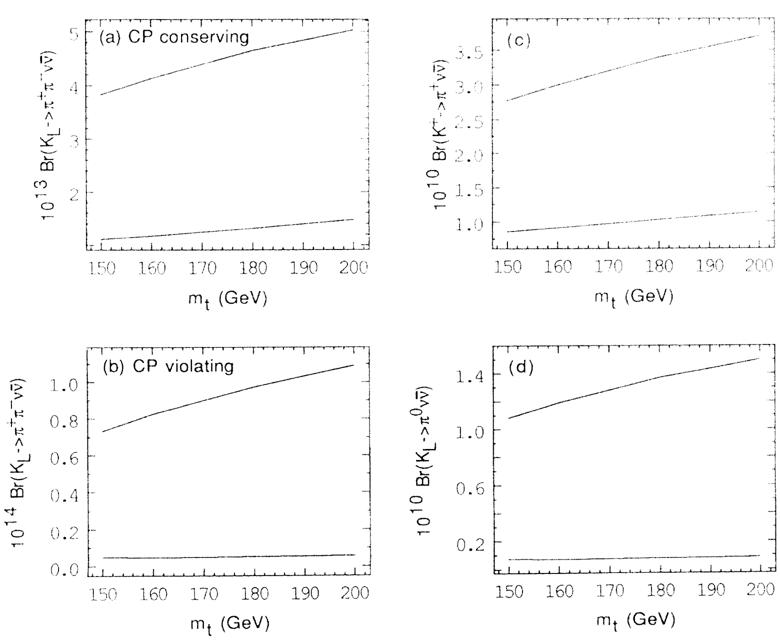

FIG. 1. Allowed branching ratios for (a) CP-conserving contribution to $K_L \to \pi^+\pi^-\nu\bar{\nu}$, (b) CP-violating contribution to $K_L \to \pi^+\pi^-\nu\bar{\nu}$, (c) $K^+ \to \pi^+\nu\bar{\nu}$; and (d) $K_L \to \pi^0\nu\bar{\nu}$ as functions of m_t at 90% C.L.

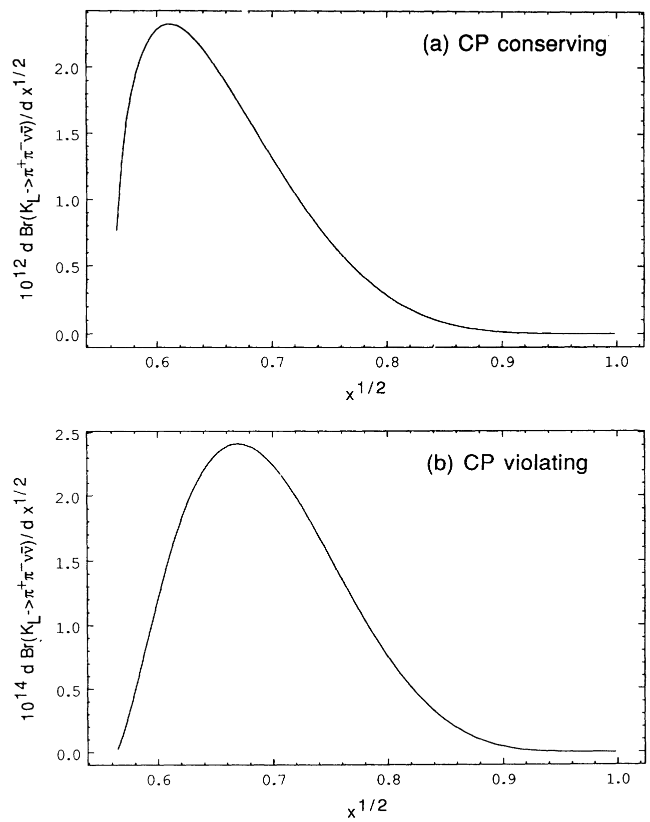

FIG. 2. The differential decay spectrum of $d\Gamma(K_L \to \pi^+\pi^-\nu\bar{\nu})/dx^{1/2}$ as a function of $x^{1/2} = \sqrt{s_\pi}/M_K$ with $m_t = 160$ GeV, $A \sim 1.0$, $\rho \sim -0.2$, and $\eta \sim 0.4$.

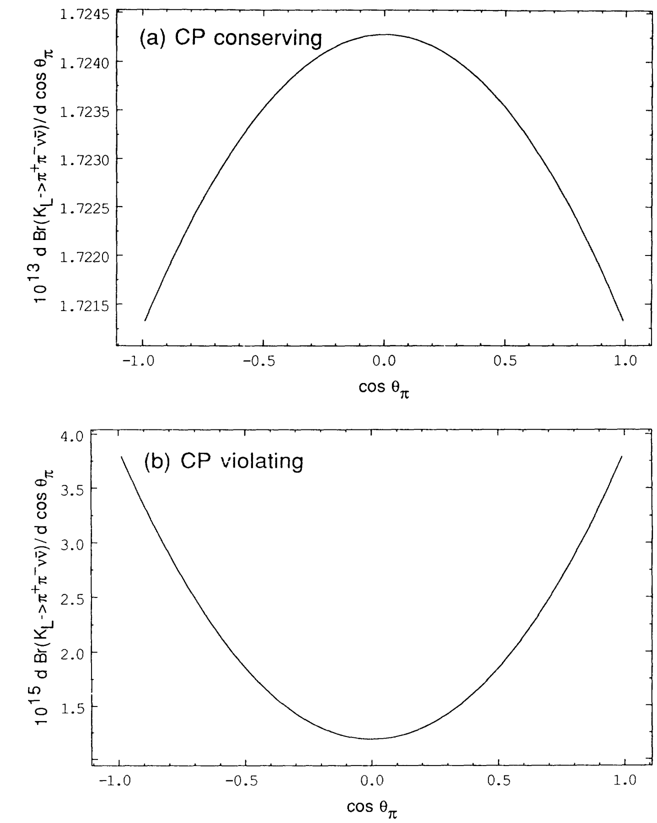

FIG. 3. The differential decay spectrum of $d\Gamma(K_L \to \pi^+\pi^-\nu\bar{\nu})/d\cos\theta_\pi$ as a function of $\cos\theta_\pi$. Legend is the same as in Fig. 2.

tio of the decay is dominated by the CP conserving part. With the updated CKM parameters, we find that the decay branching ratio is predicted to be $(1-5)\times 10^{-13}$ for $m_t \leq 200$ GeV, which could be accessible to experiments at future kaon facilities. The CP violating contribution to the branching ratio seems impossible to be measured in experiments. However, it is, in principle, possible to distinguish the CP conserving and violating contributions by measuring the spectra of the θ_π angular dependence of the differential decay rates.

ACKNOWLEDGMENTS

We thank Professor D. Bryman for suggesting that we study this decay mode. We thank D. Bryman, H. Y. Cheng, G. Ecker, T. Inagaki, Y. Kuno, W. J. Marciano, and B. Winstein for useful discussions. This work was supported in part by the National Science Council of Republic of China under Grant No. NSC-83-0208-M-007-118 (C.Q.G.) and NSC-83-0208-M-008-009 (I.-J.H. and Y.C.L.).

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